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F. Me.

Further Mechanics.

Momentum Exercise

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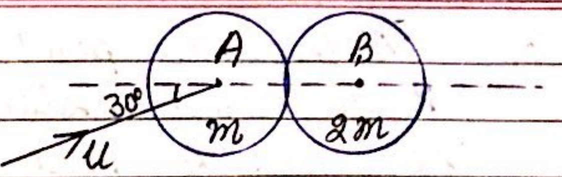
Momentum

1. Two uniform smooth spheres A and B of equal radii have masses m and $2m$ resp.



Sphere B is at rest on a smooth horizontal surface. Sphere A is moving on the surface with speed u at an angle of $30°$ to the line of centres of A and B when it collides with B. The coefficient of restitution between the spheres is e.

(a) Show that the speed of B after the collision is $\frac{\sqrt{3}}{6}u(1+e)$ and find the speed of A after the collision. ---[6]

(b) Given that $e = \frac{1}{3}$, find the loss of kinetic energy as a result of collision. $\boxed{SP\text{-}20/03/Q4}$ --[3]

2. Three uniform small spheres A, B and C have equal radii and masses $2m$, $4m$ and m respectively. The spheres are moving in a straight line on a smooth horizontal surface, with B between A and C. The coefficient of restitution between each pair of spheres is e. Spheres A and B are moving towards each other with speeds $2u$ and u respectively. The first collision is between A and B.

(i) Find the velocities of A and B after this collision. ---[3]

Sphere C is moving towards B with speed $\frac{4}{3}u$ and now collides with it. As a result of this collision, B is brought to rest.

(ii) Find the value of e. --[4]

(iii) Find the total kinetic energy lost by the three spheres as a result of the two collisions. $\boxed{S\text{-}19/21/Q3}$ --[3]

3. A particle P, of mass m, is able to move in a vertical circle on the smooth inner surface of a sphere with centre O and radius a. Points A and B are on the inner surface of the sphere and AOB is a horizontal diameter. Initially, P is projected vertically downwards with speed $\sqrt{\frac{21}{2}ag}$ from A and begins to move in vertical circle.

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3. At the lowest point of its path, vertically below O, the particle P collides with a stationary particle Q, of mass $4m$, and rebounds. The speed acquired by Q, as a result of the collision, is just sufficient for it to reach the point B.

(i) Find the speed of P and the speed of Q immediately after their collision. ---[7]

In the subsequent motion, P loses contact with the inner surface of the sphere at the point D, where the angle between OD and the upward vertical through O is θ.

(ii) Find $\cos\theta$. $\boxed{S\text{-}19/21/Q11}$ ---[5]

4. Three uniform small spheres A, B and C have equal radii and masses $3m$, m and m respectively. The spheres are at rest in a straight line on a smooth horizontal surface, with B between A and C. The coefficient of restitution between each pair of spheres is e. Sphere A is projected directly towards B with speed u.

(i) Find in terms of u and e, expressions for speeds of A, B and C after the first two collisions. --[6]

(ii) Given that A and C are moving with equal speed after these two collisions, find the value of e. $\boxed{S\text{-}19/23/Q3}$ --[3]

5. Three uniform small spheres A, B and C have equal radii and masses $5m$, $5m$ and $3m$ respectively. The spheres are at rest on a smooth horizontal surface, in a straight line, with B between A and C. The coefficient of restitution between each pair of spheres is e. Sphere A is projected directly towards B with speed u.

(i) Show that the speed of A after its collision with B is $\frac{1}{2}u(1-e)$ and find the speed of B. ---[3]

Sphere B now collides with sphere C. Subsequently there are no further collisions between any of the spheres.

(ii) Find the set of possible values of e. $\boxed{W\text{-}19/21/Q3}$ --[6]

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6. A bullet of mass m kg is fired horizontally into a fixed vertical block of material. It enters the block horizontally with speed $250\ ms^{-1}$ and emerges horizontally with speed $70\ ms^{-1}$ after $0.04\ s$. The block offers a constant horizontal resisting force of magnitude $450\ N$. Find the value of m. ---[3]

[S-18/22/Q1]

7. Two identical uniform small spheres A and B, each of mass m, are moving towards each other in a straight line on a smooth horizontal surface. Their speeds are u and ku respectively, and they collide directly. The coefficient of restitution between the spheres is e. Sphere B is brought to rest by the collision.

(i) Show that $e = \frac{k-1}{k+1}$ ---[3]

(ii) Given that 60% of the total kinetic energy is lost in the collision, find the values of k and e. [S-18/22/Q3] ---[6]

8. Two uniform small spheres A and B have equal radii and masses $4m$ and m respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides directly with sphere B which is at rest. The coefficient of restitution between the spheres is e.

(i) Show that after collision A moves with speed $\frac{1}{5}u(4-e)$ and find the speed of B. ---[4]

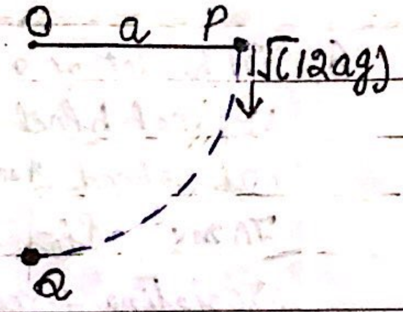
Sphere B continues to move until it collides with a fixed vertical barrier which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the barrier is $\frac{3}{4}e$. After this collision, the speeds of A and B are equal.

(ii) Find the value of e. ---[3]

The spheres A and B now collide directly again.

(iii) Determine whether sphere B collides with the barrier for a second time. [S-18/23/Q2] ---[2]

9. A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is held with the string taut and horizontal.



It is projected downwards with speed $\sqrt{(12ag)}$. At the lowest point of its motion, P collides directly with a particle Q of mass km which is at rest. In the collision, P and Q coalesce. The tension in the string immediately after the collision is half of its maximum value immediately before the collision. Find the possible values of k. --[11]

[S-18/23/Q5]

10. Two uniform small smooth spheres A and B have equal radii and masses $5m$ and $2m$ respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides directly with sphere B which is moving towards it with speed $2u$. The coefficient of restitution between the spheres is e.

(i) Show that the speed of B after the collision is $\frac{1}{7}u(1+15e)$ and find an expression for the speed of A. ---[4]

In the collision, the speed of A is halved and its direction of motion is reversed.

(ii) Find the value of e. ---[2]

(iii) For this collision, find the ratio of the loss of kinetic energy of A to the loss of K.E of B. [W-18/21/Q2] --[3]

11. Two uniform small smooth spheres A and B have equal radii and masses $2m$ and m respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides directly with sphere B which is at rest. The coefficient of restitution between the spheres is $2/3$.

(i) Find, in terms of u, the speeds of A and B after this collision; --[4]

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F.M

Momentum

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11→ Sphere B is initially at a distance d from a fixed smooth vertical wall which is perpendicular to the direction of motion of A. The coefficient of restitution between B and the wall is $\frac{1}{2}$.

(ii) Find, in terms of d and u, the time that elapses between the first and second collision between A and B. ---[5]

[W-18/22/Q2]

12. Two uniform smooth spheres A and B have equal radii and masses 5m and 2m respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides directly with sphere B which is moving towards it with speed 2u. The coefficient of restitution between the spheres is e.

(i) Show that the speed of B after the collision is $\frac{1}{7}u(1+15e)$ and find an expression for the speed of A. --[4]

In the collision, the speed of A is halved and its direction of motion reversed.

(ii) Find the value of e. ---[2]

(iii) For this collision, find the ratio of the loss of kinetic energy of A to the loss of kinetic energy of B. [W-18/23/Q2] --[3]

13. A bullet of mass 0·08 kg is fired horizontally into fixed vertical barrier. It enters the barrier horizontally with speed $300ms^{-1}$ and emerges horizontally after 0.02 s. There is a constant horizontal resisting force of magnitude 1000 N. Find the speed with which the bullet emerges from the barrier. --[3]

[S-17/21/Q1]

14. Two uniform small smooth spheres A and B have equal radii and masses 3m and m respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides directly with sphere B which is at rest. The coefficient of restitution between the spheres is e.

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14(i)→ Find, in terms of u and e, expressions for the velocities of A and B after the collision. ---[3]

Sphere B continues to move until it strikes a fixed smooth barrier which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the barrier is $3/4$. When the spheres subsequently collide, A is brought to rest.

(ii) Find the value of e. [S-17/21/Q3] ---[7]

15. Two uniform small smooth spheres A and B have equal radii and each has mass m. Sphere A is moving with speed u on a smooth horizontal surface when it collides directly with sphere B which is at rest. The coefficient of restitution between the spheres is $2/3$. Sphere B is initially at a distance d from a fixed smooth vertical wall which is perpendicular to the direction of motion of A. The coefficient of restitution between B and the wall is $1/3$.

(i) Show that the speed of B after its collision with the wall is $\frac{5}{18}u$. --[4]

(ii) Find the distance of B from the wall when it collides with A for the second time. [S-17/23/Q3] ---[6]

16. Three uniform small smooth spheres A, B and C have equal radii and masses m, km and m respectively, where k is a constant. The spheres are moving in the same directions along a straight line on a smooth horizontal surface, with B between A and C. The speeds of A, B and C are $2u$, u and $\frac{4}{3}u$ respectively. The coefficient of restitution between any pair of the spheres is $1/2$. After sphere A has collided with sphere B, sphere B collides with sphere C.

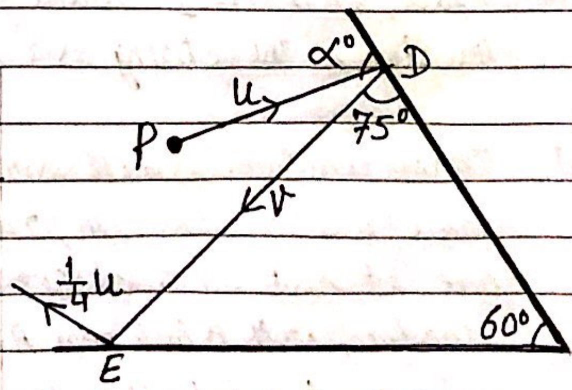
(i) Find an inequality satisfied by k. ---[5]

(ii) Given $k=2$, show that after B has collided with C there are no further collisions between any of the three spheres. [W-17/21/Q3] --[5]

17. A small smooth sphere A of mass m is moving with speed u on a smooth horizontal surface when it collides directly with an identical sphere B which is initially at rest on the surface. The coefficient of restitution between the spheres is e. Sphere B subsequently collides with a fixed vertical barrier which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the barrier is 1/2. Given that 80% of the initial kinetic energy is lost as a result of the two collisions, find the value of e. [S-16/21/Q2] --[8]

18. Three small smooth spheres A, B and C, of masses 5m, m and km respectively, are at rest on a smooth horizontal surface. The spheres lie in a straight line with B between A and C. Sphere A is projected directly towards B with speed u. The coefficient of restitution between any pair of spheres A, B and is 4/5. Show that the speed of A after the first collision is 7/10 u, and find the speed of B. --[3]
Given that the speeds of A and C are equal after the second collision.
(i) Show that the value of k is 20/7 --[4]
(ii) find the percentage loss in kinetic energy of the system as a result of the two collisions. [S-16/23/Q5] --[5]

19. Two smooth vertical walls each with their base on a smooth horizontal surface intersect at an angle of 60°. A small smooth sphere P is moving on the horizontal surface with speed u when it collides with the first vertical wall at the point D.



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19→ The angle between the direction of motion of P and the wall is $\alpha°$ before the collision and $75°$ after the collision. The speed of P after this collision is v and the coefficient of restitution between P and the first wall is e. Sphere P then collides with the second vertical wall at the point E. The speed P after this second collision is $\frac{1}{4}u$. The coefficient of restitution between P and the second wall is $3/4$.

(i) By considering the collision at E, show that $v = \frac{\sqrt{2}}{5}u$ ---[5]

(ii) Find the value of α and the value of e. $\boxed{W\text{-}16/21/Q2}$ --[5]

20. A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is held vertically above O with the string taut and then projected horizontally with speed $\sqrt{(\frac{13}{3}ag)}$. It begins to move in a vertical circle with centre O. When P is at its lowest point, it collides with a stationary particle of mass km. The two particles coalesce.

(i) Show that the speed of the combined particle immediately after the impact is $\frac{5}{\lambda+1}\sqrt{(\frac{1}{3}ag)}$. --[4]

In the subsequent motion, the string becomes slack when the combined particle is at a height of $\frac{1}{3}a$ above the level of O.

(ii) Find the value of λ. --[6]

(iii) Find, in terms of m and g, the instantaneous change in the tension in the string as a result of the collision. --[4]

$\boxed{W\text{-}16/21/Q4}$

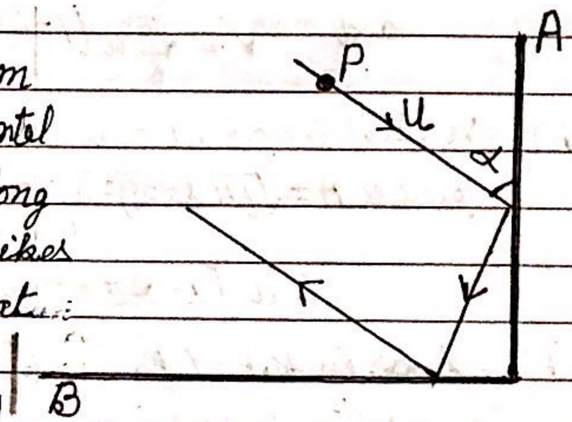
21. Three uniform small smooth spheres A, B and C have equal radii and masses $3m$, $2m$ and m respectively. The spheres are at rest in a straight line on a smooth horizontal surface, with B between A and C. The coefficient of restitution between A and B is e and the coefficient of restitution between B and C is e'. (Continued →)

(Continued→)

21→ Sphere A is projected directly towards B with speed u. Show that, after the collision between B and C, the speed of C is $\frac{2}{5}u(1+e)(1+e')$ and find the corresponding speed of B. --[7]

After this collision between B and C it is found that each of the three spheres has the same momentum. Find the values of e and e'. [S-15/21/Q5] --[5]

22. A uniform sphere P of mass m is at rest on a smooth horizontal table. The sphere is projected along the table with speed u and strikes a smooth vertical barrier A at an acute angle α. It then strikes another smooth vertical barrier B which is at right angles to A. The coefficient of restitution between P and each of the barrier is e. Show that the final direction of motion of P makes an angle $\frac{1}{2}\pi-\alpha$ with the barrier B and find the total loss in kinetic energy as a result of the two impacts. [S-15/ /Q2] --[7]



23. A small uniform sphere A, of mass $2m$, is moving with speed u on a smooth horizontal surface when it collides directly with a small uniform sphere B, of mass m, which is at rest. The spheres have equal radii and the coefficient of restitution between them is e. Find an expression for the speeds of A and B immediately after collision. --[4]

Subsequently B collides with a vertical wall which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is 0.4. After B has collided with the wall, the speeds of A and B are equal. Find e. --[2]

Initially B is at a distance d from the wall. Find the distance of B from the wall when it next collides with A. [W-15/21/Q2] --[4]

Coefficient of restitution 'e' = $\frac{\text{Relative velocity after collision}}{\text{Relative velocity before collision}}$

$V_2 - V_1 = -e(u_2 - u_1)$

$\boxed{\text{Answers}}$ continued→

1. (a) Speeds after collision.

$\underset{\rightarrow}{A},\ v \qquad \underset{\rightarrow}{B},\ w$

Momentum:

$2mw + mv = mu\cos 30^\circ$ — ①

Restitution $w - v = eu\cos 30$ — ②

solving ① & ② $w = \frac{\sqrt{3}}{6}u(1+e)$ ✓

and $v = \frac{\sqrt{3}}{6}u(1-2e)$

vertical comp. of vel. $= u\sin 30^\circ$

Speed of $A = \sqrt{(u\sin 30)^2 + \left(\frac{\sqrt{3}}{6}u(1-2e)\right)^2}$

$= u\sqrt{\frac{1-e+e^2}{3}}$ ✓

(b) Loss in K.E. (for $e = 1/3$)

$= \frac{1}{2}mu^2 - \frac{1}{2}\times 2m\left(\frac{\sqrt{3}}{6}\times\frac{4}{3}u\right)^2$

$-\frac{1}{2}m^2\times\frac{7}{27}u^2$

$= \frac{2}{9}mu^2$ ✓

using conservation of momentum for A and B

2: (i) $2mV_A + 4mV_B = 4mu - 4mu$

$\Rightarrow V_A + 2V_B = 0$ — ①

using Newton's restitution law

$V_B - V_A = e(2u + u)$

$\Rightarrow V_B - V_A = 3eu$ — ②

from ① & ② $V_A = -2eu$ ✓, $V_B = eu$ ✓

(ii) use conservation of momentum for B & C.

$4mV_B' + m\cdot V_C = 4m\cdot V_B - \frac{4}{3}mu$

$\Rightarrow 4V_B' + V_C = 4V_B - \frac{4}{3}u$

$\therefore V_B' = 0 \Rightarrow V_C = (4V_B - 4u/3)$ — ③

$V_C - V_B' = e(V_B + 4u/3)$ ... (Using Newton's Restitution Law)

Given $V_B' = 0$

$V_C = e(V_B + 4u/3)$ — ④ (continued→)

2(ii) from ③ & ④

$4V_B - 4u/3 = eV_B + \frac{4u}{3}\times e$

$4eu - \frac{4}{3}u = e\times eu + \frac{4}{3}u e$ [from (i) $V_B = eu$]

$\Rightarrow 4e - \frac{4}{3} = e^2 + 4e/3$

$\Rightarrow 3e^2 - 8e + 4 = 0$

$\Rightarrow e = 2/3$ ✓, $e = 2$ ✗

from ③ $V_C = 4u/3$

(iii) for A, KE Loss

$= \frac{1}{2}\times 2m(2u)^2 - \frac{1}{2}(2m)\left(\frac{4}{3}u\right)^2$

$= (20/9)mu^2$ — ⑤

for B, K.E. Loss

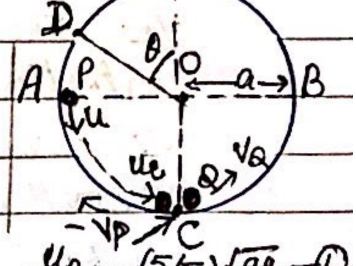
$= \frac{1}{2}4mu^2 - 0 = 2mu^2$ — ⑥

from ⑤ & ⑥ Total K.E Loss

$= (20/9)mu^2 + 2mu^2$

$= (38/9)mu^2$ ✓

3. Initial speed of P at $A = u = \sqrt{\frac{21}{2}ag}$



before collision at C, speed of $P = u_P$

$\frac{1}{2}mu_P^2 = \frac{1}{2}m\times\frac{21}{2}ag + mga$

$\Rightarrow u_P^2 = (25/2)ag \Rightarrow u_P = (5/\sqrt{2})\sqrt{ag}$ — ①

Let the speeds of P and Q after the collision are V_P & V_Q

$\therefore \frac{1}{2}(4m)V_Q^2 = (4m)ag$ (B reaches at B)

(i) $\Rightarrow V_Q = \sqrt{2ag} = 4.47\sqrt{a}$ ✓ — ②

for P $mu_P = -mV_P + 4mV_Q$

$V_P = -u_P + 4V_Q = \left(-\frac{5}{\sqrt{2}} + 4\sqrt{2}\right)\sqrt{ag}$

$V_P = \frac{3}{\sqrt{2}}\sqrt{ag} = 2.12\sqrt{ag}$ ✓

(continued→)

F.M

Answers

3(ii) V_P is the speed of P (at D') when it loses the contact

$\frac{1}{2} m V_P^2 = \frac{1}{2} m v_p^2 - mga(1+\cos\theta)$

$V_P^2 = (9/2) ag - 2ga(1+\cos\theta)$

$= (\frac{5}{2} - 2\cos\theta) ag$ — (3)

Now Radially at D' with reaction zero.

$R_D = m V_P^2/a - mg\cos\theta = 0$

$\Rightarrow V_P^2 = ag\cos\theta$ — (4)

frm (3) & (4)

$(5/2 - 2\cos\theta) ag = ag\cos\theta$

$\Rightarrow \cos\theta = 5/6$ or 0.833 ✓

4(i) Conservation of momentum for A & B

$3m V_A + m V_B = 3mu$

$\Rightarrow 3V_A + V_B = 3u$ — (i)

Using Restitution law;

$V_B - V_A = eu$ — (ii)

frm (i) & (ii) $V_A = \frac{1}{4}(3-e)u$ ✓

and $V_B = \frac{3}{4}(1+e)u$ ✓

using conservation of momentum and restitution for B & C.

$W_B + V_C = V_B$

and $V_C - W_B = eV_B$

$\Rightarrow W_B = \frac{1}{2}(1-e)V_B = \frac{3}{8}(1-e^2)u$ ✓

and $V_C = \frac{1}{2}(1+e)V_B = \frac{3}{8}(1+e)^2 u$ ✓

(ii) Given $V_A = V_C$

$\Rightarrow \frac{1}{4}(3-e) = \frac{3}{8}(1+e)^2$

$\Rightarrow 3e^2 + 8e - 3 = 0$

$\Rightarrow e = 1/3$ ✓

5. for A & B use conservation of (i) momentum & restitution law.

$5m \cdot V_A + 5m \cdot V_B = 5m \cdot u$

$\Rightarrow V_A + V_B = u$ — (i)

and $V_B - V_A = eu$ — (ii)

$\Rightarrow V_A = \frac{1}{2}(1-e)u$ ✓; $V_B = \frac{1}{2}(1+e)u$ ✓

(ii) $5m \cdot V_B' + 3m \cdot V_C = 5m \cdot V_B$ for B & C.

$\Rightarrow 5V_B' + 3V_C = V_B$ — (iii)

and $V_C - V_B' = eV_B$ — (iv)

$\Rightarrow V_B' = \frac{1}{8}(5-3e)V_B$

and $V_C = \frac{1}{8}(5+5e)V_B$

No further collisions

$\Rightarrow V_A \leq V_B'$

$\Rightarrow \frac{1}{2}(1-e)u \leq \frac{1}{8}(5-3e) \times \frac{1}{2}(1+e)u$

$\Rightarrow 3e^2 - 10e + 3 \leq 0$ $(\frac{1}{3} \leq e)$

$\frac{1}{3} \leq e \leq 1$ ✓

6. $m(250 - 70) = 450 \times 0.4$

$m = \frac{18}{180} = 0.1$ ✓

[Change in momentum = F × t]

7(i) Using conservation of momentum and restitution law.

$mV_A = mu - m \cdot ku$

$\Rightarrow V_A = (u - ku)$ — (i)

and $0 - V_A = e(u + ku)$ — (ii)

frm (i) & (ii) $-e(1+k) = (1-k)$

$\Rightarrow e = (k-1)/(k+1)$ ✓

(Continued →)

F.M

Answers

7(ii) $K.E_I - K.E_F = 0.6\,K.E_I$

$\Rightarrow$

$0.4\,K.E_I = K.E_F$

$0.4 \times \left[\frac{1}{2}mu^2 + \frac{1}{2}k^2u^2\right] = \frac{1}{2}mV_A^2$

$\Rightarrow 0.4\,\frac{1}{2}mu^2(1+k^2) = \frac{1}{2}m.u^2(1-k)^2$

$\Rightarrow .04(1+k^2) = (1-k)^2$

$\Rightarrow 0.6k^2 - 2k + 0.6 = 0$

$\Rightarrow 3k^2 - 10k + 3 = 0$

$(3k-1)(k-3) = 0$

$\Rightarrow k = 3$ or $\frac{1}{3}$ ×

for $k=3$, $e = \frac{k-1}{k+1} = \frac{2}{4} = \frac{1}{2}$

$\therefore k = 3$ ✓ and $e = \frac{1}{2}$ ✓

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8 (i) $4m.V_A + m.V_B = 4mu + 0$

$\Rightarrow 4V_A + V_B = 4u$ —— (i)

and using Newtons law of restitution

$V_B - V_A = eu$ —— (ii)

from (i) & (ii) $V_A = \frac{1}{5}(4-e)u$ ✓

$V_B = \frac{4}{5}(1+e)u$ ✓

(ii) $V_B' = -\frac{3}{4}V_B = -\frac{3}{5}e(1+e)u$ ✓

Now $V_A = V_B'$

$\Rightarrow \frac{1}{5}(4-e)u = \frac{3}{5}e(1+e)u$

$\Rightarrow 3e^2 + 4e - 4 = 0$

$\Rightarrow e = 2/3$ ✓

(iii) $4mW_A + mW_B = 3mV_A\,[=2mu]$

$W_B - W_A = e \times 2V_A = \frac{4}{3}V_A$ or $\frac{10}{9}u$

Momentum before collision is

$3mV_A = 2mu > 0$

So After collision speed of $B > 0$

$W_B > 0$, so B collides again with the barrier.

9. at the lowest point $Q \to V_1^2$

$\frac{1}{2}mV_1^2 = \frac{1}{2}mu^2 + mga$

$\Rightarrow V_1^2 = (12+2)ag = 14ag$

Now P & Q Coalesce

$MV_2 = mV_1$ with $M = (1+k)m$

$\Rightarrow V_2 = \frac{V_1}{(1+k)} = \sqrt{14ag}/1+k$ ✓

$T_1 = mV_1^2/a + mg = (14+1)mg = 15mg$

$T_2 = MV_2^2 + Mg$

$= (1+k)\{14/(1+k)^2 + 1\}mg$

or $\left\{\frac{14}{1+k} + (1+k)\right\}mg$

$\Rightarrow 14 + (1+k)^2 = 15(1+k)/2 \quad [T_2 = \frac{1}{2}T_1]$

$\Rightarrow 2k^2 - 11k + 15 = 0$

$k = 2.5$ or 3 ✓

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10. Conservation of momentum.

$5m.V_A + 2m.V_B = 5m.u - 2m.2u = mu$

$\Rightarrow 5V_A + 2V_B = u$ —— (i)

Restitution: $V_B - V_A = e(u+2u) = 3eu$ —— (ii)

from (i) & (ii) $V_A = u/7\,(1-6e)$ ✓

$V_B = u/7\,(1+15e)$ ✓ —— (iii)

(ii) $V_A = \frac{u}{7}(1-6e) = -\frac{1}{2}u \Rightarrow e = 3/4$ ✓

(iii) $K.E_A = \frac{1}{2}.5m\{u^2 - (\frac{1}{2}u)^2\} = \frac{15}{8}mu^2$

$K.E_B = \frac{1}{2} \times 2m\{(2u)^2 - (\frac{7u}{4})^2\} = \frac{15}{16}mu^2$

from (iii)

$\therefore \frac{K.E_A}{K.E_B} = \frac{15/8}{15/16} = 2:1$ ✓

F.Me

Momentus

Answers

Using momentum

11. (i) $2m.v_A + m.v_B = 2m.u$

$\Rightarrow 2v_A + v_B = 2u$ —— (i)

Using Newton's Law of restitution

$v_B - v_A = \frac{2}{3}u$ —— (ii) $(e = 2/3)$

from (i) and (ii) $v_A = 4u/9$ ✓

$v_B = 10.u/9$ ✓

(ii) $w_B = -\frac{1}{2}v_B = \pm 5u/9$

$(d-x)/v_A = d/v_B + x/w_B$

$(d-x)/4 = d/10 + x/5$

$\Rightarrow x = d/3$

$t = (d-x)/v_A = (2d/3)/(4u/9)$

$= (3d/2u)$ ✓

Using momentum

12. (i) $5m.v_A + 2m.v_B = 5m.u - 4mu$

$\Rightarrow 5v_A + 2v_B = u$ —— (i)

Using Newton's Law, $v_B - v_A = e(u+2u)$

$\Rightarrow v_B - v_A = 3eu$ —— (ii)

from (i) & (ii) $v_A = \frac{u}{7}(1-6u)$ ✓

$v_B = \frac{u}{7}(1+15e)$ ✓

(ii) $(u/7)(1-6e) = -\frac{1}{2}u \Rightarrow e = 3/4$

(iii) $K.E_A = \frac{1}{2}\times 5m\left(u^2 - \left(\frac{1}{2}u\right)^2\right) = \frac{15}{8}mu^2$

$K.E_B = \frac{1}{2}\times 2m\left\{(2u)^2 - \left(\frac{7u}{4}\right)^2\right\}$

$= \frac{15}{16}mu^2$

$\therefore \frac{KE_A}{KE_B} = \frac{15/8}{15/16} = 2:1$ ✓

13. change in momentum $= F\times t$

$0.08(300-v) = 1000\times 0.02$

$\Rightarrow v = 300 - 250 = 50ms^{-1}$ ✓

14 (i) using momentum

$3m.v_A + m v_B = 3mu$

$\Rightarrow 3v_A + v_B = 3u$ —— (i)

Newton's Law, $v_B - v_A = eu$ —— (ii)

fr (i) & (ii) $v_A = \frac{1}{4}(3-e)u$ ✓, $v_B = \frac{3}{4}(1+e)u$ ✓

(ii) $v_B' = -\frac{3}{4}v_B = -\frac{9}{16}(1+e)u$ —— (iii)

$3mV_A + mV_B = 3mV_A + mV_B'$

$V_A = 0 \Rightarrow V_B = 3(9-7e)u/16$ ✓ —— (IV)

$V_B - V_A = -e(V_B' - V_A)$

$\Rightarrow V_B = e(21+5e)u/16$ —— (V)

from (iv) & (v)

$3(9-7e) = e(21+5e)$

$\Rightarrow 5e^2 + 42e - 27 = 0$

$e = 3/5$ ✓

15 (i) $m.v_A + m v_B = mu$

$\Rightarrow v_A + v_B = u$ —— (i)

and $v_B - v_A = \frac{2}{3}u$ —— (ii)

fr (i) and (ii) $v_B = 5u/6$ ✓

After collision with the wall

$w_B = \frac{1}{3}v_B = 5u/18$ ✓

ii) fr (i) & (ii) $v_A = u/6$

$(d-x)/v_A = d/v_B + x/w_B$

$6(d-x) = 1.2d + 3.6x$

$\Rightarrow x = \frac{1}{2}d$ ✓

F.M.

Answers

16. (i) $m \cdot V_A + km \cdot V_B = m \cdot 2u + km \cdot u$

$\Rightarrow V_A + kV_B = 2u + ku$ —(i)

$V_B - V_A = \frac{1}{2}(2u - u) = \frac{1}{2}u$ ---(ii)

from (i) and (ii)

$V_B = u(2k+5)/2(k+1) = u\frac{(k+5/2)}{k+1}$

$V_A = \frac{u(k+4)}{2(k+1)}$ (speed of C)

for B collides with C, $V_B > 4u/3$

$\Rightarrow u\frac{(k+5/2)}{k+1} > \frac{4}{3}u$

$\Rightarrow k < \frac{7}{2}$ ✓

(ii) for B & C → $km \cdot W_B + mV_C = kmV_B + m \times \frac{4u}{3}$

for $k = 2 \Rightarrow 2W_B + V_C = 2 \times \frac{3u}{2} + \frac{4u}{3}$

$\Rightarrow 2W_B + V_C = \frac{13u}{3}$ —(iii)

$V_C - W_B = \frac{1}{2}\left(V_B - \frac{4u}{3}\right) = \frac{u}{12}$ —(iv)

$(k+1)W_B = (k - \frac{1}{2})V_B + 2u$

$\Rightarrow$

$3W_B = 3/2\, V_B + 2u$

for $V_B = \frac{3u}{2} \Rightarrow W_B = 17u/12$

$V_A = u,\ V_A < W_B$

and $(k+1)V_C = 3k/2\, V_B + (2-k)\frac{2u}{3}$

$\Rightarrow 3V_C = 3V_B,\ V_B = \frac{3u}{2}$

$\therefore V_C = \frac{3u}{2} > W_B$

$\therefore$ B & C can not meet again.

17. $mV_A + mV_B = mu$

$\Rightarrow V_A + V_B = u$ —(i)

and $V_B - V_A = eu$ —(ii)

$V_A = (1-e)\frac{u}{2},\ V_B = (1+e)\frac{u}{2}$

$W_B = \frac{1}{2}V_B = (1+e)u/4$

Initial 80% is lost → 20% = $\frac{1}{5}$ is ✓

$\frac{1}{2}mu^2 \times \frac{1}{5} = \frac{1}{2}mV_A^2 + \frac{1}{2}mW_B^2$

$\frac{1}{5} = (1-e)^2/4 + (1+e)^2/16$

$\Rightarrow 25e^2 - 30e + 9 = 0$

$(5e-3)^2 = 0 \Rightarrow e = \frac{3}{5}$ ✓

18. conservation of momentum.

$5m \cdot V_A + mV_B = 5m \cdot u$

$\Rightarrow 5V_A + V_B = 5u$ —(i)

Newton's law: $V_B - V_A = \frac{4}{5}u$ —(ii)

from (i) & (ii) $V_A = 7u/10,\ V_B = 3u/2$ —(iii)

(i) $mV_B' + kmV_C = mV_B$

$\Rightarrow V_B' + kV_C = V_B$ —(iv)

Newtons, $V_C - V_B' = 4/5\, V_B$ —(v)

from (iv) & (v) $(k+1)V_C = (1 + \frac{4}{5})V_B$

$(k+1) \cdot \frac{7u}{10} = \frac{9}{5} \times \frac{3u}{2}$ [$\because V_C = V_A$]

$\Rightarrow k+1 = \frac{27}{7} \Rightarrow k = 20/7$ ✓

(ii) $V_B' = -u/2$ [from (iv)]

Now

$\frac{1}{2}5mV_A^2 + \frac{1}{2}mV_B'^2 + \frac{1}{2}kmV_C^2$

$= \frac{1}{2}mu^2[49/20 + \frac{1}{4} + 7/5]$

$= \frac{41}{20}mu^2$ ✓

Now in first collision

$L_1 = \frac{1}{2}5mu^2 - \frac{1}{2}5mV_A^2 - \frac{1}{2}mV_B^2$

$L_2 = \frac{1}{2}mV_B^2 - \frac{1}{2}mV_B'^2 - \frac{1}{2}kmV_C^2$

$L_1 = \frac{3}{20}mu^2,\ L_2 = \frac{3}{10}mu^2$

(← continued)

18(ii) continued

$\therefore$ Loss in K.E

$= \frac{1}{2}5mu^2 - [(L_1 + L_2)] = \frac{9}{20}mu^2$

$\therefore$ % Loss in K.E $= \frac{9/20\, mu^2}{\frac{5}{2}mu^2} \times 100 = 18\%$ ✓

F.Me

Answers

19. (i) Finding Comps of speed after collision at E

$V\cos 45$ to wall and $\frac{3}{4}V\sin 45$ ⊥ to wall.

$\therefore \sqrt{\left\{\left(\frac{V}{\sqrt{2}}\right)^2+\left(\frac{3}{4}V/\sqrt{2}\right)^2\right\}} = \frac{1}{4}u$

$(5/4\sqrt{2})V = \frac{1}{4}u \Rightarrow V = \frac{\sqrt{2}}{5}.u$ ✓

(ii) Comp. of speed to wall after collision at D. $V\cos 75 = u\cos\alpha$

$\Rightarrow \cos\alpha = (\sqrt{2}/5)\cos 75 = 0.0732$

$\Rightarrow \alpha = 85.8°$ ✓

Comps of speed ⊥ to wall after collision at D,

$V\sin 75° = e u \sin\alpha$

$e = (\sqrt{2}/5)\sin 75°/\sin\alpha$

or $e = 0.274$ ✓

20, at the lowest point, consv. of Energy

(i) $\frac{1}{2}mv_1^2 = \frac{1}{2}mu^2 + mg.2a$ —(i)

Now conservation of momentum

$Mv_2 = mv_1 \quad (M = (\lambda+1)m)$

$\Rightarrow v_2 = v_1/(\lambda+1)$

$= \{5/(\lambda+1)\}\sqrt{(\frac{1}{3}ag)}$ ✓

(ii) F = ma and at slack T = 0

$Mv_3^2/a = Mg\cos\theta$

Now $\frac{1}{2}Mv_3^2 = \frac{1}{2}Mv_2^2 - Mg(4a/3)$

$ag/3 = v_2^2 - 8ag/3$

$3ag = \{25/(\lambda+1)^2\}ag/3$

$(\lambda+1)^2 = 25/9 \Rightarrow \lambda = 2/3$ ✓

(iii) (Continued →)

(iii) Using F = ma just before collision radially

$T_1 = mv_1^2/a + mg$

$= (25/3+1)mg = 28mg/3.$

Now F = ma, radially just after collision

$T_2 = Mv_2^2/a + Mg$

$= (3+1)(5m/3)g = 20mg/3$

∴ Change in tension

$(25mg/3)\{\lambda/(\lambda+1)\} - \lambda mg$

$= 8mg/3$ ✓

21. For A and B, conservation of Momentum

$3m.v_A + 2m.v_B = 3m.u \Rightarrow 3v_A + 2v_B = 3u$ —(i)

Using Newton's law, $v_B - v_A = eu$ —(ii)

from (i) & (ii) $v_B = 3(1+e)u/5$ —(iii)

Now for B & C using conservation of momentum

$2m.v_B' + mv_C = 2mv_B$

$\Rightarrow 2v_B' + v_C = 2v_B$ —(iv)

Now Newton's law for B & C

$v_C - v_B' = e'v_B$

$\Rightarrow v_C = 2(1+e').v_B/3$

$= 2(1+e)(1+e')u/5$ —(v)

$v_B' = (2-e').v_B/3$

$= (1+e)(2-e')u/5$ —(vi)

Now $3v_A = 2v_B' = v_C \; [=u]$

$v_A = (3-2e)u/5 = u/3$

or $v_B = \frac{1}{2}(3u-u)$ or $(\frac{1}{3}+e)u$

$= 3(1+e)u/5 \Rightarrow e = 2/3$ ✓

Now $2v_B' = v_C \Rightarrow 2(2-e') = 2(1+e')$

or $v_C = 2(1+2/3)(1+e')u/5 = u$

or $v_B' = (1+2/3)(2-e')u/5 = u/2$

$\Rightarrow e' = \frac{1}{2}$ ✓

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Answers

22. Comps of speed after I^{st} collin.
$u\cos\alpha \parallel$ to A & $eu\sin\alpha \perp$ to A
Comps of speed after 2^{nd} collision
$eu\sin\alpha \parallel B$ & $eu\cos\alpha \perp$ to
Find the final angle with B.
$\tan^{-1}(eu\cos\alpha / eu\sin\alpha) = \frac{1}{2}\pi - \alpha$ ✓

Relate angle β_1 after Ist Collision to α,

$$\left\{\begin{array}{l} V_1 \sin\beta_1 = eu\sin\alpha \\ V_1\cos\beta_1 = u\cos\alpha \end{array}\right.$$

$\Rightarrow \tan\beta_1 = e\tan\alpha$ ✓

Relate angle β_2 after 2^n colln to β_1.

$$\left.\begin{array}{l} V_2\sin\beta_2 = eV_1\cos\beta_1 \\ V_2\cos\beta_2 = V_1\sin\beta_1 \end{array}\right\}$$

$\tan\beta_2 = e/\tan\beta_1$ ✓

To find angle β_2 with barrier B
$\tan^{-1}(e/e\tan\alpha) = \frac{1}{2}\pi - \alpha$ ✓

To Find the total loss/gain in K.E
$\frac{1}{2}m\{u^2 - (eu\sin\alpha)^2 - (eu\cos\alpha)^2\}$
$= \frac{1}{2}m(1-e^2)u^2$ ✓
$u_{final} = eu$
$\therefore$ Loss in K.E
$\frac{1}{2}m(u^2 - u_{final}^2)$
$= \frac{1}{2}m(1-e^2)u^2$ ✓

23. For A & B conservation of momentum
$2m \cdot V_A + mV_B = 2mu$
$\Rightarrow 2V_A + V_B = 2u$ —— (i)
Newtons laws of restitution: $V_B - V_A = eu$ —— (ii)
fr (i) & (ii), $V_A = (2-e)u/3$ ✓
$V_B = 2(1+e)u/3$ ✓

To find e $V_A = |V_B'|$, $V_B' = [-]0.4V_B$
$\Rightarrow (2-e) = 0.4 \times 2(1+e)$
$\Rightarrow e = 2/3$ ✓

Let Req distance is x.
$(d-x)/V_A = d/V_B + x/V_B'$
$V_A = V_B' = 4u/9$, $V_B = 10u/9$
$d - x = 0.4d + x$, $x = 0.3d$
$d_A = (d/V_B)V_A = 0.4d$ ✓
$x = \frac{1}{2}(d - d_A) = 0.3d$ ✓

⟵ X —— X ⟶