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FP1

Further Pure Math 1

Vectors 3D Notes

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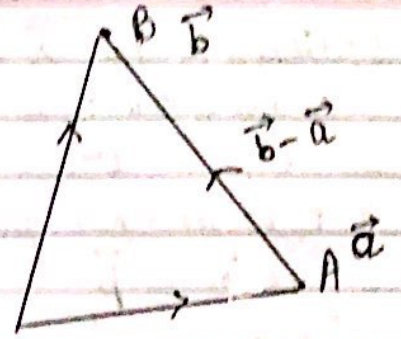
(FP1)

Vectors

- **Vectors in three dimensions:**

Given two point $\vec{a}$ and $\vec{b}$ on a line.

$$\vec{a} = a_1 i + a_2 j + a_3 j = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$



$$\vec{b} = b_1 i + b_2 j + b_3 k = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Vector $\vec{AB} = \vec{b} - \vec{a}$

$$= (b_1 - a_1)i + (b_2 - a_2)j + (b_3 - a_3)k = \begin{pmatrix} b_1 - a_1 \\ b_2 - a_2 \\ b_3 - a_3 \end{pmatrix}$$

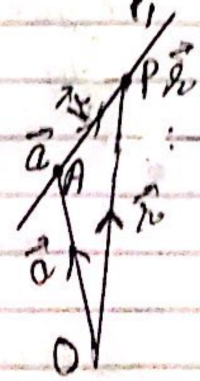
- **Equation of a line passing through a point $\vec{a}$ and its direction $\vec{v}$:**

$$\vec{r} = \vec{a} + \lambda \vec{v}$$

or

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \lambda \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}.$$

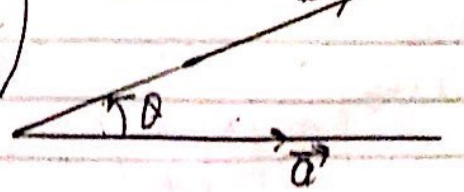
$\vec{r}$ is any (Variable) point of line l.
$\vec{a}$ is a given point.
$\vec{v} = v_1 i + v_2 j + v_3 k$.
$\vec{AP} = \vec{OA} + \vec{AP}$
$\vec{r} = \vec{a} + \lambda \vec{v}$



- **Scalar Product (Dot Product) of two vectors:**

Given $\vec{a} = a_1 i + a_2 j + a_3 k = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$

and $\vec{b} = b_1 i + b_2 j + b_3 k = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$



$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta \quad \text{---(i)}$$

or $$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} \quad \text{---(ii)}$$

$$\vec{a} \cdot \vec{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$i \cdot i = i^2 = j \cdot j = k \cdot k = 1$
$i \cdot j = j \cdot k = k \cdot i = 0$
$\vec{a}^2 = |\vec{a}|^2$ and
$\vec{a} \cdot \vec{b} = 0 \Leftrightarrow \vec{a} \perp b$
$\vec{a} \neq 0,\ \vec{b} \neq 0$

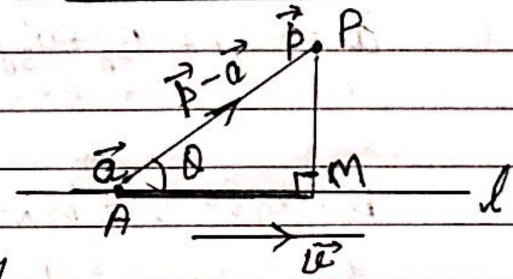
<u>Note</u>: Scalar product of two vectors is a <u>scalar</u> quantity and $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

Geometrical Application of Scalar Product:

l: Given a line $\vec{r} = \vec{a} + \lambda\vec{v}$ —— ① passing through a point $A(\vec{a})$ in direction $\vec{v}$.

Distance of a point from a line: $PM = \sqrt{(AP^2 - AM^2)}$

P is another point, position vector is $\vec{p}$



Then Geometrically Projection of segment $\overline{AP}$ on line l = AM

$(PM \perp l)$

Now $\overrightarrow{AP} = \vec{p} - \vec{a}$

Consider $\overrightarrow{AP}\cdot\vec{v} = |\overrightarrow{AP}||\vec{v}|\cos\theta$

$\left[\ \dfrac{AM}{AP} = \cos\theta \quad \therefore AM = AP\cdot\cos\theta \right.$

$\Rightarrow |\overrightarrow{AP}|\cos\theta = \dfrac{\overrightarrow{AP}\cdot\vec{v}}{|\vec{v}|}$

Projection $\overrightarrow{AP}$ along l ; $\Rightarrow AM = \overrightarrow{AP}\cdot\hat{v}$ (unit vector along l. $\hat{v} = \dfrac{\vec{v}}{|\vec{v}|}$)

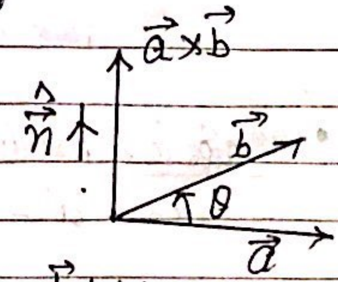
$= AM = (\vec{p} - \vec{a})\cdot\hat{v}$

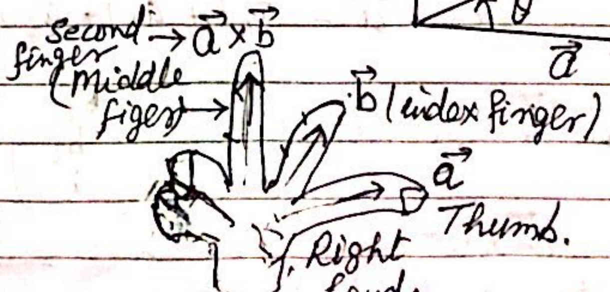
Vector Product (cross product):

Given two vectors $\vec{a}$ and $\vec{b}$, then their vector product:

$$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\cdot\hat{n}$$

where $\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$, and its sense is as per right-handed set.





Also

$$\vec{a}\times\vec{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$$

or

$$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Note 1: $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$

Note 2: $\vec{a}\times\vec{b}$ is a vector quantity.

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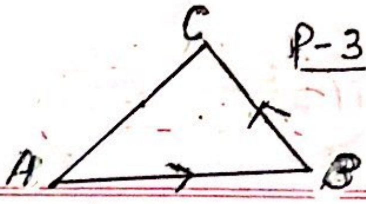


- **Properties of Vector Product:**

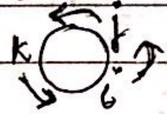
1. $\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$
2. (i) $i\times i = j\times j = k\times k = 0$
   (ii) $i\times j = k$, $j\times k = i$ and $k\times i = j$
3. $(m\vec{a})\times(n\vec{b}) = mn(\vec{a}\times\vec{b})$
4. $\vec{a}\times(\vec{b}+\vec{c}) = \vec{a}\times\vec{b} + \vec{a}\times\vec{c}$

- **Area of Triangle ABC**

$$= \frac{1}{2}|\vec{AB}\times\vec{BC}|$$

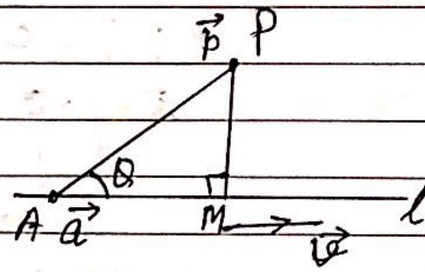
$\vec{a}\times\vec{a} = 0$ and
$\vec{a}\times\vec{b} = 0 \Leftrightarrow \vec{a} \parallel \vec{b}$, $\vec{a}\neq 0, \vec{b}\neq 0$

$i\times j = k$ but $j\times i = -k$



- **To Find the distance of a point to a line (using Vector-Product)**

Solution: Given a line $\vec{r} = \vec{a} + \lambda\vec{v}$
and a point $P(\vec{p})$
$PM \perp$ line $\Rightarrow$ To find $PM = ?$



$$\frac{PM}{AP} = \sin\theta$$

$$\Rightarrow PM = AP\sin\theta \quad \text{———} (1)$$

Now $\vec{AP} = (\vec{p}-\vec{a})$ and the direction of l is $\vec{v} = \vec{AM}$

$$|\vec{AP}\times\vec{AM}| = |\vec{AP}||\vec{AM}|\sin\theta$$

$$\Rightarrow AP\cdot\sin\theta = \frac{|\vec{AP}\times\vec{AM}|}{|\vec{AM}|} = \frac{|(\vec{p}-\vec{a})\times\vec{v}|}{|\vec{v}|} \quad \text{—} (2)$$

from (1) and (2)

$\therefore$ Required perp. distance $PM = \dfrac{|(\vec{p}-\vec{a})\times\vec{v}|}{|\vec{v}|}$ ✓

Note: To lines Intersect $\Leftrightarrow (\vec{b}-\vec{a})\cdot(\vec{V_1}\times\vec{V_2}) = 0$

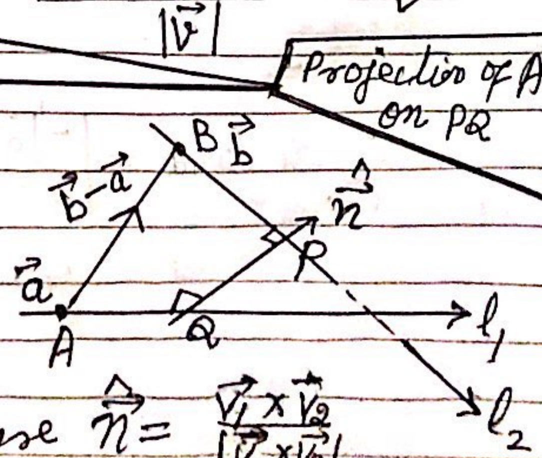
- **Shortest distance between two Skew lines:** l_1 ; $r = a + \lambda\vec{V_1}$
l_2 ; $r = b + \mu\vec{V_2}$

Let $\vec{n}$ is perp to l_1 and l_2 both

Required Shortest distance;

$$PQ = (\vec{b}-\vec{a})\cdot\frac{(\vec{V_1}\times\vec{V_2})}{|\vec{V_1}\times\vec{V_2}|}$$

Projection of AB on PQ



where $\hat{n} = \dfrac{\vec{V_1}\times\vec{V_2}}{|\vec{V_1}\times\vec{V_2}|}$

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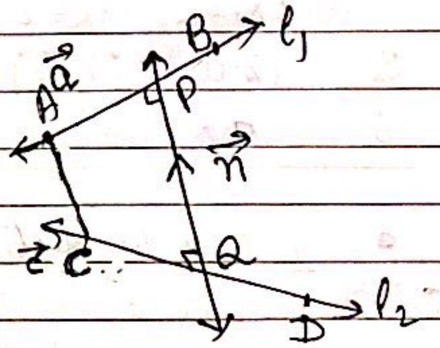
- **Example 1:** The line l_1 passes through the points $A(2,3,-5)$ and and $(8,7,-13)$. The line l_2 passes through the points $C(-2,1,8)$ and $D(3,-1,4)$.
Find the shortest distance between l_1 and l_2 [S-14 | 11 | Q 11]



Solution: l_1: $\vec{r} = \vec{a} + \lambda \vec{V_1}$

$$\vec{r} = \begin{pmatrix} 2 \\ 3 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ 4 \\ -8 \end{pmatrix} \quad \text{——} (1)$$

$$\vec{V_1} = \vec{b} - \vec{a} = \begin{pmatrix} 6 \\ 4 \\ -8 \end{pmatrix}$$

l_2: $\vec{r} = \vec{c} + \mu \vec{V_2}$ — (2)

$$\vec{r} = \begin{pmatrix} -2 \\ 1 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ -2 \\ -4 \end{pmatrix}$$

$$\vec{V_2} = \vec{d} - \vec{c} = \begin{pmatrix} 5 \\ -2 \\ -4 \end{pmatrix}$$

Required shortest distance between l_1 and l_2;

$= \text{Proj of } \vec{AC} \text{ on } \vec{PQ}$

$$= (\vec{c} - \vec{a}) \cdot \frac{\vec{n}}{|\vec{n}|} \checkmark$$

$$= \begin{pmatrix} -4 \\ -2 \\ 13 \end{pmatrix} \cdot -\frac{1}{3} \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

$$= -\frac{1}{3}(-8 - 2 + 26)$$

$$= -\frac{16}{3}$$

$$\therefore \text{distance} = \left| -\frac{16}{3} \right| = \underline{\frac{16}{3}} \checkmark$$

$\vec{PQ} \perp l_1$ $\qquad$ Let $\vec{PQ} = \vec{n}$

$\vec{PQ} \perp l_2$

$$\vec{n} = \vec{V_1} \times \vec{V_2} = \begin{vmatrix} i & j & k \\ 6 & 4 & -8 \\ 5 & -2 & -4 \end{vmatrix}$$

$$\vec{n} = -32i - 16j - 32k$$

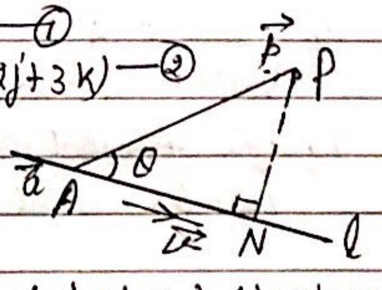
$$|\vec{n}| = 48$$

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|} = -\frac{16}{48}(2i + j + 2k)$$

and

$$\vec{c} - \vec{a} = \begin{pmatrix} -2 \\ 1 \\ 8 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ -5 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ 13 \end{pmatrix}$$

Example 2: The line 'l' has vector equation:
$r = 4i + 2j + 5k + \lambda(i + 2j + 3k)$, and a point 'P' has position vector $3i - 2j + k$. --[5]
Find the length of perpendicular from P to l. |Alevel|S-18|31|Q10(i)|



Solution: Given point P: $\vec{OP} = 3i - 2j + k$ —— ①
line l; $\vec{r} = (4i + 2j + 5k) + \lambda(i + 2j + 3k)$ —— ②
direction of line l: $\vec{v} = i + 2j + 3k$
Point A on line $\vec{OA} = 4i + 2j + 5k$ $\quad |\vec{v}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$
Now $\vec{AP} = \vec{OP} - \vec{OA} = -i - 4j - 4k$
$AN =$ Projection of AP on l $= \dfrac{\vec{AP} \cdot \vec{v}}{|\vec{v}|} = \dfrac{(-i - 4j - 4k) \cdot (i + 2j + 3k)}{\sqrt{14}}$

or $AN = \left|\dfrac{-1 - 8 - 12}{\sqrt{14}}\right| = \left|\dfrac{-21}{\sqrt{14}}\right| = \dfrac{21}{\sqrt{14}}$ —— ③

and $|\vec{AP}| = \sqrt{1^2 + 4^2 + 4^2}$
$= \sqrt{33}$

$\therefore$ The required perpendicular distance $PN = \sqrt{AP^2 - AN^2}$
or $PN = \sqrt{33 - \dfrac{441}{14}} = \sqrt{1.5} = \underline{1.22}$ ✓

$\boxed{\text{Alternate method}}$: (Using Vector Product)
$\boxed{\text{Shorter Way}}$

Length of perp. from P to l = $PN = \dfrac{|\vec{AP} \times \vec{v}|}{\vec{v}}$ —— ④

$\vec{AP} = \vec{p} - \vec{a} = (-i - 4j - 4k)$

$$\vec{AP} \times \vec{v} = \begin{vmatrix} i & j & k \\ -1 & -4 & -4 \\ 1 & 2 & 3 \end{vmatrix}$$

$= i(-12 + 8) - j(-3 + 4) + k(-2 + 4)$
$= -4i - j + 2k$

$\Rightarrow |\vec{AP} \times \vec{v}| = \sqrt{4^2 + 1^2 + 2^2} = \sqrt{21}$ —— ⑤

$|\vec{v}| = \sqrt{14}$ —— ⑥

$= \dfrac{\sqrt{21}}{\sqrt{14}}$ from ⑤ & ⑥
$= \sqrt{1.5}$
$= \underline{1.22}$ ✓

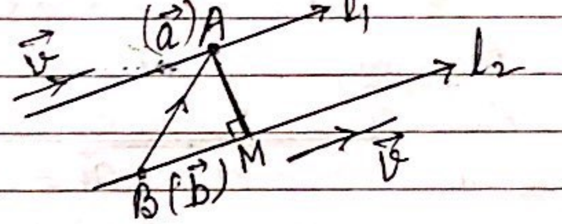
FP1

- **Shortest distance between two parallel lines:**

Given two parallel lines;

$l_1 : \vec{r} = \vec{a} + \lambda \vec{v}$

$l_2 : \vec{r} = \vec{b} + \mu \vec{v}$



$l_1 \parallel l_2$, A is point on l_1

The shortest distance between l_1 and l_2 is same as the distance of Point A to line l_2.

$$AM = \frac{|\vec{BA} \times \vec{v}|}{|\vec{v}|} = \frac{|(\vec{a} - \vec{b}) \times \vec{v}|}{|\vec{v}|}$$

- **Example 3:**

Find the shortest distance between the lines:

$l_1 : \vec{r} = (i + 2j + 3k) + \lambda(2i + 3j + 4k)$ and

$l_2 : \vec{r} = (2i + 4j + 5k) + \mu(4i + 6j + 8k)$

Solution: $l_1 : \vec{r} = (i + 2j + 3k) + \lambda(2i + 3j + 4k)$ — (1)

$l_2 : \vec{r} = (2i + 4j + 5k) + 2\mu(2i + 3j$ — (2)

lines l_1 & l_2 are parallel. $\Rightarrow$ Shortest distance $= \dfrac{|(\vec{a_2} - \vec{a_1}) \times \vec{v}|}{|\vec{v}|}$ — (3)

$\left.\begin{array}{l} \vec{a_2} - \vec{a_1} = (i + 2j + 2k) \\ \vec{v} = (2i + 3j + 4k) \end{array}\right\} \Rightarrow (\vec{a_2} - \vec{a_1}) \times \vec{v} = \begin{vmatrix} i & j & k \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{vmatrix}$

$= (2i - j)$

$\Rightarrow |\vec{a_2} - \vec{a_1}| \times \vec{v} = \sqrt{2^2 + 1^2} = \sqrt{5}$ — (4)

and $|\vec{v}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$ — (5)

$\therefore$ from (3), (4) and (5)

The required Shortest distance $= \dfrac{\sqrt{5}}{\sqrt{29}} = 0.415$

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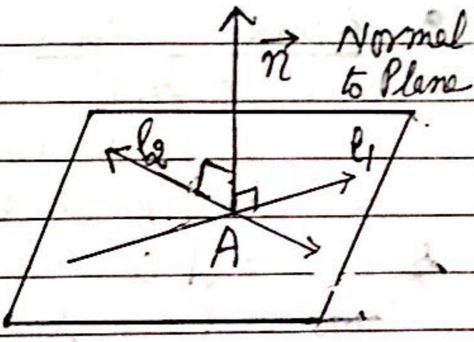
Plane

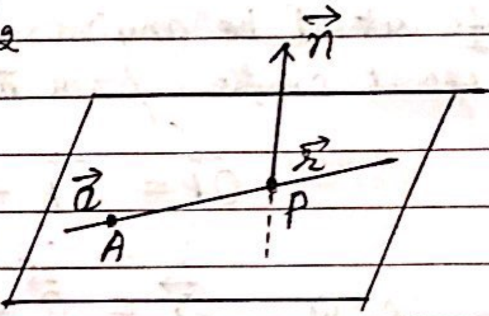
- **Plane in 3D.**



Direction of a plane is expressed in terms of its Normal $\vec{n}$ to plane, through the point of intersection 'A' of plane and normal.

$$\vec{n} \perp l_1 \text{ and } \vec{n} \perp l_2$$

- **Equation of Plane :**



Equation of Plane passing a point $A(\vec{a})$ and $\vec{n}$ is normal to the plane, Let $P(\vec{r})$ be any point (variable) on the plane.

$$\vec{AP} \perp \vec{n} \Rightarrow (\vec{r}-\vec{a})\cdot\vec{n} = 0 \quad \text{—— (1)}$$

$$\Rightarrow \vec{r}\cdot\vec{n} = \vec{a}\cdot\vec{n}$$

$(\vec{a}\cdot\vec{n} = d$ constant$)$

$$\Rightarrow \vec{r}\cdot\vec{n} = d \quad \text{—— (2)}$$

- **Cartesian Equation of plane:**

from (1) $\vec{r}\cdot\vec{n} = d$

$(xi+yj+zk)\cdot(ai+bj+ck) = 0$

or $ax+by+cz = d$ —— (3)

$\because$ $\vec{r} = xi+yj+zk$
$\vec{n} = ai+bj+ck$

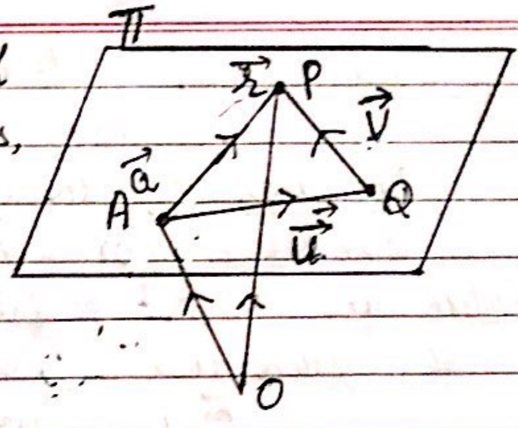
- **Equation a plane passing through a given point $A(x_1, y_1, z_1)$ and component of normal to the plane are $a, b,$ and c.**

from (1) $(\vec{r}-\vec{a})\cdot\vec{n} = 0$

Let the given point 'A'
$\vec{a} = x_1 i + y_1 j + z_1 k$

$$a(x-x_1) + b(y-y_1) + c(z-z_1) = 0 \quad \text{—— (4)}$$

Parametric form of Plane



- Equation of Plane passing through a point A, $\vec{a}$ is its position Vectors, and $\vec{u}$ and $\vec{v}$ are two non-parallel vectors on the Plane,

$$\vec{r} = \vec{a} + \lambda\vec{u} + \mu\vec{V}$$

where λ and μ are constants

Proof: Let P be any variable point on the plane π, position vector of P is $\vec{r}$.

$$\vec{OP} = \vec{OA} + \vec{AP}$$
$$\vec{r} = \vec{OA} + (\vec{AQ} + \vec{QP})$$
or $$\vec{r} = \vec{a} + \lambda\vec{u} + \mu\vec{v}$$

$\vec{AQ} = \lambda\vec{u}$
$\vec{QP} = \mu\vec{V}$

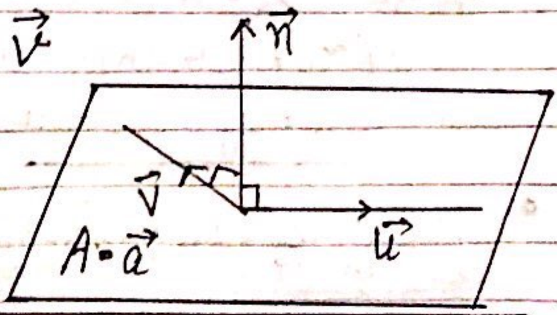
Equation of Plane AQP: $\boxed{\vec{r} = \vec{a} + \lambda\vec{u} + \mu\vec{V}}$ —— (1)

- To reduce the equation of plane (1) to $\vec{r}\cdot\vec{n} = d$ form; (or $ax+by+cz=d$)

$\vec{n}$ is normoral to the plane, must be perp. to both. the vector $\vec{u}$ and $\vec{v}$ lying in the plane,

$$\therefore \vec{n} = \vec{u} \times \vec{v}$$

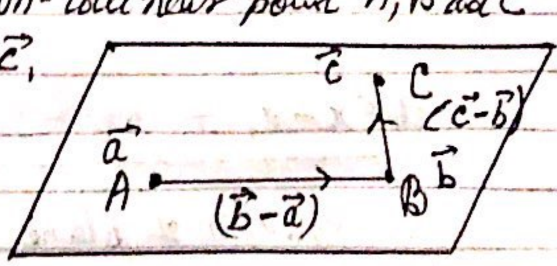
$\therefore$ Eqn of Plane:
$$(\vec{r} - \vec{a})\cdot\vec{n} = 0$$



- Equation of Plane passing through Non-collinear point A, B and C whose position vector are $\vec{a}$, $\vec{b}$ and $\vec{c}$,

from (1) Eqn of Plane ABC.

$$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a}) + \mu(\vec{c} - \vec{b})$$



FP1 Converting Equ^n of Plane in Parametric form, $\vec{r} = \vec{a} + \lambda\vec{u} + \mu\vec{v}$ to Cartesian form

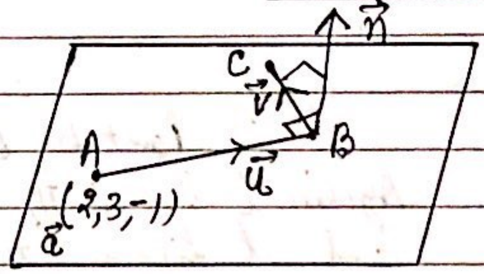
$\boxed{\text{or to } \vec{r}\cdot\vec{n} = d}$

- Example 4. The plane Π_1 has equation $\vec{r} = \begin{pmatrix}2\\3\\-1\end{pmatrix} + s\begin{pmatrix}1\\0\\1\end{pmatrix} + t\begin{pmatrix}1\\-1\\-2\end{pmatrix}$. Find a Cartesian equation of Π_1.

  The plane Π_2 has equation $2x - y + z = 10$. Find the acute angle between Π_1 and Π_2.

  Find an equation of line of intersection of Π_1 and Π_2, giving your answer in the form $\vec{r} = \vec{a} + \lambda\vec{b}$ $\boxed{W\text{-}13/11/Q8}$

Solution: $\vec{r} = \begin{pmatrix}2\\3\\-1\end{pmatrix} + s\begin{pmatrix}1\\0\\1\end{pmatrix} + t\begin{pmatrix}1\\-1\\-2\end{pmatrix}$



or $\vec{r} = \vec{a} + s\vec{u} + t\vec{v}$

The normal vector to the given plane $\vec{n}$

$$(\vec{n} = \vec{u}\times\vec{v}) \quad \vec{n} = \begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ 1 & -1 & -2 \end{vmatrix} = (i + 3j - k)$$

and point $\vec{a} = 2i + 3j - k$

$\therefore$ Equ^n of plane: $(\vec{r} - \vec{a})\cdot\vec{n} = 0$

$$\Rightarrow \vec{r}\cdot\vec{n} = \vec{a}\cdot\vec{n}$$

$$\Rightarrow \begin{pmatrix}x\\y\\z\end{pmatrix}\cdot\begin{pmatrix}1\\3\\-1\end{pmatrix} = \begin{pmatrix}2\\3\\-1\end{pmatrix}\cdot\begin{pmatrix}1\\3\\-1\end{pmatrix}$$

(Continued →)

Equ^n of Π_1 plane: $x + 3y - z = 12$ ✓ ——— (1)

another plane Π_2: $2x - y + z = 10$ ——— (2)

$\left\{ \cos\theta = \dfrac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1||\vec{n}_2|} \right.$ ⊛

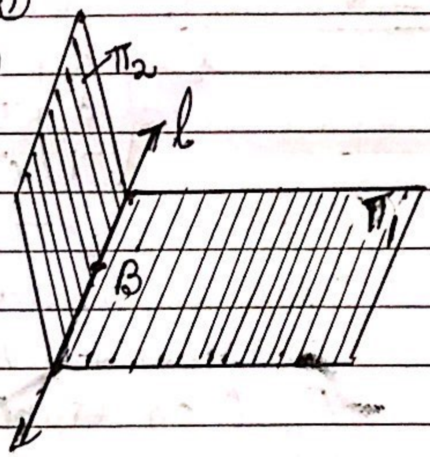
Angle between Π_1 & Π_2, is the angle between their normals $\vec{n}_1$ & $\vec{n}_2$. $\cos\theta \overset{⊛}{=} \dfrac{\begin{pmatrix}1\\3\\-1\end{pmatrix}\begin{pmatrix}2\\-1\\1\end{pmatrix}}{\sqrt{11}\cdot\sqrt{6}} = \left|\dfrac{-2}{\sqrt{66}}\right| = \dfrac{2}{\sqrt{66}}$

$\therefore \theta = \cos^{-1}\left(\dfrac{2}{\sqrt{66}}\right) = 75.7°$ ✓

FP1

(Continued→)

Example 4. Given plane π_1: $x+3y-z=12$ —①
plane π_2: $2x-y+z=10$ —②



Let l is the line of intersection of π_1 & π_2.
Let B is a point on line B.
B lies on π_1 & π_2 both.
Put $z=0$ in ① & ②

$$\begin{cases} x+3y=12 & —③ \\ 2x-y=10 & —④ \end{cases}$$

Solving ③ and ④ $x=6$, $y=2$

∴ Point $B(6, 2, 0)$ $\Rightarrow \vec{b}(6i+2j)$

Again l lies in $\pi_1 \Rightarrow l \perp \vec{n_1}$ normal of π_1
and l lies in $\pi_2 \Rightarrow l \perp \vec{n_2}$ normal of π_2.

∴ direction of line $l = \vec{v} = \vec{n_1} \times \vec{n_2}$

$$\vec{v} = \begin{vmatrix} i & j & k \\ 1 & 3 & -1 \\ 2 & -1 & 1 \end{vmatrix} = 2i+3j-7k$$

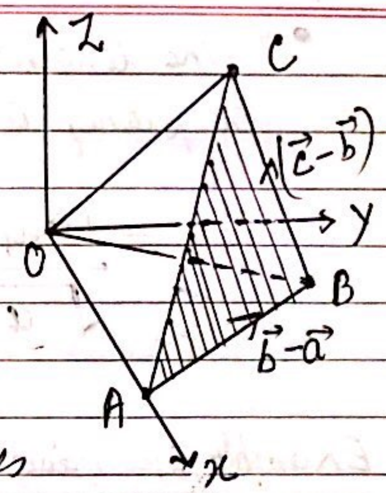
∴ Equation of line l,
Intersection of π_1 & π_2 is $\vec{r} = \vec{b} + \lambda\vec{v}$

$$\underline{\vec{r} = (6i+2j) + \lambda(2i+3j-7k)}$$

Scanned with CamScanner

Equation of a Plane passing through three points (Using Vector Product)

Example 5: The diagram shows rectangular axes Ox, Oy and Oz, and three points, A, B and C with position vectors $\vec{OA} = \begin{pmatrix}2\\0\\0\end{pmatrix}$; $\vec{OB} = \begin{pmatrix}1\\2\\0\end{pmatrix}$; $\vec{OC} = \begin{pmatrix}1\\1\\2\end{pmatrix}$



(i) Find the equation of the plane ABC, giving your answer in the form $ax+by+cz=d$

(ii) Calculate the acute angle between the planes ABC and OAB.

[A-Level/S-2007/3/Q9]

Solution: $\vec{AB} = \vec{b} - \vec{a} = \begin{pmatrix}-1\\2\\0\end{pmatrix}$, $\vec{BC} = \vec{c} - \vec{b} \begin{pmatrix}0\\-1\\2\end{pmatrix}$

$\vec{AB}$ & $\vec{BC}$ lie in plane ABC, π_1

Normal to plane ABC $\vec{n} = \vec{AB} \times \vec{BC} = \begin{vmatrix} i & j & k \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{vmatrix}$

$\vec{n_1} =$

or $\vec{n_1} = 4i + 2j + k$, ——— (1)

Position Vector of point A, $\vec{a} = 2i$

∴ Equation of plane ABC,

$(\vec{r} - \vec{a}) \cdot \vec{n} = 0 \Rightarrow \vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$

$$\Rightarrow \begin{pmatrix}x\\y\\z\end{pmatrix} \cdot \begin{pmatrix}4\\2\\1\end{pmatrix} = \begin{pmatrix}2\\0\\0\end{pmatrix} \cdot \begin{pmatrix}4\\2\\1\end{pmatrix}$$

∴ Equation of Plane ABC, $4x + 2y + z = 8$ ——— (2)

Plane OAB lies in XY Plane as B and A lie in XY Plane

$\Rightarrow$ Normal to plane OBC $\vec{n_2} = k = \begin{pmatrix}0\\0\\1\end{pmatrix}$ ——— (3)

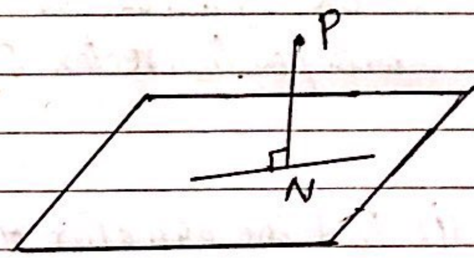
∴ Angle between Planes ABC and OAB, 'θ' : $\Rightarrow \cos\theta = \frac{1}{\sqrt{21}\sqrt{1}} \begin{pmatrix}4\\2\\1\end{pmatrix} \cdot \begin{pmatrix}0\\0\\1\end{pmatrix} = \frac{1}{\sqrt{21}} \times 1 = \frac{1}{\sqrt{21}}$

$\cos\theta = \frac{\vec{n_1} \cdot \vec{n_2}}{|\vec{n_1}||\vec{n_2}|}$

$\theta = \cos^{-1}\left(\frac{1}{\sqrt{21}}\right) = \underline{77.4^\circ}$

- The length of perpendicular from a point $P(x_1, y_1, z_1)$ to the plane, $ax+by+cz+d=0$

$$PN = \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$$



Example 6: Find the distance of the point $(2, 3, -5)$ from the plane $x+2y-2z-9=0$

Solution: distance $= \dfrac{|1\times 2 + 2\times 3 + (-2)(-5) - 9|}{\sqrt{1^2+2^2+2^2}}$

$= \dfrac{|2+6+10-9|}{3}$

$= \dfrac{9}{3} = 3$

← X —— X →