# IG 0606

# Additional Maths

# Indices and Surds Notes

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# Indices and Surds



§ Consider $2\times2\times2\times2\times2 = 2^5$ . 5 → Index or power or Exponent. 2 → base

In General. $a\times a\times a\times \cdots n\text{ times} = a^n$, $a>0$, $a \in R$ and $n \in R$.

§ Laws of Indices:

(i) $a^m \times a^n = a^{m+n}$

(ii) $\dfrac{a^m}{a^n} = a^{m-n}$

(iii) $(a^m)^n = a^{m\times n}$

(iv) $a^n \times b^n = (ab)^n$

(v) $a^n \div b^n = \left(\dfrac{a}{b}\right)^n$

(vi) $a^0 = 1$

(vii) $a^{-n} = \dfrac{1}{a^n}$

(viii) $a^{1/n} = \sqrt[n]{a}$

(ix) $a^{m/n} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$

Example 1. Express $\dfrac{(5\sqrt{q})^3}{(625p^{12}q)^{1/4}}$ in the form $5^a p^b q^c$ ---[3]

[S-17/13/Q 4]

Solution: Given, $\dfrac{(5\sqrt{q})^3}{(625p^{12}q)^{\frac{1}{4}}} = \dfrac{5^3 \cdot (q^{1/2})^3}{(5^4)^{1/4} \cdot (p^{12})^{1/4} \cdot q^{1/4}}$

$= \dfrac{5^3 \cdot q^{3/2}}{5 \cdot p^3 \cdot q^{1/4}}$

$= 5^{3-1} \cdot p^{-3} \cdot q^{3/2 - \frac{1}{4}}$

$= 5^2 \cdot p^{-3} \cdot q^{5/4}$ ✓

Example 2. Given that $\frac{p^{-2}qr^{-1/2}}{\sqrt{p^{1/3}.q^{2}.r^{-3}}} = p^{a}q^{b}r^{c}$ find the value of $a$, $b$ and $c$. [M-16/12/Q2] --[3]

Solution: Given.

$$\frac{p^{-2}qr^{-\frac{1}{2}}}{(p^{1/3}.q^{2}.r^{-3})^{1/2}} = \frac{p^{-2}qr^{-1/2}}{(p^{1/3})^{1/2}.(q^{2})^{1/2}.(r^{-3})^{1/2}}$$

$$= \frac{p^{-2}qr^{-1/2}}{p^{\frac{1}{6}}.q\,r^{-3/2}}$$

$$= p^{-2-\frac{1}{6}}q^{1-1}r^{-1/2+3/2}$$

$$= p^{-13/6}q^{0}r^{1} = p^{a}.q^{b}r^{c}$$

$$\Rightarrow \underline{a = -\frac{13}{6}, b = 0 \text{ and } c = 1}$$ ✓

Example 3. Show that $3^{0.5} \times (\sqrt{2})^{7}$ can be written in the form $a\sqrt{b}$, where $a$ and $b$ are integers and $a > b$. [S-18/11/Q10(b)] --[2]

Solution: Given $3^{0.5}.(\sqrt{2})^{7} = \sqrt{3}.2^{7/2}$

$$= \sqrt{3} \times 2^{3+\frac{1}{2}}$$

$$= \sqrt{3} \times 2^{3}.2^{1/2}$$

$$= 8\sqrt{3}\sqrt{2} = \underline{8\sqrt{6}}$$ ✓

§ **Surds:** $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ etc are surds.

In general, $\sqrt{n}$ is a surd, where $n$ is a positive integer.

Note: (i) $\sqrt{5}\cdot\sqrt{5} = 5$

(ii) $2\sqrt{5} + 7\sqrt{5} = 9\sqrt{5}$

(iii) $2\sqrt{5} \times 7\sqrt{5} = 14 \times 5 = 70$

(iv) $\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}}.\frac{\sqrt{5}}{\sqrt{5}} = \frac{1}{5}\sqrt{5}$

(v) $\sqrt{5}.\sqrt{7} = \sqrt{5 \times 7} = \sqrt{35}$

# Indices and Surds



**Example 4.** Simplify $\dfrac{5+6\sqrt{5}}{6+\sqrt{5}}$ --- [3] [S-18/11/Q10(a)]

**Solution:**

$$\frac{5+6\sqrt{5}}{6+\sqrt{5}} \times \frac{6-\sqrt{5}}{6-\sqrt{5}}$$

$$= \frac{30-5\sqrt{5}+36\sqrt{5}-30}{6^2-(\sqrt{5})^2}$$

$$= \frac{31\sqrt{5}}{31} = \underline{\sqrt{5}} \checkmark$$

**Example 5.** Solve the equation $x+\sqrt{2}=\dfrac{4}{x}$, giving your answer in the simplest surd form. --- [4] [S-18/11/Q10(c)]

**Solution:** Given, $x+\sqrt{2}=\dfrac{4}{x}$

$$\Rightarrow x^2+\sqrt{2}x-4=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$a=1, b=\sqrt{2}, c=-4$

$b^2-4ac = 2-4\times1(-4)$
$= 18$

$\sqrt{18}=\sqrt{9\times2}=3\sqrt{2}$

$$x=\frac{-\sqrt{2}\pm\sqrt{18}}{2}$$

$$=\frac{-\sqrt{2}\pm3\sqrt{2}}{2}$$

$$\underline{x=\sqrt{2}, -2\sqrt{2}} \checkmark$$

**Example 6.** By using the substitution $y=x^{1/3}$, or otherwise, Solve $x^{2/3}-4x^{1/3}+3=0$ [W-17/11/Q3] --- [4]

**Solution:**

$x^{2/3}-4x^{1/3}+3=0$

or $(x^{1/3})^2-4x^{1/3}+3=0$

or $y^2-4y+3=0$ (Put $y=x^{1/3}$)

$(y-1)(y-3)=0$

$y=1$ or $y=3$

$\therefore y=x^{1/3}=1$ or $3$

$\therefore x=1^3$ or $3^3$

$\underline{x=1; 27} \checkmark$

# Indices and Surds



**Example 7.** Solve the equation $16^{3x-1} = 8^{x+2}$ [S-16/11/Q2(a)] --[3]

**Solution:** $16^{3x-1} = 8^{x+2}$

$\Rightarrow (2^4)^{3x-1} = (2^3)^{x+2}$

$\Rightarrow 2^{4(3x-1)} = 2^{3(x+2)}$

$\Rightarrow 4(3x-1) = 3(x+2)$

$\Rightarrow 12x-4 = 3x+6 \Rightarrow 9x = 10 \Rightarrow \underline{x = \frac{10}{9}}$ ✓

**Example 8.** Solve the equations to find $p$ and $q$.

$8^{q-1} \times 2^{2p+1} = 4^7$

$9^{p-4} \times 3^q = 81$ [W-15/21/Q5(a)] ---[4]

**Solution:** $8^{q-1} \cdot 2^{2p+1} = 4^7 \Rightarrow (2^3)^{q-1} \cdot 2^{2p+1} = (2^2)^7$

$\Rightarrow 2^{3q-3} \cdot 2^{2p+1} = 2^{14}$

$\Rightarrow 2p+3q = 16$ —— ①

Also, $9^{p-4} \times 3^q = 81$

$\Rightarrow (3^2)^{p-4} \times 3^q = 3^4 \Rightarrow 3^{2(p-4)} \cdot 3^q = 3^4$

$\Rightarrow 3^{2p-8} \cdot 3^q = 3^4$

$\Rightarrow 2p+q = 12$ —— ②

Solving ① & ② $\underline{p = 5 \text{ and } q = 2}$ ✓

**Example 9.** Solve the simultaneous eqn's. → [W-14/13/Q10] --[5]

$\frac{5^x}{25^{3y-2}} = 1$ and $\frac{3^x}{27^{y-1}} = 81$

**Solution.** $\frac{5^x}{25^{3y-2}} = 1 \Rightarrow \frac{5^x}{(5^2)^{3y-2}} = 1$

$\Rightarrow \frac{5^x}{5^{6y-4}} = 1$

$\Rightarrow 5^{x-(6y-4)} = 5^0$

$\Rightarrow x = 6y-4$ —— ①

Also given, $\frac{3^x}{27^{y-1}} = 81$

$\Rightarrow \frac{3^x}{(3^3)^{y-1}} = 3^4$

$\Rightarrow \frac{3^x}{3^{3y-3}} = 3^4$

$\Rightarrow 3^{x-(3y-3)} = 3^4$

$\Rightarrow x = 3y+1$ —— ②

Solving ① & ② $\underline{x = 6;\ y = 5/3}$ ✓

# Indices and Surds



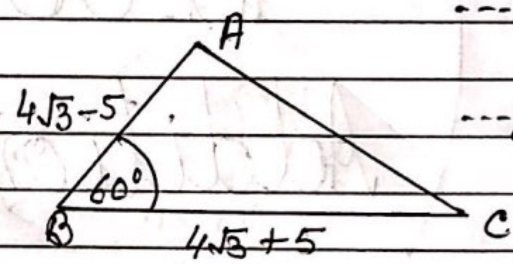
**Example 10.** The diagram shows the triangle ABC, where $AB = 4\sqrt{3}-5$, $BC = 4\sqrt{3}+5$ and angle $ABC = 60°$.
It is known that $\sin 60° = \frac{\sqrt{3}}{2}$, $\cos 60° = \frac{1}{2}$, $\tan 60° = \sqrt{3}$.

(i) Find the exact value of AC. ---[4]

(ii) Hence show that

$\operatorname{cosec} ACB = \frac{2\sqrt{p}}{q}(4\sqrt{3}+5)$, ---[4]

where p and q are integers.




[S-18/12/Q10]

**Solution** (i) $AC^2 = AB^2 + BC^2 - 2AB\cdot AC\cos 60°$ [Cosine Rule]

$$= (4\sqrt{3}-5)^2 + (4\sqrt{3}+5)^2 - 2(4\sqrt{3}-5)(4\sqrt{3}+5)\cdot\cos 60°$$

or $AC^2 = 123$

or $\underline{AC = \sqrt{123}}$ ✓ ——— ①

(ii) Using Sine rule in triangle ABC.

$$\frac{AC}{\sin 60°} = \frac{AB}{\sin ACB} \Rightarrow \frac{1}{\sin ACB} = \frac{1}{\sin 60°} \times \frac{AC}{AB}$$

$$\text{or } \operatorname{Cosec} ACB = \frac{1}{\frac{\sqrt{3}}{2}} \times \frac{\sqrt{123}}{(4\sqrt{3}-5)} \quad \text{frm ①}$$

$$= \frac{2}{\sqrt{3}} \times \frac{\sqrt{41\times 3}}{(4\sqrt{3}-5)} \times \frac{(4\sqrt{3}+5)}{(4\sqrt{3}+5)}$$

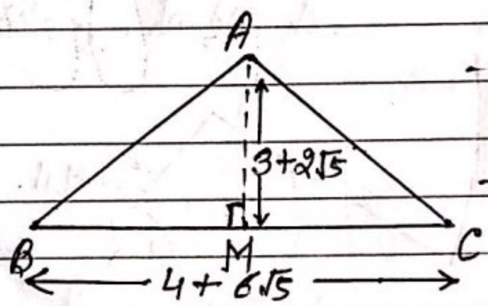
$$= \frac{2\sqrt{41}(4\sqrt{3}+5)}{(4\sqrt{3})^2 - 5^2}$$

$$= \underline{\frac{2\sqrt{41}(4\sqrt{3}+5)}{23}}$$ ✓

**Example 11.** The diagram shows an isosceles triangle $ABC$, where $AB = AC$. The point $M$ is the mid point of $BC$. Given that $AM = 3+2\sqrt{5}$ and $BC = 4+6\sqrt{5}$, find

(i) The area of triangle $ABC$. ---[2]

(ii) $\tan ABC$, giving your answer in the form $\frac{a+b\sqrt{5}}{c}$ where $a$, $b$ and $c$ are positive integers. ---[3]




**Solution:** Area of $\Delta ABC = \frac{1}{2} \times BC \times AM$

(AM ⊥ BC in an isosceles triangle median is perp. to the base.)

$$= \frac{1}{2}(4+6\sqrt{5})(3+2\sqrt{5})$$

$$= \underline{36+13\sqrt{5}} \checkmark$$

(ii) $\tan ABC = \frac{AM}{BM}$ $\qquad BM = \frac{1}{2}\cdot BC = \frac{1}{2}(4+6\sqrt{5}) = (2+3\sqrt{5}) \checkmark$

$$= \frac{3+2\sqrt{5}}{2+3\sqrt{5}} \times \frac{2-3\sqrt{5}}{2-3\sqrt{5}}$$

$$= \frac{-24-5\sqrt{5}}{-41}$$

$$= \underline{\frac{24+5\sqrt{5}}{41}} \checkmark$$

**Example 12.** Solve the equation $\sqrt{3}(1+x) = 2(x-3)$, giving your answer in the form $a+b\sqrt{5}$, where $a$ and $b$ are integers. ---[5]

[S-17/21/Q2(b)]

**Solution:** Given $\sqrt{3}(1+x) = 2(x-3)$

or $\sqrt{3}+\sqrt{3}x = 2x-6$

$\Rightarrow x(2-\sqrt{3}) = 6+\sqrt{3}$

$\Rightarrow x = \frac{6+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$= \underline{15+8\sqrt{3}} \checkmark$