FP-1

Further Pure Maths-1

Polar Coordinates Exercise.

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Polar Coordinates

Date ________

1. The curve C has polar equation:

$$r = 2 + 2\cos\theta \qquad 0 \le \theta \le \pi$$

(a) Sketch C ---[3]

(b) Find the area of the region enclosed by C and the initial line. ---[4]

(c) Show that the cartesian equation of C can be expressed as $4(x^2+y^2) = (x^2+y^2-2x)^2$ --[3]

[SP-20|01|Q3]

2. The curve C_1 has polar equation $r^2 = 2\theta$ for $0 \le \theta \le \frac{1}{2}\pi$

(i) The point on C_1 furthest from the line $\theta = \frac{1}{2}\pi$ is denoted by P. Show that, at P,

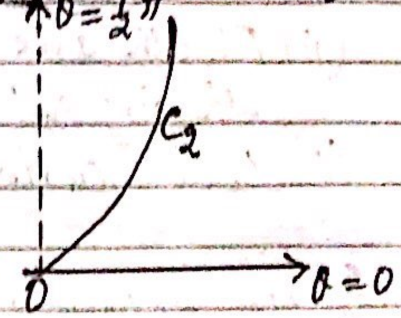
$$2\theta \tan\theta = 1$$

and verify that that equation has a root between 0.6 and 0.7. ---[5]

The curve C_2 has polar equation $r^2 = \theta\sec 2\theta$, for $0 \le \theta \le \frac{1}{8}\pi$. The curves C_1 and C_2 intersect at the pole, denoted by O, and at another point Q.

(ii) Find the exact value of θ at Q. ---[2]

(iii) The diagram below shows the curve C_2. Sketch C_1 on the diagram. ---[2]



(iv) Find, in exact form, the area of the region OPQ enclosed by C_1 and C_2. ---[5]

[S-19|11|Q11]

Date ______

3. The curve C has polar equation,
$$r^2 = \ln(1+\theta), \text{ for } 0 \leq \theta \leq 2\pi$$
(i) Sketch C ---[2]
(ii) Using substitution $u = 1+\theta$ or otherwise, find the area of the region bounded by C and the initial line, leaving your answer in exact form. --[5]
[S-19/13/Q2]

4. The curves C_1 and C_2 have polar equations, for $0 \leq \theta \leq \frac{1}{2}\pi$, as: C_1: $r = 2(e^{\theta} + e^{-\theta})$
C_2: $r = e^{2\theta} - e^{-2\theta}$

The curves intersect at point P where $\theta = \alpha$.
(i) Show that $e^{2\alpha} - 2e^{\alpha} - 1 = 0$. Hence find the exact value of α and show that the value of r at P is $4\sqrt{2}$. ---[6]
(ii) Sketch C_1 and C_2 on the same diagram. ---[3]
(iii) Find the area of the region enclosed by C_1, C_2 and the initial line, giving your answer correct to 3 significant figures. --[5]
[W-19/11/Q11]

5. The curve has polar equation,
$$r = \cos 2\theta, \text{ for } -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$$
(i) Sketch C. ---[2]
(ii) Find the area of the region enclosed by C, showing full working. --[3]
(iii) Find a cartesian equation of C. [S-18/11/Q3] -[3]

6. The curves C_1 and C_2 have polar equation as follows. C_1: $r = a$ $0 \leq \theta \leq \pi$
C_2: $r = 2a|\cos\theta|$,
where a is a positive constant. The curves intersect at the points P_1 and P_2.
(Continued)

Date

continued:

6 (i) Find the polar coordinates of P_1 and P_2. --- [2]

(ii) In a single diagram, sketch C_1, C_2 and their line of symmetry. --- [3]

(iii) The region R enclosed by C_1 and C_2 is bounded by the arcs OP_1, P_1P_2 and P_2O, where O is the pole. --- [5] Find the area of R, giving your answer in exact form.

S-18 | 13 | Q8

7. The curve C has polar equation, $r = 5\sqrt{(\cot\theta)}$, where $0.01 \leq \theta \leq \frac{1}{2}\pi$.

(i) Find the area of the finite region bounded by C and the line $\theta = 0.01$, showing full working. Give your answer correct to 1 decimal place. --- [3]

Let P be the point on C where $\theta = 0.01$.

(ii) Find the distance of P from the initial line, giving your answer correct to 1 decimal place. --- [2]

(iii) Find the maximum distance of C from initial line. --- [3]

(iv) Sketch C. --- [2]

W-18 | 11 | Q9

8. The curve C has polar equation $r = a\cos 3\theta$, for $-\frac{1}{6}\pi \leq \theta \leq \frac{1}{6}\pi$, where a is positive constant.

(i) Sketch C. --- [2]

(ii) Find the area of the region enclosed by C, showing full working. --- [3]

(iii) Using the identity $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$, find a cartesian equation of C. --- [3]

W-18 | 12 | Q3

9. The curve C has polar equation $r = a(1 + \sin\theta)$ for $-\pi \leq \theta \leq \pi$, where a is a positive constant.

(i) Sketch C. --- [2]

(ii) Find the area of the region enclosed by C. --- [4]

S-17 | 11 | Q11

Date

10. A curve C has polar equation $r = 2a\cos(2\theta + \frac{1}{2}\pi)$, for $0 \le \theta \le 2\pi$, where a is a positive constant.
(i) Show that $r = -2a\sin 2\theta$ and sketch C. --[4]
(ii) Deduce that the cartesian equation of C is;
$$(x^2+y^2)^{3/2} = -4axy$$ --[2]
(iii) Find the area of one loop of C. --[5]
(iv) Show that, at the points (other than the pole) at which a tangent to C is parallel to the initial line, $2\tan\theta = -\tan 2\theta$ [S-17/13/Q11] [3]

11. The polar equation of C is $r = a(1+\cos\theta)$ for $0 \le \theta \le 2\pi$, where a is a positive constant.
(i) Sketch C. ---[2]
(ii) Show that the cartesian equation of C is;
$$x^2+y^2 = a(x+\sqrt{(x^2+y^2)})$$ --[2]
(iii) Find the area of the sector of C between $\theta = 0$ and $\theta = \frac{1}{3}\pi$. --[4]
(iv) Find the arc length of C between the point where $\theta = 0$ and the point where $\theta = \frac{1}{3}\pi$ --[5]
[W-17/11/Q11]

12. A curve has polar equation $r^2 = 8\operatorname{cosec} 2\theta$ for $0 < \theta < \frac{\pi}{2}$,
Find a cartesian equation of C. ---[3]
Sketch C. --[2]
Determine the exact area of the sector bounded by the arc of C between $\theta = \frac{1}{6}\pi$ and $\theta = \frac{1}{3}\pi$, the half-line $\theta = \frac{1}{6}\pi$ and the half-line $\theta = \frac{1}{3}\pi$ --[3]
[It is given $\int \operatorname{cosec} x\, dx = \ln|\tan x/2| + c$] [S-16/11/4]

13. The curve C has polar equation $r = e^{4\theta}$ for $0 \le \theta \le \alpha$, where α is measured in radians. The length of C is 2015. Find the value of α. --[6] [S-15/13/Q2]

14 The curves C_1 and C_2 have polar equations:

$$C_1 : r = \frac{1}{\sqrt{2}}, \quad for\ 0 \leq \theta < 2\pi$$

$$C_2 : r = \sqrt{(\sin \tfrac{1}{2}\theta)} \quad for\ 0 \leq \theta \leq 2\pi$$

Find the polar coordinates of the point of intersection of C_1 and C_2. --- [2]

Sketch C_1 and C_2 on the same diagram --- [3]

Find the exact value of the area of the region enclosed by C_1, C_2 and the half line $\theta = 0$. --- [4]

S-15	11	Q5

15. The curve C has polar equation $r = a(1 - \cos\theta)$ for $0 \leq \theta \leq 2\pi$

(i) Sketch C, --- [2]

(ii) Find the area of the region enclosed by arc C for which $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$, the half line $\theta = \frac{\pi}{2}$ and the half line $\theta = \frac{3}{2}\pi$ --- [5]

XX (iii) Show that;

$$\left(\frac{ds}{d\theta}\right)^2 = 4a^2 \sin^2\left(\tfrac{1}{2}\theta\right)$$

where s denotes the length, and find the length of the arc C for which $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$ --- [7]

W-15	11	Q11

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Polar Coordinates

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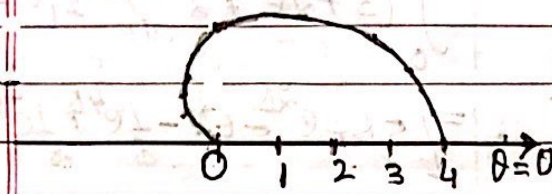
$r = 2+2\cos\theta,\ 0\le\theta\le\pi$ **Answers** continued

1.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
r	4	3.7	3.4	3	2	1	.58	.26	0

(a)



(b) Area $= \frac{1}{2}\int_0^{\pi} r^2\,d\theta = \frac{1}{2}\int_0^{\pi}(2+2\cos\theta)^2\,d\theta$

$= \frac{1}{2}\int_0^{\pi}(4+8\cos\theta+4\cos^2\theta)\,d\theta$

$= \frac{1}{2}\int_0^{\pi}\left(4+8\cos\theta+4\left(\frac{1+\cos 2\theta}{2}\right)\right)$

$= \int_0^{\pi}(3+4\cos\theta+\cos 2\theta)\,d\theta$

$= \left[3\theta+4\sin\theta+\frac{1}{2}\sin 2\theta\right]_0^{\pi}$

$= 3\pi$ ✓

(c) $r = 2+2\cos\theta$ $\quad$ $x = r\cos\theta$, $y = r\sin\theta$, $r = \sqrt{x^2+y^2}$

$\sqrt{x^2+y^2} = 2 + 2\times\frac{x}{\sqrt{x^2+y^2}}$

$2\sqrt{x^2+y^2} = x^2+y^2-2x$

$\Rightarrow 4(x^2+y^2) = (x^2+y^2-2x)^2$ ✓

2. Given $r^2 = 2\theta$ $\quad 0\le\theta\le\frac{\pi}{2}$

(i) $r = \sqrt{2}\,\theta^{1/2}$ $\quad [x = r\cos\theta$

$\Rightarrow x = \sqrt{2}\,\theta^{1/2}\cos\theta$

$\Rightarrow \frac{dx}{d\theta} = \sqrt{2}\left[-\theta^{1/2}\sin\theta + \frac{1}{2}\theta^{-1/2}\cos\theta\right] = 0$

for ① $\Rightarrow \cos\theta = 2\theta\sin\theta \Rightarrow 2\theta\tan\theta = 1$ ✓ ①

at $\theta = 0.6 \Rightarrow 2(.6)\cdot\tan(0.6) - 1 = -0.179$

at $\theta = 0.7 \Rightarrow 2(.7)\cdot\tan(0.7) - 1 = 0.179$

2(i) hence the eqn $2\theta\tan\theta = 1$ has a root between 0.6 and 0.7 as change of sign.

(ii) $C_1:\ r^2 = 2\theta$ ——— ②

$C_2:\ r^2 = \theta\sec^2\theta$ ——— ③

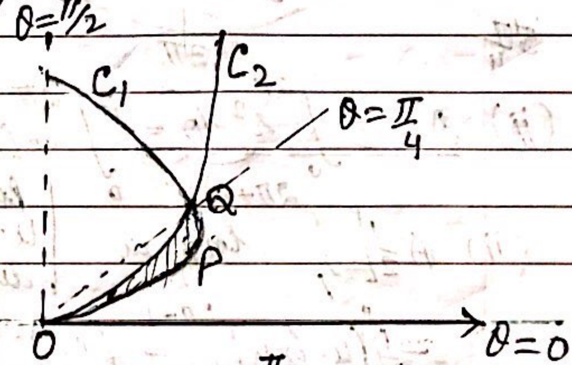
for point of intersection from ② & ③

$2\theta = \theta\sec^2\theta \Rightarrow \sec^2\theta = 2$

$\Rightarrow \sec\theta = \sqrt{2}$

$\theta = \frac{\pi}{4}$ ✓

(iii)



(iv) Area $OPQ = \frac{1}{2}\int_0^{\frac{\pi}{4}} 2\theta\,d\theta - \frac{1}{2}\int_0^{\frac{\pi}{4}}\theta\sec^2\theta\,d\theta$

$= \frac{1}{2}\int_0^{\frac{\pi}{4}}\theta(2-\sec^2\theta)\,d\theta$ $\Big\{$ Using Product Rule.

$= \frac{1}{2}\left[\theta(2\theta-\tan\theta)\right]_0^{\frac{\pi}{4}} - \frac{1}{2}\int_0^{\frac{\pi}{4}}(2\theta-\tan\theta)\,d\theta$

$= \frac{1}{2}\left[\theta(2\theta-\tan\theta)\right]_0^{\frac{\pi}{4}} - \frac{1}{2}\left[\theta^2+\ln\cos\theta\right]_0^{\frac{\pi}{4}}$

$= \frac{\pi}{8}\left[\frac{\pi}{2}-1\right] - \frac{1}{2}\left[\left(\frac{\pi}{4}\right)^2 - \frac{1}{2}\ln 2\right)$

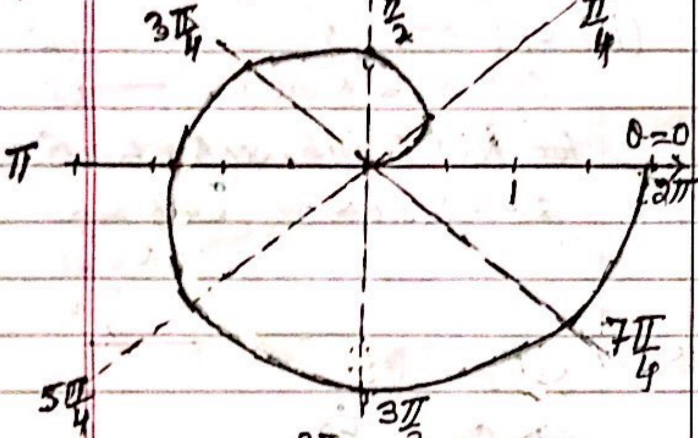
$= \frac{1}{4}\ln 2 + \frac{\pi}{8}\left[\frac{\pi}{4}-1\right]$ ✓

Answers

3. (i) Given curve C.

$r^2 = \ln(1+\theta), \quad 0 \le \theta \le 2\pi$

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
r	0	.57	.94	1.2	1.4	1.6	1.74	1.8	1.97



(ii) $A = \frac{1}{2}\int_0^{2\pi} r^2\,d\theta = \frac{1}{2}\int_0^{2\pi} \ln(1+\theta)\,d\theta$

$= \frac{1}{2}\left[\int_1^{2\pi+1} \ln u\,du\right.$ | Put $u = 1+\theta$, $du = d\theta$

$= \frac{1}{2}\left[[u\cdot\ln u]_1^{2\pi+1} - [u]_1^{2\pi+1}\right]$

$A = \left(\pi + \frac{1}{2}\right)\ln(2\pi+1) - \pi$ ✓

4. $C_1: r = 2(e^{\theta} + e^{-\theta})$ — (1) $0 \le \theta \le \frac{\pi}{2}$

$C_2: r = e^{2\theta} - e^{-2\theta}$

$\Rightarrow e^{2\alpha} - e^{-2\alpha} = 2(e^{\alpha} + e^{-\alpha})$

(i) $\Rightarrow e^{\alpha} - e^{-\alpha} = 2$

$\Rightarrow e^{2\alpha} - 1 = 2e^{\alpha}$

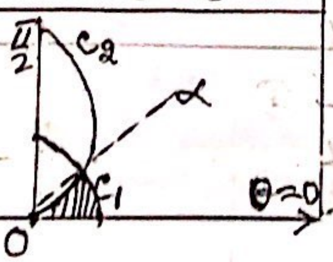
$\Rightarrow e^{2\alpha} - 2e^{\alpha} - 1 = 0$ ✓

$\Rightarrow e^{\alpha} = 1+\sqrt{2}$

$\Rightarrow \alpha = \ln(1+\sqrt{2})$ — (1)

(ii) from (1) $r = 2[(1+\sqrt{2}) + (\sqrt{2}-1)] = 4\sqrt{2}$ ✓

θ	0	.76	$\frac{\pi}{2}$
$r(C_1)$	4	5.64	10
$r(C_2)$	0	5.64	23



4 (iii) $A = \frac{1}{2}\int_0^{\alpha} 4(e^{\theta}+e^{-\theta})^2\,d\theta - \frac{1}{2}\int_0^{\alpha}(e^{2\theta}+e^{-2\theta})\,d\theta$

$= \int_0^{\alpha}\left[(5 + 2e^{2\theta} + 2e^{-2\theta}) - \frac{1}{2}(e^{4\theta} - e^{-4\theta})\right]d\theta$

$= \left[5\theta + e^{2\theta} - e^{-2\theta} - \frac{1}{8}e^{4\theta} + \frac{1}{8}e^{-4\theta}\right]_0^{\ln(1+\sqrt{2})}$

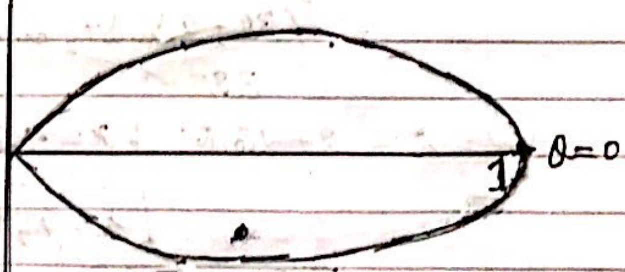
$= 5\ln(1+\sqrt{2}) + (1+\sqrt{2})^2 - (1+\sqrt{2})^{-2} - \frac{1}{8}\left[(1+\sqrt{2})^4 - (1+\sqrt{2})^{-4}\right]$

$= 5.82$ ✓

5. $r = \cos 2\theta \quad -\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$

(i)

θ	$-\frac{\pi}{4}$	$-\frac{\pi}{8}$	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$
r	0	.7	1	.7	0



(ii) $A = \frac{1}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos^2 2\theta\,d\theta$

$= \frac{1}{4}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\cos 4\theta + 1)\,d\theta$

$= \frac{1}{4}\left[\frac{1}{4}\sin 4\theta + \theta\right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$

$= \frac{\pi}{8} = 0.393$ ✓

(iii) $r = \cos 2\theta$

$r = \cos^2\theta - \sin^2\theta$

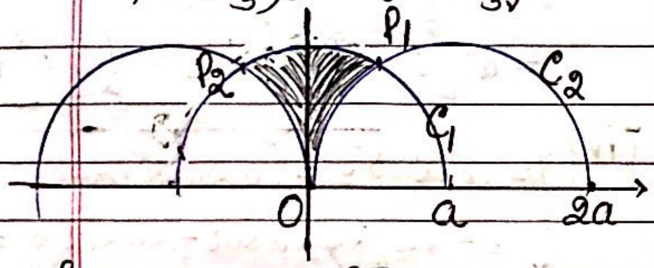
$r = \frac{x^2}{r^2} - \frac{y^2}{r^2}$

$\Rightarrow r^3 = x^2 - y^2 \Rightarrow$ | $(x^2+y^2)^{3/2} = x^2 - y^2$ ✓

Answers

6 $C_1: r=a$
$C_2: r=2a|\cos\theta|$ $\quad 0\le\theta\le\pi$, $a>0$, a is constant

(i) $a = 2a|\cos\theta| \Rightarrow \cos\theta = \pm\frac{1}{2}$
$P_1(a,\frac{\pi}{3}),\ P_2(a,\frac{2\pi}{3})$



for C_1. $=\frac{1}{2}\int_{\pi/3}^{2\pi/3} a^2\,d\theta = \frac{1}{2}a^2[\theta]_{\pi/3}^{2\pi/3}$

$= \frac{\pi a^2}{6}$ — (1)

and for C_2, $\cancel{2}\times\frac{a^2}{\cancel{2}}\int_{\pi/3}^{\pi/2} 4a^2\cos^2\theta$

$=4a^2\int_{\pi/3}^{\pi/2}\frac{(1+\cos2\theta)}{2}$

$=2a^2\left[\frac{1}{2}\sin2\theta+\theta\right]_{\pi/3}^{\pi/2}$

$=2a^2\left[\frac{\pi}{2}-\left(\frac{\sqrt{3}}{4}+\frac{\pi}{3}\right)\right]$

$=a^2\left[\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right]$ — (2)

fr (1) & (2)

Req. Area

$=\frac{\pi a^2}{6}-a^2\left[\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right]$

$=-\frac{\pi a^2}{6}+\frac{a^2\sqrt{3}}{2}$ ✓

7 $C: r=5\sqrt{(\cot\theta)}$, $0.01\le\theta\le\frac{1}{2}\pi$

(i) $A=\frac{1}{2}\int r^2\,d\theta=\frac{25}{2}\int_{0.01}^{\pi/2}\cot\theta\,d\theta$

$=\frac{25}{2}\left[\ln\sin\theta\right]_{0.01}^{\pi/2}$

$=-\frac{25}{2}\ln\sin0.01=\underline{57.6}$ ✓

(ii) $y=r\sin\theta=5\sqrt{\cot\theta}\cdot\sin\theta$

$=5\sqrt{\cos\theta\sin\theta}$

$=\frac{5}{\sqrt{2}}\sqrt{\sin2\theta}$

$\theta=0.01\Rightarrow y=0.5$ ✓

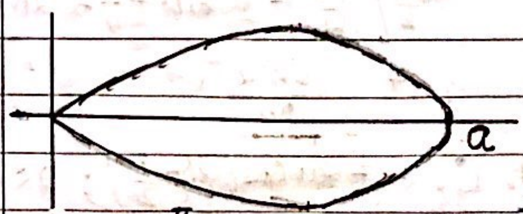
(iii) $\frac{dy}{d\theta}=\frac{5}{\sqrt{2}}\cdot\frac{1}{\cancel{2}\sqrt{\sin2\theta}}\times\cos2\theta\times\cancel{2}=0$

or Max $(\sin2\theta)=1$

$y=\frac{5\sqrt{2}}{2}=3.54$ ✓

8. $C: r=a\cos3\theta$, $-\frac{\pi}{6}\le\theta\le\frac{\pi}{6}$
and a is constant

(i)



(ii) $A=\frac{1}{2}\int_{-\pi/6}^{\pi/6} a^2\cos^2 3\theta\,d\theta$

$=\frac{a^2}{4}\int_{-\pi/6}^{\pi/6}(1+\cos6\theta)\,d\theta$

$=\frac{a^2}{4}\left[\frac{1}{6}\sin6\theta+\theta\right]_{-\pi/6}^{\pi/6}$

$=\frac{\pi a^2}{12}$

(Continued →)

(continued →) **Answers**

8(iii) $r = a\cos 3\theta$

$= a(4\cos^3\theta - 3\cos\theta)$

$= a\cos\theta(4\cos^2\theta - 3)$

$\Rightarrow r = a\left(\frac{x}{r}\right)\left[4\left(\frac{x}{r}\right)^2 - 3\right]$

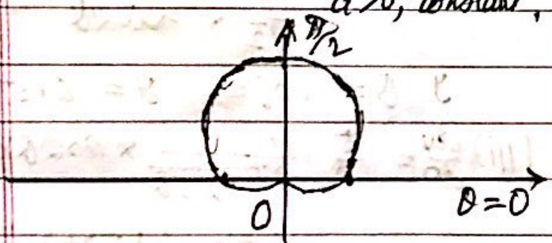
$\Rightarrow r^4 = ax(4x^2 - 3r^2)$

$\Rightarrow (x^2+y^2)^2 = ax[4x^2 - 3(x^2+y^2)]$

$\Rightarrow (x^2+y^2)^2 = ax(x^2 - 3y^2)$ ✓

9. $C: r = a(1+\sin\theta)$, $-\pi \le \theta \le \pi$
$a > 0$, constant.

(i)



(ii) $A = \frac{1}{2}a^2\int_{-\pi}^{\pi}(1 + 2\sin\theta + \sin^2\theta)\,d\theta$

$= \frac{a^2}{2}\int_{-\pi}^{\pi}\left(1 + 2\sin\theta + \frac{1}{2} - \frac{1}{2}\cos 2\theta\right)d\theta$

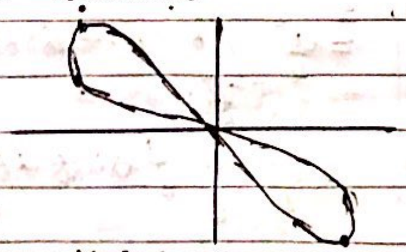
$= \frac{a^2}{2}\left[\frac{3\theta}{2} - 2\cos\theta - \frac{1}{4}\sin 2\theta\right]_{-\pi}^{\pi}$

$= \frac{3\pi a^2}{2}$ ✓

10. $C: r = 2a\cos\left(2\theta + \frac{\pi}{2}\right)$; $0 \le \theta \le 2\pi$
$a > 0$ is constant.

(i) $r = 2a\left[\cos 2\theta \cdot \cos\frac{\pi}{2} - \sin 2\theta \cdot \sin\frac{\pi}{2}\right]$

or $r = -2a\sin 2\theta$ ✓



(ii) $r = -4a\sin\theta\cos\theta = 4a\,\frac{y}{r}\cdot\frac{x}{r}$

$\Rightarrow r^3 = -4axy$

$\Rightarrow (x^2+y^2)^{3/2} = -4axy$ ✓

10 (iii) Area of one loop.

$= \frac{1}{2}\int_{\frac{\pi}{2}}^{\pi} 4a^2\sin^2 2\theta\,d\theta = a^2\int_{\frac{\pi}{2}}^{\pi} 2\sin^2 2\theta\,d\theta$

$= a^2\int_{\frac{\pi}{2}}^{\pi}(1 - \cos 4\theta)\,d\theta$

$= a^2\left[\theta - \frac{1}{4}\sin 4\theta\right]_{\frac{\pi}{2}}^{\pi} = \frac{1}{2}\pi a^2$ ✓

(iv) $y = r\sin\theta = -2a\sin 2\theta \cdot \sin\theta$

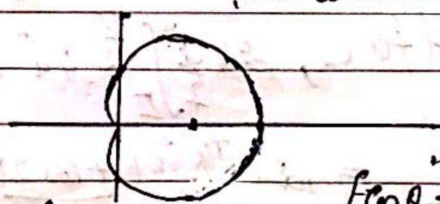
$\frac{dy}{d\theta} = -2a[2\cos 2\theta\sin\theta + \sin 2\theta\cos\theta] = 0$

$\Rightarrow 2\cos 2\theta\sin\theta = -\sin 2\theta\cos\theta$

$\Rightarrow 2\tan\theta = -\tan 2\theta$ ✓

11. $C: r = a(1+\cos\theta)$ for $0 \le \theta \le 2\pi$
a is constant.

(i)



(ii) $r = a(1+\cos\theta)$ $\left[\because \cos\theta = \frac{x}{r}\right]$

$\Rightarrow \sqrt{x^2+y^2} = a\left[1 + \frac{x}{\sqrt{x^2+y^2}}\right]$

$\Rightarrow x^2+y^2 = a\left[x + \sqrt{x^2+y^2}\right]$

(iii) Sector area $= \frac{a^2}{2}\int_0^{\frac{\pi}{3}}(1 + 2\cos\theta + \cos^2\theta)\,d\theta$

$= \frac{a^2}{2}\int_0^{\pi/3}\left(\frac{3}{2} + 2\cos\theta + \frac{\cos 2\theta}{2}\right)d\theta$

$= \frac{a^2}{2}\left[\frac{3\theta}{2} + 2\sin\theta + \frac{\sin 2\theta}{4}\right]_0^{\frac{\pi}{3}}$

$= \frac{a^2}{16}\left[4\pi + 9\sqrt{3}\right]$ ✓

(iv) arc length — × × ×

Polar Coordinates

Answers

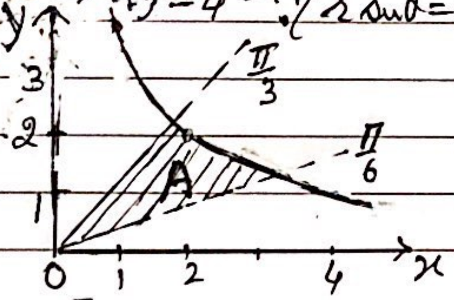
12. $r^2 = 8\operatorname{cosec} 2\theta \;;\; 0 \le \theta \le \frac{\pi}{2}$

$\Rightarrow r^2 = \dfrac{8}{\sin 2\theta}$

$\Rightarrow 2\sin\theta\cdot\cos\theta\cdot r^2 = 8$

$\Rightarrow r\cos\theta\cdot r\sin\theta = 4 \quad \{ r\cos\theta = x$

$\Rightarrow xy = 4$ ✓ $\quad r\sin\theta = y \}$



Area $A = \frac{1}{2}\int_{\pi/6}^{\pi/3} 8\operatorname{cosec} 2\theta \, d\theta$

$= \left[2\ln|\tan\theta|\right]_{\pi/6}^{\pi/3}$

$= 2\left[\ln\sqrt{3} - \ln\frac{1}{\sqrt{3}}\right] = 2\ln 3$

$\left(\text{Given } \int \operatorname{cosec} x\,dx = \ln|\tan x/2|\right)$ or $A = \ln 9$ ✓

13. $r = e^{4\theta} \qquad 0 \le \theta \le \alpha$

α is in radian.

$s = \int_0^{\alpha} \sqrt{e^{8\theta} + 16e^{8\theta}} = \int_0^{\alpha} \sqrt{17}\, e^{4\theta} d\theta$

$= \left[\frac{\sqrt{17}}{4} e^{4\theta}\right]_0^{\alpha} = \frac{\sqrt{17}}{4} e^{4\alpha} - \frac{\sqrt{17}}{4} = 2015$

$\Rightarrow e^{4\alpha} = 1955.837$

$\Rightarrow \alpha = 1.89$ ✓

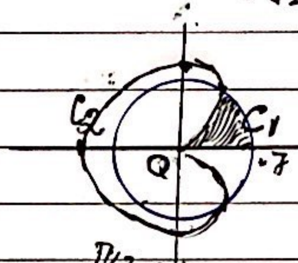
14. $C_1 : r = \frac{1}{\sqrt{2}}$ for $0 \le \theta < 2\pi$

$C_2 : r = \sqrt{(\sin\theta/2)}$ for $0 \le \theta \le 2\pi$

for point of intersection:

$\sin\theta/2 = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$.

Point $\left(\frac{1}{\sqrt{2}}, \frac{\pi}{3}\right)$ ✓



$\{ C_1$ — circle $r = \frac{1}{\sqrt{2}}$

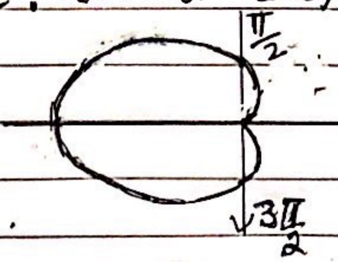
$= \frac{1}{2}\int_0^{\pi/3} \left(\frac{1}{\sqrt{2}}\right)^2 d\theta - \frac{1}{2}\int_0^{\pi/3} \sin\theta/2 \, d\theta$

$= \frac{1}{2}\times\frac{1}{2}\times\frac{\pi}{3} - \frac{1}{2}\left[-2\cos\frac{\theta}{2}\right]_0^{\pi/3}$

$= \left(\frac{1}{12}\pi + \frac{\sqrt{3}}{2} - 1\right)$ ✓

15. $C : r = a(1 - \cos\theta) \;;\; 0 \le \theta \le 2\pi$

(i)



$A = 2 \times \frac{1}{2}\int_{\pi/2}^{\pi} (1 - 2\cos\theta + \cos^2\theta)\, d\theta$

$= a^2 \int_{\pi/2}^{\pi} \left(\frac{3}{2} - 2\cos\theta + \frac{1}{2}\cos 2\theta\right) d\theta$

$= a^2 \left[\frac{3}{2}\theta - 2\sin\theta + \frac{1}{4}\sin 2\theta\right]_{\pi/2}^{\pi}$

$= a^2\left(\frac{3}{4}\pi + 2\right)$

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