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FP1

Further Pure Maths 1

Rational Functions and Graphs.

Notes.

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Rational Functions and Graphs
Asymptotes
Notes.

• **Case I.** When degree of Numerator < degree of Denominator.

Example 1. $y = f(x) = \frac{1}{x}$ —— ①

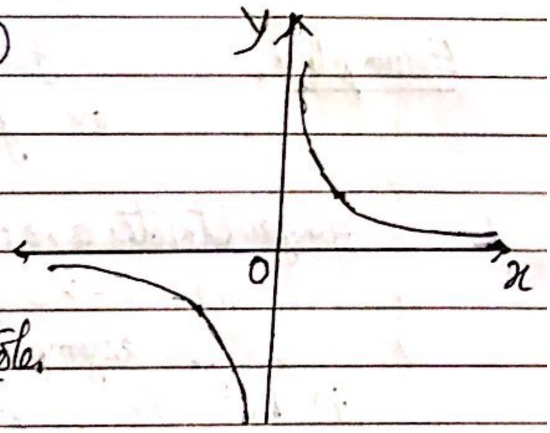
(i) $x \to 0 \Rightarrow y \to \infty$

$\therefore$ y-axis $(x=0)$ is an asymptote.

(ii) $x \to \infty \Rightarrow y \to 0$

x-axis $(y=0)$ is an asymptote.

Function ① has two asymptotes.



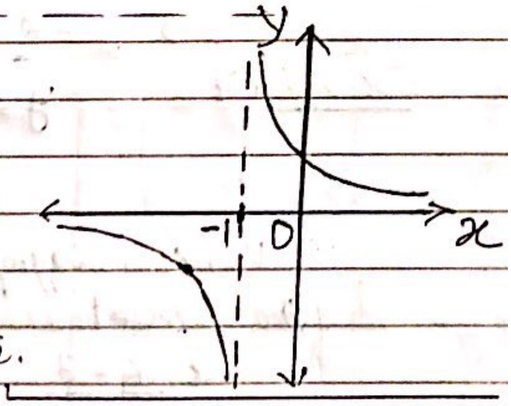
Example 2. $y = \frac{1}{(x+1)}$

(i) $x \to -1 \Rightarrow y \to \infty$

Vertical line $x=-1$ is asymptote.

(ii) $x \to \infty \Rightarrow y \to 0$

x-axis $(y=0)$ is another asymptote.



Example 3. $y = \frac{x+2}{(x+1)(x-3)}$ —— ①

(i) $x \to -1 \Rightarrow y \to \infty$

$\therefore$ Vertical line $x=-1$ is an asymptote.

(ii) $x \to 3 \Rightarrow y \to \infty$

another vertical line $x=3$ is asymptote.

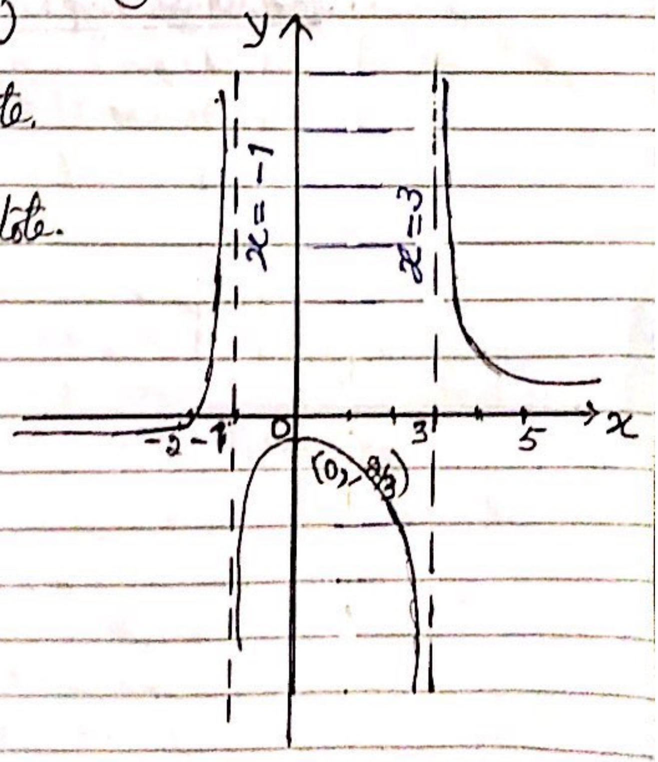
(iii) Divide N^r & D^r of ① by x^2

$$y = \frac{\frac{1}{x} + \frac{2}{x^2}}{\left(1+\frac{1}{x}\right)\left(1-\frac{3}{x}\right)}$$

$x \to \infty$, $\frac{1}{x} \to 0$

$y \to \frac{0}{1} = 0$

$\therefore$ x-axis $(y=0)$ is the third asymptote.



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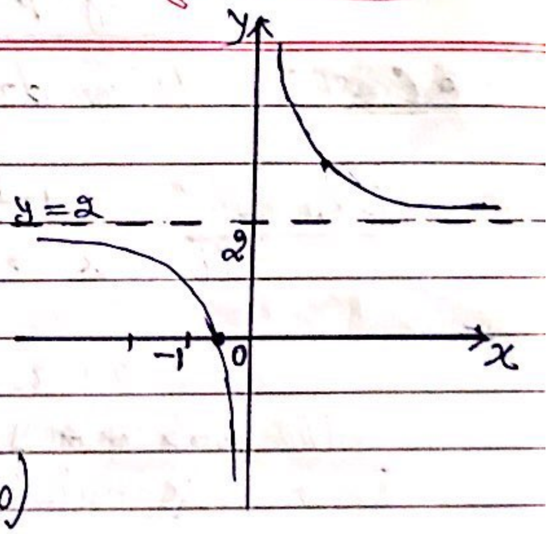
Asymptotes

- **Case II** $\quad$ Deg N^r = Deg D^r

Example 4. $y = \frac{2x+1}{x}$

or $y = 2 + \frac{1}{x}$



Asymptotote are:

(i) $x \to 0 \Rightarrow y \to \infty$

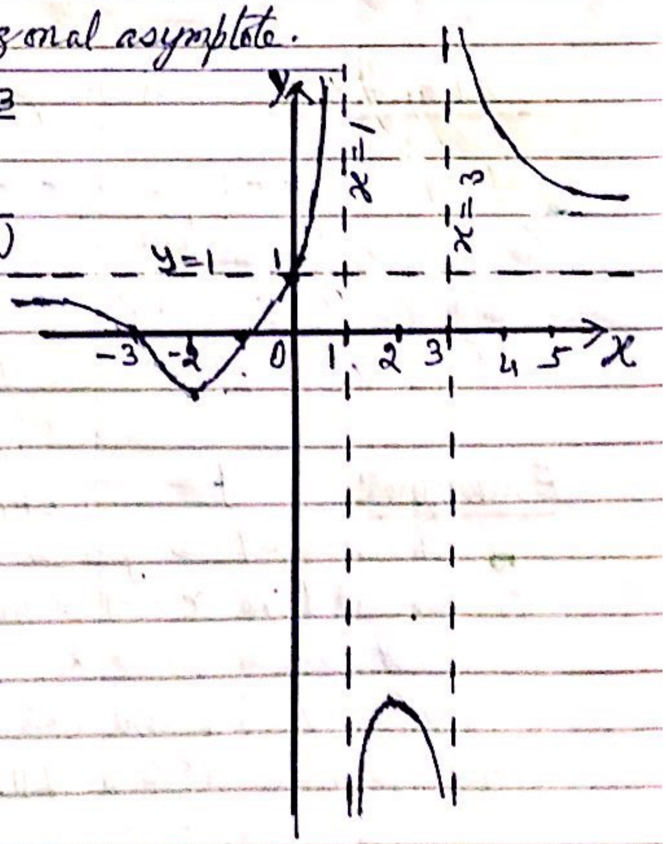
$\therefore$ asymptote is y-axis (line $x=0$)

(ii) But when $x \to \infty$, $y \to 2+0$

$\Rightarrow$ $y=2$ is another horizonal asymptote.

Example 5: $y = \frac{x^2+4x+3}{x^2-4x+3}$

$= 1 + \frac{8x}{(x-1)(x-3)}$



(i) Vertical Asymptote;

(ii) $x=1$

and $x=3$

(iii) $x \to \infty$, $y \to 1$

$\therefore$ Horizontal asyptote $y=1$ ✓

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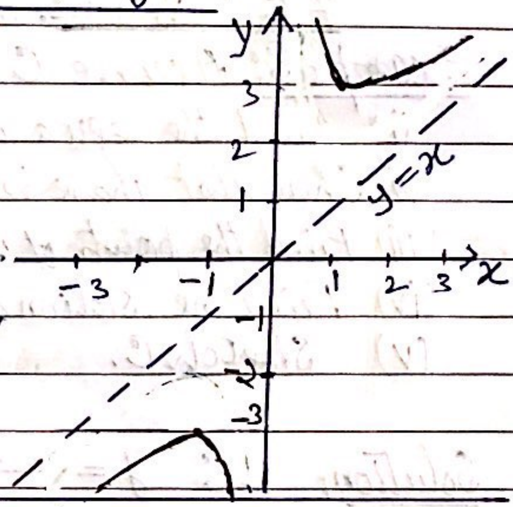
FP1 Asymptotes

- **Case III (Oblique Asymptotes)** Deg N^r > Deg D^r



Example 6: $y = \frac{x^2+2}{x}$

or $y = x + \frac{2}{x}$

(i) $x \to 0 \Rightarrow y \to \infty$

$\Rightarrow$ y-axis (or $x=0$) is a vertical asymptote.

(ii) $x \to \infty$, $y \to x$

$\therefore$ Oblique asymptote: $y = x$

Example 7: $y = \frac{x^2-5x+6}{x-1}$

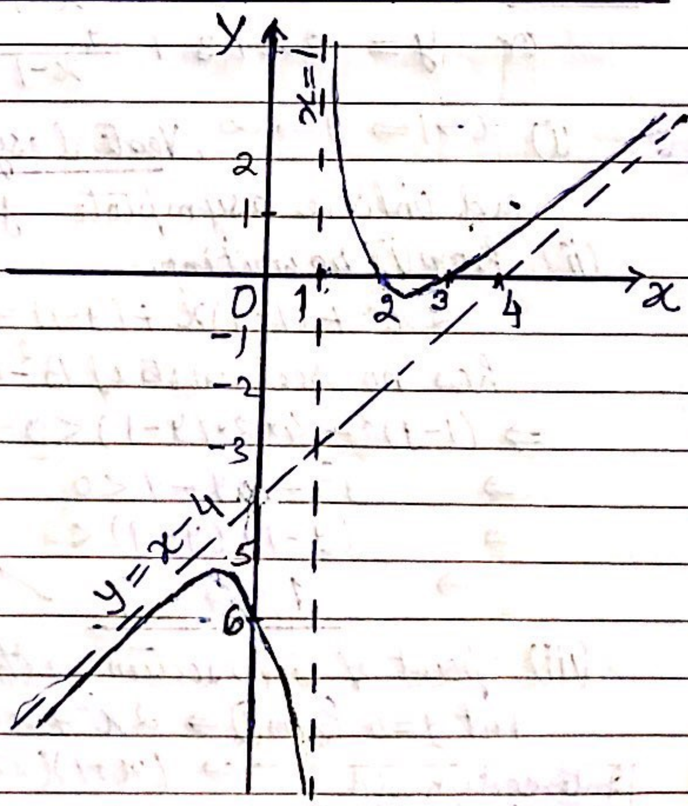
Divide:

or $y = (x-4) + \frac{2}{x-1}$

(i) $x \to 1 \Rightarrow y = \infty$

$\Rightarrow$ $x=1$ is a vertical asymptote.

(ii) $y = (x-4)$ is another asymptote. (oblique)

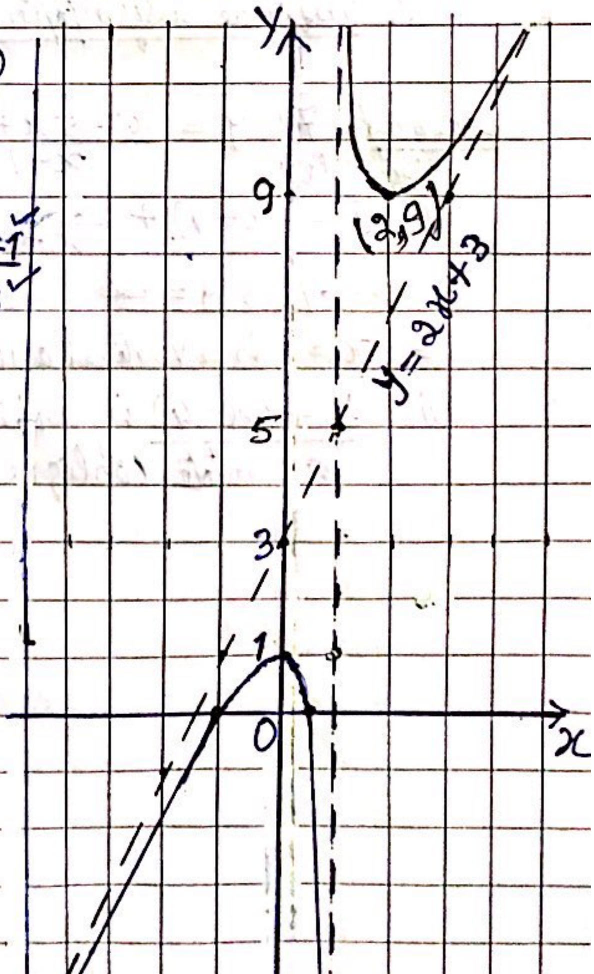


Rational Functions and Graphs.

- To find (i) the range of a ration function (ii) points of intersect of C with axes. (iii) turning points.

Example 8: A curve C has equation $y = \frac{2x^2+x-1}{x-1}$.
(i) Find the equations of the asymptotes of C.
(ii) Show that there is no point on C for which $1 < y < 9$.
(iii) Find the point of intersections of C with axis.
(iv) Find the stationary points (or turning points) of C.
(V) Sketch C.

W-14 | 11 | Q4



Solution: C : $y = \frac{2x^2+x-1}{x-1}$ — ①

or $y = 2x+3+\frac{2}{x-1}$ — ②

(i) $x \to 1 \Rightarrow y \to \infty$, Vertical asymptote $x=1$ ✓
and Oblique asymptote $y = 2x+3$ ✓

(ii) from ① rewriting.
$$2x^2+(1-y)x+(y-1)=0$$
has no real roots if $B^2-4AC<0$
$$\Rightarrow (1-y)^2-4\times2\times(y-1)<0$$
$$\Rightarrow y^2-10y+9<0$$
$$\Rightarrow (y-1)(y-9)<0$$
$$\Rightarrow 1<y<9 \checkmark$$

(iii) point of intersection with X-axis
Put $y=0$ from ① $\Rightarrow 2x^2+x-1=0$
$$\Rightarrow (x+1)(2x-1)=0$$
$$\Rightarrow x=-1, \tfrac{1}{2} \checkmark$$
$\therefore$ Points on X-axis $(-1,0);(\frac{1}{2},0)$

Intersection with Y-axis (0,1) ✓

(iv) for stationary points $\frac{dy}{dx}=0$
$$\Rightarrow \frac{(x-1)(4x+1)-(2x^2+x-1)}{(x-1)^2}=0$$
$$\Rightarrow 2x^2-4x=0 \Rightarrow 2x(x-2)=0$$
$$\Rightarrow x=0, x=2$$
$\therefore$ Turning points are $(0,0)$ and $(2,9)$ ✓

Rational Functions and Graphs.

Example 9: The curve C has equation $y = \frac{2x^2+kx}{x+1}$, where k is a constant. Find the set of values of k for which C has no stationary points. [5]

For the case $k=4$, find the equations of the asymptotes of C, Sketch C, indicating the coordinates of the points where C intersects the coordinate axes. [W-15|11|Q8] [6]

Solution: for stationary point $\frac{dy}{dx}=0$

(i) $\Rightarrow (4x+k)(x+1) - (2x^2+kx)\times 1 = 0$

$\Rightarrow \quad 2x^2+4x+k=0$

For No. Stationary points $B^2-4AC<0$

$\Rightarrow 16-8k<0 \Rightarrow \underline{k>2}$ ✓

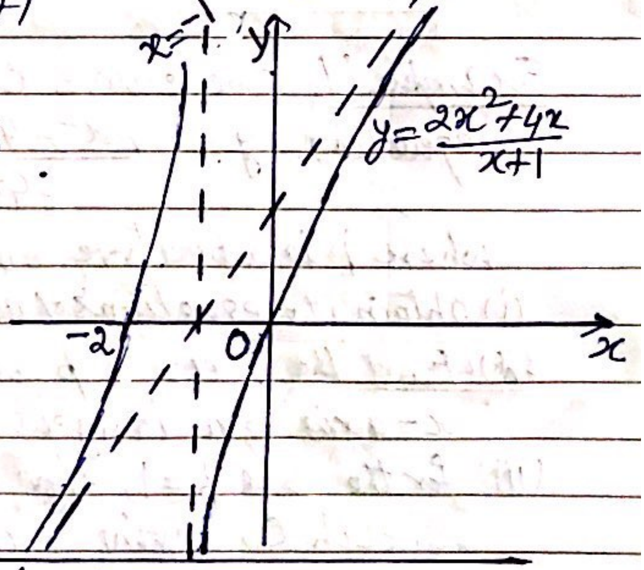
(ii) For $k=4$; C: $y = \frac{2x^2+4x}{x+1}$ ——— ①

or $y = 2x+2 - \frac{2}{x+1}$ ——— ②

(a) The curve has a Vertical asymptote at $x=-1$

(b) and an oblique asymptote at $y=2x+2$

(c) The curve ① intersects x-axis, when $y=0 \Rightarrow x=0$ or $x=-2$
$\Rightarrow$ at $(0,0)$, $(-2,0)$ ✓
Intersects y-axis at $(0,0)$ only.



Example 10. The curve C has equation $y = \lambda x + \frac{x}{x-2}$ where λ is a non-zero constant. Find

(i) the equations of the asymptotes of C,

(ii) Show that C has no turning point if $\lambda<0$

(iii) Sketch C in case $\lambda=-1$, stating the coordinates of the intersections with the axes. [W-12|11|Q7]

Solution: (i) $y = \lambda x + \frac{x}{x-2}$ ——— ①

or $y = \lambda x + 1 + \frac{2}{(x-2)}$ ——— ②

The asymptotes are $x=2$ (Vertical) and $y=\lambda x+1$, oblique.

(Continued→)

FP1

(continued→)

10(ii) for turning point diff. ② ⇒ $\frac{dy}{dx} = \lambda - \frac{2}{(x-2)^2}$

for ∴ turning point, $\frac{dy}{dx} = 0 \Rightarrow \lambda - \frac{2}{(x-2)^2} = 0$

$\Rightarrow \lambda(x-2)^2 = 2 \Rightarrow (x-2)^2 = \frac{2}{\lambda}$ —— ③

for no turning point $\frac{2}{\lambda} < 0 \Rightarrow \underline{\lambda < 0}$ ✓

Given $\lambda = -1$

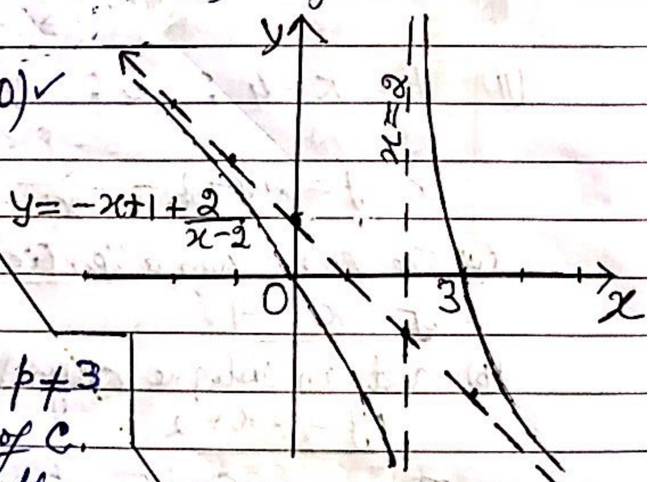
(iii) $C:$ $y = -x+1 + \frac{2}{x-2}$ —— ④

∴ asymptotes are $x = 2$ ✓ and $y = 1-x$ ✓

Intersects x-axis at $y=0 \Rightarrow x = 0, 3$ from ④

∴ (0,0) and (3,0)

On y-axis $x=0 \Rightarrow y=0 \Rightarrow (0,0)$ ✓



Example 11. The curve C has equation $y = \frac{px^2+4x+1}{x+1}$

where p is a positive constant and $p \neq 3$.

(i) Obtain the equations of asymptotes of C.

(ii) Find the value of p for which the x-axis is a tangent to C, and sketch C in this case.

(iii) for the case $p=1$, show that C has no turning points and sketch C, giving exact coordinates of the points of intersection of C with x-axis.

W-13/11/Q10

Solution: (i) $C;$ $y = \frac{px^2+4x+1}{x+1}$ —— ①

or $y = px+4-p + \frac{p-3}{x+1}$ —— ②

Asymptotes are $x=-1$ ✓ and $y = px+4-p$ ✓

(ii) for X-axis be tangent to the curve

from ① $y=0 \Rightarrow px^2+4x+1=0$ —— ③

and should have only one root $B^2-4AC=0$

$16-4p=0 \Rightarrow p=4$ ✓

for $p=4$ in ③ $x=-\frac{1}{2}$, tangent at $(-\frac{1}{2}, 0)$ ✓

(continued→)

FP1

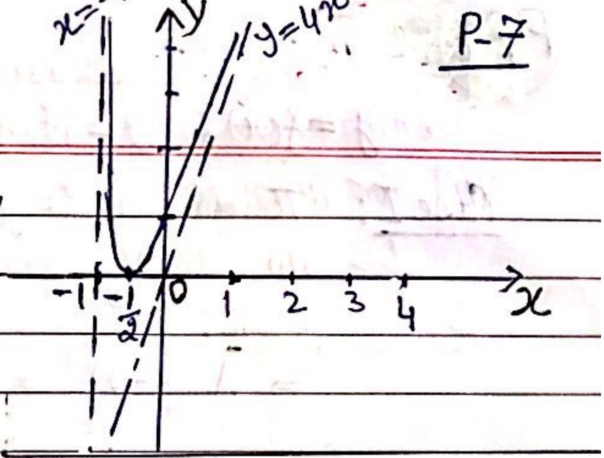
(Continued→)

11(ii) Now for $p=4$,

C: $y = 4x + \frac{1}{x+1}$ $\quad y = \frac{4x^2+4x+1}{x+1}$

Curve touches X-axis at $(-\frac{1}{2}, 0)$

Asymptotes are $x=-1$ & $y=4x$



(iii) for $p=1$; $y = x+3 + \frac{-2}{x+1}$

or $y = \frac{x^2+4x+1}{x+1}$ — ③

Asymptotes are $x=-1$ and $y=x+3$

from ③ $\frac{dy}{dx} = \frac{(x+1)(2x+4)-(x^2+4x+1)}{(x+1)^2}$

for Turning point $\frac{dy}{dx}=0$

$\Rightarrow x^2+2x+3=0$

$B^2-4AC = 4-12 = -8 < 0$

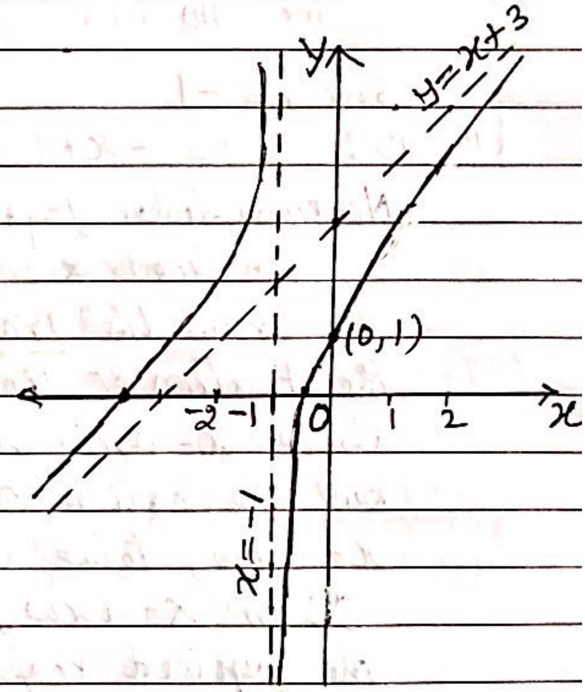
$\therefore$ No turning point.

C intersects X-axis for $y=0$ in ③

$\Rightarrow x^2+4x+1=0$

$x = \frac{-4 \pm \sqrt{12}}{2} = (-2\pm\sqrt{3}, 0)$

Intersects Y axis at $x=0$, $y=1 \Rightarrow (0,1)$

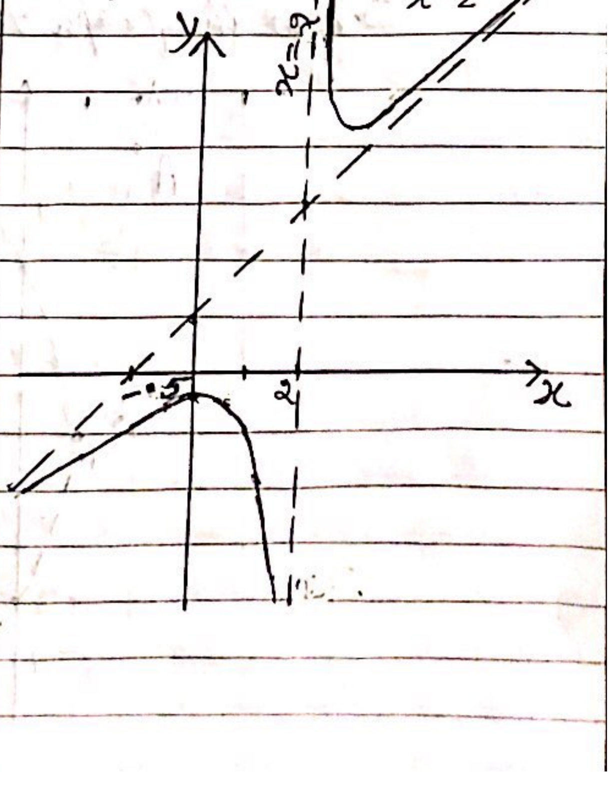


Example 12: The curve C has equation, $y = \frac{x^2+px+1}{x-2}$, where p is a constant. Given that C has two asymptotes, find the equation of each asymptote. Find the set of values of p for which C has two distinct turning points. Sketch C in case $p=-1$. Your sketch should indicate the coordinates of any intersections with the axes, but need not show the coordinates of any turning points.

|W-11|11|Q7|

(Try Yourself)

for $p=-1$; $y = \frac{x^2-x+1}{x-2}$



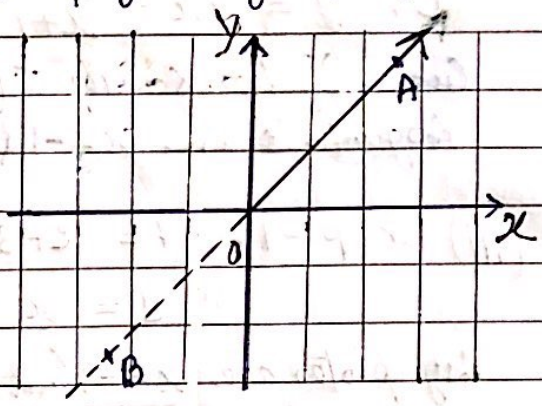
Ans: Vertical asymptote $x=2$, oblique asymptote: $y = x+p+2$; $p > -\frac{5}{2}$

FP1

Using the relationship between the graphs of $y=f(x)$; $y=|f(x)|$; $y=f(|x|)$; $y^2=f(x)$ and $y=\frac{1}{f(x)}$

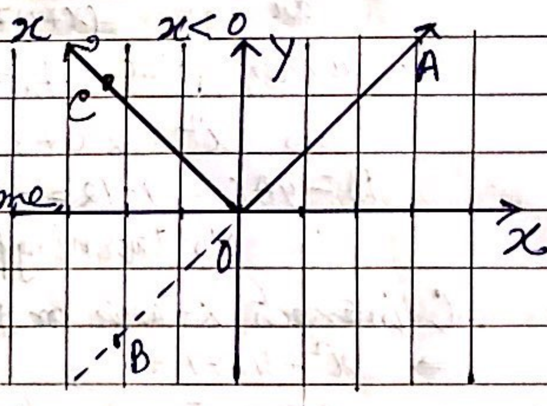
Case I: To draw the graph of $y=|f(x)|$, given $y=f(x)$.

Example 13: Given $y=f(x)=x$



Now consider $y=|f(x)|=|x|=\begin{cases} x, & x\geq 0 \\ -x, & x<0 \end{cases}$

To draw the graph of $|x|$ the portion of the graph $\vec{OA}$ above x-axis remain the same. But the portion $\vec{OB}$ below the x-axis, take its reflection $\vec{OC}$ in x-axis. $\vec{OA}\cup\vec{OC}$ is the required graph of $y=|x|$.

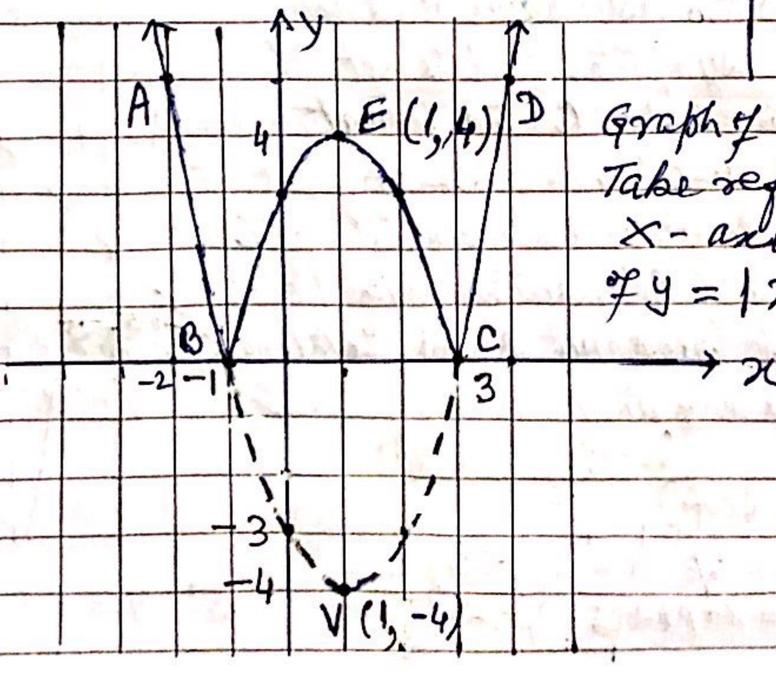


Example 14: Given $y=x^2-2x-3 = (x-3)(x+1)$

Draw the graph of $y=|x^2-2x-3|$

Intersects x-axis at $(3,0)$, $(-1,0)$

Vertex $(1,-4)$



Graph of $y=f(x)$ is ABVCD

Take reflection of $\widehat{BVC}$ in x-axis gives the graph of $y=|x^2-2x-3|$ is "ABECD"

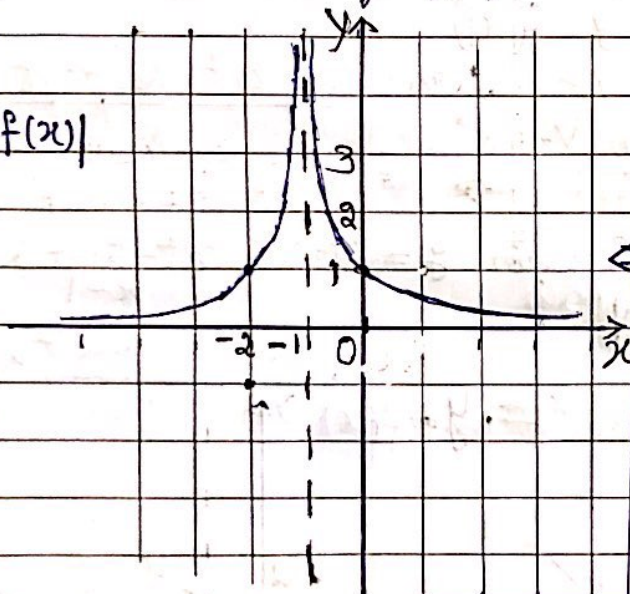
FP1

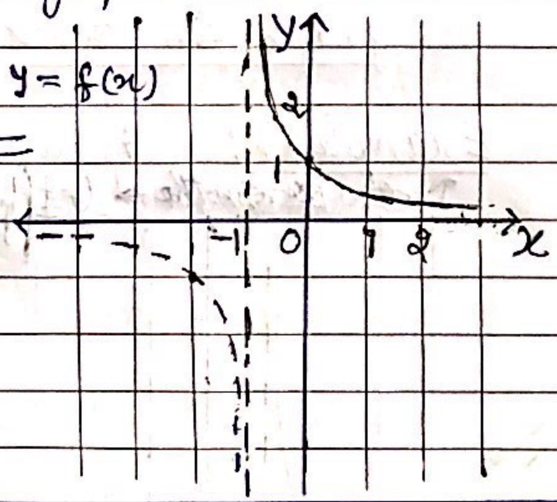
Case I. Graph of $y=|f(x)|$

Example 15: Draw the graph of $y=|f(x)|$; Given $f(x)=\frac{1}{x+1}$

(See Example 2)

asymptote at $x=-1$





Example 16: Draw the graph of $y=|f(x)|$

Given $y=f(x)=\frac{x+2}{(x+1)(x-3)}$ (See example 3)

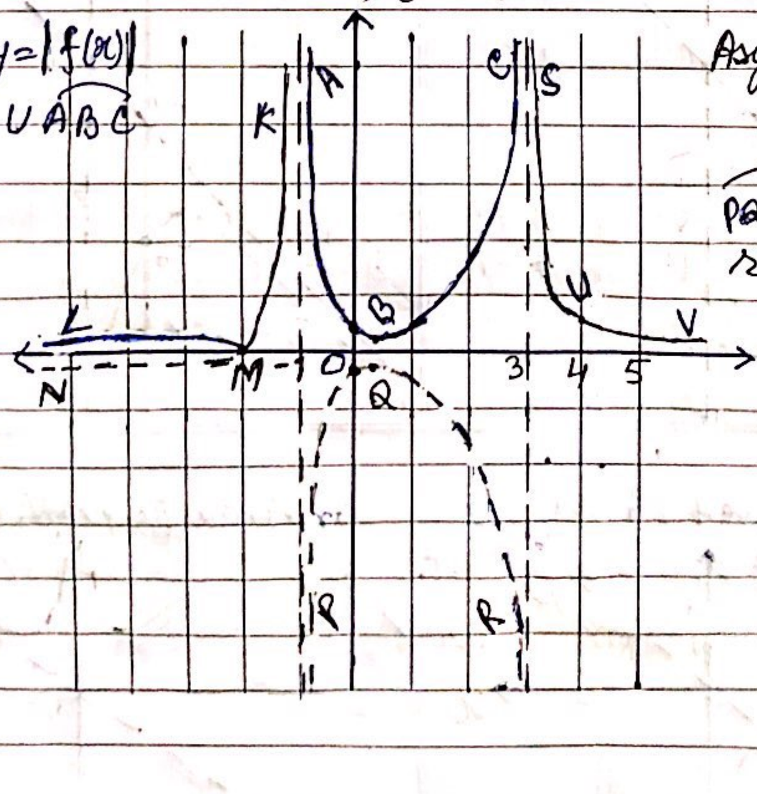
Graph of $y=|f(x)|$ is $\overset{\frown}{KML} \cup \overset{\frown}{ABC} \cup \overset{\frown}{SUV}$

Asymptotes, $x=-1$

$x=3$

$y=0$ (x-axis)

$\overset{\frown}{PQR}$ and $\overset{\frown}{MN}$ is refelected in the X-axis.



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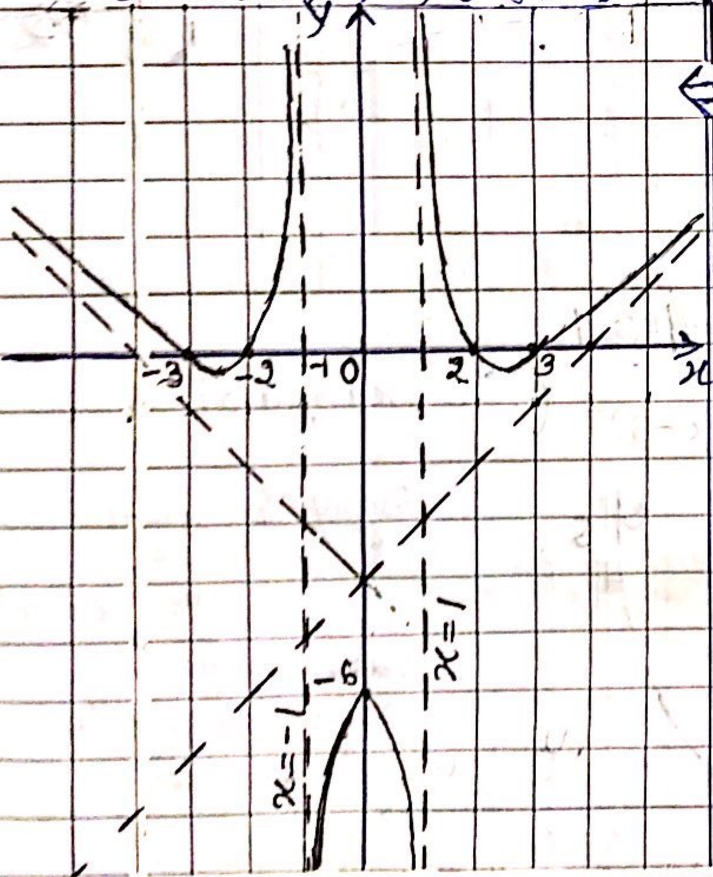
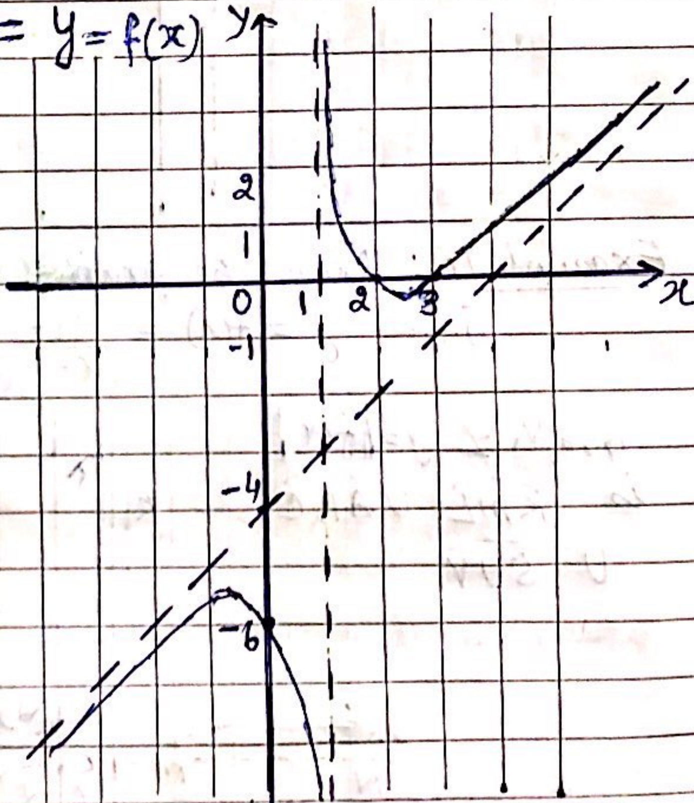
Case II. To draw the graph of $y = f|x|$, Given the Function $y = f(x)$.

§ Two draw the graph of $y = f(|x|)$
draw the graph of $y = f(x)$ and reflect the part of the graph for $x > 0$ in y-axis.

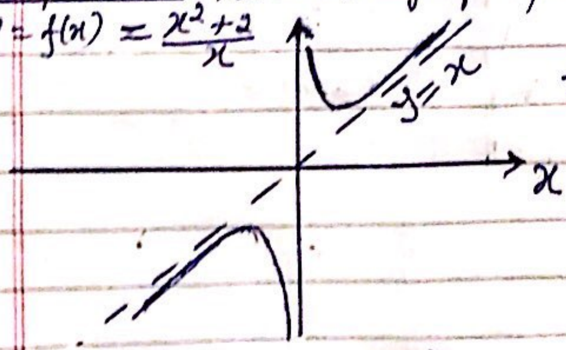
Example 17: Given a function $y = f(x) = \frac{x^2 - 5x + 6}{x - 1}$
Draw the Graph of $y = f(|x|)$



⟸ $y = f(x)$ (See example 7)

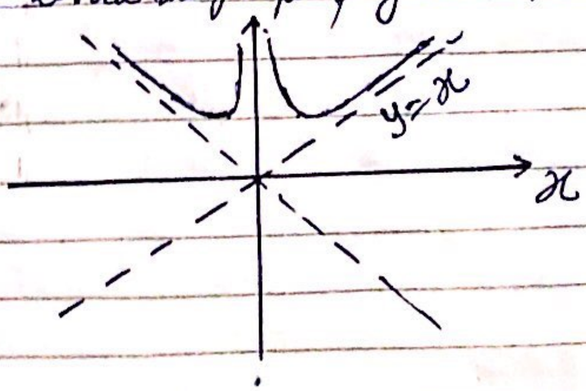


Example 18 Given the graph of $y = f(x) = \frac{x^2 + 2}{x}$



(See Example 6)

⟹ Draw the graph of $y = f(|x|)$



FP1 Case III

Given the graph of a function $y=f(x)$,

To draw the graph of $\frac{1}{f(x)}$

The following steps may be followed:

(i) If the given function $y=f(x)$ has x-intercepts (or roots) $x=x_1$, $x=x_2$, ... Then the graph of $y=\frac{1}{f(x)}$ will have vertical asymptotes at $x=x_1$, $x=x_2$ ...

(ii) $y=f(x)=\pm 1 \Rightarrow$ at the same points $\frac{1}{f(x)}=\pm 1$, i.e, (at $y=\pm 1$) these points will be common to the graphs of $y=f(x)$; $y=\frac{1}{f(x)}$

(iii) $f(x)$ and $\frac{1}{f(x)}$ have the same sign in an interval.

(iv) If $f(x)$ has a vertical asymptote at some point $x=a$, then $\frac{1}{f(x)}$ is not defined at that point.

(v) $\frac{d}{dx}\left[\frac{1}{f(x)}\right]=\frac{-f'(x)}{[f(x)]^2} \Rightarrow f(x)$ and $\frac{1}{f(x)}$ have the turning points (stationary points) for the same value of x.

(a) If (x_1, y_1) is Max of $f(x) \Rightarrow (x_1, \frac{1}{y_1})$ is min of $\frac{1}{f(x)}$

(b) if (x_2, y_2) is Min of $f(x) \Rightarrow (x_2, \frac{1}{y_2})$ is max of $\frac{1}{f(x)}$.

(VI) If $f(x)$ is increasing in an interval then $\frac{1}{f(x)}$ is decreasing in that interval and if $f(x)$ is decreasing then $\frac{1}{f(x)}$ is increasing in that interval.

(VII) (a) If $x=x_1$ is an asymptote of $y=f(x)$; Then $x=x_1$ is a discontinuity of $y=\frac{1}{f(x)}$

(b) If $y=y_1$ is an asymptote of $y=f(x)$; Then $y=\frac{1}{y_1}$ is an asymptote of $y=\frac{1}{f(x)}$.

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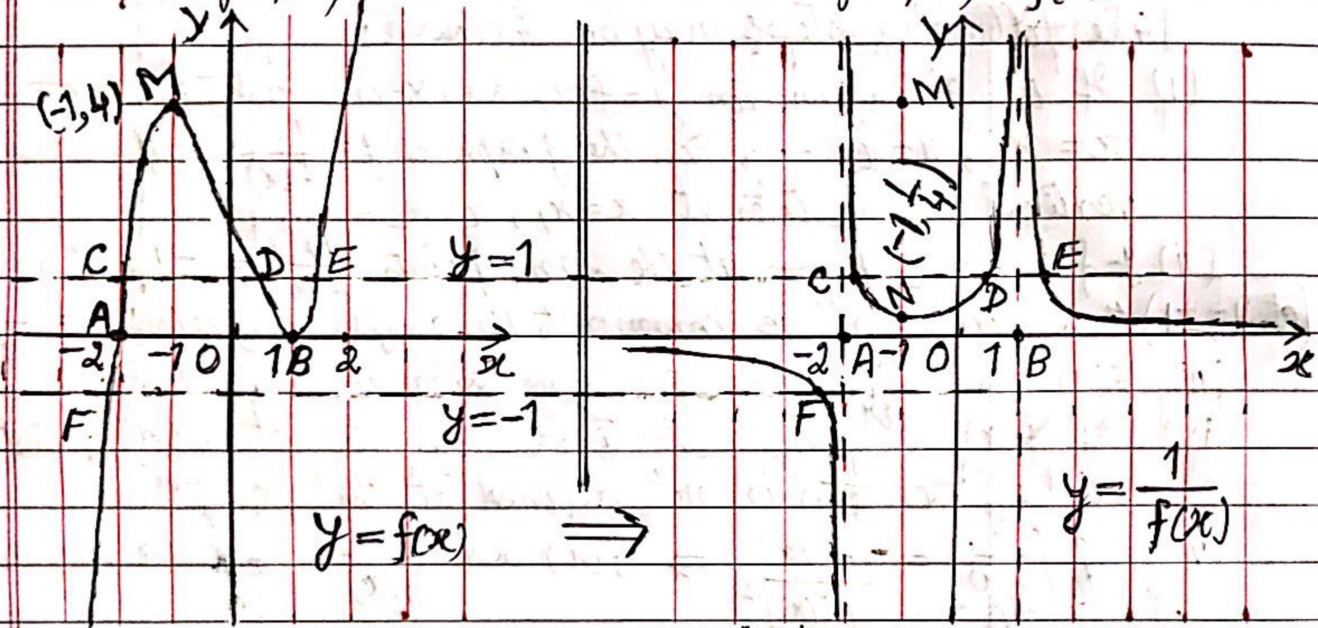
Case III

Example 19:

Given the graph of $y=f(x)$; Sketch the graph of $\frac{1}{f(x)}$



(i) $y=f(x)$ has roots $x=-2$ and $x=-1 \Rightarrow y=\frac{1}{f(x)}$ has asymptotes at $x=-2$ and $x=1$

(ii) Horizonal line $y=+1$ intersects $y=f(x)$ at point C, D, E $\Rightarrow$ The graph of $y=\frac{1}{f(x)}$ will pass through C, D, E.
also $y=-1$ intersects $y=f(x)$ at F $\Rightarrow$ $y=\frac{1}{f(x)}$ will also pass through F.

(iii) The graph to $y=f(x)$ corresponds to + values on $y=\frac{1}{f(x)}$

(iv) $y=f(x)$ has a Max at at $M(-1,4) \Rightarrow y=\frac{1}{f(x)}$ will have a minimum at $N(-1,\frac{1}{4})$

(v) $f(x)$ is inc. $x\le 2 \Rightarrow \frac{1}{f(x)}$ is decreasing for $x<-2$
$f(x)$ is also increasing, $x\ge 1 \Rightarrow \frac{1}{f(x)}$ is dec. for $x>1$

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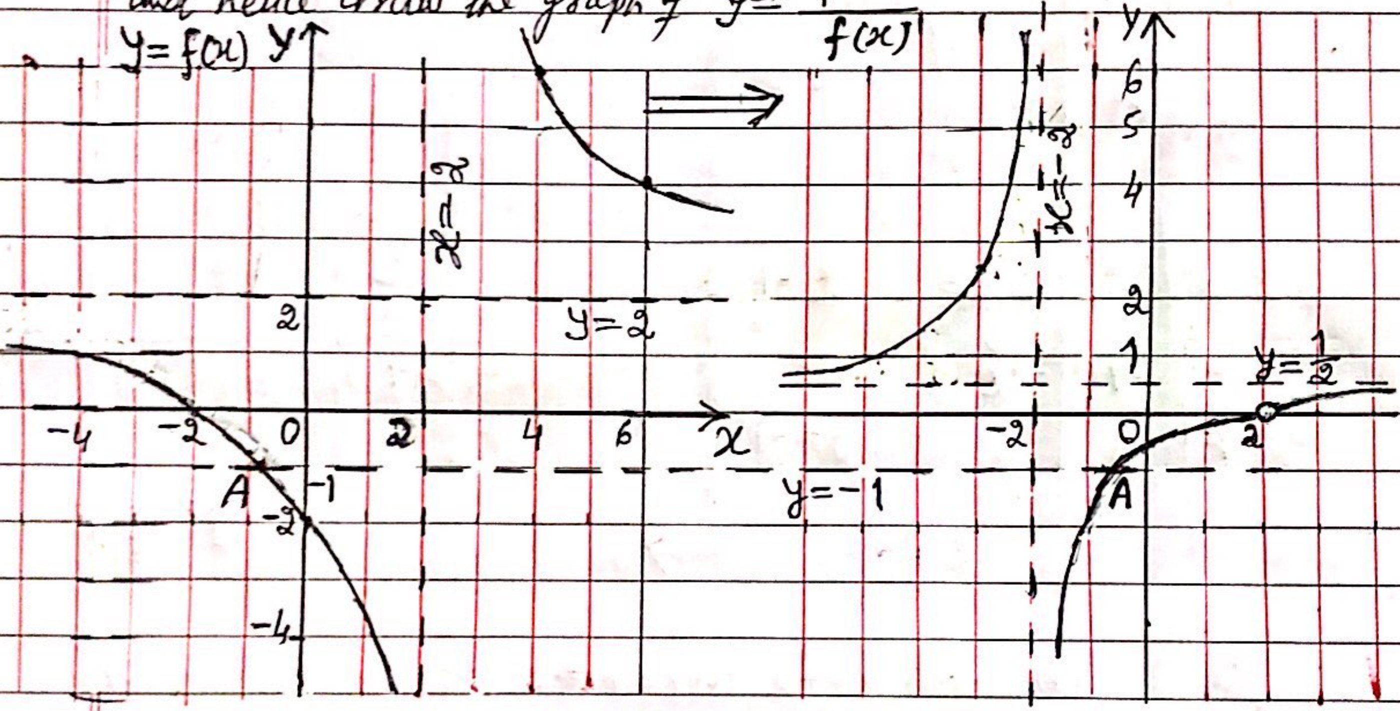
Case III

Example 20: Given $y = f(x) = \frac{2x+4}{x-2}$ draw the graph of $y=f(x)$, and hence draw the graph of $y = \frac{1}{f(x)}$



$y = \frac{2x+4}{x-2}$, $y = 2 + \frac{8}{x-2}$, { has an asymptote $y = 2$ (horizontal), and another asymptote $x = 2$, Intercepts $(-2, 0)$, $(0, -2)$ }

or $y = \frac{1}{f(x)}$, { $y = \frac{1}{2}$ is an asymptote and $(2, 0)$ is point of discontinuity; $x = -2$ is an asymptote. Line $y = -1$ intersect both the graphs at the same point $A(-\frac{2}{3}, -1)$ }

No stationary point on $y = f(x)$ or $y = \frac{1}{f(x)}$

for $x < 2$ { $y = f(x)$ is decreasing; $y = \frac{1}{f(x)}$ is increasing }

for $x > 2$ { $y = f(x)$ is decreasing; $y = \frac{1}{f(x)}$ is increasing }

Case III

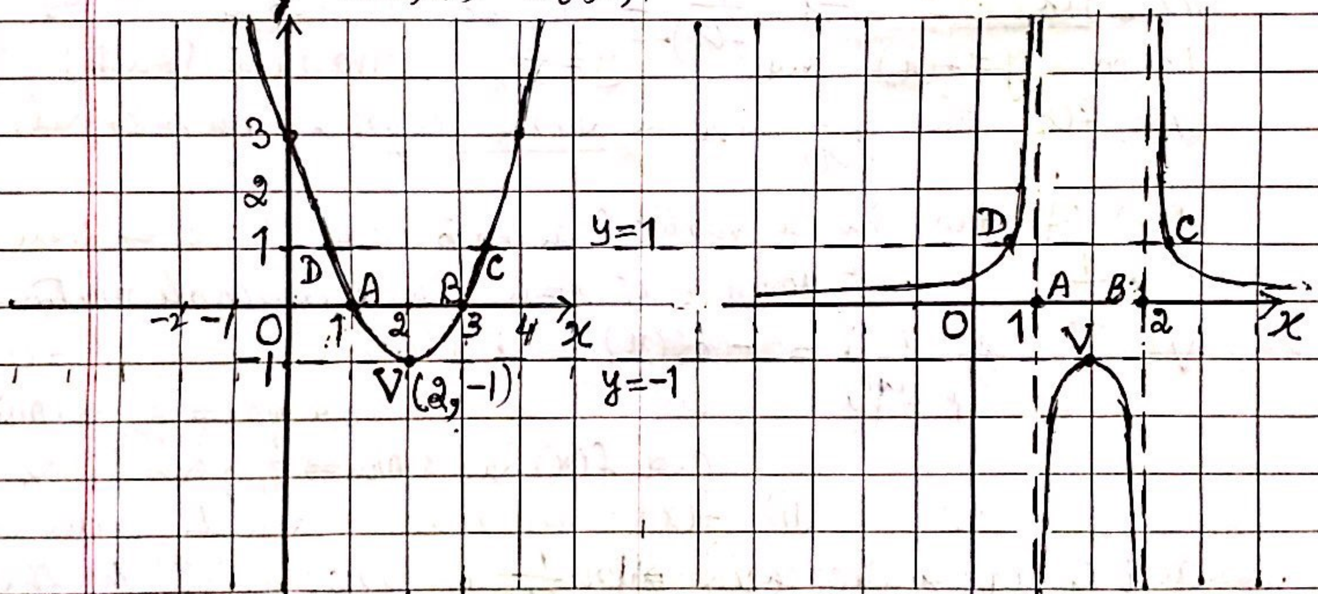
Example 20:

Given a function $f(x) = x^2 - 4x + 3$

To draw the graph of $\frac{1}{f(x)}$

Given, $y = f(x) = x^2 - 4x + 3$
$= (x-3)(x-1)$
has root of $f(x)$ at $x=3$ and $x=1$
$A(3,0)$, $B(1,0)$

$\Rightarrow \frac{1}{f(x)}$ has vertical asymptotes at $x=1$ and $x=3$.



The point on $f(x)$ at $y=1$ are C and D will also be the there on the graph of $\frac{1}{f(x)}$; and similarly the point V at $y=-1$ is common to both $y=f(x)$ & $y=\frac{1}{f(x)}$

The function $f(x)$ has a max at $V(2,-1)$, and $y=\frac{1}{f(x)}$ has a minimum at V.

for $x \leq 1$ $f(x)$ is increasing $\Rightarrow \frac{1}{f(x)}$ is decreasing.
for $x \geq 3$ $f(x)$ is inc. $\Rightarrow \frac{1}{f(x)}$ is decreasing.
and between $1 \leq x \leq 3$ the graph is reversed.

FP1

Case III

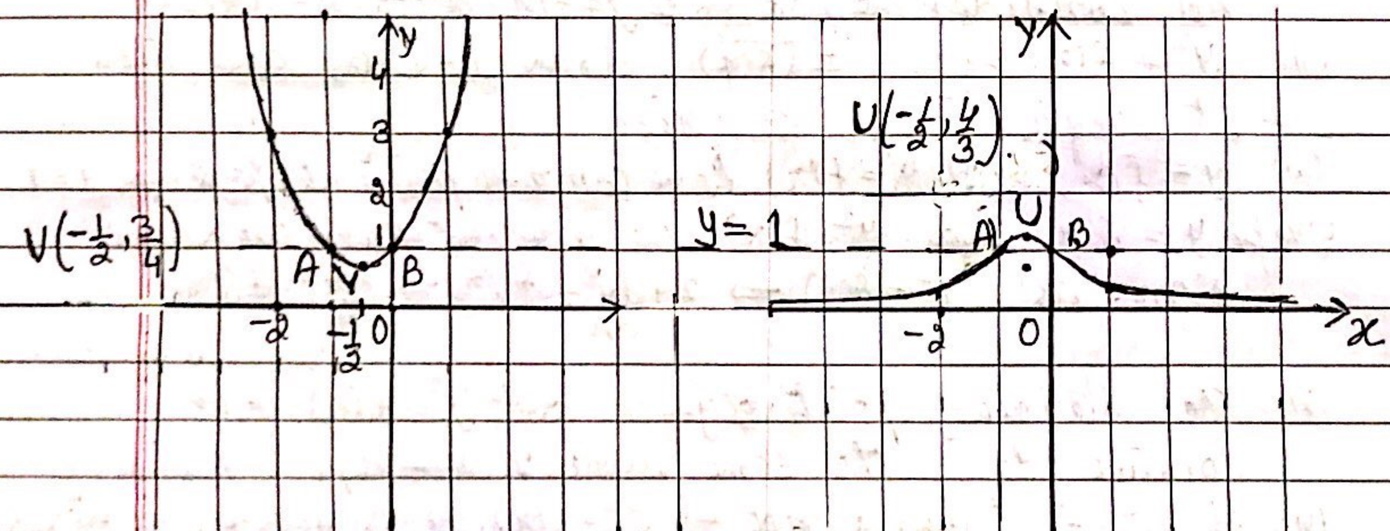
When $y=f(x)$ does not has x-Int. (or roots)

Example 21. Given $y=f(x)=x^2+x+1$; Draw the graph of $y=\frac{1}{f(x)}$

$y=f(x)=x^2+x+1$

$b^2-4ac=-3<0$

$f(x)$ has no real roots. $\Rightarrow$ $y=\frac{1}{f(x)}$ has no asymptotes.



The points A and B on the line $y=1$, will be there on both the graphs, $y=f(x)$ and $y=\frac{1}{f(x)}$ common points.

$V(-\frac{1}{2},\frac{3}{4})$ is Min. of $y=f(x)$

$\therefore$ $U(-\frac{1}{2},\frac{4}{3})$ is Max of $y=\frac{1}{f(x)}$

$x\le-\frac{1}{2}$ $\begin{cases} f(x) \text{ is decreasing} \\ \Rightarrow \frac{1}{f(x)} \text{ is increasing} \end{cases}$

$x\ge-\frac{1}{2}$ $\begin{cases} f(x) \text{ is increasing} \\ \frac{1}{f(x)} \text{ is decreasing.} \end{cases}$

To find the range of $y=\frac{1}{f(x)}$

or $y=\frac{1}{x^2+x+1}$

$\Rightarrow y x^2+y.x+y-1=0$

for real roots $b^2-4ac\ge0$

$y^2-4y(y-1)\ge0$

$-3y^2+4y\ge0$

$y(4-3y)\ge0$ $\begin{cases} y=0 \\ y=4/3 \end{cases}$

$\Rightarrow 0\le y\le 4/3$

FP$_1$ Case(iv)

Given the graph of $y=f(x)$; To draw the graph of $y^2=f(x)$.

The following points should be noted:

(i) Given the graph of $y=f(x)$
Now when we draw the graph of $y^2=f(x)\geq 0$, we should not consider the portion of $y=f(x)$ below x-axis.

(ii) $y^2=f(x) \Rightarrow y=\pm\sqrt{f(x)}$, draw $y=\sqrt{f(x)}$ and then its reflection in x-axis.

(iii) $y=f(x)$ and $y^2=f(x)$ have common point at $y=0$ and $y=1$.

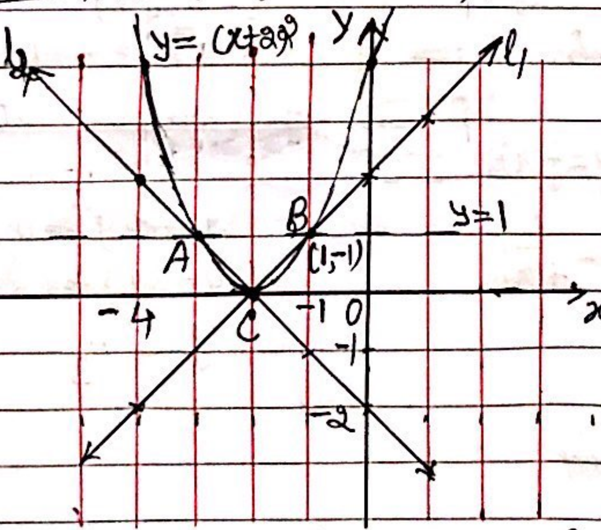
(iv) for $y=f(x)$ and $y^2=f(x)$ the gradient have the same sign. as $y^2=f(x) \Rightarrow 2y\frac{dy}{dx}=f'(x) \Rightarrow \frac{dy}{dx}=\frac{f'(x)}{2y}$

(v) The stationary points of $y=f(x)$ and $y^2=f(x)$ are located for the same value of $x=x_1$, $(x_1, \sqrt{y_1})$

(vi) If $y=f(x)$ has a root $\Rightarrow$ $y^2=f(x)$ graph passes vertically through it.

VII any vertical asymptote remains the same / and the horizontal asymptote $y=k$ changes to $y=\sqrt{k}$

Example 22: Given $f(x)=(x+2)^2$, sketch $y=f(x)$ and hence $y^2=f(x)$.



Graph of $y^2=f(x)$ is $l_1 \cup l_2$

$y=(x+2)^2$; $y^2=f(x)=(x+2)^2$
$\Rightarrow y=(x+2)$ Draw the graph. Now $y=1$
say line l_1 $\Rightarrow x=-1$
Then take the reflection of l_1 in X-axis

Both the curves have the common point at $y=1$, A & B
; also at $y=0$ point C.

FP1

Case (iv)

Date P-17

Example 23: Given $y=f(x)=\dfrac{x+3}{x+1}$, $x\neq -1$; Draw the graph of $y^2=f(x)$.

Solution: Given $y=\dfrac{x+3}{x+1}$

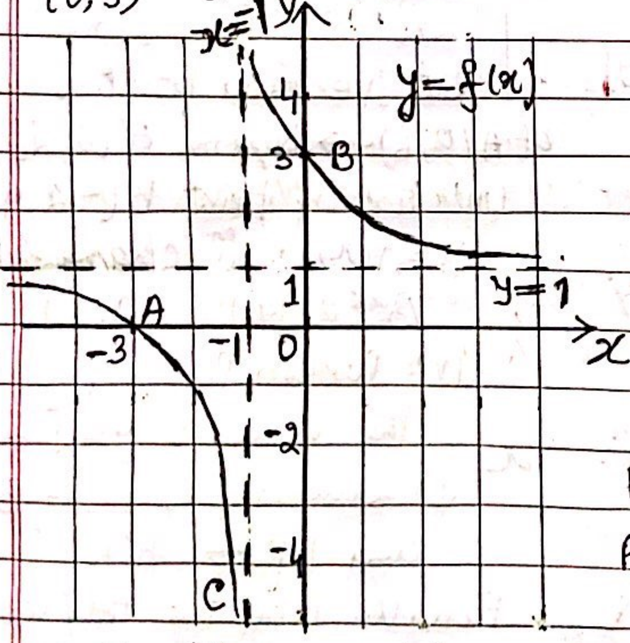
Draw the graph.

or $y=1+\dfrac{2}{(x+1)}$

has asymptotes $y=1$ and $x=-1$

Intersects X-axis at $x=-3$ $(-3,0)$

Intersects y-axis at $x=0$, $(0,3)$



No stationary point.

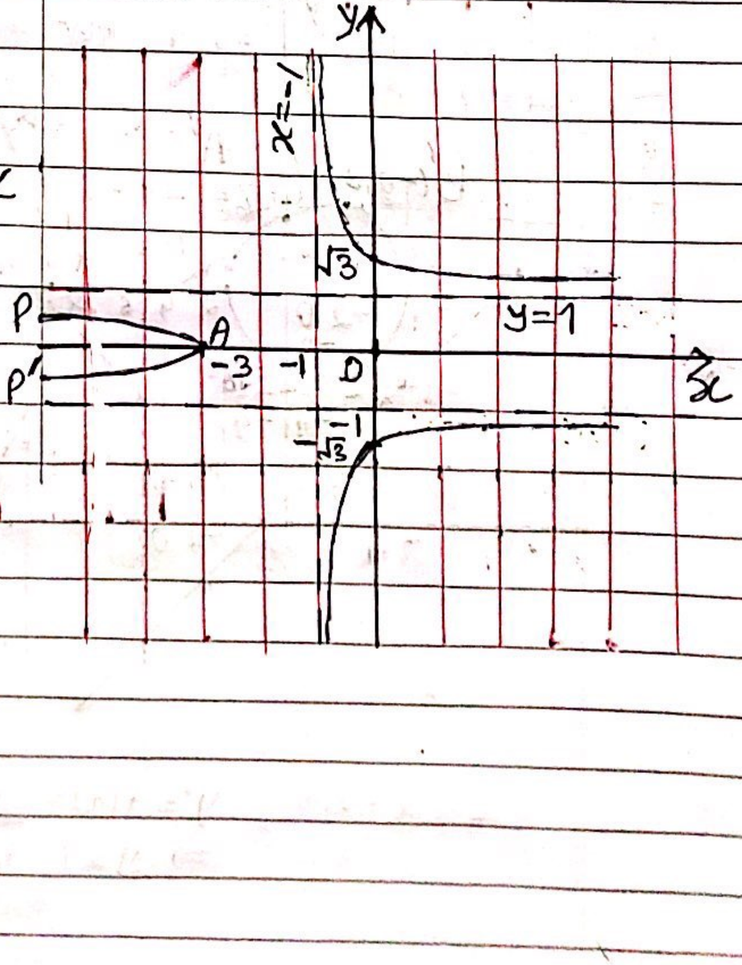
Now $y^2=f(x)=\dfrac{x+3}{x+1}$

$$y=\pm\sqrt{\frac{x+3}{x+1}}$$

(i) Horizontal intercept of $y=f(x)$ $(-3,0)$ will also be there for $y^2=f(x)$ but vertical intercept $(0,3)$ of $y=f(x)$ will now B $(0,\sqrt{3})$ for $y^2=f(x)$.

(ii) the line $y=1$ will yet be asymp.

(iii) The portion $\overset{\frown}{AC}$ of $y=f(x)$ will not be there.

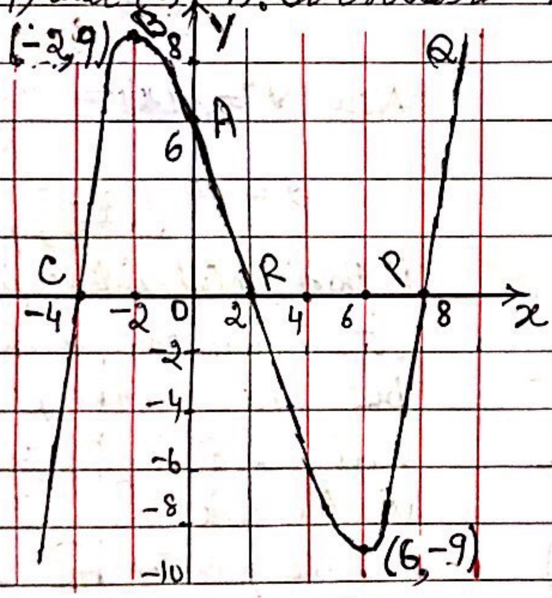


FP1

Case (iv)

Example 24: Given is a function $y = f(x)$, The curve has turing points $(-2, 9)$ and $(6, -9)$. It crosses the x-axis at $(-4,0)$, $(2,0)$, $(8,0)$ and y-axis at $(0,6)$.

$y = f(x)$



To Draw the graph of $y^2 = f(x)$

(I) Do not consider the graph of $y = f(x)$ below x-axis, as $y^2 = f(x) \geq 0$

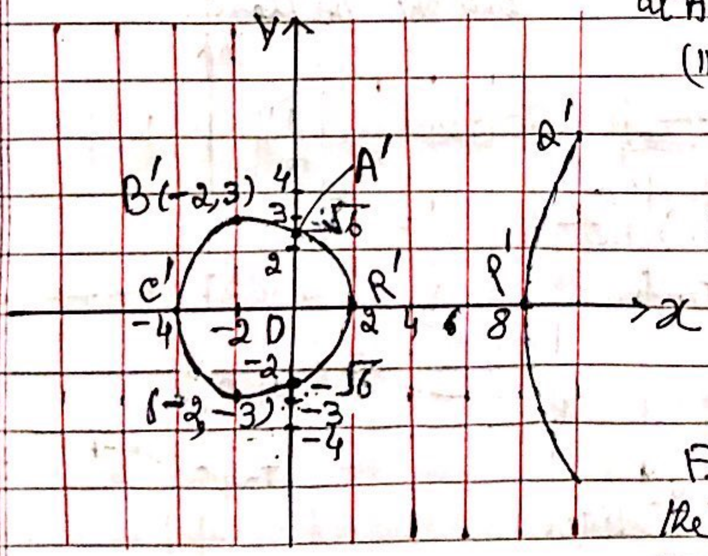
(II) The horizontal points of intersection $(-4,0)$, $(2,0)$ and $(8,0)$ will remain same But vertical point of intersect at $A(0,6) \to$ become $A'(0, \sqrt{6})$

(III) turning point $B(-2,9)$ on $y = f(x)$ will change to $B'(2, \sqrt{9})$ or $(2,3)$.

(IV) Portion $PQ \to P'Q'$ but the values of $y = \sqrt{f(x)}$

(V) Similarly of $BC \to B'C'$ and $AR \to A'R'$

Finally take the reflection the graph drawn in X-axis.



FP1

To Solve the inequalities by sketching the graphs of $y = f(x)$

Example 25: solving the inequalities, using the graphs.

(i) $\frac{x+2}{(x+1)(x-3)} \leqslant 0$

(ii) $\frac{x^2+4x+3}{x^2-4x+3} \geqslant 1$

(iii) $\frac{x^2+2}{x} \geqslant 3$

(iv) $1 < \frac{2x^2+x-1}{x-1} < 9$

Solution (i) $\frac{x+2}{(x+1)(x-3)} \leqslant 0$

Example 3. (Page 1), from graph:

$\underline{x \leqslant -2 \quad \text{or} \quad -1 < x < 3}$ ✓

(ii) $\frac{x^2+4x+3}{x^2-4x+3} \geqslant 1$, see Graph → Example 5. (Page 2)

$\underline{0 \leqslant x < 1 \quad \text{or} \quad x > 3}$ ✓

(iii) $\frac{x^2+2}{x} \geqslant 3$, see Graph → Example 6. (Page 3)

$\underline{x > 0}$ ✓

(iv) $1 < \frac{2x^2+x-1}{x-1} < 9$, see Graph → Example 8 (Page 4)

<u>No Solution</u>

Example 26. Sketch the graph of the following and solve.

(i) $\left|\frac{1}{x+1}\right| \leq 1$

(ii) $-1 < \frac{x^2-5x+6}{x-1} \leq 0$

(iii) $\frac{(|x|^2-5|x|+6)}{|x|-1} \leq -6$

Solution (i) $\left|\frac{1}{x+1}\right| \leq 1$, for Graph → Example 15 (Page 9)

$x \leq -2$ or $x \geq 0$

(ii) $-1 < \frac{x^2-5x+6}{(x-1)} \leq 0$, for Graph → Example 7 (Page 3)

$\underline{2 \leq x \leq 3}$ ✓

(iii) $\frac{|x|^2-5|x|+6}{|x|-1} \leq -6$; for graph, Example 17 (Page 10)

$\underline{-1 < x < 1}$ ✓

← X —— X —— X →