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FP-2

Further Pure Maths-2

Differential Equations Exercise.

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Differential equations

1. Find the general solution of the differential equation.

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = 7 - 2t^2$$

[SP-20/02/Q1] [6]

2. Find the solution of the differential equation

$$x\frac{dy}{dx} + 3y = \frac{\sin x}{x}$$

for which $y = 0$ when $x = \frac{1}{2}\pi$.
Give your answer in the form $y = f(x)$ [SP-20/02/Q3] [8]

3. Find the solution of the differential equation:

$$\frac{dy}{dx} + 5y = e^{-7x}$$

for which $y = 0$ when $x = 0$. Give your answer in the form $y = f(x)$. [S-20/21/Q1] ---[6]

4. It is given that $x = t^3 y$ and

$$t^3\frac{d^2y}{dt^2} + (4t^3 + 6t^2)\frac{dy}{dt} + (13t^3 + 12t^2 + 6y)y = 61e^{\frac{1}{2}t}.$$

(a) Show that, $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = 61e^{\frac{1}{2}t}$ --[4]

(b) Find the general solution for y in terms of t. ---[7]

[S-20/21/Q7]

5. Find the general solution of the differential equation:

$$\frac{d^2x}{dt^2} - 8\frac{dx}{dt} - 9x = 9e^{8t}$$

---[6]

[S-20/23/Q1]

6.(a) Show that an appropriate integrating factor for

$$(x^2+1)\frac{dy}{dx} + y\sqrt{x^2+1} = x^2 - x\sqrt{x^2+1}$$

is $x + \sqrt{x^2+1}$ ---[4]

(b) Hence find the solution of the differential equation

$$(x^2+1)\frac{dy}{dx} + y\sqrt{x^2+1} = x^2 - x\sqrt{x^2+1}$$

for which $y = \ln 2$ when $x = 0$. Give your answer in the form $y = f(x)$. [S-20/23/Q7] ---[7]

7. Find the particular solution of the differential equation,
$$10\frac{d^2x}{dt^2}+3\frac{dx}{dt}-x=t+2.$$
given that when $t=0$, $x=0$ and $\frac{dx}{dt}=0$ --[10]
[S-19/11/Q7]

8. Find the particular solution of the differential equation:
$$9\frac{d^2x}{dt^2}+6\frac{dx}{dt}+x=50\sin t$$
given that when $t=0$, $x=0$ and $\frac{dx}{dt}=0$ ---[10]
[S-19/13/Q8]

9. It is given that $w=\cos y$ and
$$\tan y.\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+2\tan y\frac{dy}{dx}=1+e^{-2x}\sec y$$
(i) Show that: $\frac{d^2w}{dx^2}+2\frac{dw}{dx}+w=-e^{-2x}$ ---[4]

(ii) Find the particular solution for y in terms of x, given that when $x=0$, $y=\frac{1}{3}\pi$ and $\frac{dy}{dx}=\frac{1}{\sqrt{3}}$ --[10]
[W-19/11/Q11(i)]

10. Find the particular solution of the differential equation:
$$49\frac{d^2y}{dx^2}+14\frac{dy}{dx}+y=49x+735$$
given that:
$x=0$, $y=0$ and $\frac{dy}{dx}=0$ [S-18/11/Q7] ---[10]

11. The variables x and y are such that $y=-1$ when $x=0$
and $\left(x+\frac{dy}{dx}\right)^3=y^2+x$

(i) Find the value $\frac{dy}{dx}$ when $x=0$ --(1)

(ii) Find also the value of $\frac{d^2y}{dx^2}$ when $x=0$ --[4]

[S-18/13/Q1]

12. It is given that $t \neq 0$ and
$$t\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 9tx = 3t^2+1$$

(i) Show that if $y = tx$ then
$$\frac{d^2y}{dt^2} + 9y = 3t^2+1 \quad \text{---} [3]$$

(ii) Find x in terms of t, given that,
$$x = \frac{1}{9}\pi \text{ and } \frac{dx}{dt} = 2/3 \text{ when } t = \frac{1}{3}\pi \quad \text{---} [9]$$

[S-18/13/Q10]

13(i) Find the particular solution of the differential equation:
$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 10x = 37\sin 3t,$$
given that $x = 3$ and $\frac{dx}{dt} = 0$ when $t = 0$ --- [10]

(ii) Show that, for large positive values of t and for any initial conditions, $x \approx \sqrt{(37)}\sin(3t - \phi)$,
where constant ϕ is such that $\tan\phi = 6$ --- [3]

[W-18/11/Q10]

14(i) Find the general solution of the differential equation:
$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 4\sin t \quad \text{---} [7]$$

(ii) State an approximate solution for large positive value of t. --- [1]

[W-18/12/Q4]

15. Find the solution of the differential equation
$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 18t^2 + 6t + 1$$
given that, when $t = 0$, $x = 3$ and $\frac{dx}{dt} = 0$ --- [10]

[S-17/13/Q8]

16. Find the general solution of the differential equation.
$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 4 - 5t^2 \quad \text{---} [6]$$

[W-17/11/Q2]

17. (i) Find the value of constant k such that $y = kx^2 e^{2x}$ is a particular integral of the differential equation.

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 4e^{2x} \quad (*) \quad \text{---[4]}$$

(ii) Hence find the general solution of $(*)$ --[3]

(iii) Find the particular solution of $(*)$ such that $y = 3$ and $\frac{dy}{dx} = -2$ when $x = 0$ [S-16/11/Q9] --[4]

18. Find the general solution of the differential equation.

$$\frac{d^2x}{dt^2} + 7\frac{dx}{dt} + 10x = 116\sin 2t \quad \text{---[8]}$$

State an approximate solution for large positive values of t. --[1]

[W-16/11/Q.6]

19. (i) Show that the substitution $v = \frac{1}{y}$ reduce the differential equation. $\frac{2}{y^3}\left(\frac{dy}{dx}\right)^2 - \frac{1}{y^2}\frac{d^2y}{dx^2} - \frac{2}{y^2}\frac{dy}{dx} + \frac{5}{y} = 17 + 6x - 5x^2$

to the differential equation. $\frac{d^2v}{dx^2} + 2\frac{dv}{dx} + 5v = 17 + 6x - 5x^2$ --[4]

(ii) Hence find y in terms of x, given that when $x = 0$, $y = \frac{1}{2}$ and $\frac{dy}{dx} = -1$. [S-15/11/Q11] --[10]

20. Find the particular solution of the differential equation.

$$\frac{d^2x}{dt^2} - 3\frac{dx}{dt} - 10x = 2\sin t - 3\cos t$$

given that, when $t = 0$, $x = 3.3$ and $\frac{dx}{dt} = 0.9$ --[11]

[S-15/13/Q9]

21. Find the general solution of the differential equation,

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = 7 - 2t^2 \quad \text{--[6]}$$

[W-15/11/Q2]

Answers

1. $m^2+4m+4=0$; $m=-2,-2$

$x=(At+B)e^{-2t}$

$x=P+Qt+Rt^2$, $\frac{dx}{dt}=Q+2Rt$

$\frac{d^2x}{dt^2}=2R$

Substitute and equate the coeff of the powers of t.

$P=1$, $Q=1$, $R=-\frac{1}{2}$

∴ The required gen. solution:

$x=(At+B)e^{-2t}+1+t-\frac{1}{2}t^2$ ✓

2. $\frac{dy}{dx}+3\frac{y}{x}=\frac{\sin x}{x^2}$

$I.F=e^{\int\frac{3}{x}dx}=e^{3\ln x}=x^3$

Solution is

$y(I.F)=\int\frac{\sin x}{x^2}(I.F)\,dx$

$\Rightarrow yx^3=\int\frac{\sin x}{x^2}x^3\,dx$

$=\int x\sin x\,dx$

$yx^3=-x\cos x+\sin x+C$

for $y=0$, $x=\frac{\pi}{2}$

$\Rightarrow C=-1$

$yx^3=-x\cos x+\sin x-1$

$\Rightarrow y=\frac{(-x\cos x+\sin x-1)}{x^3}$ ✓

3. $I.F=e^{\int 5\,dx}=e^{5x}$

∴ $y\cdot IF=\int e^{-7x}(I.F)\,dx$

$\Rightarrow ye^{5x}=\int e^{-2x}\,dx$

$\Rightarrow ye^{5x}=-\frac{1}{2}e^{-2x}+C$

$x=0$, $y=0\Rightarrow C=\frac{1}{2}$

$e^{5x}\cdot y=-\frac{1}{2}e^{-2x}+\frac{1}{2}$

$\Rightarrow y=-\frac{1}{2}e^{-7x}+\frac{1}{2}e^{-5x}$ ✓

$x=t^3y$

4(i) $\frac{dx}{dt}=t^3\frac{dy}{dt}+3t^2y$

$\frac{d^2x}{dt^2}=t^3\frac{d^2y}{dt^2}+6t^2\frac{dy}{dt}+6ty$

∴ $\frac{d^2x}{dt^2}+4\frac{dx}{dt}+13x$

$=t^3\frac{d^2y}{dt^2}+6t^2\frac{dy}{dt}+6ty+4t^3\frac{dy}{dt}$

$+12t^2y+13t^3y=61e^{\frac{1}{2}t}$ ✓

(ii) Given

$t^3\frac{d^2y}{dt^2}+(4t^3+6t^2)\frac{dy}{dt}+(13t^3+12t^2+6y)t$

$=61e^{\frac{1}{2}t}$

$m^2+4m+13=0\Rightarrow m=-2\pm 3i$

$x=e^{-2t}(A\cos 3t+B\sin 3t)$

$x=ke^{\frac{1}{2}t}\Rightarrow\frac{dx}{dt}=\frac{1}{2}ke^{\frac{1}{2}t}\Rightarrow\frac{d^2x}{dt^2}=\frac{1}{4}ke^{t/2}$

$\Rightarrow\frac{1}{4}k+2k+13k=61\Rightarrow k=4$

∴ $t^3y=e^{-2t}(A\cos 3t+B\sin 3t)+4e^{\frac{1}{2}t}$

$\Rightarrow y=t^{-3}e^{-2t}(A\cos 3t+B\sin 3t)+4t^{-3}e^{\frac{1}{2}t}$ ✓

5. $m^2-8m-9=0\Rightarrow m=-1,9$

$x=Ae^{-t}+Be^{9t}$

$x=ke^{8t}\Rightarrow\frac{dx}{dt}=8ke^{8t}\Rightarrow\frac{d^2x}{dt^2}=64ke^{8t}$

∴ $64ke^{8t}-64ke^{8t}-9ke^{8t}=9e^{8k}\Rightarrow -9k=9$

$\Rightarrow k=-1$

∴ $x=Ae^{-t}+Be^{9t}-e^{8t}$ ✓

6(a) $(x^2+1)\frac{dy}{dx}+y\sqrt{x^2+1}=(x^2-x\sqrt{x^2+1})$

$\Rightarrow\frac{dy}{dx}+\frac{1}{\sqrt{x^2+1}}y=\frac{x}{x^2+1}(x-\sqrt{x^2+1})$

$I.F=e^{\int\frac{1}{\sqrt{x^2+1}}dx}=e^{\sinh^{-1}x}$

$=e^{\ln(x+\sqrt{x^2+1})}$

$\Rightarrow I.F=x+\sqrt{x^2+1}$

(Continued →)

(continued→)

Answers

Rep. Soln.

6(b). $y.I.F = \int \frac{x}{(x^2+1)}(x-\sqrt{x^2+1}) \times I.F\,dx$

$\Rightarrow y(x+\sqrt{x^2+1}) = \int \frac{x(x-\sqrt{x^2+1})(x+\sqrt{x^2+1})}{(x^2+1)}dx$

$= -\int \frac{x}{x^2+1}dx = -\frac{1}{2}\ln(x^2+1)+c$

Now $y = \ln 2$ for $x=0$

$\Rightarrow c = \ln 2$

$\therefore y = \frac{\ln 2 - \frac{1}{2}\ln(x^2+1)}{(x+\sqrt{x^2+1})}$

or $y = (x-\sqrt{x^2+1})\ln\left(\frac{2}{\sqrt{x^2+1}}\right)$ ✓

7. $10u^2+3u-1=0 \Rightarrow (2u+1)(5u-1)=0$

CF: $x = Ae^{-\frac{1}{2}t} + Be^{\frac{1}{5}t}$

PI: $x = p+qt \Rightarrow \frac{dx}{dt} = q \Rightarrow \frac{d^2x}{dt^2}=0$

$3q-p-qt = t+2$

$\Rightarrow q=-1,\ p=-5$

GS: $x = Ae^{-\frac{1}{2}t} + Be^{\frac{1}{5}t} - t - 5$

$\frac{dx}{dt} = -\frac{1}{2}Ae^{-\frac{1}{2}t} + \frac{1}{5}Be^{\frac{1}{5}t} - 1$

for $t=0$, $x=0$ and $\frac{dx}{dt}=0$

$\Rightarrow \left.\begin{array}{l} A+B=5 \\ -\frac{1}{2}A+\frac{1}{5}B=1 \end{array}\right\} \Rightarrow A=0,\ B=5$

$x = 5e^{\frac{1}{5}t} - t - 5$ ✓

Given $\tan y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 2\tan y\frac{dy}{dx} = 1+e^{-2x}$.

9(i) $W = \cos y \Rightarrow \frac{dw}{dx} = -\sin y\frac{dy}{dx}$

$\frac{d^2w}{dx^2} = -\sin y\frac{d^2y}{dx^2} - \cos y\left(\frac{dy}{dx}\right)^2$

$\therefore \frac{d^2w}{dx^2} + 2\frac{dw}{dx} + w$

$= -\sin y\frac{d^2y}{dx^2} - \cos y\left(\frac{dy}{dx}\right)^2 - 2\sin y\frac{dy}{dx} + \cos y$

$= -\cos y(e^{-2x}\sec y) = -e^{-2x}$ ✓

(ii) $m^2+2m+1=0 \Rightarrow m=-1$

C.F: $W = (Ax+B)e^{-x}$

PI: $W = ke^{-2x} \Rightarrow \frac{dw}{dx} = -2ke^{-2x}$

$\Rightarrow \frac{d^2w}{dx^2} = 4ke^{-2x}$

$4k-4k+k=-1 \Rightarrow k=-1$.

$W = (Ax+B)e^{-x} - e^{-2x}$

$x=0,\ y=\pi/3,\ W=\frac{1}{2} \Rightarrow B=3/2$

$\frac{dw}{dx} = -(Ax+B)e^{-x} + Ae^{-x} + 2e^{-2x}$

$x=0,\ y=\frac{\pi}{3},\ y'=\frac{\sqrt{3}}{3},\ \frac{dw}{dx} = -\frac{1}{2}$

$\Rightarrow -\frac{1}{2} = -\frac{3}{2}+A+2 \Rightarrow A=-1$

$y = \cos^{-1}\left(\left(\frac{3}{2}-x\right)e^{-x} - e^{-2x}\right)$ ✓

8. $9u^2+6u+1=0 \Rightarrow (3u+1)^2=0$

CF: $x = (At+B)e^{-\frac{1}{3}t}$

$x = p\sin t + q\cos t$

$\frac{dx}{dt} = p\cos t - q\sin t$

$\frac{d^2x}{dt^2} = -p\sin t - q\cos t$

$\left.\begin{array}{l} -9p-6q+p=50 \\ -9q+6p+q=0 \end{array}\right\}$

$\Rightarrow q=-3,\ p=-4$

$x = (At+B)e^{-\frac{t}{3}} - 4\sin t - 3\cos t$ continued→

$\frac{dx}{dt} = -\frac{1}{3}(At+B)e^{-\frac{1}{3}t} + Ae^{-\frac{1}{3}t} - 4\cos t + 3\sin t$

$\Rightarrow B=3$

$-\frac{1}{3}B+A-4=0 \Rightarrow A=5$

$\therefore x = (5t+3)e^{-\frac{1}{3}t} - 4\sin t - 3\cos t$.

Answers

10. The auxiliary equation: $\left(n+\frac{1}{7}\right)^2=0$

$\Rightarrow n=-\frac{1}{7}$

C.F: $y=(A+Bx)e^{-\frac{1}{7}x}$

P.I: $y=px+q$, $\frac{dy}{dx}=p$; $\frac{d^2y}{dx^2}=0$

$14p+px+q=49x+735$

$\Rightarrow p=49,\ q=49$

$\therefore y=(A+Bx)e^{-\frac{1}{7}x}+49(x+1)$

$y=0$ when $x=0 \Rightarrow A+49=0 \Rightarrow A=-49$

$\frac{dy}{dx}=-\frac{1}{7}(A+Bx)e^{-\frac{1}{7}x}+Be^{-\frac{1}{7}x}+49$

$\frac{dy}{dx}=0$ when $x=0 \Rightarrow -\frac{1}{7}A+B+49=0$

$\Rightarrow B=-56$

$\therefore y=-(49+56x)e^{-\frac{1}{7}x}+49(x+1)$ ✓

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11. $\left(x+\frac{dy}{dx}\right)^3=y^2+x$ ①

(i) when $x=0$ and $y=-1$ $\Rightarrow \left(0+\frac{dy}{dx}\right)^3=1+0$

$\Rightarrow \frac{dy}{dx}=1$ ✓

(ii) diff ① $3\left(x+\frac{dy}{dx}\right)^2\left(1+\frac{d^2y}{dx^2}\right)=2y\frac{dy}{dx}+1$

$\Rightarrow 3\left(1+\frac{d^2y}{dx^2}\right)=-2+1 \Rightarrow \frac{d^2y}{dx^2}=-\frac{4}{3}$ ✓

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13. (i) $\frac{d^2x}{dt^2}+2\frac{dx}{dt}+10x=37\sin 3t$

$m^2+2m+10=0 \Rightarrow m=-1\pm 3i$

$\Rightarrow x=e^{-t}(A\cos 3t+B\sin 3t)$

$x=p\cos 3t+q\sin 3t$

$\Rightarrow \frac{dx}{dt}=-3p\sin 3t+3q\cos 3t$

$\frac{d^2x}{dt^2}=-9p\cos 3t-9q\sin 3t$

$(p+6q)\cos 3t+(q-6p)\sin 3t=37\sin 3t$

$\Rightarrow p=-6,\ q=1$

$x=e^{-t}(A\cos 3t+B\sin 3t)-6\cos 3t+\sin 3t$

$x=3$, when $t=0 \Rightarrow 3B-A+3=0 \Rightarrow B=2$

$x=e^{-t}(9\cos 3t+2\sin 3t)-6\cos 3t+\sin 3t$ ✓

(ii) As $t\to\infty$, $e^{-t}\to 0 \Rightarrow x\approx -6\cos 3t+\sin 3t$

$\sin 3t-6\cos 3t=\sqrt{37}\left(\frac{1}{\sqrt{37}}\sin 3t-\frac{6}{\sqrt{37}}\cos 3t\right)$

$=\sqrt{37}\sin(3t-\tan^{-1}6)$ ✓

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12. Given $t\frac{d^2x}{dt^2}+2\frac{dx}{dt}+9tx=3t^2+1$ ①

(i) $y=tx \Rightarrow \frac{dy}{dt}=x+t\frac{dx}{dt}$

and $\frac{d^2y}{dt^2}=\frac{dx}{dt}+\frac{dx}{dt}+t\frac{d^2x}{dt^2}$

$\therefore \frac{d^2y}{dt^2}+9y=2\frac{dx}{dt}+t\frac{d^2x}{dt^2}+9tx$

$=3t^2+1$ ✓ from ①

(ii) Auxiliary eqn: $m^2+9=0 \Rightarrow m=\pm 3i$

$CF=A\cos 3t+B\sin 3t$

PI: $y=at^2+bt+c \Rightarrow \frac{dy}{dt}=2at+b$

and $\frac{d^2y}{dt^2}=2a$.

Continued

$a=\frac{1}{3},\ b=0,\ c=\frac{1}{27}$

$y=A\cos 3t+B\sin 3t+\frac{1}{3}t^2+\frac{1}{27}$

$x=\frac{\pi}{9}$ when $t=\frac{\pi}{3} \Rightarrow A=\frac{1}{27}$

$\frac{dy}{dt}=-3A\sin 3t+3B\cos 3t+\frac{2}{3}t$

$\frac{dx}{dt}=\frac{2}{3}$ when $t=\frac{\pi}{3} \Rightarrow B=-\frac{\pi}{27}$

$\Rightarrow$

$x=\frac{\cos 3t-\pi\sin 3t+9t^2+1}{27t}$ ✓

Answers

14(i) $\frac{d^2x}{dt^2}+2\frac{dx}{dt}+x=4\sin t$

$m^2+2m+1=0 \Rightarrow (m+1)^2=0 \Rightarrow m=-1$

CF: $(A+Bt)e^{-t}$

PI: $x=p\sin t+q\cos t$

$\Rightarrow \frac{dx}{dt}=p\cos t-q\sin t,$ $\quad \frac{d^2x}{dt^2}=-p\sin t-q\cos t$

$-p\sin t-q\cos t+2(p\cos t-q\sin t)+p\sin t+q\cos t=4\sin t$

$\Rightarrow 2p=0 \Rightarrow p=0,\ -2q=4 \Rightarrow q=-2$

GS: $x=(A+Bt)e^{-t}-2\cos t$ ✓

(ii) $x \approx -2\cos t$ ✓ ($\because$ as $t\to\infty$, $e^{-t}\to 0$)

15. $\frac{d^2x}{dt^2}+6\frac{dx}{dt}+9x=18t^2+6t+1$

$m^2+6m+9=0 \Rightarrow (m+3)^2=0 \Rightarrow m=3$

CF: $x=Ae^{-3t}+Bte^{-3t}$

$x=pt^2+qt+r \Rightarrow \frac{dx}{dt}=2pt+q$ and $\frac{d^2x}{dt^2}=2p$

$\Rightarrow 2p+6(2pt+q)+9(pt^2+qt+r)=18t^2+6t+1$

$\Rightarrow 9pt^2+(9q+12p)t+(2p+6q+9r)=18t^2+6t+1$

$\Rightarrow p=2,\ q=-2,\ r=1$

$\therefore$ PI $=2t^2-2t+1$

GS: $x=Ae^{-3t}+Bte^{-3t}+2t^2-2t+1$

$x=3$, when $t=0 \Rightarrow A=2$

$\frac{dx}{dt}=-3Ae^{-3t}-3Bte^{-3t}+Be^{-3t}+4t-2$

$x=0$, when $t=0 \Rightarrow B=8$

$\therefore x=2e^{-3t}+8te^{-3t}+2t^2-2t+1$ ✓

16. $\frac{d^2x}{dt^2}+2\frac{dx}{dt}+5x=4-5t^2$

CF: $m^2+2m+5=0 \Rightarrow m=-1\pm 2i$

$e^{-t}(A\cos 2t+B\sin 2t)$

PI: $x=pt^2+qt+r \Rightarrow \frac{dx}{dt}=2pt+q$

$\Rightarrow \frac{d^2x}{dt^2}=2p$

$2p+4pt+2q+5pt^2+5qt+5r=4-5t^2$

$\Rightarrow p=-1,\ q=4/5,\ r=22/25$

GS: $x=e^{-t}(A\cos 2t+B\sin 2t)+\frac{22}{25}+\frac{4}{5}t-t^2$ ✓

17(i) $y=Kx^2e^{2x}$ — (1)

and $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=4e^{2x}$ — (2)

diff (1) $\frac{dy}{dx}=2Kxe^{2x}+2Kx^2e^{2x}$

$\frac{d^2y}{dx^2}=2Ke^{2x}+8Kxe^{2x}+4Kx^2e^{2x}$

from (2) $2Ke^{2x}+8Kxe^{2x}+4Kx^2e^{2x}-8Kxe^{2x}-8Kx^2e^{2x}+4Kx^2e^{2x}=4e^{2x}$

$\Rightarrow 2K=4 \Rightarrow K=2$ ✓

(ii) $m^2-4m+4=0 \Rightarrow (m-2)^2=0 \Rightarrow m=2$

CF: $y=Ae^{2x}+Bxe^{2x}$

GS: $y=Ae^{2x}+Bxe^{2x}+2x^2e^{2x}$

(iii) $y=3$, when $x=0 \Rightarrow A=3$

$\frac{dy}{dx}=2Ae^{2x}+B(e^{2x}+2xe^{2x})+(4xe^{2x}+4x^2e^{2x})$

$\frac{dy}{dx}=-2$ when $x=0$ and $A=3$ $\Rightarrow B=-8$

$\Rightarrow y=3e^{2x}-8xe^{2x}+2x^2e^{2x}$ ✓

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Answers

18. $\frac{d^2x}{dt^2} + 7\frac{dx}{dt} + 10x = 116\sin 2t$

$m^2 + 7m + 10 = 0 \Rightarrow (m+2)(m+5) = 0$

$\Rightarrow m = -2$ or -5

CF: $Ae^{-2t} + Be^{-5t}$

PI: $x = p\sin 2t + q\cos 2t$

$\Rightarrow \frac{dx}{dt} = 2p\cos 2t - 2q\sin 2t$

$\Rightarrow \frac{d^2x}{dt^2} = -4p\sin 2t - 4q\cos 2t$

Substituting $\left.\begin{aligned} 14p + 6q &= 0 \\ 6p - 14q &= 116 \end{aligned}\right\}$

$\Rightarrow p = 3, q = -7 \Rightarrow x = 3\sin 2t - 7\cos 2t$

GS. $x = Ae^{-2t} + Be^{-5t} + 3\sin 2t - 7\cos 2t$

for large values of t, $t \to \infty$, $e^{-2t} \to 0$

$x \approx 3\sin 2t - 7\cos 2t$ ✓

19. (i) $V = \frac{1}{y} \Rightarrow \frac{dV}{dx} = -\frac{1}{y^2}\frac{dy}{dx}$ — ①

$\frac{d^2V}{dx^2} = -\frac{1}{y^2}\frac{d^2y}{dx^2} + \frac{2}{y^3}\left(\frac{dy}{dx}\right)^2$ — ②

Now $\frac{2}{y^3}\left(\frac{dy}{dx}\right)^2 - \frac{1}{y^2}\frac{d^2y}{dx^2} - \frac{2}{y^2}\frac{dy}{dx} + \frac{5}{y}$

$= 17 + 6x - 5x^2$

for ① & ②

$\Rightarrow \frac{d^2V}{dx^2} + 2\frac{dV}{dx} + 5V = 17 + 6x - 5x^2$ ✓

(ii) $m^2 + 2m + 5 = 0 \Rightarrow m = -1 \pm 2i$

$\Rightarrow$ CF: $e^{-x}(A\cos 2x + B\sin 2x)$

PI: $V = px^2 + qx + r \Rightarrow \frac{dV}{dx} = 2px + q$

$\frac{d^2V}{dx^2} = 2p$

Equating the coefficients:

$5p = -5$, $4p + 5q = 6$,

$2p + 2q + 5r = 17$

$\Rightarrow p = -1, q = 2, r = 3$

G.S. $V = e^{-x}(A\cos 2x + B\sin 2x)$

$+ 3 + 2x - x^2$ ✓

When $x = 0, y = \frac{1}{2} \Rightarrow V = 2$ and $\frac{dy}{dx} = -1$

$\Rightarrow \frac{dV}{dx} = 4$ ✓

20. $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} - 10x = 2\sin t - 3\cos t$

$m^2 - 3m - 10 = 0 \Rightarrow (m-5)(m+2) = 0$

$m = 5, -2$

CF: $x = Ae^{5t} + Be^{-2t}$

PI: $x = p\sin t + q\cos t$

$\Rightarrow \frac{dx}{dt} = p\cos t - q\sin t \Rightarrow \frac{d^2x}{dt^2} = -p\sin t - q\cos t$

$\Rightarrow -11p + 3q = 2$, $3p + 11q = 3$

$\Rightarrow p = -0.1, q = 0.3$

GS: $x = Ae^{5t} + Be^{-2t} + 0.3\cos t - 0.1\sin t$

Initial conditions. $t = 0$, $x = 3.3$, $\frac{dx}{dt} = 0.9$

$\Rightarrow A + B + 0.3 = 3.3 \Rightarrow A + B = 3$ ✓

And $\frac{dx}{dt} = 5Ae^{5t} - 2Be^{-2t} - 0.3\sin t$

$-0.1\cos t$

$\Rightarrow 5A - 2B - 0.1 = 0.9$

$\Rightarrow 5A - 2B = 1$ ✓

Solving $A = 1, B = 2$

$\Rightarrow x = e^{5t} + 2e^{-2t} + 0.3\cos t - 0.1\sin t$ ✓

21. $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = 7 - 2t^2$

$m^2 + 4m + 4 = 0 \Rightarrow (m+2)^2 = 0 \Rightarrow m = -2$

CF: $x = pt^2 + qt + r \Rightarrow \frac{dx}{dt} = 2pt + q$

$\frac{d^2x}{dt^2} = 2p$

$\Rightarrow 2p + 8pt + 4q + 4pt^2 + 4qt + 4r$

$= 7 - 2t^2$

$\Rightarrow p = -\frac{1}{2}, q = 1, r = 1$

GS: $x = Ae^{-2t} + Bte^{-2t} - \frac{1}{2}t^2 + t + 1$ ✓

← X — X →