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F. Me

Further Mechanics

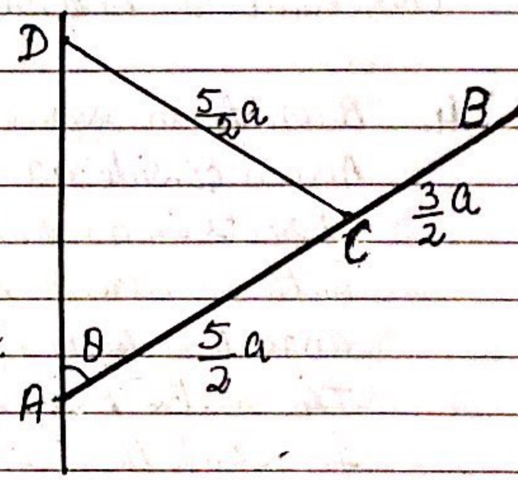
Equilibrium of a Rigid Body

Exercise

Suresh Goel
Noida, Delhi N.C.R.
INDIA.

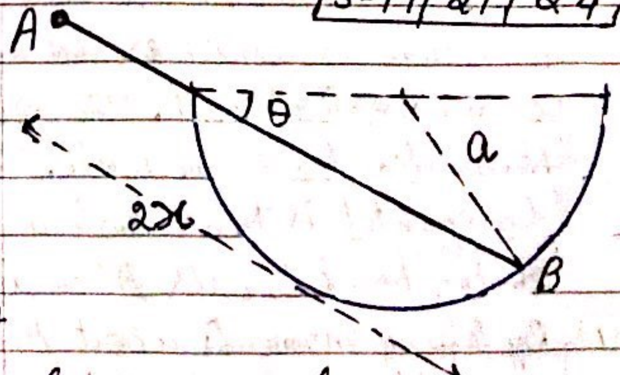
1. A child's toy consists of a uniform solid circular cone, of vertical height $3r$ and radius r, and a uniform solid hemisphere of radius r. The circular bases of the cone and the hemisphere are joined together so that they coincide. The cone and the hemisphere are made of the same material. Show that the centre of mass of the toy is at a distance $\frac{27}{10}r$ from the vertex of the cone. [SP-20/03/Q1]...[4]

2. A uniform rod AB of length $4a$ and weight W rests with the end A in contact with a rough vertical wall. A light inextensible string of length $\frac{5}{2}a$ has one end attached to the point C on the rod, where $AC = \frac{5}{2}a$. The other end of the string is attached to a point D on the wall, vertically above A. The vertical plane containing the rod AB is perpendicular to the wall. The angle between the wall and the rod is θ, where $\tan\theta = 2$. The end A of the rod is on the point of slipping down the wall and the coefficient of friction between the rod and the wall is μ. Find the tension in the string and the value of μ. ---[10]



[S-19/21/Q4]

3. A uniform rod AB of length $2x$ and weight W rests on the smooth rim of fixed hemispherical bowl of radius a. The end B of the rod is in contact with the rough inner surface of the bowl. The coefficient of friction between the rod and the bowl at B is $\frac{1}{3}$. (continued→)



(continued→)

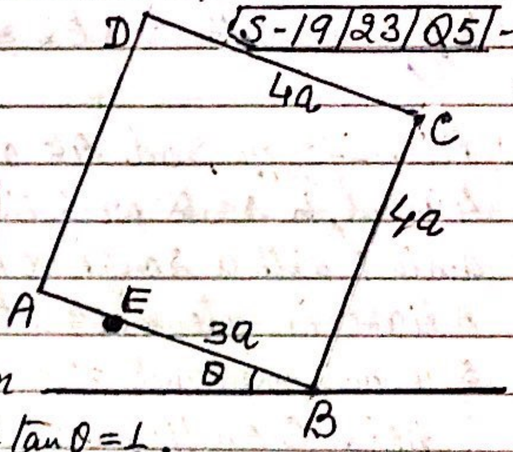
3→ A particle of weight $\frac{1}{4}W$ is attached to the end A of the rod. The end B is about to slip upwards when AB is inclined at an angle θ to the horizontal, where $\tan\theta = 3/4$

(i) By resolving parallel to the rod, show that the normal component of the reaction of the bowl on the rod at B is $\frac{3}{4}W$. --[5]

(ii) Find, in term of W, the reaction between the rod and the smooth rim of the bowl. [S-19/23/Q5] ---[4]

(iii) Find x in terms of a. --[3]

4. A uniform squre lamina ABCD of side $4a$ and weight W rests in a vertical plane with the edge AB inclined at an angle θ to the horizontal, where $\tan\theta = \frac{1}{3}$.



The vertex B is in contact with a rough horizontal surface for which the coefficient of friction is μ. The lamina is supported by a smooth peg at the point E on AB, where $BE = 3a$.

(i) Find an expression in terms of W for the normal reaction forces at E and B. ---[5]

(ii) Given that the lamina is about to slip, find the value of μ.--[3]

[W-19/21/Q2]

5. A uniform rod AB has length $2a$ and weight W. The end A rests on rough horizontal ground and the end B rests against a smooth vertical wall. The rod is in vertical plane that is perpendicular to the wall. The angle between the rod and the horizontal is θ. A particle of weight 5W hangs from the rod at the point C, with $AC = xa$, where $0 < x < 1$.

(i) By taking moments about A, show that the magnitude of the normal reaction at B is $\frac{W(5x+1)}{2\tan\theta}$ ---[3]

(continued→)

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(continued→)

5→ The particle of weight $5W$ is now moved a distance a up the rod, so that $AC=(x+1)a$. This results in the magnitude of the normal reaction at B being double its previous value. The system remains in equilibrium with the rod at angle θ with horizontal.

(ii) Show that $x = 4/5$ --[3]

The coefficient of friction between the rod and the ground is $2/3$.

(iii) Given that the rod is about to slip when the particle of weight $5W$ is in its second position, find the value of $\tan\theta$. $\boxed{S-18/22/Q4}$ --[5].

6. A uniform rod AB has length $2a$ and weight W. The end A rests on rough horizontal ground and the end B rests against a smooth vertical wall. The angle between the rod and the horizontal is θ, where $\tan\theta = 4/3$. One end of a light inextensible rope is attached to a point C on the rod. The other end is attached to a point where the vertical wall and the horizontal ground meet. The rope is taut and perpendicular to the rod. The rope and rod are in a vertical plane perpendicular to the wall.

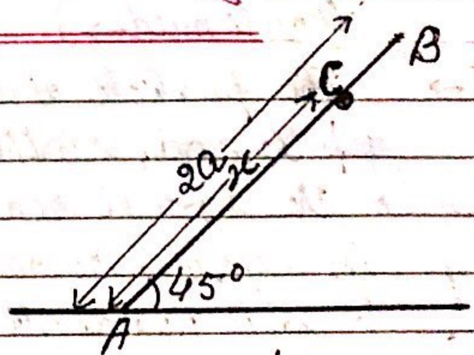
(i) Show that $AC = 18/25\, a$ ---[2]

The magnitude of the frictional force at A is equal to one quarter of the magnitude of the normal reaction force at A.

(ii) Show that the tension in the rope is $\frac{1}{4}W$. ---[6]

(iii) Find expressions, in terms of W, for the magnitudes of the normal reaction forces at A and B. $\boxed{S-18/23/Q4}$ ---[2]

7. A uniform rod AB of length 2a and weight W rests against a smooth horizontal peg at a point C on the rod, where AC = x. The lower end A of the rod rests on rough horizontal ground. The rod is in equilibrium inclined at an angle of 45° to the horizontal. The coefficient of friction between the rod and the ground is μ. The rod is about to slip at A.

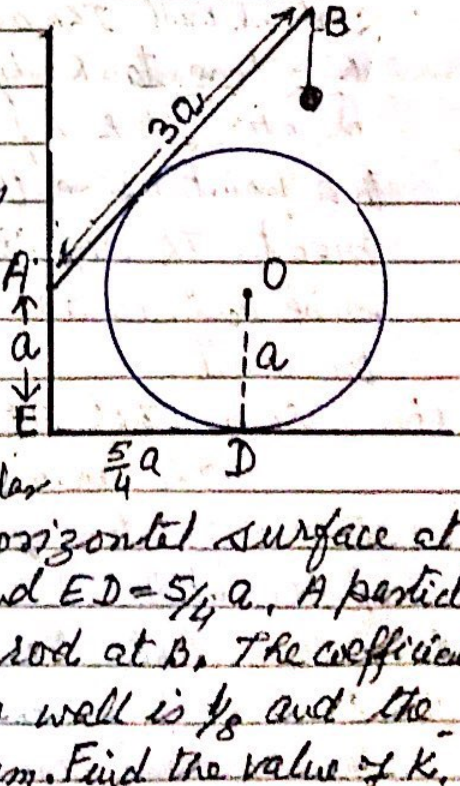


(i) Find an expression for x in terms of a and μ. ---[5]

(ii) Hence show that $\mu \geq 1/3$ ---[2]

(iii) Given that $x = 3/2\, a$, find the value of μ and the magnitude of the resultant force on the rod at A. [W-18/22/Q4] ---[4]

8. A uniform smooth disc with centre O and radius a is fixed at the point D on a horizontal surface. A uniform rod of length 3a and weight W rests on the disc with its end A in contact with a rough vertical wall. The rod and the disc lie in a vertical plane that is perpendicular to the wall. The wall meets the horizontal surface at the point E such that AE = a and ED = 5/4 a. A particle of weight KW is hung from the rod at B. The coefficient of friction between the rod and the wall is 1/8 and the system is in limiting equilibrium. Find the value of K. --[8]

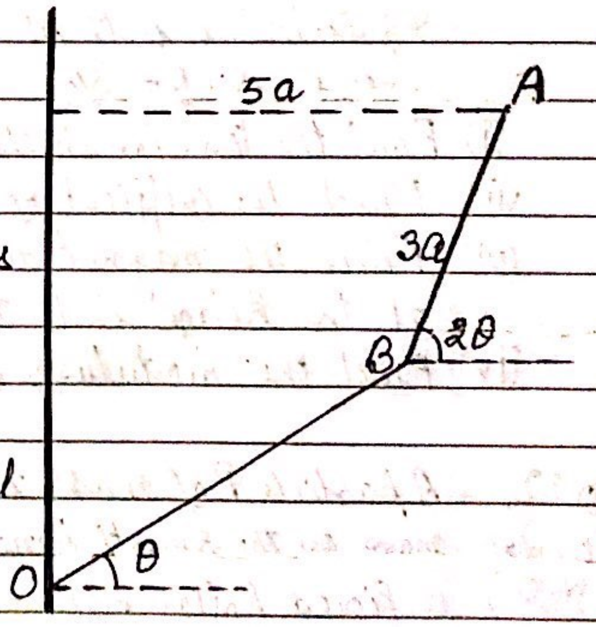


[S-17/21/Q2]

9. A particle P of mass 3m is attached to one end of a light elastic string of natural length a and modulus of elasticity kmg. The other end of the string is attached to fixed point O on a smooth plane that is inclined to the horizontal at an angle α, where $\sin\alpha = 2/3$. The system rests in equilibrium with P on the plane at the point E. The length of the string in this position is $5/4\,a$.

(i) Find the value of k [S-17/21/Q11] --[3]

10. A uniform rod AB of length 3a and weight W is freely hinged to a fixed point at the end A. The end B is below the level of A and is attached to one end of a light elastic string of natural length 4a. The other end of the string is attached to a point O on a vertical wall. The horizontal distance between A and the wall is 5a.



The string and the rod make angles θ and 2θ respectively with the horizontal. The system is in equilibrium with the rod and the string in the same vertical plane. It is given that $\sin\theta = 3/5$ and you may use the fact that $\cos 2\theta = 7/25$

(i) Find the tension in the string in terms of W. ---[3]

(ii) Find the modulus of elasticity of the string in terms of W --[4]

(iii) Find the angle that the force acting on the rod at A makes with the horizontal. [S-17/23/Q4] ---[3]

11. A small ring P of weight W is free to slide on a rough horizontal wire, one end of which is attached to vertical wall at Q. The end A of a thin uniform rod AB of length 2a and weight $5/2\,W$ is freely hinged --- (Continued →)

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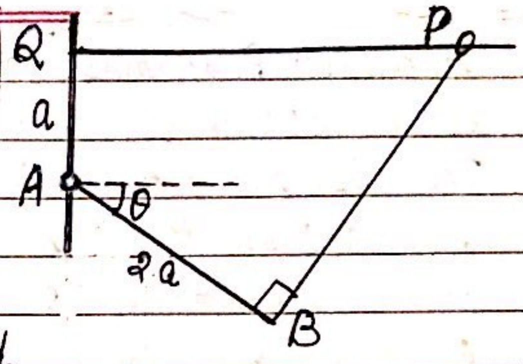
11→ to the wall at the point A which is a distance a vertically below Q. A light elastic string of natural length 2a has one end attached to the ring P and the other end attached to the rod



at B. The string is at right angles to the rod and A, B, P and Q lie in a vertical plane. The system is in limiting equilibrium with AB making an angle θ with the horizontal, where $\sin\theta = 3/5$

(i) Find the tension in the string in terms of W. --[2]

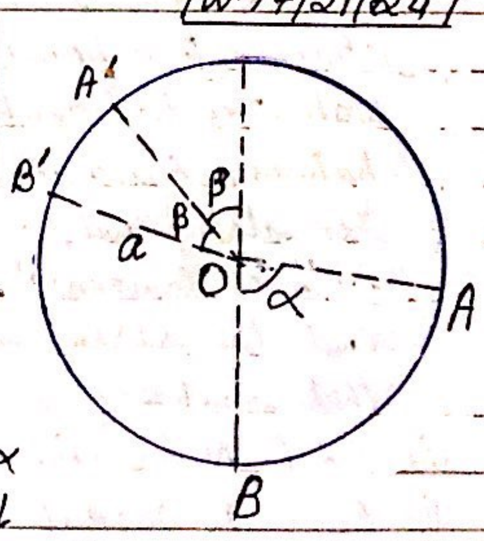
(ii) Find the coeffient of friction between the ring and the wire. --[2]

(iii) Find the magnitude of the resultant force on the rod at the hinge in terms of W. --[3]

(iv) Find the modulus of elasticity of the string in terms of W --[3]

[W-17/21/Q4]

12. (Circular Motion) A particle P of mass m is free to move on the smooth inner surface of a fixed hollow sphere of radius a. The centre of the sphere is O. The points A and A′ are on the inner surface of the surface, on the opposite sides of the vertical through O, the radius OA makes an angle α with the downward vertical and



the radius OA′ makes an angle β with the upward vertical. The point B is on the inner surface of the sphere, vertically below O. The point B′ is on the inner surface of the sphere and such that OB′ makes an angle 2β with the upward vertical through O. It is given that $\cos\alpha = 1/16$

(Continued→)

(Continued→)

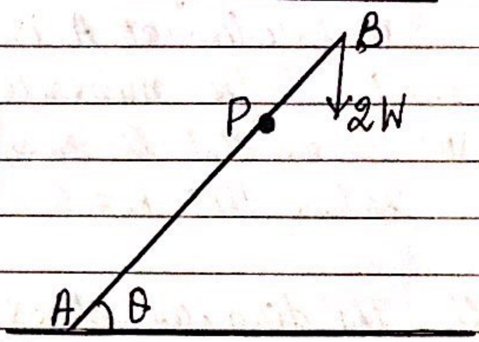
12(i)→ P is projected from A with speed u along the surface of the sphere downwards towards B. Subsequently it loses contact with the sphere at A'. Show that $u^2 = \frac{1}{8}ag(1+24\cos\beta)$ ---[5]

(ii) P is now projected from B with speed u along the surface of the sphere towards B'. Subsequently it loses contact with the sphere at B'. Find $\cos\beta$. ---[6]

(iii) In part (i), the reaction of the sphere on P when it is initially projected at A is R. Find R in terms of m and g. ---[3]

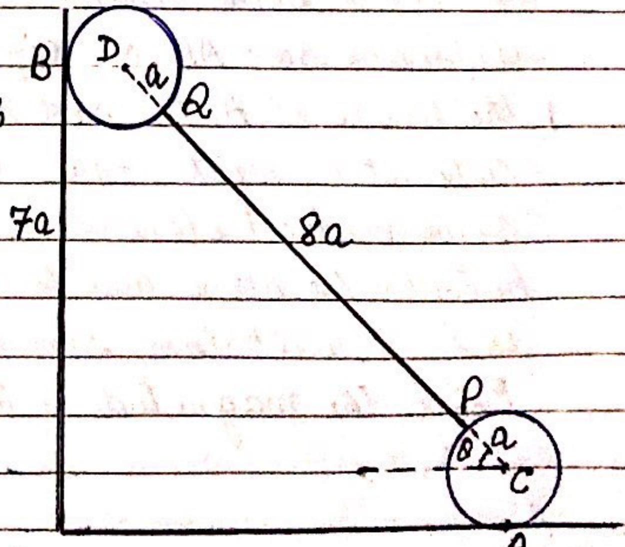
[W-17/21/Q 11]

13. A uniform rod AB of length 2a and weight W rests with its lower end A in contact with a rough horizontal surface. The rod rest on a smooth peg at the point P. A particle of weight 2W is suspended from end B of the rod.



The system is in limiting equilibrium with the angle between the rod and the horizontal surface equal to θ, whose $\tan\theta = \frac{4}{3}$. The coefficient of friction between the rod and the surface is 2/3. Find the distance AP. [S-16/23/Q2] --[8]

14. The end P of a uniform rod PQ, of weight KW and length 8a, is rigidly attached to a point on the surface of a uniform sphere with centre C, weight W and radius a. The end Q is rigidly attached to a point on the surface of an identical sphere with centre D. The points C, P, Q and D are in a straight line. (Continued→)



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14→ The object consisting of the rod and two spheres rests with one sphere in contact with a rough horizontal surface, at the point A, and the other sphere in contact with a smooth vertical wall, at the point B. The angle between CD and the horizontal is θ. The point B is at a height of 7a above the base of the wall. The points A, B, C, D, P and Q are all in the same vertical plane.

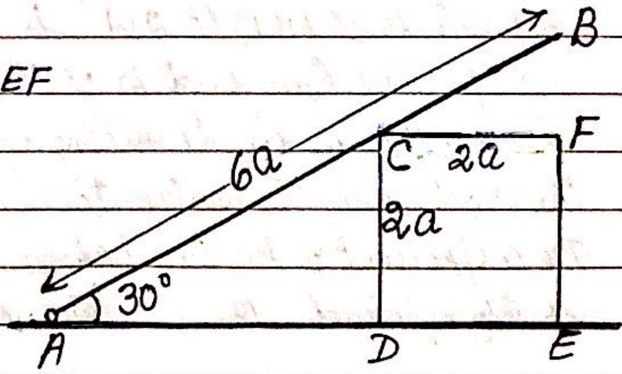
(i) Show that $\sin\theta = 3/5$ --[1]

The object is in limiting equilibrium and the coefficient of friction at A is μ.

(ii) Find the numerical value of μ. --[7]

(iii) Given that the resultant force on the object A is $W\sqrt{65}$, show that $k = 5$. [W-16/21/Q3] --[3]

15. The diagram shows a central cross-section CDEF of a uniform solid cube of weight W and with edges of length 2a. The cube rests on a rough horizontal plane. A thin



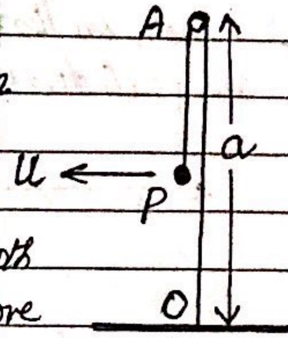
uniform rod AB, of weight W and length 6a, is hinged to the plane at A. The rod rests in smooth contact with the cube at C, with angle CAD equal to 30°. The rod is in the same vertical plane as CDEF. The coefficient of friction between the plane and the cube is μ. Given that the system is in equilibrium, show that $\mu \geqslant \frac{3}{25}\sqrt{3}$. --[6]

Find the magnitude of the force acting on the rod at A. --[4]

[S-15/23/Q4]

Not in this Unit ⊗

16. One end of a light inextensible string of length $\frac{3}{2}a$ is attached to a fixed point O on a horizontal surface. The other end of the string is attached to a particle P of mass m. The string passes over a small fixed smooth peg A which is at a distance a vertically above O. The system is in equilibrium with P hanging vertically below A and string taut. The particle is projected horizontally with speed u. When P is at the same horizontal level as A, the tension in the string is T. Show that



$$T = \frac{2m}{a}(u^2 - ag) \quad \text{---[3]}$$

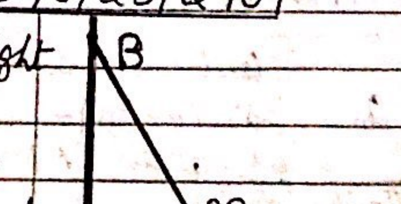
The ratio of the tensions in the string immediately before and immediately after, the string loses contact with the peg is 5:1,

(i) Show that $u^2 = 5ag$ ---[6]

(ii) Find, in terms of m and g, the tension in the string when P is next at the same horizontal level as A. ---[5]

S-15/23/Q10

17. A uniform ladder AB, of length 3a and weight W, rests with the end A in contact with smooth horizontal ground and the end B against a smooth vertical wall. One end of a light inextensible rope is attached to the ladder at the point C, where AC = a. The other end of the rope is fixed to the point D at the base of the wall and the rope DC is in the same vertical plane as the ladder AB. The ladder rests in equilibrium in a vertical plane perpendicular to the wall with the ladder making an angle θ with the horizontal and the rope making an angle α with the horizontal. It is given that $\tan\theta = 2\tan\alpha$.



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17. Find, in terms of W and α, the tension in the rope and the magnitudes of the forces acting on the ladder at A and at B. --[9]

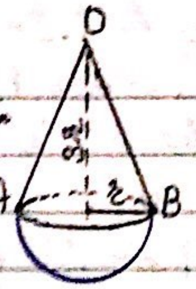
[W-15/21/Q1]

← X — X →

Answers (continued →)

1.

	Volume	COM from Vertex
Cone	$\frac{1}{3}\pi r^2 \times 3r$	$\frac{3}{4} \times 3r$
Hemisph.	$\frac{2}{3}\pi r^3$	$3r + \frac{3}{8}r$
Combined	$\frac{5}{3}\pi r^3$	$\bar{x}$



Taking moments about Vertex.

$\frac{5}{3}\pi r^3 \times \bar{x} = \pi r^3 \times \frac{9}{4}r + \frac{2}{3}\pi r^3 \times \frac{27}{8}r$

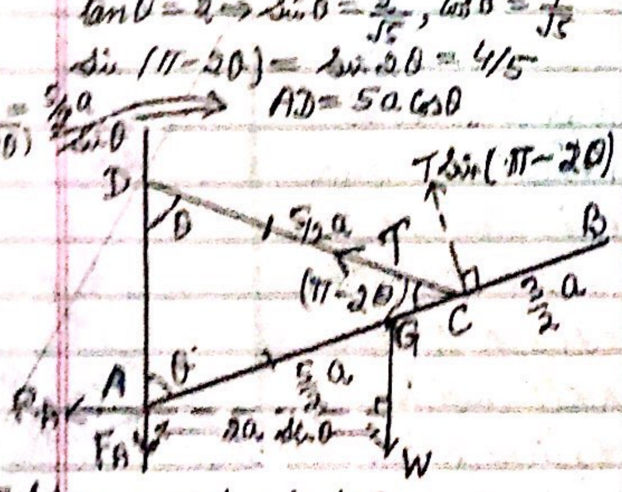
$\Rightarrow \bar{x} = \frac{27}{10}r$

∴ Distance of COM from vertex $= \frac{27}{10}r$ ✓

2. $\tan\theta = 2 \Rightarrow \sin\theta = \frac{2}{\sqrt{5}}, \cos\theta = \frac{1}{\sqrt{5}}$

$\sin(\pi - 2\theta) = \sin 2\theta = 4/5$

$\frac{AD}{\sin(\pi-2\theta)} = \frac{5/2 a}{\sin\theta} \Rightarrow AD = 5a\cos\theta$



Taking moments about A.

$T\sin(\pi - 2\theta)\cdot\frac{5a}{2} - W \times 2a\sin\theta = 0$

$\Rightarrow T = \frac{2W\sin\theta}{\frac{5}{2}\sin 2\theta} = \frac{2W}{5\cos\theta}$

$\Rightarrow T = 2W/\sqrt{5} = 0.894W$

(Continued →)

2. Taking moment about D.

$R_A \times 5a\cos\theta - W \times 2a\sin\theta =$

$\Rightarrow R_A = \frac{W \times 2a\sin\theta}{5a\cos\theta} = \frac{2W\tan\theta}{5}$

$R_A = \frac{4}{5}W$ ✓ — (1)

Taking moment about C.

$F_A \times \frac{5a}{2}\sin\theta - R_A \times \frac{5a}{2}\cos\theta - W \times \frac{a}{2}\sin\theta = 0$

$\Rightarrow F_A \cdot \frac{5a}{2}\sin\theta - \frac{4}{5}W \times \frac{5a}{2}\cos\theta - \frac{Wa}{2}\sin\theta = 0$

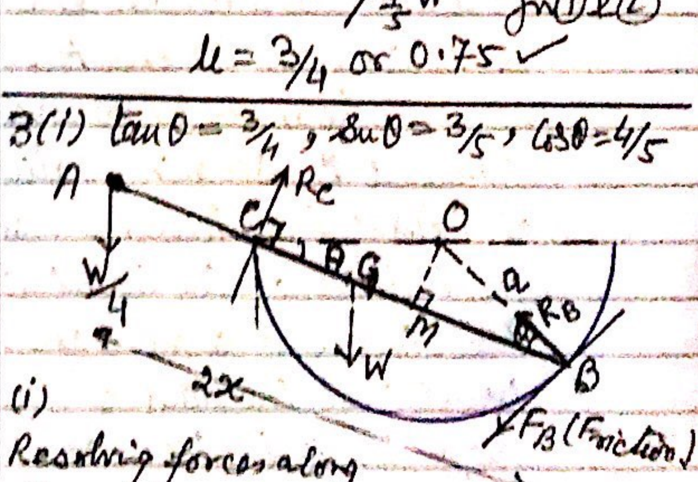
$F_A = 3/5\,W$ — (2)

Now $F_A = \mu \cdot R_A \Rightarrow \mu = \frac{F_A}{R_A}$

$\Rightarrow \mu = \frac{3/5\,W}{4/5\,W}$ from (1) & (2)

$\mu = 3/4$ or 0.75 ✓

3(i) $\tan\theta = 3/4$, $\sin\theta = 3/5$, $\cos\theta = 4/5$



(i)

Resolving forces along the rod.

$R_B\cos\theta + F_B\sin\theta = W\sin\theta + \frac{W}{4}\sin\theta$ — (1)

$\Rightarrow F_B = \frac{1}{3}R_B \quad (F = \mu R)$ — (2)

from (1) & (2)

$(\frac{4}{5} + \frac{1}{3} \times \frac{3}{5}) \cdot R_B = (1 + \frac{1}{4}) \cdot \frac{3}{5}W \Rightarrow R_B = \frac{3}{4}W$ ✓

$\Rightarrow F_B = W/4$ and $R_B = \frac{3}{4}W$ ✓ (continued →)

(Continued→) **Answers**

3 (ii) Resolving the forces ⊥ to rod. C denotes rim.

$R_C = F_B\cos\theta - R_B\sin\theta + W\cos\theta + \frac{1}{4}W\cos\theta$

$= (\frac{1}{3}\times\frac{3}{4}\times\frac{4}{5})W - (\frac{3}{4}\times\frac{3}{5})W + (\frac{5}{4}\times\frac{4}{5})W$

$= (\frac{1}{5} - \frac{9}{20} + 1)W = \frac{3}{4}W$ ✓

(iii) Let the rod's centre is denoted by G.

$AC = 2x - 2\times BM = 2x - 2a\cos\theta$

$\Rightarrow AC = 2x - 8a/5$ ✓

and $GC = x - AC = x - [2x - 8a/5]$

$\Longrightarrow CG = 8a/5 - x$ ✓

Taking moment about A.

$R_C\times AC + (R_B\sin\theta - F_B\cos\theta)\times 2x - W\cos\theta\times x = 0$

$\Rightarrow \frac{3}{4}W(2x - \frac{8a}{5}) + (\frac{3}{4}W\times\frac{3}{5} - \frac{W}{4}\times\frac{4}{5})\times 2x - W\times\frac{4}{5}\times x = 0$

$\Rightarrow x = a$ ✓

4. Taking moment about B; $GB = 2\sqrt{2}a$

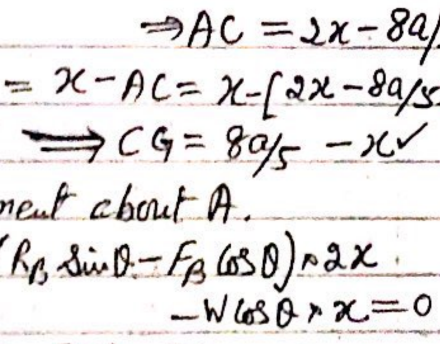
(i) $R_E\times 3a = W\times NB$

$= W\times 2\sqrt{2}a\sin(\frac{\pi}{4} - \theta)$ —(1)

$\tan\theta = \frac{1}{3}$

$\sin\theta = \frac{1}{\sqrt{10}}$

$\cos\theta = \frac{3}{\sqrt{10}}$



Now $\sin(\frac{\pi}{4} - \theta) = \frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta$

$= \frac{1}{\sqrt{2}}\left[\frac{3}{\sqrt{10}} - \frac{1}{\sqrt{10}}\right]$

$= 2/\sqrt{10}\times\sqrt{2}$

from (1) $R_E\times 3a = 2\sqrt{2}aW\times\frac{2}{\sqrt{2}\sqrt{10}}$

$\Rightarrow R_E = 4W/3\sqrt{10}$ ✓ —(2)

(Continued→)

4(i) continued→

Now resolving forces vertically

$R_B = W - R_E\cos\theta$

$= W - \frac{4W}{3\sqrt{10}}\times\frac{3}{\sqrt{10}} = \frac{3W}{5}$

$\Rightarrow R_B = \frac{3}{5}W$ ✓ —(3)

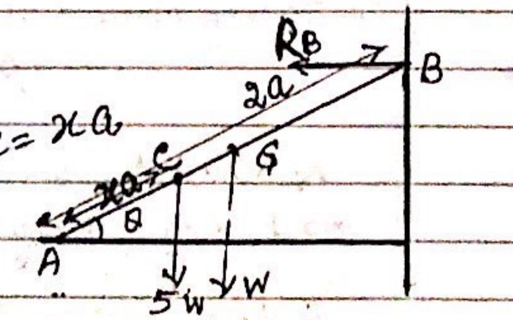
(ii) $F_B = R_B\sin\theta = 2W/15$

$\mu = F_B/R_B = (2/15)/\frac{3}{5} = \frac{2}{9}$ ✓

$\Rightarrow \mu = 2/9$ ✓

5.

(i)



Taking moment about A.

$R_B\times 2a\sin\theta = Wa\cos\theta + 5Wax\cos\theta$

$\Rightarrow R_B = W(5x+1)/2\tan\theta$ ✓ —(1)

(ii) Replace $xa \rightarrow (x+1)a$ in (1)

$R_B' = W\left(\frac{5x+5+1}{2\tan\theta}\right)$ —(2)

Given $R_B' = 2R_B$ —(2)

from (1) & (2) $5x + 6 = 2(5x+1) \Rightarrow x = \frac{5}{4}$ ✓

(iii) $R_A = 6W$, $F_A' = R_B' = 2R_B$

$\Rightarrow F_A' = \frac{10W}{2\tan\theta} = 4W$

$\Rightarrow \frac{F_A'}{R_A} = 4W/6W = 2/3$ (Given $\mu = 2/3$)

$= (5W/\tan\theta)/6W$

$\Rightarrow \tan\theta = 5/4$ or 1.25 ✓

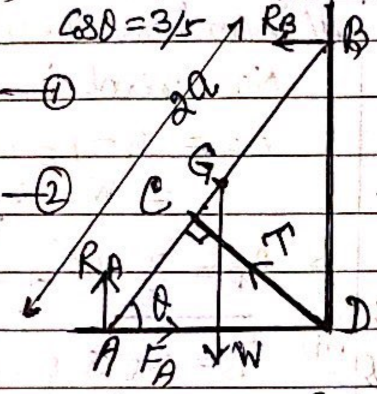
Answers

6(i) $\tan\theta = \frac{4}{3}$, $\sin\theta = 4/5$, $\cos\theta = 3/5$

$AC = AD\cos\theta$ — (1)

$AD = AB\cos\theta$

or $AD = 2a\cos\theta$ — (2)



fr (1) & (2)

$AC = 2a\cos^2\theta = 2a\left(\frac{3}{5}\right)^2$

$\Rightarrow AC = 18a/25$ ✓

(ii) Taking Moment about A

(Given $F_A = \frac{1}{4}R_A$ or $\mu = \frac{1}{4}$)

$R_B \times 2a\sin\theta - W \times a\cos\theta - T \times AC = 0$

$\Rightarrow R_B \times \frac{8a}{5} - W \times \frac{3a}{5} - T \times \frac{18a}{25} = 0$

$\Rightarrow 40R_B - 15W - 18T = 0$ — (1)

Horizontally $R_B - F_A = T\sin\theta = \frac{4T}{5}$ — (2)

Vertically $R_A - W = T\cos\theta = \frac{3T}{5}$ — (3)

fr (2) $R_B - \frac{1}{4}R_A = \frac{4T}{5}$ — (4)

~~(3)+4×(4)~~ fr (3) & (4) $4R_B - W = 19T/5$ — (5)

$4R_B = W + 19T/5$ — (6)

fr (1) & (6)

$10W + 38T - 15W - 18T = 0$

$20T = 5W$

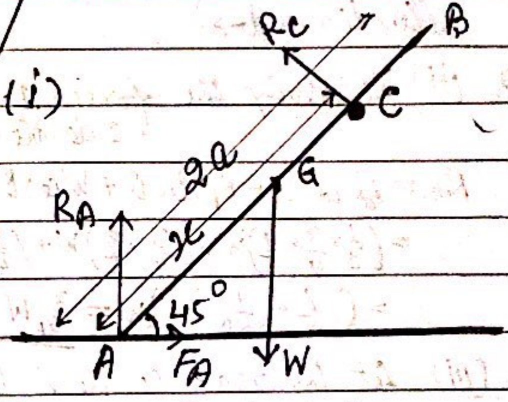
$\Rightarrow T = W/4$ ✓

(iii) fr (3) $R_A = W + \frac{3}{5} \times \frac{W}{4} = \frac{23W}{20}$ ✓

fr (6) $4R_B = W + \frac{19}{5} \times \frac{W}{4} = \frac{39}{20}W$

or $R_B = \frac{39}{80}W$ ✓

7. (i)



Taking moment about A,

$R_c \times x - W \times a\cos 45^\circ = 0$

$\Rightarrow R_c \cdot x = aW/\sqrt{2}$

$\Rightarrow R_c = aW/x\sqrt{2}$ — (1)

Horizontally: $F_A - R_c\cos 45^\circ = 0$ — (2)

Vertically $R_A + R_c\cos 45^\circ - W = 0$ — (3)

fr (1) & (2) $F_A = aW/2x$ — (4)

fr (1) & (3) $R_A = W - \frac{aW}{2x}$ — (5)

fr (4) & (5) $\frac{F_A}{R_A} = \mu$

$\Rightarrow \frac{aW/2x}{W\left(1 - \frac{a}{2x}\right)} = \mu$

$\Rightarrow x = \frac{a(1+\mu)}{2\mu}$ ✓

(ii) Given $x < 2a$

$\Rightarrow \frac{\cancel{a}(1+\mu)}{2\mu} \le 2\cancel{a}$

$\Rightarrow 1 + \mu \le 4\mu \Rightarrow \mu \le \frac{1}{3}$ ✓

(iii) Given $x = 3/2\,a$

$\Rightarrow \frac{a(1+\mu)}{2\mu} = \frac{3a}{2} \Rightarrow \mu = \frac{1}{2}$ ✓

from (4) $F_A = aW/2 \times \frac{3a}{2} = \frac{W}{3}$ ✓

& (3) and $R_A = W - aW/2 \times \frac{3a}{2} = \frac{2W}{3}$

∴ Magnitude of Resultant force N_A at A, $N_A = \sqrt{F_A^2 + R_A^2} = \frac{\sqrt{5}}{3}W$ ✓

Answers.

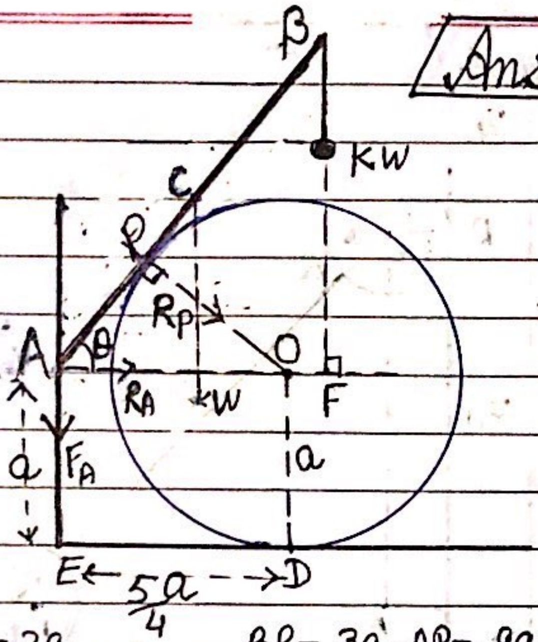
8.

$AP = \frac{3a}{4}$

$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

$\tan\theta = \frac{4}{3}$



$AB = 3a \qquad BP = 3a - AP = 9a/4$

$AC = BC = 3a/2$

Taking moment about A for rod.

$R_P \times AP - KW \times 3a\cos\theta = W \times \frac{3a}{2}\cos\theta$

$R_P \times \frac{3a}{4} - KW \times \frac{9a}{5} = W \times \frac{9a}{10}$

$\Rightarrow 15R_P - 36KW = 18W.$

$R_P = \frac{6}{5}W(1+2K)$ — ①

Horizontally $R_A = R_P\sin\theta = \frac{4}{5}R_P$ — ②

vertically $F_A + (K+1)W = R_P\cos\theta = \frac{3}{5}R_P$ — ③

$F_A = -\mu R_A \Rightarrow F_A = -\frac{1}{8}R_A$ — ④

fr ① & ② $R_A = \frac{24}{25}W(1+2K)$ — ⑤

fr ① & ③ $F_A = \frac{1}{25}W(11K-7)$ — ⑥

for ④, ⑤ and ⑥

$\frac{1}{25}W(11K-7) = -\frac{1}{8}\times\frac{24}{25}(1+2K)W$

$\Rightarrow K = \frac{4}{17}$ ✓

9. $T = 3mg\sin\alpha = 2mg$ $[\sin\alpha = 2/3]$ — ①

$T = \frac{\lambda e}{L} = Kmg(\frac{5a}{4} - a)/a = \frac{1}{4}Kmg$ — ②

$\therefore$ fr ① & ② $\frac{1}{4}Kmg = 2mg$

$\Rightarrow K = 8$ ✓

10.(i) $\sin\theta = 3/5$, $\cos\theta = \frac{4}{5}$

$\cos 2\theta = 7/25.$

Taking moment for rod about A

$T \times 3a\sin\theta = W \times 1.5a\cos 2\theta$

$\Rightarrow T = 7W/30$ ✓ — ①

(ii) $OB = (5a - 3a\cos 2\theta)/\cos\theta = 26a/5$

$T = \lambda(OB - 4a)4a$

$T = \lambda(6a/5)/4a = 3\lambda/10$ — ②

fr ① and ② $3\lambda/10 = 7W/30$

$\Rightarrow \lambda = 7W/9$ ✓

(iii) Finding horizontal component of the force X acting at A

$X = T\cos\theta = 14W/75$

Vertical component

$Y = T\sin\theta + W = 57W/50$

$\Rightarrow \tan\phi = Y/X =$

$\phi = \tan^{-1}(Y/X) = \tan^{-1}\left(\frac{171}{28}\right)$

$= 80.7°$ ✓

11(i) $\sin\theta = 3/5$, $\cos\theta = 4/5$

Taking moment for rod about A

$T \times 2a = (5W/2) \times a\cos\theta$

$T = \frac{1}{2}(5W/2) \times \frac{4}{5} = W$ — ①

(ii) $F_P = \mu R_P$ at P

$T\sin\theta = \mu(W + T\cos\theta)$

$\Rightarrow \frac{3}{5}W = \mu(1 + \frac{4}{5})W = \mu(9/5)W$

$\Rightarrow \mu = 1/3$ ✓

(Continued →)

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(continued →)

Answers

11 (iii) Horizontal component at A,

$X = T\sin\theta = 3W/5$ ✓

Vertical comp. $Y = 5W/2 - T\cos\theta = \frac{17}{10}W$ ✓

$\therefore R_A^2 = \sqrt{X^2+Y^2} = \frac{13W^2}{4}$

$\Rightarrow R_A = \sqrt{13}\cdot W/2$ ✓

(iv) $PB = (a + 2a\sin\theta)/\cos\theta$

$= 5a/4 + \frac{3a}{2} = 11a/4$

$e = 3a/4$

$T = \lambda\frac{(PB-2a)}{2a} \Rightarrow \lambda = 8W/3$ ✓

12 - Circular Motion

13. Taking moment for rod about A,

$R_P \times AP = W \cdot a\cos\theta + 2W \times 2a\cos\theta$

$= 5Wa\cos\theta = 5Wa \times 3/5$

or $R_P \times AP = 3Wa$ —— (1)

$\tan\theta = 4/3$, $\sin\theta = 4/5$, $\cos\theta = 3/5$

Resolve on rod horizontally

$F_A = R_P\sin\theta = 4R_P/5$ —— (2)

Vertically $3W - R_A = R_P\cos\theta = 3R_P/5$ —— (3)

fr (3) & (2) $\Rightarrow 3W - R_A = \frac{3}{5} \times \frac{5}{4}F_A$

$\left[F_A = \mu R_A = \frac{2}{3}R_A\right]$

$\Rightarrow 3W - R_A = \frac{3}{4} \times F_A$

$\Rightarrow 3W - R_A = \frac{3}{4} \times \frac{2}{3}R_A$

$\Rightarrow \frac{3}{2}R_A = 3W$

$\Rightarrow R_A = 2W$ —— (4)

$\boxed{F_A = \frac{2}{3} \times 2W = \frac{4}{3}W \text{ —— (5)}}$

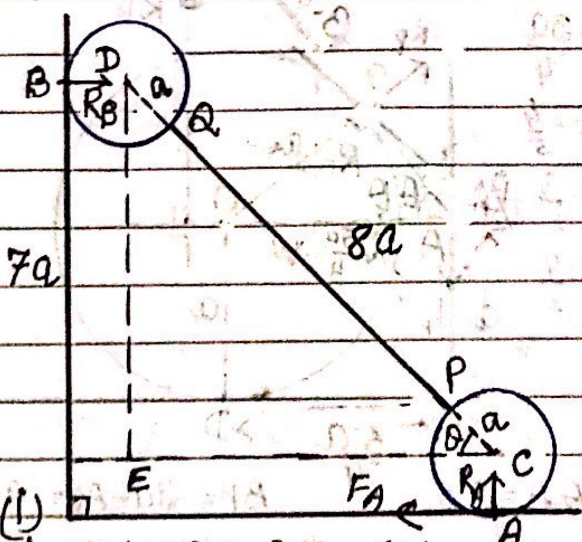
fr (5) & (2) $\frac{4}{3}W = \frac{4}{5}R_P \Rightarrow R_P = \frac{5}{3}W$ —— (6)

fr (1) & (6)

$\frac{5}{3}W \times AP = 3Wa$

$\Rightarrow AP = 9a/5$ ✓

14.



(i) In rt Δ CDE, $\sin\theta = 6a/10a = 3/5$ ✓

$\cos\theta = 4/5$

(ii) Resolving forces on obj. vertically.

$R_A = (K+2)W$ —— (1)

Taking moment about B.

$F_A \times 7a + Wa + KWa(1 + 5\cos\theta)$

$+ (W - R_A)a(1 + 10\cos\theta) = 0$

$\Rightarrow F_A \times 7a + KW \cdot 5a\cos\theta + (W - R_A)10a\cos\theta = 0$

$\Rightarrow 7F_A = 4W(K+2)$ —— (2)

fr (1) and (2)

$7F_A = 4R_A \Rightarrow \mu = \frac{F_A}{R_A} = \frac{4}{7}$ ✓

$\therefore \mu = 4/7$ ✓

(iii) Equating resultant force at A to $W\sqrt{65}$;

$(K+2)^2 + \left(\frac{4}{7}\right)^2(K+2)^2 = 65$

$\Rightarrow K = 5$ ✓

Answers

15. Taking moment for rod at A,

$R_C \times 2a/\sin 30° = W \times 3a\cos 30$

$\Rightarrow R_C = 3\sqrt{3}W/8$ — (1)

Horizontally $F = R_C \sin 30°$

from (1) $\Rightarrow F = 3\sqrt{3}W/16$ — (2)

Reaction R on the cube, taking vertically

$R = W + R_C\cos 30 = 25W/16$ — (3)

$F \le \mu R$

$\frac{3\sqrt{3}}{16} \le \frac{25}{16}\mu \Rightarrow \mu \ge \frac{3\sqrt{3}}{25}$ ✓

Now Horizontal force X at A

$X = F = 3\sqrt{3}W/16$ — (4)

and Vertical force Y at A,

$Y = 2W - R = 2W - 25W/16$

$\Rightarrow Y = 7W/16$ — (5)

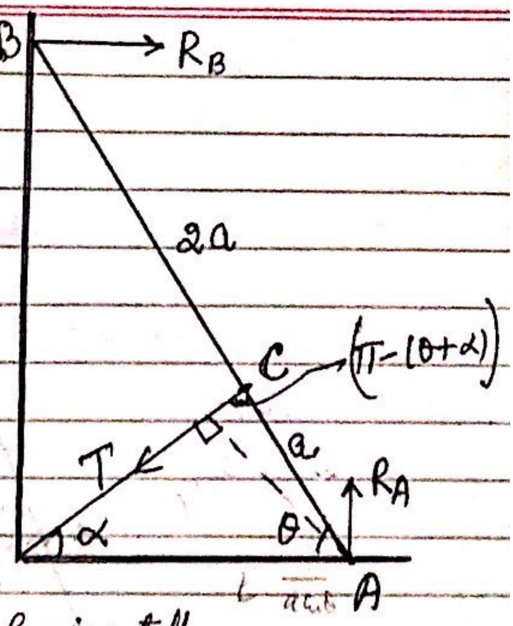
∴ Magnitude of force A, from (4) & (5)

$= \sqrt{X^2 + Y^2}$

$= \left(\frac{\sqrt{19}}{8}\right)W$ ✓

(16) not from this unit xx

17. $\tan\theta = 2\tan\alpha$ — (1)



Resolving horizontally.

$R_B = T\cos\alpha$ — (2)

Vertically, $R_A = W + T\sin\alpha$ — (3)

Taking moment about A,

$R_B \times 3a\sin\theta = W \times \frac{3a}{2}\cos\theta + Ta(\sin\theta+\alpha)$

$\{$or $Ta(\sin\alpha\cos\theta + \cos\alpha\sin\theta)\}$

$\Rightarrow R_B \cdot 3a\sin\theta = W \times \frac{3a}{2}\cos\theta + Ta \times 3\cos\theta\sin\alpha$ (from (1))

$R_B \times 3a = W \times \frac{3a}{2} \times \frac{\cos\theta}{\sin\theta} + 3aT\frac{\cos\theta\sin\alpha}{\sin\theta}$

$R_B = \frac{W}{2\tan\theta} + \frac{T\sin\alpha}{\tan\theta}$

$R_B = \frac{W}{2\times 2\tan\alpha} + \frac{T\sin\alpha}{2\tan\alpha}$

$R_B = \frac{W}{2\times 2\tan\alpha} + \frac{T\cos\alpha}{2}$

$R_B = \frac{W}{4\tan\alpha} + \frac{R_B}{2}$ from (2)

$\Rightarrow R_B = \frac{W}{2\tan\alpha}$ ✓ — (4)

from (2) & (4) $T = W/2\sin\alpha$ ✓ — (5)

from (3) & (5) $R_A = \frac{3W}{2}$ ✓

← x —— x →