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# Further Mechanics

## Hook's Law Exercise

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(FM) Hook's Law (P-1)

1. A light elastic string has natural length $a$ and modulus of elasticity $24mg$. One end of the string is attached to a fixed point A. The other end of the string is attached to a particle of mass $2m$.

(a) Find, in terms of $a$, the extension of the string when the particle hangs freely in equilibrium below A. ---[2]

(b) The particle is released from rest at A.
Find, in terms of $a$, the distance of the particle A when it first comes to instantaneous rest. $\boxed{\text{SP-20/03/Q2}}$ ---[6]

2. A light spring has natural length $a$ and modulus of elasticity $kmg$. The spring lies on a smooth horizontal surface with one end attached to a fixed point O. A particle P of mass $m$ is attached to the other end of the spring. The system is in equilibrium with $OP = a$. The particle is projected towards O with speed $u$ and comes to instantaneous rest when $OP = \frac{3}{4}a$.

(i) Use an energy method to show that $k = 16u^2/ag$ --- [2]

× (ii) Show that P performs simple harmonic motion and find the period of this motion, giving your answer in terms of $u$ and $a$. --[4]

(iii) Find, in terms of $u$ and $a$, the time that elapses before P first loses 25% of its initial kinetic energy. $\boxed{\text{S-19/23/Q11}}$ -- [6]

3. The points A and B are a distance 1·2 m apart on a smooth horizontal surface. Particle P of mass $\frac{2}{3}$ kg is attached to one end of a light spring of natural length 0·6 m and modulus of elasticity 10 N. The other end of the spring is attached to the point A. A second light spring, of natural length 0·4 m and modulus of elasticity 20 N, has one end attached to P and the other end attached to B.

(i) Show that when P is in equilibrium $AP = 0{\cdot}75$ m ---[3]

The particle P is displaced by 0·05 m from the equilibrium position towards A and then released from rest.

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× 3(ii) Show that P performs S.H.M and state the period of motion. --- [6]

× (iii) Find the speed of P when it passes through the equilibrium position. --- [2]

× (iv) Find the speed of P when its acceleration is equal to half of its maximum value. [W-19/21/Q11] --- [3]

4. A uniform rod AB of length 4a and weight W is smoothly hinged to a vertical wall at the end A. The rod is held at an angle θ above the horizontal by a light elastic string. One end of the string is attached to the point C on the rod, where AC = 3a. The other end of the string is attached to a point D on the wall, with D vertically above A and such that angle ACD = 2θ. A particle of weight $\frac{1}{2}W$ is attached to the rod at B. It is given that $\tan\theta = 8/15$.

(i) Show that the tension in the string is $\frac{17}{12}W$. --- [4]

(ii) Find the magnitude and direction of the reaction at the hinge. --- [5]

(iii) Given that the natural length of the string is 2a, find its modulus of elasticity. [W-18/21/Q4] --- [2]

5. The fixed points A and B are on a smooth horizontal surface with AB = 2.6 m. One end of a light elastic spring, of natural length 1.25 m and modulus of elasticity λ N is attached to A. The other end is attached to a particle P of mass 0.4 kg. One end of a second light elastic spring, of natural length 1.0 m and modulus of elasticity 0.6λ N, is attached to B; its other end is attached to P. The system is in equilibrium with P on the surface at point E.

Show that AE = 1.4 m. [W-18/21/Q5] -- [4]

× 6. (Same as Q4.) A uniform rod AB of length 4a and weight W is smoothly hinged to vertical wall at the end A. The rod is held at an angle θ above the horizontal by a light elastic string.

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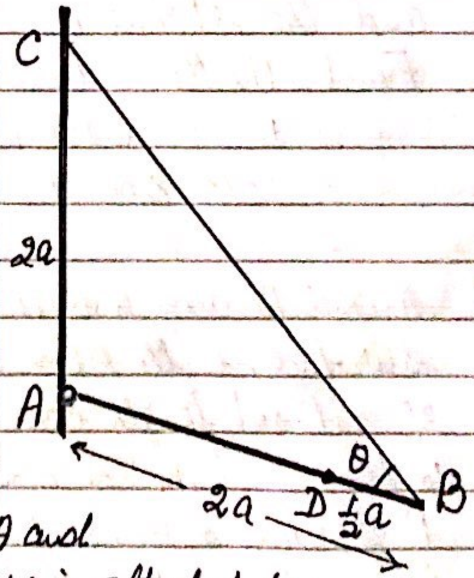
6 → One end of the string is attached to the point C on the rod, where AC = 3a. The other end of the string is attached to a point D on the wall, with D vertically above A and such that angle ACD = 2θ. A particle of weight $\frac{1}{2}W$ is attached to the rod at B. It is given that $\tan\theta = 8/15$

(i) Show that the tension in the string is $17/12$ W. ---[4]

(ii) Find the magnitude and direction of the reaction at the hinge. ---[5]

(iii) Given that the natural length of the string is 2a, find its modulus of elasticity. $\boxed{W-18/23/Q4}$ ---[2]

7. The end A of uniform rod AB, of length 2a and weight W, is freely hinged to a vertical wall. The end B of the rod is attached to light elastic string of natural length 3/3 a and modulus of elasticity 3W. The other end of the string is attached to the point C on the wall, where C is vertically above A and AC = 2a. A particle of weight 2W is attached to the rod at the point D, where DB = $\frac{1}{2}$a. The angle ABC is equal to θ. Show that $\cos\theta = 3/4$ and find the tension in the string in terms of W. ---[10]




Find the magnitude of the reaction force at the hinge. ---[4]

$\boxed{S-16/21/Q11}$

8. The fixed points A and B are such that AB = 3.2m and A is vertically above B. One end of a light elastic string, of natural length 0.8m and modulus of elasticity 8N, is attached to a particle P of mass 0.2kg. The other end of the string is attached to A. One end of the second light elastic string, of natural length 1.2m and modulus of elasticity 4N, is attached to P and the other end is attached to B. $\boxed{S-16/23/Q11}$

Find the length of AP when the particle is in equilibrium. --[4]

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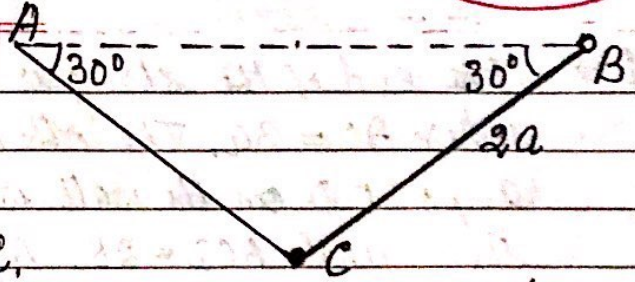
9. A uniform rod BC of length 2a and weight W is hinged at B. A particle of weight 3W is attached to the rod at C.




The system is held in equilibrium by a light elastic string of natural length $\frac{3}{5}a$ in the same vertical plane as the rod. One end of the elastic string is attached to the rod at C and the other end is attached to a fixed point A which is at the same horizontal level as B. The rod and the string each make an angle 30° with the horizontal.
Find (i) the modulus of elasticity of the string. --[5]
(ii) the magnitude and direction of the force acting on the rod at B. [S-15/21/Q4] ---[5]

10. A and B are two fixed points on a smooth horizontal surface, with AB = 3a m. One end of a light elastic string of natural length a m and modulus of elasticity mg N, is attached to the point A. The other end of this string is attached to a particle P of mass m kg. One end of a second light elastic string, for natural length ka m and modulus of elasticity 2mg N, is attached to B. The other end of this string is attached to P. Given that the system is in equilibrium when P is at M, the mid point of AB, find the value of k. [W-15/21/Q3] ---[3]

⟵ X —— X ⟶

(Answers- on the next page)

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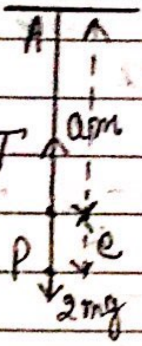
T = Tension in elastic string
e = extension produced.
L = the natural length of the string.
$\lambda$ = modulus of elasticity

# Hook's Law



## Answers

**1.** $L = a,\ \lambda = 24mg$

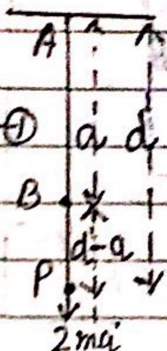



(a) $T = \frac{\lambda}{L} \times e$

$\Rightarrow T = 2mg = \frac{24mg \cdot e}{a}$

$\Rightarrow e = a/12$ ✓

(b) Let $d$ is the distance below A.




G.P.E of Particle at A $= 2mgd$ — ①

E.P.E at A $= \frac{\lambda e^2}{2L} = \frac{24mg(d-a)^2}{2a}$ — ②

No change in K.E at A and E.

at P, loss in GPE = Gain in EPE

$\Rightarrow 2mgd = 24mg(d-a)^2/2a$

$\Rightarrow 6(d-a)^2 = ad$

$\Rightarrow d = \frac{3}{2}a$ or $\frac{2}{3}a$ (as $d > a$)

$\therefore$ Required distance $= \frac{3}{2}a$ ✓

**2.** (i) $\frac{1}{2}mu^2 = \frac{1}{2}Kmg\left(\frac{1}{4}a\right)^2/a$

$\Rightarrow K = 16u^2/ag$ — ①

(ii) $\frac{1}{2}mv^2 = \frac{3}{4} \times \frac{1}{2}mu^2 \Rightarrow v^2 = \frac{3}{4}u^2$

$\Rightarrow v = \frac{\sqrt{3}}{2}u$ — ②

$x = \left(\frac{1}{4}a\right)\sin\omega t,\ v = \left(\frac{1}{4}a\right)\omega\cos\omega t$
or $u\cos\omega t$

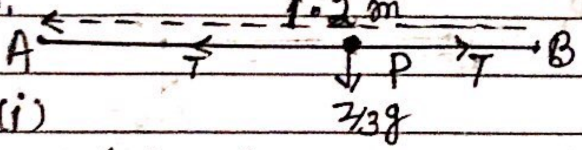
$\cos\omega t = \frac{\sqrt{3}}{2}u / \left(\frac{1}{4}a\right)\omega = \frac{\sqrt{3}}{2}$ ✓

$t = \frac{\pi}{6} / \frac{4u}{a} = \pi a/24u$

or $t = 0.0417\pi a/u$

$t = 0.131a/u$ ✓

**3.** $L_1 = 0.6m,\ \lambda_1 = 10N$
$L_2 = 0.4m,\ \lambda_2 = 20N$



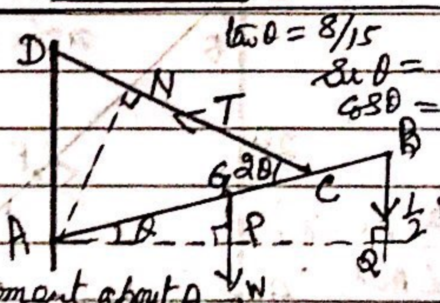

(i) $T = \frac{10(AP - 0.6)}{0.6} = \frac{20(1.2 - AP - 0.4)}{0.4}$

$\Rightarrow 4AP - 2.4 = 9.6 - 12AP$

$\Rightarrow AP = 0.75m$ ✓

**4.** (i) $AB = 4a$, $AC = 3a$, $AG = 2a$

$\tan\theta = 8/15$
$\sin\theta = 8/17$
$\cos\theta = 15/17$




Taking moment about A,

$T \times AN = W \times AP + \frac{1}{2}W \times AQ$

$\Rightarrow T \times 3a\sin 2\theta = W \times 2a\cos\theta + \frac{1}{2}W \times 4a\cos\theta$

$\Rightarrow 6a \cdot T\sin\theta\cos\theta = 4aW\cdot\cos\theta$

$T = (2W/3)/\sin\theta = \left(\frac{2W}{3}\right)/\frac{8}{17}$

$T = 17W/12$ ✓ — ①

(ii) The horizontal comp of X of force at A

$X = T\cos\theta = \frac{5}{4}W$ — ②

Vertically, $Y = W + \frac{1}{2}W - T\sin\theta = \frac{5}{6}W$ — ③

for ② & ③ $F = \sqrt{X^2 + Y^2} = (5\sqrt{13}/12)W$ ✓

upward at an angle to horizontal

$\tan^{-1}\frac{Y}{X} = \tan^{-1}2/3 = 33.7°$ ✓

(iii) $T = \lambda(DC - 2a)/2a$ — ④

$DC = AC = 3a$ [$\because \angle A = \angle D = 90 - \theta$]

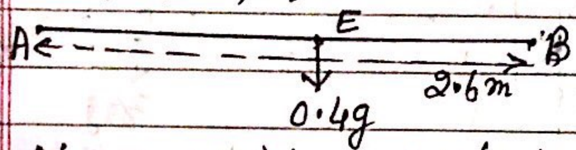
for ④ & ①

$\Rightarrow \frac{17W}{12} = \lambda(3a - 2a)/2a$

$\Rightarrow \lambda = 17W/6$ ✓

# Answers

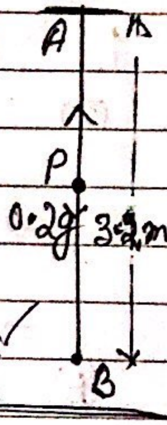
5. $L_1 = 1.25\,m,\ \lambda_1 = \lambda$
$L_2 = 1.0\,m,\ \lambda_2 = 0.6\lambda$




$\lambda(AE - 1.25)/1.25 = 0.6\lambda(2.6 - AE - 1.0)/1.0$

$\Rightarrow \underline{AE = 1.4}$ ✓

8. $L_1 = 0.8\,m,\ \lambda_1 = 8N$
$L_2 = 1.2\,m,\ \lambda_2 = 4N$

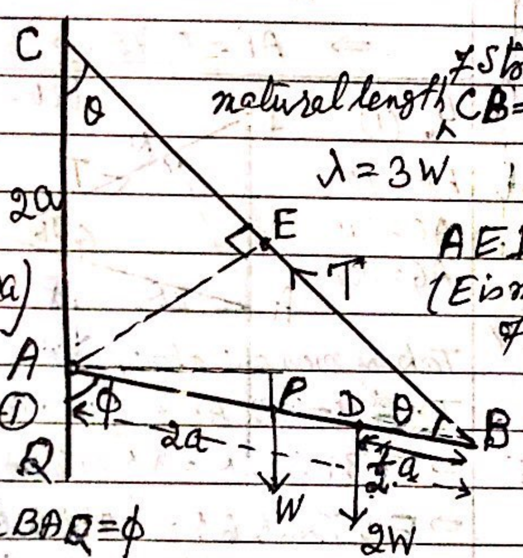



$8(AP - 0.8)/0.8 = 4(2 - AP)/1.2 + 0.2g$

$\Rightarrow AP = 5/4$ or $\underline{1.25\,m}$ ✓

7. String natural length $CB = \frac{3}{2}a$

$\lambda = 3W$




$AE \perp BC$ (E is mid point of BC)

$(AB = BC = 2a)$
$\Rightarrow \angle ACB = \theta$
$\Rightarrow \phi = 2\theta$ — (1)

Let angle $BAQ = \phi$

Taking moment about A.

$T \times AE = W \times a\sin\phi + 2W \times \frac{3a}{2}\sin\phi$

$(\phi = 2\theta)$ fr (1)

$\Rightarrow T \times 2a\sin\theta = W \times a\sin 2\theta + 2W \times \frac{3a}{2}\sin 2\theta$

$\Rightarrow T = 2W\sin 2\theta/\sin\theta$

$\Rightarrow T = 4W\cos\theta$ — (2)

Using Hook's Law

$\frac{\lambda e}{L} = T = 3W(BC - 3a/2)/(3a/2)$

$T = W(8\cos\theta - 3)$ — (3)

fr (2) & (3) $\underline{\cos\theta = 3/4}$ ✓

$(\sin\theta = \sqrt{7}/4)$ $\therefore \underline{T = 3W}$ ✓

Now Horizontal force at A, $X = T\sin\theta = 3W \times \sqrt{7}/4$

Vertically $Y = 3W - T\cos\theta = 3W/4$

$F = \sqrt{X^2 + Y^2} = (3/\sqrt{2})W$

or $\underline{\frac{3\sqrt{2}}{2}W}$ ✓

9. Taking moment about B,

$W.a\cos 30 + 3W \times 2a\cos 30^\circ = T.2a\cos 30^\circ$ (i)

$\Rightarrow T = 7W/2$ ✓

Hooks law $T = \lambda(2a - 3a/5)/(3a/5)$

$\Rightarrow \lambda = \underline{3W/2}$ ✓

Horizontal comp of force at B.

$X = T\cos 30^\circ = \frac{7\sqrt{3}}{4}W$ — (1)

Vertically.

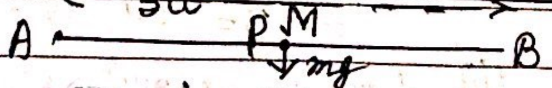
$Y = 4W - T\sin 30^\circ = 9W/4$ — (2)

$F = \sqrt{X^2 + Y^2} = \underline{(\sqrt{57}/2)W}$ ✓

upward force at angle to AB.

$\tan^{-1} Y/X = \tan^{-1} 3\sqrt{3}/7 = \underline{36.6^\circ}$ ✓

10.




$L_1 = a\,m,\ \lambda_1 = mg\,N$

$L_2 = ka\,m,\ \lambda_2 = 2mg\,N$

$mg(3a/2 - a)/a = 2mg(3a/2 - ka)/ka$

$\Rightarrow k = 6/5 = \underline{1.2\,m}$ ✓

← X — X →