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F. Me

Further Mechanics

Circular Motion Exercise

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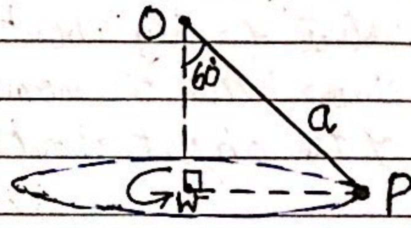
FM

Circular Motion

1. A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O.

(a) The particle P moves in a circle with a constant angular speed ω with the string inclined at $60°$ to the downward vertical through O. Show that $\omega^2 = 2g/a$ ---[4]



(b) The particle now hangs at rest a distance a, vertically below O. It is then projected horizontally so that it begins to move in a vertical circle with centre O. When the string makes an angle of $60°$ with the downward vertical through O, the angular speed of P is $\sqrt{2g/a}$. The string first goes slack when OP makes an angle θ with the upward vertical through O. Find the value of $\cos\theta$. [SP-20/03/Q5] --[6]

2. A particle P moves along an arc of a circle with centre O, and radius 2m. At time t seconds, the angle POA is θ, where $\theta = 1 - \cos 2t$, and A is a fixed point on the arc of the circle.

(i) Show that the magnitude of the radial component of the acceleration of P when $t = \frac{1}{6}\pi$ is $6\,\text{ms}^{-2}$. ---[2]

(ii) Find the magnitude of the transverse component of the acceleration of P when $t = \frac{1}{6}\pi$ [S-19/21/Q1] --[2]

3. ✗✗ [Not in this Unit] A thin uniform rod AB has mass KM and length 2a. The end A of the rod is rigidly attached to the surface of a uniform hollow sphere with centre O, mass KM and radius 2a. The end B of the rod is rigidly attached to the circumference of a uniform ring with centre C, mass M and radius a. The points C, B, A, O lie in a straight line. The horizontal axis L passes through the mid-point of the rod and is perpendicular to the rod and in the plane of the ring.

(continued→)

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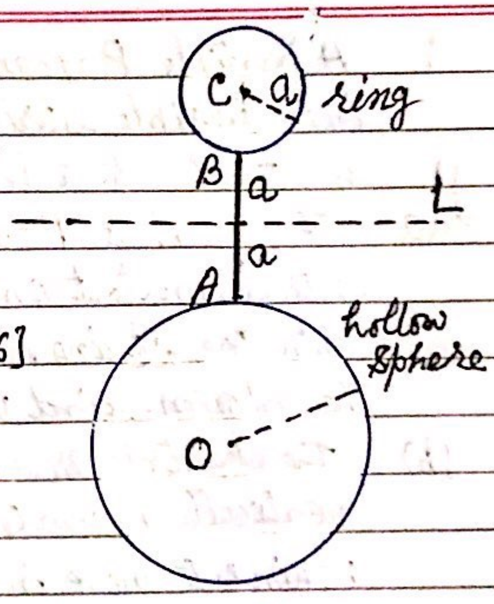
3. The object consisting of the rod, the ring and the hollow sphere can rotate freely about L.



(i) Show that the moment of inertia of the object about L is,

$$\frac{3}{2}(8k+3)Ma^2 \quad \text{---[6]}$$

The object performs small oscillations about L, with the ring above the sphere as shown in the diagram.

(ii) Find the set of possible values of k and the period of these oscillations in terms of k. [S-19/21/Q5] ---[6]

4. A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle P is moving in a complete vertical circle about O. The point A and B are on the circle, at the opposite ends of a diameter, and such that OA makes an acute angle α with the upward vertical through O. The speed of P as it passes through A is $\frac{3}{2}\sqrt{ag}$. The tension in the string when P is at B is four times the tension in the string when P is at A.

(i) Show that $\cos\alpha = 3/4$ [S-19/23/Q2] --[6]

(ii) Find the tension in the string when P is at B. --[2]

5. A particle P is moving in a circle of radius 2m. At time t seconds, its velocity is $(t-1)^2$ ms^{-1}. At a particular time T seconds, where $T>0$, the magnitude of the radial component of the acceleration of P is $8ms^{-2}$. Find the magnitude of the transverse component of the acceleration of P at this instant. [W-19/21/Q1] [5]

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6. A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O and P is held with string taut and horizontal. The particle P is projected vertically downwards with speed $\sqrt{(2ag)}$ so that it begins to move along a circular paths. The string becomes slack when OP makes an angle θ with the upwards vertical through O.

(i) Show that $\cos\theta = 2/3$ --- [5]

(ii) Find the greatest height, above the horizontal through O, reached by P it its subsequent motion. [W-19/21/Q4] --- [4]

7. A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is held so that the string is taut, with OP horizontal. The particle is projected downward with speed $\sqrt{(\frac{2}{5}ag)}$ and begin to move in a vertical circle. The string breaks when its tension is equal to $\frac{11}{5}mg$.

(i) Show that the string breaks when OP makes an angle θ with the downward vertical through O. where $\cos\theta = 3/5$. Find the speed of P at this instant. ---- [6]

(ii) For the subsequent motion after the string breaks, [6] find the distance OP when the particle P is vertically below O.

[S-18/22/Q11]

8. A particle P is moving in a fixed circle of radius 0.8 m. At time t s its velocity is $(t^2 - t + 2)\ ms^{-1}$. Find the magnitude of the radial and transverse components of the acceleration of P when $t = 2$. -- [3]

Radial Component ---

Transverse Component ---

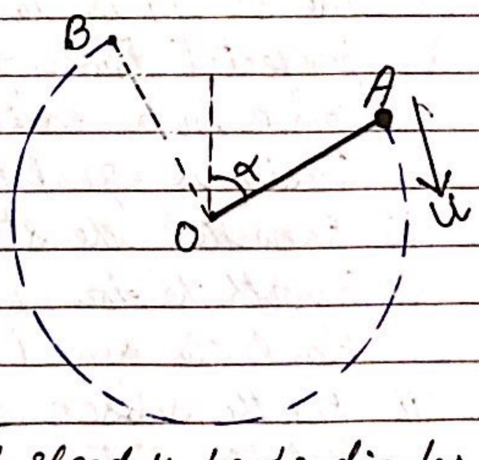
[S-18/23/Q1]

9. A particle P of mass m is free to move on the smooth inner surface of a fixed hollow sphere of radius a. The centre of the sphere is O and point C is on the inner surface of the sphere, vertically below O. The points A and B on the inner surface of the sphere are the ends of a diameter of the sphere. The diameter AOB makes an acute angle α with the vertical, where $\cos\alpha = \frac{4}{5}$, with A below the horizontal level of B. The particle is projected from A with speed u, and moves along the inner surface of the sphere towards C. The normal reaction forces on the particle at A and C are in the ratio 8:9.

(i) Show that $u^2 = 4ag$ ---[6]

(ii) Determine whether P reaches B without losing contact with the inner surface of the sphere. $\boxed{W-18/21/Q11}$ --[6]

10. A particle of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The point A is such that OA = a and OA makes an angle α with the upward vertical, where $\tan\alpha = 12/5$. The particle is projected downwards from A with speed u perpendicular to the string and move in vertical plane. The string becomes slack after the string has rotated through 270° from its initial position, with the particle now at the point B.

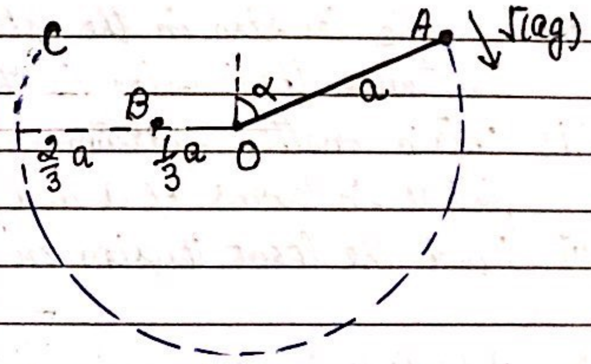


(i) Show that $u^2 = 2ag$ ---[5]

(ii) Find the maximum tension in the string as the particle moves from A to B. ---[4]

$\boxed{W-18/22/Q3}$

11\. A particle of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The point A is such that $OA = a$ and OA makes an angle α with the upward vertical through O. The particle is held at A and then projected downwards with speed $\sqrt{(ag)}$ so that it begins to move in a vertical circle with centre O. There is a small smooth peg at the point B, which is at the same horizontal level as O and at a distance $\frac{1}{3}a$ from O on the opposite side of O to A.

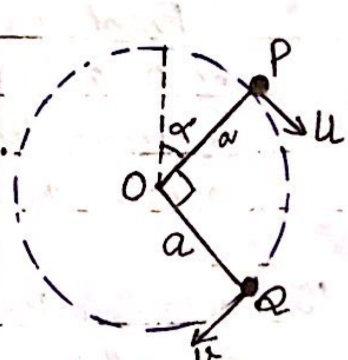


(i) Show that, when the string first makes contact with the peg, the speed of the particle is $\sqrt{(ag(1+2\cos\alpha))}$. ---[2]

The particle now begins to move in a vertical circle with centre B. When the particle is at the point C where angle $CBO = 150°$, the tension in the string is the same as it was when the particle was at the point A.

(ii) Find the value of $\cos\alpha$. [S/17/21/Q5] [10]

12\. A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is moving in complete vertical circles with the string taut. When the particle is at the point P, where OP makes an angle α with the upward vertical through O, its speed is u. When the particle is at Q, where angle $QOP = 90°$, its speed is v. It is given that $\cos\alpha = 4/5$



(i) Show that $v^2 = u^2 + \frac{14}{5}ag$ ---[2]

(Continued →)

(Continued →)

12→ The tension in the string when the particle is at Q is twice the tension in the string when the particle is at P.

(ii) Obtain another equation relating u^2, v^2, a and g and hence find u in terms of a and g. ---[5]

(iii) Find the least tension in the string during the motion. ---[3]

[S-17/23/Q5]

13. A particle P is moving in a circle of radius 0.8 m. At time ts its velocity is $(8 - pt + t^2)\,ms^{-1}$, where p is a constant. The magnitude of the transverse component of the acceleration of P when t = 2 is zero. Find the magnitude of the radial component of the acceleration of P when t = 2. [W-17/21/Q1] ---[4]

14. A particle is at rest at the lowest point on the smooth inner surface of a hollow sphere with centre O and radius a. The particle is projected horizontally with speed u and begins to move in a vertical circle on the inner surface of the sphere. The particle loses contact with the sphere at the point A, where OA makes an angle θ with the upward vertical through O. Given that the speed of P at A is $\sqrt{(\frac{3}{5}ag)}$, find u in terms of a and g. ---[5]

Find in terms of a, the greatest height above the level of O achieved by P in its susequent motion (You may assume that P achieves its greatest height before it makes any further contact with the sphere.) ---[5]

[S-16/21/Q4]

15. A particle P is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle moves a complete vertical circles with centre O. The tension in the string when P is at its lowest point is twice the tension in the string when P is at its highest point. Find, in terms of a and g, the greatest speed of P during the motion. [S-16/23/Q7] ---[6]

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16. A particle P of mass 0.2 kg is moving in circle of radius 0.5 m. At time t s its velocity is $(t^2+ct+d)\,ms^{-1}$, where c and d are constants. The magnitude of the resultant force acting on P when t=3 is $\frac{2}{5}\sqrt{17}$ N, the transverse component of the acceleration of P when t=3 is $2\,ms^{-2}$. Find the value of c and d. [S-16/23/Q3] --[8]

17. A particle P is moving in a circle of radius 0.25 m. At time t seconds, its velocity is $(2t^2-4t+3)\,ms^{-1}$. At a particular time T seconds, where $T>0$, the magnitude of the transverse component of the acceleration of P is $6\,ms^{-2}$. Find the magnitude of the radial component of the acceleration of P at this instant. [S-15/21/Q1] --[4]

18. A particle P, of mass m, is placed at the highest point of a fixed solid smooth sphere with centre O and radius a. The particle P is given a horizontal speed u and it moves in a part of a vertical circle, with centre O, on the surface of the sphere. When OP makes an angle θ with the upward vertical, and P is still in contact with the surface of the sphere, the speed of P is v and the reaction of the sphere on P has magnitude R.
Show that $R = mg(3\cos\theta - 2) - mu^2/a$. --[5]
The particle loses contact with the sphere at the instant when $v = 2u$, find u in terms of a and g. --[4]
[S-15/21/Q3]

19. A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. When P is hanging at rest vertically below O, it is projected horizontally. In the subsequent motion P completes a vertical circle. The speed of P when it is at its highest point is u.
(Continued→)

(Continued →)

19 → Show that the least possible value of u is $\sqrt{(ag)}$ --[2]

It is now given that $u=\sqrt{(ag)}$. when P passes through the lowest point of its path, it collides with, and coalesces with, a stationary particle of mass $\frac{1}{4}m$. Find the speed of the combined particle immediately after a collision. --[4]

In the subsequent motion, when OP makes an angle θ with the upward vertical the tension in the string is T. Find an expression for T in terms of m, g and θ. --[5]

Find the value of $\cos\theta$ when the string becomes slack. --[2]

[W-15 / 21 / Q4]

← × — × →

(Answers— on the next page.)

Circular Motion

(In vertical plane)

Answers

1.(a) Vertically, $T\cos 60 = mg$ —(1)

Horizontally, $T\sin 60 = mr\omega^2$

or $T\sin 60 = m.a\sin 60.\omega^2$ —(2)

(2) ÷ (1) $\omega^2 = \frac{2g}{a}$ —(3) ✓

(b) Energy from 60° with downward vertical to point when string goes slack,

$$\frac{1}{2}m\left(a\sqrt{\frac{2g}{a}}\right)^2 - \frac{1}{2}mv^2 = mg\underbrace{(a\cos 60 + a\cos\theta)}_{(h_1+h_2)}$$

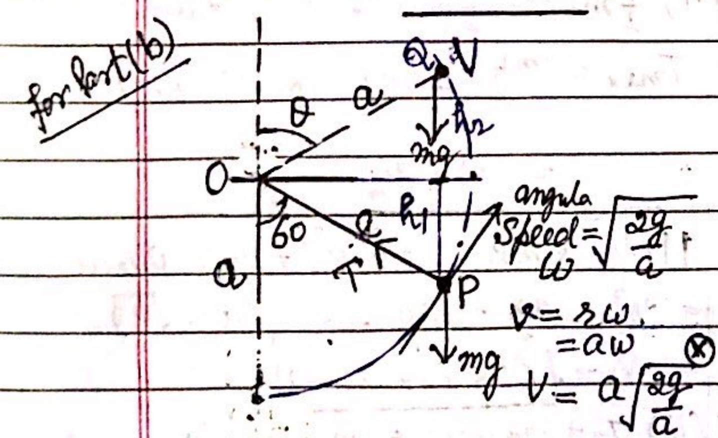
$\Rightarrow v^2 = ga - 2ga\cos\theta$ —(4)

when string goes slack, tension is zero, $mg\cos\theta = \frac{mv^2}{a}$ —(5)

from (4) & (5)

$\cos\theta = \frac{1}{3}$ ✓

for part (b)



2(i) radial acc. $a_R = r\omega^2 = 2\left(\frac{d\theta}{dt}\right)^2$

$\theta = 1-\cos 2t$

$\Rightarrow \frac{d\theta}{dt} = 2\sin 2t$ $\Rightarrow a_R = 2(2\sin 2t)^2$

at $t = \frac{\pi}{6} \Rightarrow a_R = 2\left(2\sin\frac{\pi}{3}\right)^2$

$= 2(\sqrt{3})^2 = 6\,ms^{-2}$ ✓

(ii) Transverse comp, $a_T = r\frac{d^2\theta}{dt^2} = 2\times(4\cos 2t)$

$\therefore$ at $t = \frac{\pi}{6} \Rightarrow a_T = 2\left(4\cos\frac{\pi}{3}\right) = 4\,ms^{-2}$

3. × Not included in this unit.

4. at A, tension T_A from $F = ma$ radially

$T_A = mV_A^2/a - mg\cos\alpha$

or $T_A = m(9ag/4)/a - mg\cos\alpha$ —(1)

at B, tension T_B radially from $F = ma$

$T_B = mV_B^2/a + mg\cos\alpha$ —(2)

Conservation of energy at B

$\frac{1}{2}mV_B^2 = \frac{1}{2}mV_A^2 + 2mga\cos\alpha$

$\Rightarrow V_B^2 = ag\left(\frac{9}{4} + 4\cos\alpha\right)$ —(3)

Now given $T_B = 4T_A$

$\Rightarrow V_B^2 = 4V_A^2 - 5ag\cos\alpha$

or $V_B^2 = \frac{\cancel{4}\times 9ag}{\cancel{4}} - 5ag\cos\alpha$

$V_B^2 = 9ag - 5ag\cos\alpha$ —(4)

for (3) & (4)

$\frac{9ag}{4} + 4ag\cos\alpha = 9ag - 5ag\cos\alpha$

$\Rightarrow 9\cancel{ag}\cos\alpha = 27\times\frac{\cancel{ag}}{4}$

$\Rightarrow \cos\alpha = 3/4$ ✓

(ii) $T_B = 4T_A = (9 - 4\cos\alpha)mg = 6mg$ ✓

5. $a_R = \frac{V^2}{r} = \frac{(T-1)^4}{2} = 8$

$\Rightarrow T-1 = 2 \Rightarrow T = 3$ | $\because V = (t-1)^2$

Now $a_{Tran} = \frac{dV}{dt} = 2(T-1)$

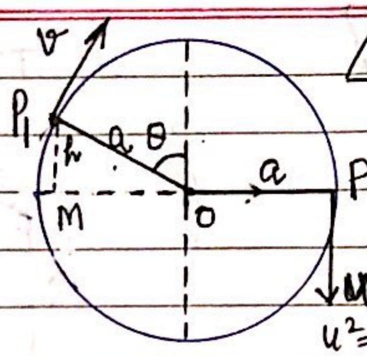
$\Rightarrow$ at $T=3$; $a_{Tran} = 2(3-1)$

$= 4\,ms^{-2}$ ✓

F. M2

Answers

6.



$u = \sqrt{2ag}$

$u^2 = 2ag$

$P_1M = a\cos\theta$

(i) Conservation of energy to slack point P_1

$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mg \cdot a\cos\theta$ — ①

Equating tension at P_1 to 0 and $F = ma$

$T = \frac{mv^2}{a} - mg\cos\theta = 0$ —

$\Rightarrow v^2 = ag\cos\theta$ — ②

fr ① $v^2 = u^2 - 2ga\cos\theta$ — ③

from ② and ③ $[u^2 = 2ag]$

$2ag - 2ag\cos\theta = ag\cos\theta$

$\Rightarrow \cos\theta = 2/3$ ✓, $\sin\theta = \frac{\sqrt{5}}{3}$

(ii) Vertical speed V_v at P_1

$V_v = v\sin\theta = \sqrt{\frac{2ag}{3}} \times \frac{\sqrt{5}}{3}$

$V_v^2 = \frac{10}{27}ag$ $\longrightarrow 0 = V_v^2 - 2gh$

$\Rightarrow h = V_v^2/2g = \frac{5}{27}a$

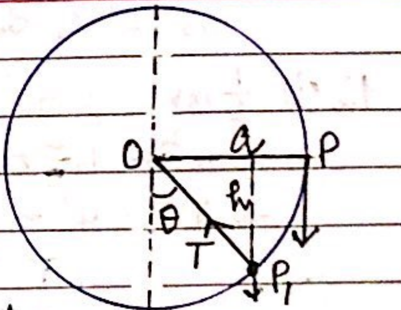
∴ Greatest height above the horizontal POM.

$H = h + a\cos\theta$

$= \frac{5}{27}a + \frac{2}{3}a$

$= \frac{2}{27}a = 0.852a$ ✓

7,



$(h = a\cos\theta)$

(i)

Using conservation of energy.

$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mg \cdot a\cos\theta$ — ①

finding tension at P_1

$T = \frac{mv^2}{a} + mg\cos\theta$

fr ①

$T = \frac{mu^2}{a} + 2mg\cos\theta + mg\cos\theta$

$T = \frac{mu^2}{a} + 3mg\cos\theta$ — ②

Put $T = 11/5\, mg$; $u = \sqrt{(2ag/5)}$

fr ②

$\frac{11}{5}mg = 2mg/5 + 3mg\cos\theta$

$\Rightarrow \cos\theta = 3/5$ ✓

$\tan\theta = 4/3$

∴ fr ①

$v^2 = u^2 + 2ag\cos\theta$

$= (\frac{2}{5} + \frac{6}{5})ag = 8ag/5$

$v = 4\sqrt{a}$ ✓ — ③

(ii) for horizontal motion, time t to P_2,

$t = \frac{a\sin\theta}{v\cos\theta} = \frac{4a}{3v} = \frac{4a}{3 \times 4\sqrt{a}} = \frac{1}{3}\sqrt{a}$ ✓

Now Vertically

$h = (v\sin\theta)t + \frac{1}{2}gt^2$

$= \frac{16a}{15} + \frac{5a}{9} = \frac{73a}{45}$

$OP_2 = h + a\cos\theta$

$= \frac{73a}{45} + \frac{3a}{5} = \frac{20a}{9}$ ✓

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Answers

8. $r = 0.8\,m$, $(V = t^2 - t + 2)$

$a_R = \frac{V^2}{r} = \frac{(t^2 - t + 2)^2}{r}$

at $t = 2$, $a_R = \frac{(2^2 - 2 + 2)^2}{0.8} = \underline{20\,ms^{-2}}$ ✓

and $a_{Tran} = \left(\frac{dV}{dt}\right)_{t=2} = (2t - 1)_{t=2}$

$= \underline{3\,ms^{-2}}$ ✓

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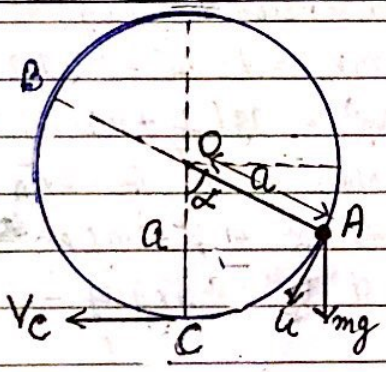
9. $\cos\alpha = \frac{4}{5}$, $\sin\alpha = \frac{3}{5}$



(i) Conservation of energy

$\frac{1}{2} m V_C^2 = \frac{1}{2} m u^2 + mga(1 - \cos\alpha)$

$\Rightarrow V_C^2 = u^2 + \frac{2}{5} ag$ —— ①

R_A at A using $F = ma$

$R_A = mg\cos\alpha + \frac{mu^2}{a}$ —— ②

and $R_C = mg + mV_C^2/a$ —— ③

Given $\frac{R_C}{R_A} = 9/8$

$\Rightarrow 8R_C = 9R_A$

$8[ag + u^2 + \frac{2}{5} ag] = 9\{\frac{4}{5} ag + u^2\}$

$\Rightarrow u^2 = (8 + 16/5 - \frac{36}{5}) ag$

$u^2 = \underline{4ag}$ ✓

(Continued →)

9(ii) $\frac{1}{2} m V_B^2 = \frac{1}{2} m u^2 - 2mga\cos\alpha$

$\Rightarrow V_B^2 = (4 - \frac{16}{5}) ag = \frac{4}{5} ag$ ✓ ≥ 0

So P reaches height of B.

$R_B = mV_B^2/a - mg\cos\alpha = 4mg/5 - 4\frac{mg}{5} = 0$

≥ 0 so P still (just) in contact at B.

10.(i) $\sin\alpha = 12/13$, $\cos\alpha = 5/13$

$\frac{1}{2} m V_B^2 = \frac{1}{2} m u^2 - mga(\sin\alpha - \cos\alpha)$

$\Rightarrow V_B^2 = u^2 - \frac{14}{13} ag$ —— ①

$T_B = mV_B^2/a - mg\sin\alpha = 0$

$\Rightarrow V_B^2 = 12/13\, ag$ —— ②

from ① and ②

$u^2 = (3\sin\alpha - 2\cos\alpha) ag$

$u^2 = (36/13 - \frac{10}{13}) ag = \underline{2ag}$ ✓

(ii) $\frac{1}{2} m V_C^2 = \frac{1}{2} m u^2 + mga(1 + \cos\alpha)$

$T_{max} = \frac{mV_C^2}{a} + mg = \frac{62mg}{13} + mg$

$= 75mg/13$ ✓

11(i) $\frac{1}{2} m V_1^2 = \frac{1}{2} m u^2 + mg \cdot a\cos\alpha$

$\Rightarrow V_1^2 = ag + 2ag\cos\alpha$ [$\because u = \sqrt{ag}$]

$\Rightarrow V_1 = \sqrt{(ag(1 + 2\cos\alpha))}$ ✓

(ii) $T_A + mg\cos\alpha = m(\sqrt{ag})^2/a$

$\Rightarrow T_A = mg(1 - \cos\alpha)$ —— ①

at C, $\frac{1}{2} m V_2^2 = \frac{1}{2} m u^2 + mg\,a\cos\alpha - mg \times \frac{2}{3} a \cos 60^\circ$

$\Rightarrow V_2^2 = ag + 2ag\cos\alpha - \frac{2}{3} ag$

or $V_2^2 = (\frac{1}{3} + 2\cos\alpha) ag$.

$T_C + mg\cos 60^\circ = mV_2^2/\frac{2}{3}a = 3mV_2^2/2a$

$\Rightarrow T_C = 3mg\cos\alpha$ —— ②

from ① & ② $mg(1 - \cos\alpha) = 3mg\cos\alpha$

$\Rightarrow \underline{\cos\alpha = 1/4}$ ✓

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P-12

$\cos\alpha = 4/5$, $\sin\alpha = 3/5$

Answers

12 (i) $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga(\cos\alpha + \sin\alpha)$

$\Rightarrow v^2 = u^2 + 2ag(4/5 + 3/5)$

$\Rightarrow v^2 = u^2 + 14ag/5$ ✓ — (1)

(ii) $T_P = mu^2/a - mg\cos\alpha$

$T_Q = mv^2/a + mg\sin\alpha$

Given $T_Q = 2 \cdot T_P$

$\Rightarrow v^2 = 2u^2 - ag(2\cos\alpha + \sin\alpha)$

$\Rightarrow v^2 = 2u^2 - 11ag/5$ — (2)

from (1) & (2) $u = \sqrt{(5ag)}$

(and $v^2 = 29ag/5$)

(iii) $T_{min} + mg = mV^2/a$

$\frac{1}{2}mV^2 = \frac{1}{2}mu^2 - mga(1-\cos\alpha)$

$\Rightarrow V^2 = (5 - \frac{2}{5})ag = 23ag/5$

$T_{min} = 23mg/5 - mg$

$= 18mg/5$ ✓

13. $V = (8 - pt + t^2)$, $r = 0.8m$

$\frac{dv}{dt} = -p + 2t$

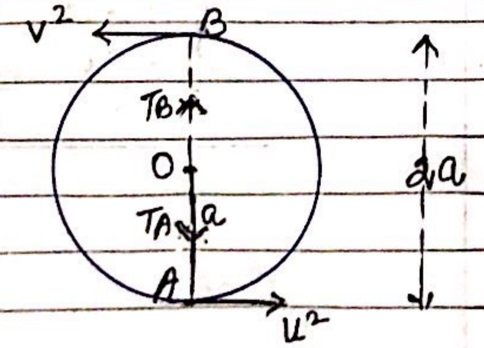
$a_{Tran} = \left(\frac{dv}{dt}\right)_{t=2} = -p + 4 = 0$

$\Rightarrow p = 4$ — (1)

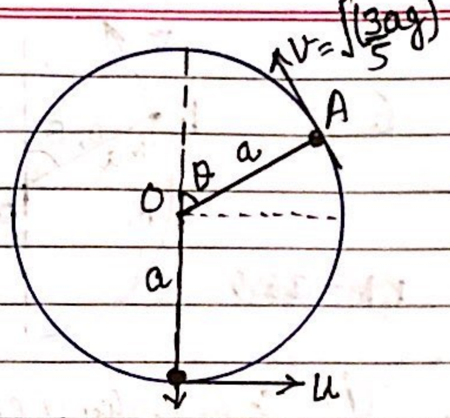
$\therefore a_R = \frac{V^2}{r} = \frac{(8 - pt + t^2)^2}{0.8}$

$= (8 - 4t + t^2)^2 / 0.8$

at $t = 2$, $a_R = 4^2/0.8 = 20\, ms^{-2}$



14.



(i) $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mga(1 + \cos\theta)$

$\Rightarrow v^2 = u^2 - 2ga(1 + \cos\theta)$ — (1)

a_R radially $F = ma$

$\frac{mv^2}{a} = mg\cos\theta$ [T=0]

$\Rightarrow v^2 = ag\cos\theta$ — (2)

from (1) & (2) eliminate $\cos\theta$

$\Rightarrow 3v^2 = u^2 - 2ga$

$\frac{9}{5}ag = u^2 - 2ga$ [Given $v^2 = \frac{3}{5}ag$] — (4)

$\Rightarrow u^2 = 19ag/5$

$u = \sqrt{(19ag/5)}$ ✓

from (2) & (4)

$\cos\theta = 3/5$ ✓, $\sin\theta = \frac{4}{5}$

(ii) Vertical comp. of speed at A

$V = v\sin\theta = \frac{4}{5}v$

$h_A = \frac{V^2}{2g} = \frac{24a}{125}$ [Height above A = h_A]

$\therefore h_O = h_A + a\cos\theta$

$= 99a/125$ ✓

15. $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - 2mga$

$\Rightarrow v^2 = u^2 - 4ag$ — (1)

at A, $T_A = mu^2/a + mg$

at B, $T_B = mv^2/a - mg$

Given $T_A = 2T_B \Rightarrow mu^2/a + mg = 2[mv^2/a - mg]$

$\Rightarrow 2v^2 = u^2 + 3ag$ — (2)

from (1) and (2) $u^2 = 11ag$

$\Rightarrow u = \sqrt{(11ag)}$ ✓ and $v = \sqrt{(7ag)}$ ✓

Circular Motion

Answers

16. $v = t^2 + ct + d$

Transverse acc, $a_T = \frac{dv}{dt} = 2t + c + 0$

at $t=3$, $a_T = 2\times 3 + c = 2$ (given)

$\Rightarrow C = -4$ ✓ —①

Radial force $F_R = mv^2/r$

at $t=3$, $= 0.2(3^2+3c+d)^2/0.5$

$F_R = 0.4(d-3)^2$ —②

Transverse force at $t=3$

$F_T = ma = 0.2\times 2 = 0.4$ —③

frm ② & ③ $F_T^2 + F_R^2 = (2\sqrt{17}/5)^2$ Given

$\Rightarrow 0.4^2 + 0.4^2(d-3)^4 = \frac{68}{25}$

$\Rightarrow 1 + (d-3)^4 = \frac{68}{25}\times\frac{1}{0.4^2}$

$1 + (d-3)^4 = \frac{68}{25\times 0.16} = \frac{68}{4} = 17$

$\Rightarrow (d-3)^4 = 16$

$d - 3 = \pm 2$

$\Rightarrow d = \pm 2 + 3 = 5$ or 1

$\therefore$ $C = -4$ ✓ & $d = 5$ or 1 ✓

17. $r = .25\,m$; $v = 2t^2 - 4t + 3$

$\frac{dv}{dt} = 4t - 4$

at time T, $a_T = \left(\frac{dv}{dt}\right)_T = 4T - 4 = 6$ given

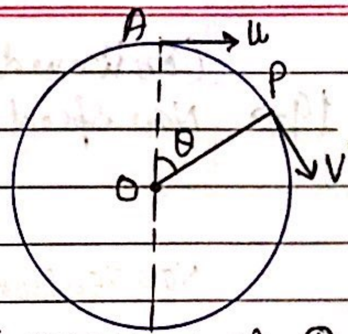
$\Rightarrow T = 2.5 = 5/2$

Now Radial acc. $a_R = \frac{v^2}{r} = (2T^2 - 4T + 3)^2/0.25$

at $T = 2.5$; $a_R = \left(\frac{11}{2}\right)^2 \times 4$

$= 121\ ms^{-2}$ ✓

18.



$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga(1-\cos\theta)$ —①

$R = mg\cos\theta - \frac{mv^2}{a}$ —②

from ① and ②

$R = mg(3\cos\theta - 2) - mu^2/a$ ✓

Now when $R = 0$

$u^2 = ag(3\cos\theta - 2)$ —③

and $v^2 = ag\cos\theta$ —④

given $v = 2u$

or $v^2 = 4u^2$ —⑤

fr ③ ④ & ⑤ $ag\cos\theta = 4ag(3\cos\theta - 2)$

$\Rightarrow \cos\theta = 8/11$ ✓

fr ③ $u^2 = ag\left[3\times\frac{8}{11} - 2\right] = 2ag/11$

$\Rightarrow u = \sqrt{(2ag/11)}$ ✓

19. Tension at top $F = ma$

$T = mu^2/a - mg$ —①

for u_{min} $T \geq 0 \Rightarrow u^2/a - g \geq 0$

$\Rightarrow u = \sqrt{(ag)}$ —② ✓

Speed at bottom,

$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mg\times 2a$

$\Rightarrow v^2 = ga + 4ag \Rightarrow v = \sqrt{5ag}$

Now new speed V at bottom

$(m + \frac{1}{4}m)V = mv \Rightarrow V = \frac{4}{5}v = 4\sqrt{\frac{ag}{5}}$

or $V = \frac{4}{5}\sqrt{(5ag)}$ ✓ —③

(Continued→)

(Continued→)

Answers

19→ Now speed w^2 at angle θ; $\frac{1}{2}m'w^2 = \frac{1}{2}m'V^2 - m'ga(1+\cos\theta)$

$\Rightarrow w^2 = ag(6/5 - 2\cos\theta)$ ⟵ (4)

Now Tension T'; $F=ma$; $T' = m'w^2/a - m'g\cos\theta$

$= m'V^2/a - m'g(2+3\cos\theta)$

$= (5mg/4)(6/5 - 3\cos\theta)$

$T' = 3mg/2 - \frac{15}{4}mg\cos\theta$ — (5)

When string becomes slack $\Rightarrow T' = 0$

$\Rightarrow \underline{\cos\theta} = \frac{1}{3} \times 6/5 = \underline{2/5}$ ✓

⟵ X —— X ⟶