FP-1

Further Pure Math-1

Roots of Polynomials. Exercise.

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Roots of polynomial equations

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1. The cubic equation $z^3 - z^2 - z - 5 = 0$ has roots α, β and γ.
(a) Show that the value of $\alpha^3 + \beta^3 + \gamma^3$ is 19. ---[4]
(b) Find the value of $\alpha^4 + \beta^4 + \gamma^4$ --[2]
(c) Find a cubic equation with roots $\alpha+1$, $\beta+1$ and $\gamma+1$, give your answer in the form $px^3 + qx^2 + rs + s = 0$, where p, q, r and s are constants to be determined. --[3]

[SP-20/01/Q4]

2. The equation, $x^3 - x + 1 = 0$ has roots α, β and γ.
(i) Use the relation $x = y^{1/3}$ to show that the equation,
$$y^3 + 3y^2 + 2y + 1 = 0$$
has roots α^3, β^3, γ^3. Hence write down the value of, $\alpha^3 + \beta^3 + \gamma^3$. ---[3]
Let $S_n = \alpha^n + \beta^n + \gamma^n$
(ii) Find the value of S_{-3}. ---[2]
(iii) Show that $S_6 = 5$ and find the value of S_9. --[4]

[S-19/11/Q6]

3. A cubic equation $x^3 + bx^2 + cx + d = 0$ has real roots α, β and γ such that, $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{-5}{12}$,
$$\alpha\beta\gamma = -12$$
$$\alpha^3 + \beta^3 + \gamma^3 = 90$$
(i) Find the values of c and d. ---[3]
(ii) Express $\alpha^2 + \beta^2 + \gamma^2$ in terms of b. ---[2]
(iii) Show that $b^3 - 15b + 126 = 0$ --[4]
(iv) Given that $3 + i\sqrt{(12)}$ is a root of, $y^3 - 15y + 126 = 0$, deduce the value of b. --[2]

[S-19/13/Q9]

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4. The equation $x^3+2x^2+x+7=0$ has roots α, β, γ.
(i) Use the relation $x^2=-7y$ to show that the equation
$$49y^3+14y^2-27y+7=0$$
has roots $\frac{\alpha}{\beta\gamma}, \frac{\beta}{\gamma\alpha}, \frac{\gamma}{\alpha\beta}$ --- [4]

(ii) Show that $\frac{\alpha^2}{\beta^2\gamma^2}+\frac{\beta^2}{\gamma^2\alpha^2}+\frac{\gamma^2}{\alpha^2\beta^2}=\frac{58}{49}$ --- [3]

(iii) Find the exact value of $\frac{\alpha^3}{\beta^3\gamma^3}+\frac{\beta^3}{\gamma^3\alpha^3}+\frac{\gamma^3}{\alpha^3\beta^3}$ -- [2]

W-19/11/Q7

5. It is given that the equation $x^3-21x^2+kx-216=0$, where k is a constant, has real roots a, ar and ar^{-1},
(i) Find the numerical values of the roots. --- [6]
(ii) Deduce the value of k. S-18/11/Q4 -- [2]

6. The equation $9x^3-9x^2+x-2=0$
has roots α, β, γ.
(i) Use substitution $y=3x-1$ to show that $3\alpha-1$, $3\beta-1$, $3\gamma-1$ are the roots of the equation,
$$y^3-2y-7=0$$ --- [2]
The sum $(3\alpha-1)^n+(3\beta-1)^n+(3\gamma-1)^n$ is denoted by S_n
(ii) Find the value of S_3. --- [2]
(iii) Find the value of S_{-2} -- [4]

S-18/13/Q6

7. The roots of the equation $x^3+px^2+qx+r=0$ are $\alpha, 2\alpha, 4\alpha$, where p, q, r are α are non-zero real constants.
(i) Show that: $2p\alpha+q=0$ --- [4]
(ii) Show that: $p^3r-q^3=0$ -- [2]

W-18/11/Q2

8. The cubic equation,
$2x^3-3x^2+4x-10=0$ has roots α, β, and γ.
(i) Find the value of $(\alpha+1)(\beta+1)(\gamma+1)$ -- [4]
(ii) Find the value of $(\beta+\gamma)(\gamma+\alpha)(\alpha+\beta)$ W-17/11/Q4 -- [4]

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9. The roots of the cubic equation,
$x^3-5x^2+13x-4=0$ are α, β, γ.
(i) Find the value of $\alpha^2+\beta^2+\gamma^2$ ---[3]
(ii) Find the value of $\alpha^3+\beta^3+\gamma^3$ ---[2]
[W-18|12|Q1]

10. By finding a cubic equation whose roots are α, β, and γ, solve the set of simultaneous equations.
$$\alpha+\beta+\gamma=-1$$
$$\alpha^2+\beta^2+\gamma^2=29$$
$$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=-1$$
[S-17|11|Q7] ---[8]

11. The roots of the cubic equation,
$x^3+2x^2-3=0$ are α, β and γ.
(i) using the substitution $y=\frac{1}{x^2}$, find the cubic equation with roots $\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}$. ---[3]
(ii) Hence find,
the value of $\frac{1}{\alpha^2}, \frac{1}{\beta^2}, \frac{1}{\gamma^2}$ ---[1]
(iii) Find also the value of, $\frac{1}{\alpha^2\beta^2}+\frac{1}{\beta^2\gamma^2}+\frac{1}{\alpha^2}$ ---[1]
[S-17|13|Q1]

12. The roots of cubic equation:
$2x^3+x^2-7=0$ are α, β and γ. Using the substitution $y=1+\frac{1}{x}$, or otherwise, find the cubic equation whose roots are $1+\frac{1}{\alpha}$, $1+\frac{1}{\beta}$ and $1+\frac{1}{\gamma}$, giving your answer in the form $ay^3+by^2+cy+d=0$, where a, b, c and d are the constants to be determined ---[4]
[S-16|11|Q1]

13. The cubic equation $z^3-z^2-z-5=0$ has roots α, β and γ. Show that the value of $\alpha^3+\beta^3+\gamma^3$ is 19. --[4]
Find the value of $\alpha^4+\beta^4+\gamma^4$ --[2]
Show that the cubic equation with roots $\frac{\alpha-1}{\alpha}$, $\frac{\beta-1}{\beta}$ and $\frac{\gamma-1}{\gamma}$ may be found using the substitution →
(Continued →)

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13. $z = \frac{1}{1-x}$, and find this equation, giving your answer in the form $px^3+qx^2+rx+s=0$, where p, q, r and s are constants to be determined. --- [4] [S-16/13/Q8]

14. Find the cubic equation with root α, β and γ such that,

$$\alpha+\beta+\gamma = 3$$
$$\alpha^2+\beta^2+\gamma^2 = 1$$
$$\alpha^3+\beta^3+\gamma^3 = -30$$

giving your answer in the form $x^3+px^2+qx+r=0$, where p, q and r are integers to be found [W-16/11/Q2] --[6]

15. The roots of cubic equation $x^3-7x^2+2x-3=0$ are α, β and γ. Find the value of,

(i) $\frac{1}{(\alpha\beta)(\beta\gamma)(\gamma\alpha)}$ (ii) $\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha}$

(iii) $\frac{1}{\alpha^2\beta\gamma}+\frac{1}{\alpha\beta^2\gamma}+\frac{1}{\alpha\beta\gamma^2}$ --[6]

Deduce a cubic equation, with integer coefficient, having roots $\frac{1}{\alpha\beta}$, $\frac{1}{\beta\alpha}$ and $\frac{1}{\gamma\alpha}$ [S-15/11/Q4] [2]

16. The quartic equation $x^4-px^2+qx-r=0$, where p, q and r are real constants, has two pairs of equal roots, show that $p^2+4r=0$ and state the value of q. [S-15/13/Q7] --[6]

17. The cubic equation x^3+px^2+qx+r, where p, q and r are integers, has roots α, β, and γ, such that,

$\alpha+\beta+\gamma = 15$; $\alpha^2+\beta^2+\gamma^2 = 83$

Write down the value of p and the value of q. --[3]

Given that α, β and γ are all real and that $\alpha\beta+\gamma\alpha = 36$, find α and hence find the value of r. [W-15/11/Q5] --[5]

Answers

1.
Let S_n denote $\alpha^n+\beta^n+\gamma^n$

$S_2 = S_1^2 - 2\Sigma\alpha\beta = 1^2 - 2(-1) = 3$

(a) $\therefore S_3 = S_1 \times S_2 - S_1 \cdot \Sigma\alpha\beta + 3\alpha\beta\gamma$
$= 1\times 3 - 1\times(-1) + 3\times 5$
$= 19$ ✓

or

$\Sigma\alpha^3 = (\Sigma\alpha)^3 - 3(\Sigma\alpha)(\Sigma\alpha\beta) + 3\alpha\beta\gamma$
$= 1^3 - 3\times 1\times(-1) + 3\times 5$
$= 19$ ✓

or

$\Sigma\alpha^3 = (\Sigma\alpha)[\Sigma\alpha^2 - \Sigma\alpha\beta] + 3\alpha\beta\gamma$
$= 1[3-(-1)] + 3\times 5$
$= 19$ ✓

(b) $\alpha^4+\beta^4+\gamma^4 = (\Sigma\alpha^2)^2 - 2[(\Sigma\alpha\beta)^2 - 2(\Sigma\alpha)\cdot\alpha\beta\gamma]$
$= 3^2 - 2[(-1)^2 - 2\times 1\times 5]$
$= 9+18 = 27$ ✓

or

$S_4 = S_3 + S_2 + 5S_1$
$= 19+3+5\times 1 = 27$ ✓

(c) Put $x = z+1 \Rightarrow z = x-1$

Req Eqn: $(x-1)^3 - (x-1)^2 - (x-1) - 5 = 0$

$\Rightarrow x^3 - 4x^2 + 4x - 6 = 0$

2(i) Given $x^3 - x + 1 = 0$ —①

Put $x = y^{1/3}$ in ①

$y - y^{1/3} + 1 = 0 \Rightarrow y^{1/3} = y+1$

$\Rightarrow y = (y+1)^3$

$\Rightarrow y = y^3 + 3y^2 + 3y + 1$

$\rightarrow y^3 + 3y^2 + 2y + 1 = 0$ —②

② has roots $\alpha^3, \beta^3, \gamma^3$

$\Rightarrow \alpha^3+\beta^3+\gamma^3 = -3$ ✓

(ii) $S_{-3} = \frac{1}{\alpha^3} + \frac{1}{\beta^3} + \frac{1}{\gamma^3}$
$= \frac{\Sigma\alpha^3\beta^3}{\alpha^3\beta^3\gamma^3} = \frac{2}{-1} = -2$

2(iii) $S_6 = \alpha^6+\beta^6+\gamma^6$
$= (\alpha^3)^2 + (\beta^3)^2 + (\gamma^3)^2$
$= (\alpha^3+\beta^3+\gamma^3)^2 - 2\Sigma\alpha^3\beta^3$
$= (-3)^2 - 2\times 2 = 5$ ✓

3(i) $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma} = \frac{\Sigma\alpha\beta}{\alpha\beta\gamma} = \frac{c}{-d} = \frac{-5}{12}$

and $\alpha\beta\gamma = -12 = -d$
$\Rightarrow d = 12$ ✓

$\frac{c}{-12} = \frac{-5}{12} \Rightarrow c = 5$ ✓

(ii) $\alpha^2+\beta^2+\gamma^2 = (\Sigma\alpha)^2 - 2\Sigma(\alpha\beta)$
$= (-b)^2 - 2\times c$
$= b^2 - 2\times 5$
$= b^2 - 10$ ✓

(iii) Given $x^3 + bx^2 + cx + d = 0$

$\Rightarrow \Sigma\alpha^3 + b\Sigma\alpha^2 + c(\Sigma\alpha) + 3d = 0$

$\Rightarrow 90 + b(b^2-10) + 5(-b) + 3\times 12 = 0$

$\Rightarrow b^3 - 15b + 126 = 0$ ✓

(iv) Real root is $b = -6$ ✓

4(i) $x^3 + 2x^2 + x + 7 = 0$ —①

$\alpha\beta\gamma = -7$ —④

Put $x^2 = -7y \Rightarrow x = \sqrt{-7y}$ —②

and $y = \frac{x^2}{-7}$ —③

fr ① & ②

$\sqrt{-7y}(-7y) + 2(-7y) + \sqrt{-7y} + 7 = 0$

$\Rightarrow \sqrt{-7y}(-7y+1) = 14y - 7$

Sq, $-7y(-7y+1)^2 = (14y-7)^2$

$\Rightarrow 49y^3 + 14y^2 - 27y + 7 = 0$ —⑤

fr ③ $y = \frac{x^2}{-7} = \frac{x^2}{\alpha\beta\gamma}$ fr ④.

So roots are $\frac{\alpha^2}{\alpha\beta\gamma}, \frac{\beta^2}{\alpha\beta\gamma}, \frac{\gamma^2}{\alpha\beta\gamma}$

or $\frac{\alpha}{\beta\gamma}, \frac{\beta}{\alpha\gamma}, \frac{\gamma}{\alpha\beta}$ ✓

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Answers

4(ii) $\frac{\alpha}{\beta\gamma}, \frac{\beta}{\gamma\alpha}, \frac{\gamma}{\alpha\beta}$ are roots of (5)

$\therefore \Sigma\frac{\alpha}{\beta\gamma} = \frac{-14}{49} = -\frac{2}{7}$ — (6)

$\Sigma\frac{\alpha}{\beta\gamma}\times\frac{\beta}{\gamma\alpha} = -\frac{27}{49}$

$\Rightarrow \Sigma\frac{1}{\gamma^2} = -\frac{27}{49}$ — (7)

Now

$\frac{\alpha^2}{\beta^2\gamma^2}+\frac{\beta^2}{\alpha^2\gamma^2}+\frac{\gamma^2}{\alpha^2\beta^2}$

$= \left(\Sigma\frac{\alpha}{\beta\gamma}\right)^2 - 2\left(\Sigma\frac{1}{\gamma^2}\right)$

$= \left(-\frac{2}{7}\right)^2 - 2\left(-\frac{27}{49}\right) = \frac{58}{49}$ ✓

from (6) & (7)

(iii) from (5)

$49\left(\Sigma\frac{\alpha^3}{\beta^3\gamma^3}\right) = -14\left(\Sigma\frac{\alpha^2}{\beta^2\gamma^2}\right) + 27\left(\Sigma\frac{\alpha}{\beta\gamma}\right) - 21$

$= -14\left(\frac{58}{49}\right) + 27\left(-\frac{2}{7}\right) - 21$

$\therefore \left(\Sigma\frac{\alpha^3}{\beta^3\gamma^3}\right) = -\frac{317}{343}$ ✓

5(i) $\alpha\beta\gamma = a\times ar\times\frac{a}{r} = 216 \Rightarrow a = 6$ ✓

$\Sigma\alpha = a + ar + \frac{a}{r} = 21$

$6\left(1 + r + \frac{1}{r}\right) = 21$

$2r^2 - 5r + 2 = 0 \Rightarrow r = 2, \frac{1}{2}$

Roots are $6, 12, 3$ ✓

(ii) $k = \alpha\beta + \beta\gamma + \gamma\alpha$

$= 6\times12 + 12\times3 + 3\times6$

$= 126$ ✓

6(i) Substitute $x = \frac{y+1}{3}$

we get $y^3 - 2y - 7 = 0$ — (1)

(ii) from (1) $y^3 = 2y + 7$

$\Rightarrow S_3 = 2S_1 + 3 = 2\times0 + 7\times3$

$= 21$ ✓

6(iii) $S_{-1} = \frac{1}{3\alpha-1} + \frac{1}{3\beta-1} + \frac{1}{3\gamma-1}$

$= \frac{\Sigma(3\alpha-1)(3\beta-1)}{(3\alpha-1)(3\beta-1)(3\gamma-1)}$

$= -2/7$ — (2)

from (1)

$7S_{-2} = S_1 - 2S_{-1}$

$S_{-2} = \frac{1}{7}\left(0 - 2\left(-\frac{2}{7}\right)\right)$

$= \frac{4}{49}$ ✓

7 $\alpha + 2\alpha + 4\alpha = -p \Rightarrow -p = 7\alpha$ — (1)

(i) $\alpha\times2\alpha + 2\alpha\times4\alpha + 4\alpha\times\alpha = q = 14\alpha^2$ — (2)

(2) ÷ (1) $\frac{14\alpha^2}{7\alpha} = -\frac{q}{p}$

$\Rightarrow 2p\alpha + q = 0$ ✓ — (3)

(ii) $\alpha\times2\alpha\times4\alpha = r$

or $8\alpha^3 = -r$ | from (3)

$(2\alpha)^3 = -r$ | $2\alpha = -\frac{q}{p}$

$\left(-\frac{q}{p}\right)^3 = -r$

$\Rightarrow p^3r - q^3 = 0$ ✓

8(i) $\alpha+\beta+\gamma = 3/2$, $\alpha\beta\gamma = \frac{10}{2} = 5$

$\alpha\beta+\beta\gamma+\gamma\alpha = 2$ — (1)

Now $(\alpha+1)(\beta+1)(\gamma+1)$

$= \alpha\beta\gamma + (\alpha\beta+\beta\gamma+\gamma\alpha) + 1 + (\alpha+\beta+\gamma)$

$= 5 + 2 + 3/2 + 1 = 9\frac{1}{2}$ ✓

(ii) $(\beta+\gamma)(\gamma+\alpha)(\alpha+\beta)$

from (1) $= \left(\frac{3}{2}-\alpha\right)\left(\frac{3}{2}-\beta\right)\left(3/2-\gamma\right)$

$= \frac{27}{8} - \frac{9}{4}(\alpha+\beta+\gamma) - \alpha\beta\gamma + 3/2(\alpha\beta+\beta\gamma+\gamma\alpha)$

$= \frac{27}{8} - \frac{9}{4}\times\frac{3}{2} + 3/2\times2 - 5$

$= -2$ ✓

Roots of Polynomial Equations

Answers

Given $x^3-5x^2+13x-4=0$ — (1)

9. (i) $\alpha+\beta+\gamma=5$, $\alpha\beta+\beta\gamma+\gamma\alpha=13$

$\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$

$=5^2-2\times13=-1$ ✓

(ii) from (1)

$\alpha^3+\beta^3+\gamma^3=5(\alpha^2+\beta^2+\gamma^2)-13(\alpha+\beta+\gamma)+4\times3$

$=5(-1)-13\times5+12=-58$ ✓

Alt: $\Sigma\alpha^3=(\Sigma\alpha)(\Sigma\alpha^2-\Sigma\alpha\beta)+3\alpha\beta\gamma$

$=5[-1-13]+3\times4=-58$ ✓

10. $2\Sigma\alpha\beta=(\Sigma\alpha)^2-\Sigma\alpha^2$

$=(-1)^2-29$

$\Rightarrow \Sigma\alpha\beta=-14$ — (1)

$\dfrac{\Sigma\alpha\beta}{\alpha\beta\gamma}=-1\Rightarrow\alpha\beta\gamma=14$

$\therefore$ Eqn. $x^3-(\Sigma\alpha)x^2+(\Sigma\alpha\beta)x-\alpha\beta\gamma=0$

or $x^3+x^2-14x-14=0$

$\Rightarrow (x+1)(x^2-14)=0$

$\Rightarrow x=-1$ or $x=\pm\sqrt{14}$ ✓

11. $y=\frac{1}{x^2}\Rightarrow x=\frac{1}{\sqrt{y}}$

(i) $\dfrac{1}{y\sqrt{y}}+\dfrac{2}{y}-3=0$

$\Rightarrow \dfrac{1}{y\sqrt{y}}=3-\dfrac{2}{y}$

(ii) $\Rightarrow 9y^3-12y^2+4y-1=0$

has roots $\frac{1}{\alpha^2}, \frac{1}{\beta^2}, \frac{1}{\gamma^2}$

$\therefore$ Sum of roots

$\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}=+\frac{12}{9}=\frac{4}{3}$ ✓

(iii) $\dfrac{1}{\alpha^2\beta^2}+\dfrac{1}{\beta^2\gamma^2}+\dfrac{1}{\gamma^2\alpha^2}$

$=\Sigma\frac{1}{\alpha^2}\times\frac{1}{\beta^2}=4/9$ ✓

Put $y=1+\frac{1}{x}\Rightarrow x=\frac{1}{y-1}$

12. $\Rightarrow \dfrac{2}{(y-1)^3}+\dfrac{1}{(y-1)^2}-7=0$

$\Rightarrow 2+(y-1)-7(y-1)^3=0$

$\Rightarrow 7y^3-21y^2+20y-8=0$ ✓

13. Let S_n denote $\alpha^n+\beta^n+\gamma^n$

$S_2=S_1^2-2\Sigma\alpha\beta=1^2-2(-1)=3$.

Given $z^3=z^2+z+5$

$\therefore S_3=S_2+S_1+5\times3$

or $\Sigma\alpha^3=3+1+15=19$ ✓

from (1) $S_4=S_3+S_2+5S_1$

$=19+3+5\times1=27$ ✓

Alt

$\alpha^4+\beta^4+\gamma^4=(\alpha^2+\beta^2+\gamma^2)^2-2\Sigma\alpha^2\beta^2$

$=3^2-2[(\Sigma\alpha\beta)^2-2(\Sigma\alpha)\alpha\beta\gamma]$

$=9-2[1-2\times1\times5]=27$ ✓

Now $\because$ $x=\frac{z-1}{z}\Rightarrow z=\frac{1}{1-x}$

$\therefore \dfrac{1}{(x-1)^3}-\dfrac{1}{(1-x)^2}-\dfrac{1}{(1-x)}-5=0$

$\Rightarrow 5x^3-16x^2+18x-6=0$ ✓

14. $2\Sigma\alpha\beta=(\Sigma\alpha)^2-\Sigma\alpha^2=3^2-1=8$

$\Rightarrow \Sigma\alpha\beta=4$ — (1) ✓

$\Sigma\alpha^3-3\alpha\beta\gamma=\Sigma\alpha(\Sigma\alpha^2-\Sigma\alpha\beta)$

$\Rightarrow -30-3\alpha\beta\gamma=3[1-4]$

$\Rightarrow \alpha\beta\gamma=-7$ ✓

Req. Cubic Eqn

$x^3-(\Sigma\alpha)x^2+(\Sigma\alpha\beta)x-\alpha\beta\gamma=0$

$x^3-3x^2+4x+7=0$ ✓

(PF1) Roots of Polynomial Equations

Answers

15. $\alpha+\beta+\gamma=7$

$\Sigma\alpha\beta=2, \quad \alpha\beta\gamma=3$

(i) $\dfrac{1}{(\alpha\beta)(\beta\gamma)(\gamma\alpha)}=\dfrac{1}{(\alpha\beta\gamma)^2}=\dfrac{1}{9}$ ✓

(ii) $\dfrac{1}{\alpha\beta}+\dfrac{1}{\beta\gamma}+\dfrac{1}{\gamma\alpha}=\dfrac{\Sigma\alpha}{\alpha\beta\gamma}=\dfrac{7}{3}$ ✓

(iii) $\dfrac{1}{\alpha^2\beta\gamma}+\dfrac{1}{\alpha\beta^2\gamma}+\dfrac{1}{\alpha\beta\gamma^2}$

$=\dfrac{1}{\alpha\beta\gamma}\left[\dfrac{\Sigma\alpha\beta}{\alpha\beta\gamma}\right]=\dfrac{2}{9}$ ✓

$x^3-\frac{7}{3}x^2+\frac{2}{9}x-\frac{1}{9}=0$

$\Rightarrow 9x^3-21x^2+2x-1=0$ ✓

16. (i) $2\alpha+2\beta=0 \Rightarrow \beta=-\alpha$

(ii) $\alpha^2+4\alpha\beta+\beta^2=-p$

(iii) $2\alpha^2\beta+2\alpha\beta^2=-q$

(iv) $\alpha^2\beta^2=-r$

fr (i) Put $\beta=-\alpha$ in (ii), (iii), (iv)

$\Rightarrow p=2\alpha^2$ and $\alpha^4=-r$

$\Rightarrow p^2+4r=0$ ✓

and $q=0$ ✓

17. $\alpha+\beta+\gamma=-p=15 \Rightarrow p=-15$ ✓

$\alpha^2+\beta^2+\gamma^2=(\Sigma\alpha)^2-2\Sigma\alpha\beta$

$83=(-15)^2-2q$

$\Rightarrow q=71$ ✓

$\alpha\beta+\alpha\gamma=36$ — ①

$\Rightarrow \beta+\gamma=\dfrac{36}{\alpha}$

$\Rightarrow 15-\alpha=\dfrac{36}{\alpha}$ $\left[\begin{array}{l}\alpha+\beta+\gamma=15\\ \therefore \beta+\gamma=15-\alpha\end{array}\right.$

$\alpha^2-15\alpha+36=0 \Rightarrow \alpha=3$ ✓

$\Sigma\alpha\beta=\alpha\beta+\beta\gamma+\gamma\alpha=q=71$ $\left[\begin{array}{l}\alpha\neq 12\\ \because 12^2>83\end{array}\right]$

fr ① $\gamma\beta+36=71$

$\Rightarrow \beta\gamma=35 \Rightarrow r=-\alpha\beta\gamma=-3\times35=-105 \Rightarrow r=-105$ ✓