IG-Maths 0580

Algebra-1

Contents

—x—x—x—x—

Note: Following topics are in Algebra-2.

1. Direct and inverse varial.
2. Graphs in practical situation
3. Graphs of functions.
4. Simple functions and composite functions.

Algebra

Algebra an introduction :

In the primary classes you have been doing 'Arithmetic' and 'Geometry'
Now you will study 'Algebra', which ~~is~~ more in a generalised form.

Example: Father's age is 20 years more than twice the son's age, if the son's age is 15 year, find the age of father.

Father's age $= 20 + 2 \times 15 = 50$ years

But if the statement chaged as:
Father's age is 20 years more than twice the son's age, find the age of father and the age of son.
This statement can mathematically be written as:

$F = 20 + 2S$ --① where symbol 'F' denotes father's age.
and symbol 'S' denotes son's age.

No doubt if $S = 15$ then $F = 50$
but there are many more posibilities: $S = 20$ then $F = 60$
$S = 30$, then $F = 80$ ----

here we find that the values of 'F' and 'S' keep varying.

In algebra we use :

§ Constants: $2, \frac{3}{4}, 7, \pi$ (absolute constants)
or may be a, b, c, α, β, -- (arbitrary constants)

and

§ Variables: x, y, z, θ, ϕ, --

Note : Here a, b, c and x, y, z, -- both can be replaced by numerical values. Then what is the difference ?

Example: $y = mx + c$
here x and y are variables and m and c are arbitrary contants.
Then what is the difference ?
(This we shall discuss later - Unit: "Functions and Graphs")

Algebra

§ **Algebraic Expression:**

A expression which is a combination of variables and constants connected by operations of addition, subtraction, multiplication and division.

Example:

$3a$; $\frac{5}{3}x^2+7$; $a^2+3xy+7$ ---

Terms: Given an algebric expression $x^2+2xy-y^2$ then x^2, $2xy$, $-y^2$ are its terms.

§ **Finding the Value of an Algebric Expression:**

Example 1: $y=\sqrt{8+4/x}$, Find the value of y when $x=2$, --[2] give your answer correct to 4 decimal places. | S-13/23/Q20 |

Solution $y=\sqrt{8+4/x}=\sqrt{8+4/2}$ when $x=2$

$=\sqrt{8+2}=\sqrt{10}=3.162277=\pm 3.1623$ ✓

Example 2: $y=p^2+qr$ --[2]

Find the value of y when $p=-5$, $q=3$ and $r=-7$ | W-16/21/Q18 |

Solution $y=p^2+qr=(-5)^2+(3)\times(-7)$ [when $p=-5$, $q=3$ and $r=-7$

$=25-21$

$=4$ ✓

Example 3: Find the value of s, when $u=2$ and $t=3$ --[2]

for $s=ut+16t^2$ | M-17/22/Q5 |

Solution: $s=ut+16t^2$

$=2\times3+16\times3^2$ [when $u=2$ and $t=3$

$=6+144$

$=150$ ✓

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Algebra

§ Changing the subject of a formula:

Example 1: Make q the subject of the formula. $p = 2q^2$ --- [2]

S-17/22/Q15

Solution: $p = 2q^2 \Rightarrow q^2 = \frac{p}{2}$

$\Rightarrow q = \pm\sqrt{\frac{p}{2}}$ ✓

Example 2: Make p the subject of the formula: $rp + 5 = 3p + 8r$ --- [3]

S-16/22/Q10

Solution: Given. $rp + 5 = 3p + 8r$

or $rp - 3p = 8r - 5$

or $p(r-3) = 8r - 5 \Rightarrow p = \frac{8r-5}{r-3}$ ✓

Example 3. Rearrange $y = \sqrt{8 + 4/x}$ to make x the subject. --- [4]

S-13/23/Q

Solution: $y = \sqrt{8 + \frac{4}{x}}$

squaring $y^2 = 8 + \frac{4}{x}$

$\Rightarrow \frac{4}{x} = y^2 - 8$

$\Rightarrow \frac{x}{4} = \frac{1}{(y^2-8)}$

$\Rightarrow x = \frac{4}{(y^2-8)}$ ✓

Example 4: Make x the subject of the formula $y = (x-4)^2 + 6$ -- [3]

S-14/21/Q 7

Solution: Given,

$y = (x-4)^2 + 6$

or $(x-4)^2 = y - 6$

$x - 4 = \pm\sqrt{y-6}$ (Taking sq root)

or $x = 4 \pm \sqrt{y-6}$ ✓

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Algebra

§ Like and Unlike terms:

The terms having the same symbol (a, b, c, - x, y, z) factors are like terms.

Example: (i) $3ab$, $7ab$ (ii) $7x^2y$; $\frac{2}{3}x^2y$
(iii) $5abc^2$, $2abc^2$

are pairs of like terms.

but $3ab$; $5bc$ are unlike terms.

§ Operation on Algebric Expressions:

§ Addition of Algebric expressions:

Example 1. Add $3x^2y - 5y^2$ and $7x^2y + 3y^2$

Solution:

$$(3x^2y - 5y^2) + (7x^2y + 3y^2)$$
$$= 3x^2y + 7x^2y - 5y^2 + 3y^2$$
$$= 10x^2y - 2y^2$$

or

$$\begin{array}{r} 3x^2y - 5y^2 \\ +\ 7x^2y + 3y^2 \\ \hline 10x^2y - 2y^2 \end{array}$$

Example 2: Simplify $1 - 2u + u + 4$ --- [2]

[W-15 | 23 | Q6]

Solution:

$$1 - 2u + u + 4$$
$$= -2u + u + 4 + 1$$
$$= -u + 5 \quad \text{or} \quad (5 - u)$$

Example 3: Expand the bracket and simplify. --- [2]

$4(5w+3) - 2(w-1)$

[M-17 | 22 | Q1]

Solution:

$$4(5w+3) - 2(w-1)$$
$$= 4\times5w + 4\times3 - (2w - 2)$$
$$= 20w + 12 - 2w + 2$$
$$= 20w - 2w + 12 + 2$$
$$= 18w + 14$$

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Example 4. Expand and simplify: $x(2x+3)+5(x-7)$ --[2]

Solution: consider. [S-15/21/Q4]

$$x(2x+3)+5(x-7)$$
$$= 2x\cdot x+3x+5x-35 \qquad [x\cdot x = x^2]$$
$$= 2x^2+8x-35$$

Example 5: Expand the bracket and simplify. $(5-n)(3+n)$ --[2]

Solution [W-17/22/Q12]

$$(5-n)(3+n) = 5(3+n)-n(3+n)$$
$$= 15+5n-3n-n^2$$
$$= \underline{15+2n-n^2}$$

Some Algebric Identities

§

1. $(a+b)^2 = a^2+2ab+b^2$
2. $(a-b)^2 = a^2-2ab+b^2$
3. $(a+b)(a-b) = a^2-b^2$

$\because (a+b)(a+b)$
$= a(a+b)+b(a+b)$
$= a\cdot a+ab+ba+b\cdot b$
$= a^2+ab+ab+b^2$
$= \underline{a^2+2ab+b^2}$

$\because (a+b)(a-b)$
$= a(a-b)+b(a-b)$
$= a\cdot a-a\cdot b+b\cdot a-b\cdot b$
$= a^2-\cancel{ab}+\cancel{ab}-b^2$
$= a^2-b^2$

Algebra

§ Factorising:

Example 1. Factorise Completely:

(a) $15c^2 - 5c$ --- [2]

(b) $2kp - km + 6p - 3m$ [M-17 | 22 | Q13] -- [2]

Solution: (a) $15c^2 - 5c$

$= 5c(3c-1)$ ✓ $[\because ab - a = a(b-1)]$

(b) $2kp - km + 6p - 3m$

$= k(2p-m) + 3(2p-m)$

$= (2p-m)(k+3)$ ✓

Example 2. Factorise $14x - 21y$ [S-17 | 22 | Q6] --- [1]

$= 7(2x-3y)$ ✓

Example 3. Factorise completely.

(a) $9t^2 - u^2$

(b) $2c - 4d - pc + 2pd$

Solution: (a)

$9t^2 - u^2$

$= (3t)^2 - (u)^2$ $[\because a^2 - b^2 = (a+b)(a-b)]$

$= (3t+u)(3t-u)$ ✓

(b) $2c - 4d - pc + 2pd$

$= 2(c-2d) - p(c-2d)$ $[\because -ab + ac$

$= (c-2d)(2-p)$ ✓ $[\quad -a(b-c)$

Example 4.

Factorise. $18x + 27y$ [W-17 | 23 | Q5] --- [1]

$= 9(2x+3y)$ ✓

Example 5: Factorise:

(a) $2a + 4 + ap + 2p$ --- [2]

$= 2(a+2) + p(a+2) = (a+2)(2+p)$ ✓ [S-16 | 21 | Q24]

(b) $162 - 8t^2$

$= 2(81 - 4t^2)$ --- [2]

$= 2[9^2 - (2t)^2]$ $[\because a^2 - b^2 = (a+b)(a-b)$

$= 2(9+2t)(9-2t)$ ✓

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Algebra

§ Factorising: Quadratic Expression

$$ax^2 + bx + c$$

When $a = 1$: $x^2 + bx + c$

When $a \neq 1$: $ax^2 + bx + c$

When $a = 1$

Case I When c is +

Take two factors of c whose sum is b

Example 1. $x^2 + 7x + 12$

$= x^2 + 4x + 3x + 12$ $\quad \{ \because 12 = 4\times 3$ and $4+3 = 7$

$= x(x+4) + 3(x+4)$

$= (x+4)(x+3)$ ✓

Example 2. $x^2 - 13x + 42$

$= x^2 - 7x - 6x + 42$ $\quad \{ 7\times 6 = 42,\ 7+6 = 13$

$= x(x-7) - 6(x-7)$

$= (x-7)(x-6)$ ✓

Case II When constant c is −ve

Take two factors of c whose numerical difference is b

Example 3: $x^2 + 4x - 12$

$= x^2 + 6x - 2x - 12$ $\quad [\, 6\times 2 = 12,\ 6-2 = 4$

$= x(x+6) - 2(x+6)$

$= (x+6)(x-2)$ ✓

Example 4. $x^2 - x - 42$

$= x^2 - 7x + 6x - 42$ $\quad [\, 7\times 6 = 42,\ 7-6 = 1$

$= x(x-7) + 6(x-7)$

$= (x-7)(x+6)$ ✓

When $a \neq 1$

Case I Consider the product 'ac' > 0

Rearrage 'ac' such that the product is ac but sum is b.

Example 1'. $2x^2 + 7x + 6$ $\quad \{$ Consider $2\times 6 = 12$ ✓ Now $4\times 3 = 12$, $4+3 = 7$

$= 2x^2 + 4x + 3x + 6$

$= 2x(x+2) + 3(x+2)$

$= (x+2)(2x+3)$ ✓

Example 2': $3x^2 - 13x + 14$ $\quad \{$ Consider $3\times 14 = 42$ Now $7\times 6 = 42$, $7+6 = 13$

$= 3x^2 - 7x - 6x + 14$

$= x(3x-7) - 2(3x-7)$

$= (3x-7)(x-2)$

Case II Consider the product $ac < 0$

'ac' such that the product is 'ac' but numerical difference is b.

Example 3': $12x^2 + 4x - 1$ $\quad \{$ Consider $12\times(-1) = -12$ Now $6\times 2 = 12$, $6-2 = 4$

$= 12x^2 + 6x - 2x - 1$

$= 6x(2x+1) - 1(2x+1)$

$= (2x+1)(6x-1)$ ✓

Example 4'. $7x^2 - x - 6$ $\quad [\, 7\times 6 = 42,\ 7-6 = 1$

$= 7x^2 - 7x + 6x - 6$

$= 7x(x-1) + 6(x-1)$

$= (x-1)(7x+6)$ ✓

Algebra

§ Factorising:

Example 5. Factorise completely: [W-17/22/Q25]

(a) $x^2 - x - 132$ --- [2]

$= x^2 - 12x + 11x - 132$

$= x(x-12) + 11(x-12)$

$= (x-12)(x+11)$

$\because$ $12 \times 11 = 132$ and $12 - 11 = 1$

(b) $x^3 - 4x$ --- [2]

$= x(x^2-4)$

$= x[x^2 - 2^2]$ $\because a^2-b^2 = (a+b)(a-b)$

$= x(x+2)(x-2)$

Example 6. Factorise: [W-16/22/Q13]

(a) $m^3 + m$ --- [1]

$= m(m^2+1)$

(b) $25 - y^2 = 5^2 - y^2$ --- [1]

$= (5+y)(5-y)$

(c) $x^2 + 3x - 28$

$= x^2 + 7x - 4x - 28$

$= x(x+7) - 4(x+7)$

$= (x+7)(x-4)$

$7 \times 4 = 28$
$7 - 4 = 3$

Example 7. Factorise: $2x^2 - 5x - 3$ [S-15/23/Q5] --- [2]

$= 2x^2 - 6x + x - 3$

$= 2x(x-3) + 1(x-3)$

$= (x-3)(2x+1)$

$2 \times (-3) = -6$
Now $-6 \times 1 = -6$
$-6 + 1 = -5$

Example 8. Factorise:

$18x^2 - 287x - 323$ --- [2] [W-16/41/Q8]

$= 18x^2 - 306x + 19x - 323$

$= 18x(x-17) + 19(x-17)$

$= (x-17)(18x+19)$

$18 \times 323 = 18 \times 17 \times 19$ [$\because 323 = 17 \times 19$]
$= 306 \times 19$
and $306 - 19 = 287$

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Algebra

§ Algebric Fractions:

Write as a single fraction in its simplest form.

Example1(a) $\frac{x^2-3x}{x^2-9}$ --- [3] S-17/21/Q22

$= \frac{x\cancel{(x-3)}}{(x+3)\cancel{(x-3)}}$

$\left\{ \because x^2-9 = x^2-3^2 = (x+3)(x-3) \right.$

$= \frac{x}{(x+3)}$ ✓

(b) $\frac{3}{x-4} + \frac{2}{2x+5}$ ---[3]

$= \frac{3(2x+5)+2(x-4)}{(x-4)(2x-5)}$

$= \frac{6x+15+2x-8}{(x-4)(2x-5)}$

$= \frac{8x+7}{(x-4)(2x-5)}$ ✓

Example2: $\frac{5}{x-3} + \frac{3}{x+7} + \frac{1}{2}$ ---[4] W-17/21/Q19

$= \frac{5\times2(x+7)+3\times2(x-3)+(x-3)(x+7)}{2(x-3)(x+7)}$

$= \frac{10x+70+6x-18+x(x+7)-3(x+7)}{2(x-3)(x+7)}$

$= \frac{16x+52+x^2+7x-3x-21}{2(x-3)(x+7)}$

$= \frac{x^2+20x+31}{2(x-3)(x+7)}$ ✓

Example3. $\frac{x^3y+2xy^3}{x^2y^2}$ W-16/21/Q7 ---[2]

$= \frac{xy(x^2+2y^2)}{x^2y^2}$

$\left[\because \frac{xy}{x^2y^2} = \frac{1}{xy} \right.$

$= \frac{x^2+2y^2}{xy}$ ✓

§ Algebric Frations:

Example 4 (b) Simplify. $\dfrac{4x^2-16x}{2x^2+6x-56}$ [W-14/22/Q21] ---[4]

$= \dfrac{4x(x-4)}{2(x^2+3x-28)}$

$= \dfrac{4x\cancel{(x-4)}}{2(x+7)\cancel{(x-4)}}$

$= \dfrac{2x}{(x+7)}$

Consider $2\times 56 = 2\times 4\times 14$

$\because 2x^2+6x-56$
$= 2x^2+x-8x-56$
$= 2x(x+7)-8(x+7)$
$= (x+7)(2x-8)$
$= (x+7)\cdot 2(x-4)$

Example 5 (b). Simply: [SP-15/04/Q2] ---(4)

$\dfrac{x^2+3x-10}{x^2-25}$

$= \dfrac{(x+5)(x-2)}{x^2-5^2}$

$= \dfrac{\cancel{(x+5)}(x-2)}{\cancel{(x+5)}(x-5)}$

$= \dfrac{x-2}{x-5}$ ✓

$\because x^2+3x-10$
$= x^2+5x-2x-10$
$= x(x+5)-2(x+5)$
$= (x+5)(x-2)$

Example 6: Write as a single fraction in its simplest form. ---[5]
[M-16/42/Q8]

$\dfrac{x-2}{x+1} - \dfrac{x+3}{x-1}$

$= \dfrac{(x-2)(x-1)-(x+3)(x+1)}{(x+1)(x-1)}$

$= \dfrac{x(x-1)-2(x-1)-\{x(x+1)+3(x+1)\}}{x^2-1}$

$= \dfrac{x^2-x-2x+2-\{x^2+x+3x+3\}}{x^2-1}$

$= \dfrac{x^2-3x+2-(x^2+4x+3)}{x^2-1}$

$= \dfrac{\cancel{x^2}-3x+2-\cancel{x^2}-4x-3}{x^2-1} = \dfrac{-7x-1}{x^2-1}$ ✓

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Algebra

§ Algebric Indices

§ Laws of Indices:

(i) $a^m \times a^n = a^{m+n}$

(ii) $\frac{a^m}{a^n} = a^{m-n}$

(iii) $(a^m)^n = a^{m.n}$

(iv) $a^0 = 1$

(v) $a^{-n} = \frac{1}{a^n}$

(vi) $(ab)^n = a^n . b^n$

(vii) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

(viii) $a^{1/n} = \sqrt[n]{a}$

(ix) $a^{m/n} = (a^{1/n})^m$

$= (\sqrt[n]{a})^m$

or $= (a^m)^{1/n}$

$= \sqrt[n]{a^m}$

Example 1. Work out the value of x. $\boxed{S-16/42/Q2}$

(i) $3^x = 243$ ---[1]

or $3^x = 3^5$

$\therefore x = 5$ ✓

(ii) $16^x = 4$ ---[1]

or $(4^2)^x = 4$

or $4^{2x} = 4^1 \Rightarrow 2x = 1$

$\Rightarrow x = 1/2$ ✓

(iii) $8^x = 32$ ---[2]

or $(2^3)^x = 2^5$

or $2^{3x} = 2^5 \Rightarrow 3x = 5$

$\Rightarrow x = 5/3$ ✓

(iv) $27^x = \frac{1}{9}$ ---[2]

$(3^3)^x = \frac{1}{3^2}$

$\Rightarrow 3^{3x} = 3^{-2} \Rightarrow 3x = -2$

$\Rightarrow x = -2/3$ ✓

Example 2: Simplify: (i) $x^0 = 1$ ✓ --(1) $\boxed{W-17/43/Q2(b)}$

(ii) $x^7 \times x^3 = x^{10}$ ✓ --(1)

(iii) $\frac{(3x^6)^2}{x^{-4}} = 3^2 . (x^6)^2 . x^{-(-4)}$ --- [2]

$= 9 . x^{12} . x^4 = 9 x^{12+4} = 9x^{16}$ ✓

Algebra

Algebric Indices

M-17/22/Q15

Example 3. Workout: (a) $t^{24} \div t^4$ ---[1]

$= t^{24-4} = \underline{t^{20}}$ ✓

(b) $(x^5)^2$ ---[1]

$= x^{5\times 2} = \underline{x^{10}}$ ✓

(c) $(81m^8)^{3/4}$ ---[2]

$= (3^4.(m^2)^4)^{3/4} = (3^4)^{3/4}.((m^2)^4)^{3/4}$

$= 3^{\cancel{4}\times 3/\cancel{4}}.(m^2)^{\cancel{4}\times 3/\cancel{4}}$

$= 3^3 . (m^2)^3$

$= 27 . m^{2\times 3} = \underline{27m^6}$ ✓

Example 4: Simplify. (a) $(m^5)^2$ ---[1] W-17/21/Q13

$= m^{5\times 2} = m^{10}$ ✓

(b) $4x^3y \times 5x^2y$ ---[2]

$= 4\times 5\, x^3 . x^2\, y . y \quad = 20\, x^{3+2} . y^{1+1}$

$= 20\, x^5 . y^2$ ✓

Example 5 (a) Find the value of x, $3^x = \sqrt[4]{3^5}$ ---[1]

W-13/21/Q13

$\Rightarrow 3^x = 3^{5/4}$

$\Rightarrow x = \frac{5}{4}$ ✓

(b) Simplify: $(32y^{15})^{2/5}$ ---[2]

$= (2^5 y^{15})^{2/5} = (2^5)^{2/5} . (y^{15})^{2/5}$

$= 2^{5\times 2/5} . y^{15\times \frac{2}{5}}$

$= 2^2 . y^6$

$= 4y^6$ ✓

Example 6: $5^7 \div 5^9 = p^2$ find p ---[2]

$\Rightarrow 5^{7-9} = p^2 \Rightarrow p^2 = 5^{-2} \Rightarrow p^2 = \frac{1}{5^2} = (\frac{1}{5})^2$

$\Rightarrow p = \frac{1}{5}$ ✓

W-13/23/Q14(b)

Algebraic Indices

Example 7. Simplify. $\left(\frac{16}{9x^4}\right)^{-3/2}$ ---[2]

S-15/42/Q9(c)

$$= \frac{(4^2)^{-3/2}}{(3^2 x^4)^{-3/2}}$$

$$= \frac{(4^2)^{-3/2}}{(3^2)^{-3/2} \times (x^4)^{-3/2}}$$

$$= \frac{4^{2\times(-\frac{3}{2})}}{3^{2\times(-3/2)} \cdot x^{4\times(-3/2)}}$$

$$= \frac{4^{-3}}{3^{-3} \cdot x^{-6}} = \frac{3^3 \cdot x^6}{4^3}$$

$$= \frac{27}{64} \cdot x^6 \checkmark$$

Example 8. Find the Value of x

W-15/41/Q2(b)

(i) $2^x = 128 \Rightarrow 2^x = 2^7 \Rightarrow x = 7 \checkmark$ ---[1]

(ii) $2^x \times 2^9 = 2^{13} \Rightarrow 2^{x+9} = 2^{13} \Rightarrow x+9 = 13$ ---[1]

$\Rightarrow x = 4 \checkmark$

(iii) $2^9 \div 2^x = 4 \Rightarrow 2^{9-x} = 2^2 \Rightarrow 9-x = 2 \Rightarrow x = 7 \checkmark$ ---[1]

(iv) $2^x = \sqrt[3]{2} \Rightarrow 2^x = 2^{1/3} \Rightarrow x = 1/3 \checkmark$ ---[1]

Example 9. Simplify,

W-14/43/Q6

(i) $x^3 \div \frac{3}{x^5}$

$= x^3 \times \frac{x^5}{3}$

$= \frac{x^{3+5}}{3}$

$= \frac{x^8}{3}$

(ii) $5xy^8 \times 3x^6 \cdot y^{-5}$ ---[2]

$= 5 \times 3 \times x \cdot x^6 \cdot y^8 \cdot y^{-}$

$= 15 \cdot x^{1+6} \cdot y^{8-}$

$= 15\, x^7 y^3$

(iii) $(64x^{12})^{2/3}$ ---[2]

$= 64^{2/3} \cdot (x^{12})^{2/3}$

$= (4^3)^{2/3} \cdot x^{12\times 2/3}$

$= 4^{3\times 2/3} \cdot x^8 = 4^2 x^8 = 16x^8 \checkmark$

Algebra

§ Solution of Linear equation in one variable:

Example 1. Solve: $2-x = 5x+1$ ---[2] S-17/22/Q10

or $5x+1 = 2-x$

or $5x+x = 2-1$

or $6x = 1 \Rightarrow x = 1/6$

Example 2 (a) Solve: $7-3n = 11n+2$ ---[2] W-17/22/Q24

$\Rightarrow -3n-11n = 2-7$

$\Rightarrow -14n = -5$

$\Rightarrow n = \frac{+5}{+14} = \frac{5}{14}$ ✓

(b) Solve: $\frac{p-3}{5} = 3$ ---[2]

or $p-3 = 3\times5$

or $p = 15+3$

or $p = 18$ ✓

Example 3. Solve: $\frac{x+5}{x} = \frac{7}{3}$ ---[3] W-14/21/Q10

or $7x = 3(x+5)$

or $7x = 3x+15$

or $7x-3x = 15$

or $4x = 15$

or $x = \frac{15}{4}$ or $3\frac{3}{4}$ ✓ or 3.75 ✓

Example 4. Solve. $\frac{3}{2x} + \frac{1}{x+1} = 0$ ---[3] S-14/22/Q12

or $\frac{3(x+1)+2x}{2x(x+1)} = 0$

$\left\{ \frac{a}{b} = 0 \Rightarrow a = 0 \right.$ if $b \neq 0$

or $3(x+1)+2x = 0$

or $3x+3+2x = 0$

or $5x = -3$

or $x = -3/5$ ✓

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Algebra.

§ Solution of Linear Equation in One Variable:

Example 5: (i) Solve (i) $\frac{2x}{3}+5=-7$ --- [3] [S-17/41/Q6(b)]

or $\frac{2x}{3}=-7-5$

or $2x=-12\times3$

or $x=-\frac{36}{2}=-18$

$\therefore x=-18$ ✓

Solve (ii) $4x+9=3(2x-7)$

or $4x+9=6x-21$

or $4x-6x=-21-9$

or $-2x=-30$

or $x=+\frac{30}{+2}=15$ $\therefore x=15$ ✓

Example 6. (i) Solve, $2(3x-7)=13$ ---[3] [S-13/43/Q(a)(i)(ii)]

or $6x-14=13$

or $6x=13+14$

or $6x=27$

or $x=\frac{27}{6}=\frac{9}{2}$ or $4\frac{1}{2}$

$\therefore x=4\frac{1}{2}$ or 4.5 ✓

(ii) Solve, $\frac{3x-2}{5}+\frac{x+2}{10}=4$ ... [4]

or $\frac{2(3x-2)+x+2}{10}=4$ L.C.M of 5 & 10 = 10

or $6x-4+x+2=4\times10$

or $7x-2=40$

$7x=40+2$

$x=42/7=6$

$\therefore x=6$ ✓

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Algebra.

§ Solving Simultaneous Linear Equations in Two Unknowns: Elimination Method:

Example 1. Solve the simultaneous equations. $\boxed{S\text{-}14|22|Q3}$ --- [2]

$2x - y = 7$ ---(i)

$3x + y = 3$ ---(ii)

add equation (i) and (ii) we get, $5x = 10$ [∵ $-\cancel{y}+\cancel{y}$ cancells

$\Rightarrow x = \frac{10}{5} = 2$ ✓

Now put $x = 2$ in equ^n (1)

$2 \times 2 - y = 7$

$4 - y = 7 \Rightarrow -y = 7 - 4$

$-y = 3$

$\Rightarrow y = -3$

∴ Req. solution: $x = 2$ and $y = -3$

Example 2: Solve the simultaneous equation: $\boxed{S\text{-}13|23|Q10}$ ---- [4]

$3x + 5y = 48$ ---(i) and hence find the value of $2x+y$.

$2x - y = 19$ ---(2) ×5

multiply equation (2) by 5.

$10x - \cancel{5y} = 95$ ---(iii)

from (i) $3x + \cancel{5y} = 48$ ---(i)

add equations (i) and (iii)

$13x = 143$

or $x = \frac{143}{13} = 11$ ✓

Now put $x = 11$ in equation (i)

$3 \times 11 + 5y = 48$

or $5y = 48 - 33 = 15$

or $y = \frac{15}{5} = 3$ ✓

∴ $x = 11$ and $y = 3$

Now for these values of x and y,
the value of the expression, $2x + y$

$= 2 \times 11 + 3$

$= 22 + 3 = \underline{25}$ ✓

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Algebra

§ Solving simultaneous Linear Equations:

Example 3. Find the coordinates of the point of intersection of the two lines: W-13/22/Q15 --- [3]

$$2x - 7y = 2 \quad \text{---(i)} \quad \times 5$$
$$4x + 5y = 42 \quad \text{---(ii)} \quad \times 7$$

multiplying equation (i) by 5 and equⁿ (ii) by 7,

$$10x - 35y = 10 \quad \text{---(iii)}$$
$$28x + 35y = 294 \quad \text{---(iv)}$$

add equations (iii) and (iv), $38x = 304$

$$x = \frac{304}{38} = 8 \checkmark$$

Put $x = 8$ in equation (ii)

$$4 \times 8 + 5y = 42$$
$$5y = 42 - 32$$
$$y = \frac{10}{5} = 2 \checkmark$$

∴ The coordinates of the point of intersection $(8, 2)$. ✓

Example 4: Solve the simutaneous equations: --- [3]

W-15/42/Q5(b)

$$3x - 2y = 23 \quad \text{---(i)}$$
$$-4x - y = -5 \quad \text{---(ii)} \quad \times 2$$

multiply equation (ii) by 2

$$-8x - 2y = -10 \quad \text{---(iii)}$$

from (i) $\quad 3x - 2y = 23 \quad$ ---(i)

Subtracting equation (i) from (iii)
(Sign's of (i) will change)

we get $-11x = -33$

or $x = -33/-11 = 3 \checkmark$

Now put $x = 3$ in (i)

$$3 \times 3 - 2y = 23$$
$$-2y = 23 - 9$$
$$\text{or} \quad -2y = +14$$
$$y = +\frac{14}{-2} = -7 \checkmark$$

∴ $x = 3$ and $y = -7$ ✓

Altertemate-method:

multiply eqⁿ (i) by 4 and (ii) by 3,

$$12x - 8y = 92 \quad \text{--(iv)}$$
$$-12x - 3y = -15 \quad \text{--(v)}$$

add equations (iv) and (v)

$$-11y = 77$$
$$\text{or} \quad y = \frac{77}{-11} = -7 \checkmark$$

Now put $y = -7$ in equⁿ (i)

$$3x - 2(-7) = 23$$
$$3x + 14 = 23$$
$$3x = 23 - 14 = 9$$
$$\therefore x = 9/3 = 3 \checkmark$$

∴ $x = 3$ and $y = -7$ ✓

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Algebra

§ Solving Simultaneous Equations:

Substitution Method

Example 5: Solve the simultaneous equations: ---[3]

W-17|22|Q18

$$y = x/2 \quad \text{---(i)}$$
$$2x - y = 1 \quad \text{---(ii)}$$

Put $y = x/2$ in equation (ii),

$$2x - \frac{x}{2} = 1$$

multiply by 2,

$$4x - x = 2$$
$$3x = 2$$
$$\text{or } x = 2/3 \checkmark$$

Put $x = 2/3$ in equation (i)

$$y = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3} \checkmark$$

$$\left[\because y = x/2 = \frac{1}{2} \times x \right.$$

$$\therefore x = \frac{2}{3} \text{ and } y = 1/3 \checkmark$$

Example 6: Solve the simultaneous equations. ---[3]

W-14/21/Q12

$$0.4x - 5y = 27 \quad \text{---(i)}$$
$$2x + 0.2y = 9 \quad \text{---(ii)}$$

from equation (ii) $2x + \frac{2}{10}y = 9$

$$\text{or } 2x + \frac{1}{5}y = 9$$
$$10x + y = 45 \quad \text{(Multiply by 5)}$$
$$\text{or } y = (45 - 10x) \quad \text{---(iii)}$$

Put $y = (45 - 10x)$ in equation (i)

$$\frac{4}{10}x - 5(45 - 10x) = 27$$
$$\text{or } 4x - 50(45 - 10x) = 270 \quad \text{(multiply by 10)}$$
$$\text{or } 4x - 2250 + 500x = 270$$
$$\text{or } 4x + 500x = 270 + 2250$$
$$504x = 2520$$
$$x = \frac{2520}{504} = 5 \checkmark$$

Put $x = 5$ in (iii) $\Rightarrow y = 45 - 10 \times 5$

$$= 45 - 50 = -5 \checkmark$$

$$\left. \begin{aligned} \therefore x &= 5 \\ y &= -5 \end{aligned} \right\} \checkmark$$

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Algebra

Solving Simultaneous Equations:

Example 7: Solve the simultaneous equations. $\boxed{W-16/22/8}$ ---[3]

$\frac{1}{2}x + y = 8$ ---(i)

$x - 2y = 2$ ---(ii)

from equation (i) $y = (8 - \frac{1}{2}x)$ ---- (iii)

Put $y = (8 - \frac{1}{2}x)$ in equation (ii)

we get, $x - 2(8 - \frac{1}{2}x) = 2$

or $x - 16 + x = 2$

or $2x = 2 + 16$

$x = 18/2 = 9$ ✓

Put $x = 9$ in equation (iii)

$y = 8 - \frac{1}{2} \times 9$

$= 8 - \frac{9}{2}$

$= 8 - 4.5$

$y = 3.5$ ✓

$\therefore x = 9$ and $y = 3.5$

$\frac{1}{2}x + y = 8$ ---(i)

$x - 2y = 2$ ---(ii)

Alternate Method:

multiply equ^n (i) by 2

$x + 2y = 16$ ---(iv)

from (ii) $x - 2y = 2$ ---(ii)

add. equ^n (ii) and (iv)

$2x = 18$

$x = 18/2 = 9$ ✓

Put $x = 2$ in equation (iv)

$+ 2y = 16$

or $2y = 16 - 9 = 7$

$\therefore y = \frac{7}{2} = 3.5$ ✓

$\therefore x = 9$ and $y = 3.5$

Example 8: solve: $\boxed{S-16/22/Q16}$ ---[3]

$3x + 4y = 14$ ---(i)

$5x + 2y = 21$ ---(ii) ×2

multiply equation (ii) by 2

$10x + 4y = 42$ ---(iii)

from (i) $\underline{-3x \mp 4y = -14}$ ---(i)

Subtract the equation (i) from (iii) (change the signs in (i))

$7x = 28$

$x = 4$ ✓

Put $x = 4$ in (i)

$3 \times 4 + 4y = 14$

$4y = 14 - 12$

$y = \frac{2}{4} = \frac{1}{2}$ ✓

$\therefore x = 4$ and $y = \frac{1}{2}$

Note:

To solve two linear equations by elimination-method:

1. The coeff. of y (or x) should be equal.

2. (i) If both coeff of y are positive and equal - then subtract.

(ii) If the coeff. of y are equal but opposite sign then add.

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Algebra

§ Solving Quadratic Equations by Factorisation:

Example 1. Solve by factorising. W-17/23/Q14 ---[3]

$$3x^2-7x-20=0$$

or $3x^2-12x+x-20=0$

$3x(x-4)+5(x-4)=0$

$(x-4)(3x+5)=0$

consider: $3\times 20=-60$, $-3\times4\times5$, $-12\times5=-60$ and $-12+5=-7$

$\therefore$ either $x-4=0$ or $3x+5=0$

$\therefore$ $x=4$ ✓ or $x=-\frac{5}{3}$ ✓

or $x=4$ or $-5/3$ ✓

Example 2. Solve. $3x^2+2x-8=0$ ---[3] S-14/23/Q12

or $3x^2+6x-4x-8=0$

$3x(x+2)-4(x+2)=0$

$(x+2)(3x-4)=0$

$-3\times8=-24$, $-3\times4\times2$

$6\times(-4)=-24$ and $6-4=2$

$\therefore$ either $x+2=0$ or $3x-4=0$

$x=-2$ or $x=4/3=1\frac{1}{3}$

$\therefore x=-2$ or $1\frac{1}{3}$ ✓

Example 3: Solve. $\frac{2}{x+3}+\frac{1}{12}=\frac{3}{2x-1}$ ---[7] W-16/43/Q11

or $\frac{2}{x+3}-\frac{3}{2x-1}=-\frac{1}{12}$

or $\frac{2(2x-1)-3(x+3)}{(x+3)(2x-1)}=-\frac{1}{12}$

or $\frac{4x-2-3x-9}{2x^2-x+6x-3}=-\frac{1}{12}$

$\frac{x-11}{2x^2+5x-3}=\frac{-1}{12}$

$\Rightarrow 12(x-11)=-(2x^2+5x-3)$

$\Rightarrow 12x-132=-2x^2-5x+3$

or $2x^2+5x+12x-132-3=0$

or $2x^2+17x-135=0$

or $2x^2+27x-10x-135=0$

$x(2x+27)-5(2x+27)=0$

$(2x+27)(x-5)=0$

$\Rightarrow 2x+27=0$ or $x-5=0$

$\Rightarrow x=-\frac{27}{2}$ or $x=5$

$\therefore x=5$ or $-\frac{27}{2}$ ✓

Solving Quadratic Equation by Factorisation:

Solving one linear and one quad. equation simultaneously. (Sy-2020)

*Example 4: Solve the simultaneous equations. [SP-20 | 02 | Q28] ---- [5]

$y = 4x+1$ --- (i)

$y = 5x^2 + 4x - 19$ --- (ii)

Equating the values of y from (i) and (ii)

$$5x^2 + 4x - 19 = 4x + 1$$

or $5x^2 + \cancel{4x} - \cancel{4x} - 19 - 1 = 0$

or $5x^2 - 20 = 0$

$5(x^2 - 4) = 0$

$\cancel{5}(x+2)(x-2) = 0$

$\because x^2 - 4 = x^2 - 2^2 = (x+2)(x-2)$

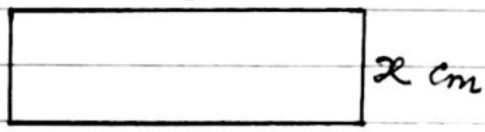
$\therefore$ either $x + 2 = 0$ or $x - 2 = 0$

$\Rightarrow x = -2$ or $x = 2$

from Eqn (i) when $x = -2$, $y = -7$ and $x = 2$, $y = 9$

$\therefore$ Required Solutions $(2, 9)$ and $(-2, -7)$.

Example 5: The perimeter of a rectangle is 80 cm. The area of rectangle is A cm².



(i) Show that $x^2 - 40x + A = 0$ --- [3]

(ii) When $A = 300$, solve the equation $x^2 - 40x + A = 0$ --- [3]

[SP-20 | 04 | Q5]

Solution: Let the length of rect = l cm.

(i) $\therefore$ Area $= l \cdot x = A$ given

$\Rightarrow l = \frac{A}{x}$

$\therefore$ Perimeter = 2 (length + breadth)

$= 2\left(\frac{A}{x} + x\right) = 80$ given.

$\Rightarrow \frac{A}{x} + x = 40$

$\Rightarrow A + x^2 = 40x$

or $x^2 - 40x + A = 0$ ✓ --- (i)

(ii) When $A = 300$

from ① we get

$x^2 - 40x + 300 = 0$

$x^2 - 30x - 10x + 300 = 0$

$x(x - 30) - 10(x - 30) = 0$

$(x - 30)(x - 10) = 0$

either $(x - 30) = 0$ or $(x - 10) = 0$

$x = 30$ or $x = 10$ ✓

§ Solving Quadratic Equations by Factorisation:

Example 6. On the first part of a journey, Alan drove a distance of x Km and his car used 6 litres of fuel. The rate of fuel used by his car was $600/x$ litres per 100 Km.

(a) Alan then drove another $(x+20)$ Km and his car used another 6 litres of fuel.

(i) Write down an expression, in terms of x, for the rate of fuel used by his car on this part of the journey. Give your answer in litres per 100 Km. --- [1]

(ii) On this part of the journey the rate of fuel used by his car decreased by 1.5 litres per 100 Km. Show that $x^2+20x-8000=0$ [4]

(b) Solve the equation $x^2+20x-8000=0$ --- [3]

(c) Find the rate of fuel used by Alan's car for the complete journey. Give your answer in litres per 100 Km. $\boxed{\text{S-15/42/Q3}}$ --- [2]

Solution (i) $(x+20)$ Km used $= 6$ lit

1 Km --- used $= \dfrac{6}{(x+20)}$ lit

100 Km --- used $= \dfrac{6\times 100}{(x+20)}$ lit

(ii) rate of fuel is decreased by 1.5 lit

$\therefore \quad \dfrac{600}{x} - \dfrac{600}{(x+20)} = \dfrac{15}{10}$

$\Rightarrow \dfrac{600(x+20)-600x}{x(x+20)} = \dfrac{3}{2}$

$\Rightarrow \dfrac{\cancel{600x}+12000-\cancel{600x}}{(x^2+20x)} = \dfrac{3}{2}$

$\Rightarrow 3(x^2+20x) = 2\times 12000$

$\rightarrow x^2+20x = \dfrac{24000}{3}$

$\Rightarrow x^2+20x = 8000$

$\Rightarrow x^2+20x-8000=0$ ✓

(b) Solve. $x^2+20x-8000=0$

$x^2+100x-80x-8000=0$

$x(x+100)-80(x+100)=0$

$(x+100)(x-80)=0$

$(x+100)=0$ or $(x-80)=0$

$\therefore x=-100$ ✓ or $x=80$ ✓

(C) distance $x = 80$ Km ✓

Total distance $= x+(x+20)$

$= 2x+20$

$= 2\times 80+20 \quad [\because x=80]$

$= 180$ Km

Total fuel used $= 6+6 = 12$ litres

$\therefore$ fuel used for 1 Km $= \dfrac{12}{180}$

$\therefore$ fuel used for 100 Km $= \dfrac{12}{180}\times 100$

$= \underline{\underline{6.67}}$ lit per 100 Km.

Algebra

§ Solving Quadratic Equation by Using Formula:
(Quadratic formula)

Given a quadratic equation:

$ax^2+bx+c=0$ — a, b, c are real no, $a \neq 0$

Then $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Here (b^2-4ac) is called discriminant. denoted by D.

or $D=(b^2-4ac)$

Case I: If $D>0$ then the quad equation has two real and distict roots.

II If $D=0$ two coincident roots (i,e only one root)

III if $D<0$ then no real roots (solutions)

Example 1. Solve the equation: --- [4]

S-17/23/Q21

$5x^2+10x+2=0$

$a=5,\ b=10,\ c=2$

$b^2-4ac = 10^2-4\times5\times2$
$= 100-40$
$= 60$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{-10 \pm \sqrt{60}}{2\times5}$

$= \dfrac{-10 \pm 7.746}{10}$

$= \dfrac{-10}{10} \pm \dfrac{7.746}{10}$

$= -1 \pm 0.7746$

$= -0.2254$ or -1.7746

$= \underline{-0.23}$; $\underline{-1.77}$ (2 d.p)

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Algebra

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§ Solving Quadratic Equation using Formula:

Example 2. Solve the equation. [M-16/22/Q17] ---[4]

$3x^2 - 11x + 4 = 0$

$a = 3,\ b = -11,\ c = 4$

$b^2 - 4ac = \sqrt{(-11)^2 - 4\times3\times4}$

$= \sqrt{121 - 48}$

$= \sqrt{73}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-11) \pm \sqrt{73}}{2\times3}$$

$$= \frac{11 \pm 8.544}{6}$$

$$= \frac{11 + 8.544}{6} \text{ or } \frac{11 - 8.544}{6}$$

$$= \frac{19.544}{6} \text{ or } \frac{2.456}{6}$$

$$= 3.257 \text{ or } 0.409$$

$$= 3.26 \checkmark \text{ or } 0.41 \checkmark \quad (2\ d.p)$$

Example 3: Solve the equation: [S-13/22/Q15] ---[4]

$2x^2 + 7x - 3 = 0$

$a = 2,\ b = 7,\ c = -3$

$b^2 - 4ac = 7^2 - 4\times2\times(-3)$

$= 49 + 24$

$= 73$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-7 \pm \sqrt{73}}{2\times2}$$

$$= \frac{-7 \pm 8.544}{4}$$

$$= \frac{-7 + 8.544}{4} \text{ or } \frac{-7 - 8.544}{4}$$

$$= \frac{1.544}{4} \text{ or } \frac{-15.544}{4}$$

$$= 0.386 \text{ or } -3.886$$

$$= 0.39 \checkmark \text{ or } -3.89 \checkmark \quad (2dp)$$

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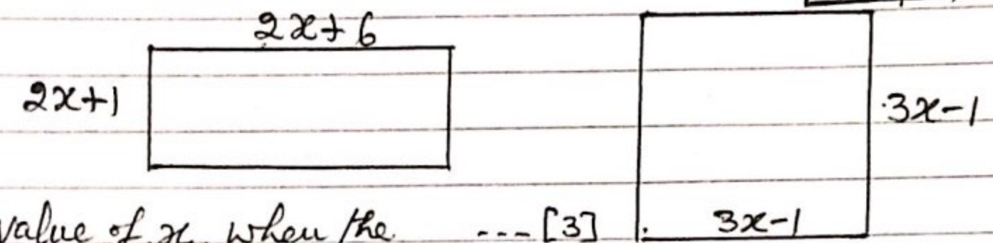
Algebra

§ Solving Quadratic Equations Using Formula:

Example 4. (a) In this part, all lengths are in centimetres. [S-17/42/Q7]



(i) Find the value of, x, when the perimeter of the rectangle is equal to the perimeter of the square. ---[3]

(ii) Find the value of, x, when the area of the rectangle is equal to the area of the square. ---[7]

Solution:

(i) Perimeter of rectangle $= 2(l+b) = 2[2x+6+2x+1]$
$= 2(4x+7)$
$= 8x+14$ ---(i)

and Perimeter of Square $= 4l = 4(3x-1)$
$= 12x-4$ ---(ii)

fr (i) and (ii) perimeters are equal $\therefore$ $8x+14 = 12x-4$

or $8x-12x = -4-14$

or $-4x = -18$

$x = \frac{-18}{-4} = \frac{9}{2} = 4\frac{1}{2}$ ✓

(ii) Area of rectangle = Area of Square

$\therefore$ $(2x+6)(2x+1) = (3x-1)^2$

or $2x(2x+1)+6(2x+1) = (3x)^2 - 2\times3x\times1+1$

or $4x^2+2x+12x+6 = 9x^2-6x+1$

or $4x^2+14x+6-9x^2+6x-1=0$

or $-5x^2+20x+5=0$

or $5x^2-20x-5=0$

or $x^2-4x-1=0$

To Solve it Using formula

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $\quad a=1,\ b=-4,\ c=-1$

$b^2-4ac = (-4)^2-4\times1\times(-1) = 16+4=20$

$x = \frac{-(-4)\pm\sqrt{20}}{2\times1}$

or $x = \frac{4\pm4.472}{2}$

or $x = \frac{8.472}{2}$ or $\frac{-0.472}{2}$

$x = 4.236$ ✓ or -0.236 ✗

$x = 4.24$ ✓ (2dp)

[Note $x \neq -0.236$ which will make $(3x-1)$ negative]

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Algebra

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§ Solving Quadratic Equation Using Formula:

Example 5. Alfonso runs 10 Km at an average speed of x Km/h.
The next day he runs 12 Km at an average speed of $(x-1)$ Km/h.
The time taken for the 10 Km run is 30 minutes less than the time taken for the 12 Km run. [S-16 | 43 | Q7(a)]

(a)(i) Write down an equation in x and show that, its simplifies to $x^2 - 5x - 20 = 0$ ---[4]

(ii) Use quad. formula to solve the equation $x^2 - 5x - 20 = 0$ ---[4]
give your answers correct to 2 decimal places.

(iii) Find the time that Alfonso takes to complete the 12 km run. ---[2]
Give your answer in hours and minutes, correct to the nearest minute.

Solution: Time for 10 Km run $= 10/x$ hr

(i) Time for 12 Km run $= \frac{12}{(x-1)}$

Given $\frac{12}{(x-1)} - \frac{10}{x} = \frac{30}{60}$

or $\frac{12x - 10(x-1)}{x(x-1)} = \frac{1}{2}$

or $\frac{12x - 10x + 10}{(x^2 - x)} = \frac{1}{2}$

or $x^2 - x = 2(2x + 10)$

$x^2 - x = 4x + 20$

$x^2 - x - 4x - 20 = 0$

or $x^2 - 5x - 20 = 0$ ✓

(ii) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1$
$b = -5$
$c = -20$
$b^2 - 4ac$
$= (-5)^2 - 4 \times 1($
$= 25 +$
$=$

$x = \frac{-(-5) \pm \sqrt{105}}{2 \times 1}$

$= \frac{+5 \pm 10.247}{2}$

$= \frac{15.247}{2}, \frac{-5.247}{2}$

$= 7.623, -2.623$

$= 7.62; -2.62$ ✓ (2dp)

(iii) Time for 12 Km $= \frac{12}{x-1}$

$= \frac{12}{7.62 - 1}$ $[x = 7.62]$

$= \frac{12}{6.62}$

$= 1.8126$ hr

$[.8126 \times 60$ min$]$

$= 1$ hr 48.76 min

$= 1$ hr 49 min

§ Solving Quadratic Equation by Completing the Square:

Note:
$x^2+2ax+a^2=(x+a)^2$
(i) Coefficient of $x^2=1$
and
Constant $a^2=\left(\frac{\text{Coeff of } x}{2}\right)^2$

Example 1 (i) $x^2-12x+a=(x+b)^2$ --- [3]
Find the value of a and the value of b. | S-17 | 43 | Q7(b) |

Solution: Consider $x^2-12x+a$

Constant $a=\left(\frac{\text{Coefficient of } x}{2}\right)^2$

$=\left(\frac{12}{2}\right)^2=6^2$

$\therefore\ a=36$ ✓

Now given expression:

$x^2-12x+a=x^2-12x+6^2$
$=(x-6)^2$ $\quad\left[\because x^2-2ax+a^2=(x-a)^2\right.$

$\therefore\ (x+b)^2=(x-6)^2$
$\Rightarrow b=-6$ ✓

(ii) Solve the equation by completing the square.
$x^2-12x+7=0$
or $x^2-12x=-7$
add $\left(\frac{12}{2}\right)^2$ or 6^2 on the sides [See Part (i)]
$x^2-12x+6^2=-7+6^2$
$\Rightarrow (x-6)^2=29$
$\Rightarrow x-6=\pm\sqrt{29}$
$\Rightarrow x=6\pm\sqrt{29}=6\pm 5.39$
$\therefore x=11.39$ ✓ or -0.61 ✓

§ Solving Quadratic Equation by completing the Square:

Example 2. $f(x) = x^2 + 4x - 6$

(a) $f(x)$ can be written in the form $(x+m)^2 + n$
Find the value of m and. ---[2]

(b) Use your answer to part (a) to find the positive solution to $x^2 + 4x - 6 = 0$ $\boxed{S-15/22/Q21}$ ---[2]

Solution: Consider.

$x^2 + 4x - 6$ (add and subtract 2^2)

$= x^2 + 4x + 2^2 - 2^2 - 6$

$= (x+2)^2 - 4 - 6$

$= (x+2)^2 - 10 \equiv (x+m)^2 + n$

for completing the square constant $= \left(\frac{\text{coeff of } x}{2}\right)^2$

$\therefore$ $= \left(\frac{4}{2}\right)^2 = 2^2$

$\therefore$ $m = 2$ and $n = -10$ ✓

(b) To solve. $x^2 + 4x - 6 = 0$

or $(x+2)^2 - 10 = 0$ [from part (a)

or $(x+2)^2 = 10$

$x + 2 = \pm\sqrt{10}$

$\therefore$ $x = -2 \pm \sqrt{10}$

$= -2 + 3.162$ for positive solution

$= 1.162$

$= 1.16$ (2d.p) ✓

Example 3. Solve:

$2x^2 - x - 6 = 0$

or $x^2 - \frac{1}{2}x - 3 = 0$ (Divide by 2)

or $x^2 - \frac{1}{2}x = 3$

or $x^2 - \frac{1}{2}x + \left(\frac{1}{4}\right)^2 = 3 + \left(\frac{1}{4}\right)^2$

or $\left(x - \frac{1}{4}\right)^2 = \frac{49}{4}$

$x - \frac{1}{4} = \pm\sqrt{\frac{49}{4}}$

$x = \frac{1}{4} \pm \frac{7}{4}$

$\therefore$ constant $= \left(\frac{1}{2} \times \text{coeff of } x\right)^2$

$= \left(\frac{1}{2} \times \frac{1}{2}\right)^2$

$= \left(\frac{1}{4}\right)^2$

add on both the sides $\left(\frac{1}{4}\right)^2$

$\therefore$ $x = \frac{1}{4} + \frac{7}{4}$ or $\frac{1}{4} - \frac{7}{4}$

$x = 2$ or $-\frac{3}{2}$ ✓

§ Solving Linear Inequalities:

Rules: Given a linear inequality, it remain unchanged if you (i) add / or subtract any real number on both sides.

$a > b$ (or $a < b$) $a, b, C \in R$

Then $a + C > b + C$ (or $a + C < b + C$)

(ii) If you multiply/or divide both sides on an inequility by a positive real number, it remains unchaged.

(I) $a > b \Rightarrow aC > bC$ $C > 0$ +ve

(II) $a > b \Rightarrow a/C > b/C$

(iii) But if you muliply (ordivide) both sides of an inequality by a negative r… number, the sign of inequality reverses.

(I) $a > b \Rightarrow aC < bC$ if $C < 0$ -ve

(II) $a > b \Rightarrow \frac{a}{C} < b/C$

Example (i) $5 > 2$

multiply by -1, $-5 \not> -2$ but $-5 < -2$

(ii) $-4 < -1$

divide by -1, $\frac{-4}{-1} \not< \frac{-1}{-1}$ but $4 > 1$

Example 1 Solve the inequality: $3n - 11 > 5n - 18$ ---[2]

$\boxed{S\text{-}17/22/Q13}$

or $3n - 5n > -18 + 11$

or $-2n > -7$

or $\frac{-2n}{-2} < \frac{-7}{-2}$ (divide by -2, change the inequality)

or $n < \frac{7}{2}$ ✓

or $n < 3.5$ ✓

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Algebra

Solving Linear Inequities

Example 2. Solve. $n+7 < 5n-8$ --- [2] [SP-20/02/Q13]

or $n-5n < -8-7$

or $-4n < -15$

Divide by -4, $n > \frac{-15}{-4}$ [∵ Dividing by -4, changes the sign of inequality

or $n > \frac{15}{4}$ or $n > 3.75$ ✓

Example 3. Solve. $5t+23 < 17-2t$ [S-14/23/Q9] .. [2]

or $5t+2t < 17-23$

or $7t < -6$ (Divide by 7)

or $t < -\frac{6}{7}$ ✓

Example 4. Solve. $6n+3 > 8n$ --- [2] [M-16/22/Q4]

or $6n-8n > -3$

or $-2n > -3$

Divide by -2 $\Rightarrow$ $n < \frac{+3}{+2}$ {divide by -2 Change the sign of inequality.

or $n < 1.15$ ✓

※ Example 5. Solve, for positive integral value of x; $\frac{21+x}{5} > x+1$ --- [4] [S-14/21/Q15]

or $21+x > 5x+5$

or $x-5x > 5-21$

or $-4x > -16$ {Divide both sides by -4, changes the sign of inequality

or $x < \frac{-16}{-4}$

or $x < 4$

$\Rightarrow$ $x = 1, 2, 3$ ✓✓ [∵ x is a positive integer.

Example 6: Solve. $\frac{x}{2} + \frac{x-2}{3} < 5$

$3x + 2(x-2) < 30$

or $3x+2x-4 < 30$

or $5x < 34$

or $x < \frac{34}{5}$ $\Rightarrow$ $x < 6.8$ --- [4] [W-13/22/Q16]

Solving Linear Inequalities:

Example 7: Find the integer values of, t, which satisfy the inequalities: $4t + 7 < 39 \leq 7t + 2$

Solve it in two parts:

$4t+7<39$ and $39 \leq 7t+2$

or $4t < 32$ and $7t \geq 37$

or $t<8$ and $t \geq \frac{37}{7} = 5.3$

or $5.3 \leq t < 8$

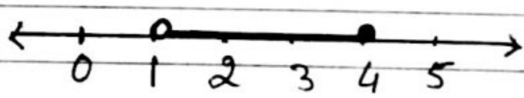
$\therefore$ $t = 6, 7$ ✓ as t is an integer.

Example 8: Solve the inequality and represent the solution on the number line;

$5 < 2t + 3 \leq 11$

or $2 < 2t \leq 8$

or $1 < t \leq 4$



Algebra

§ Graph of Linear Inequalities in two variable (In Plane)

Example 1: Draw the graph of inequality.

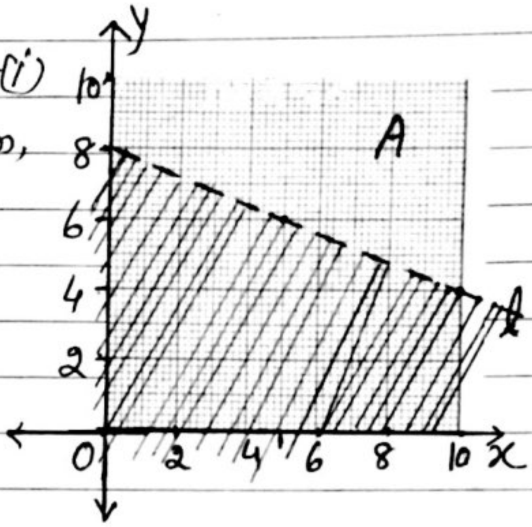
$2x + 5y < 40$ --- (i)

consider the corresponding equation,

$2x+5y=40$

or $5y = 40-2x$

or $y = \frac{(40-2x)}{5}$



x	0	5	10
y	8	6	4

Draw points (0,8), (5,6), (10,4)

line 'l' is drawn dotted as inequality (i) is only < (and not ≤)

Now line 'l' divides the plane into two half plane,

'O'-side of line 'l' and 'A' side of l.

How to decide which side is the graph of inequality (i)

Chose a point say O(0,0) (But not on line l)

Put (0,0) in (i) $\Rightarrow$ $0+0<40$ True

$\therefore$ O-side of line 'l' is required graph.

shade it.

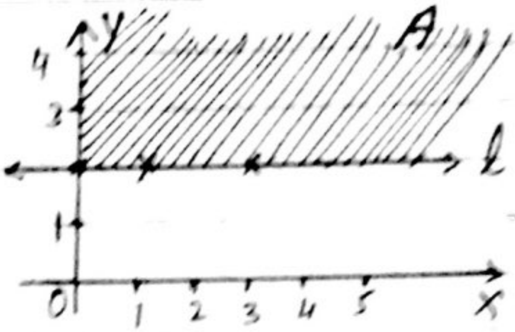
(Note 1. some time in question. you are asked to shade unwanted region)

2. If $x \geq 0$ and $y \geq 0$
Then shade only the Ist quad. region.

Graph of Linear Inequalities in two variable (In Plane).

Example 2. Draw the graph of inequality

$y \geq 2$ ---- (i)



Consider the equation $y = 2$

graph of $y = 2$ is line 'l' ∥ x-axis.

Points (0, 2), (1, 2), (3, 2)

Now to test, which side of line 'l' to shade, put O(0,0) in (i)

$0 \geq 2$ false

{Given $x \geq 0$, $y \geq 0$

∴ We shade the side opposite to O-side. (A-side of 'l')

Example 3: Draw the graph of inequality:

$x < 3$ ---- (i)

Consider equation $x = 3$

Take point (3, 1), (3, 2), (3, 4)....

graph is line ∥ y-axis.

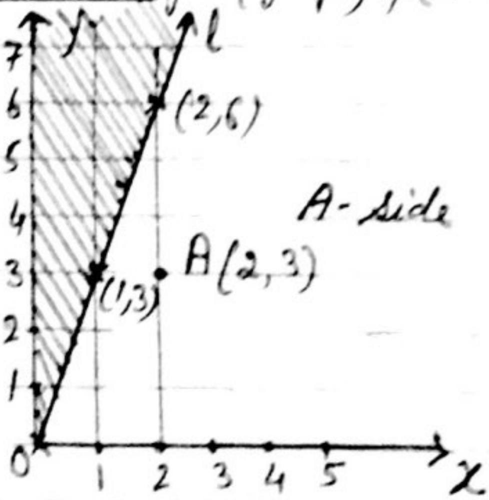
Put O(0,0) in ① ⇒ $0 < 3$ True

{'l' is Broken line (∵ $x < 3$)

∴ O-side of the line is the required region (graph) of (i)

Example 4: Draw the graph of inequality:

$y \geq 3x$ ---- (i)



Consider the equation:

$y = 3x$, (0,0), (1,3), (2,6).

Draw line 'l' (complete line)

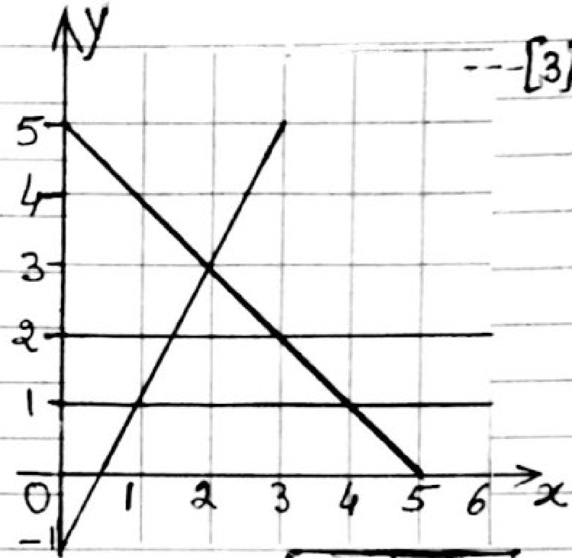
Now to check that which side of line 'l' is the required graph

we cannot take O(0,0), as line 'l' passes through O. Put A(2,3) in (i) ⇒ $3 \geq 3 \times 2$ false.

∴ the side opposite to A-side of 'l' is the required graph.

§ Graph of Linear Inequalities:

---[3]

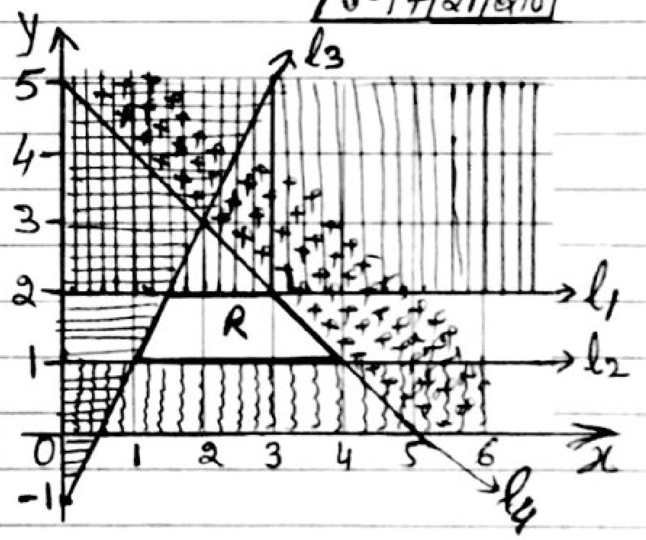


Example 5: By shading the unwanted regions of the grid, find and label the region R that satisfies the following four inequalities:

$y \leq 2$, $y \geq 1$

$y \leq 2x-1$, $y \leq 5-x$

S-17/21/Q10



Solution: $y \leq 2$ —— (i)

consider eqn $y=2$ is line 'l_1'

for O(0,0) in (i) $0 \leq 2$ True

∴ O-side is req. region wanted

Unwanted opp. to -O-Side be shaded.

Now $y \geq 1$ ---- (ii)

Consider $y=1$ is line "l_2"

for O(0,0) in (ii) $0 \geq 1$ is false

the required is side opp. to O.

∴ Unwanted is O-side of l_2

Again $y \leq 2x-1$ --- (iii)

consider eqn $y=2x-1$, graph is line "l_3"; for O(0,0) in (iii) $0 \leq -1$ is false. So unwanted is O-Side of l_3.

and finally. $y \leq 5-x$ —— (iv)

consider $y=5-x$, graph is line 'l_4", at O(0,0) in (iv)

$0 \leq 5$ is True, ∴ Unwanted is opposite to O-side of l_4.

Trapezium R. is the required region.

Note	Students you don't need to give all this season (explaination) in exam. for a 3 marks question. Just Shade

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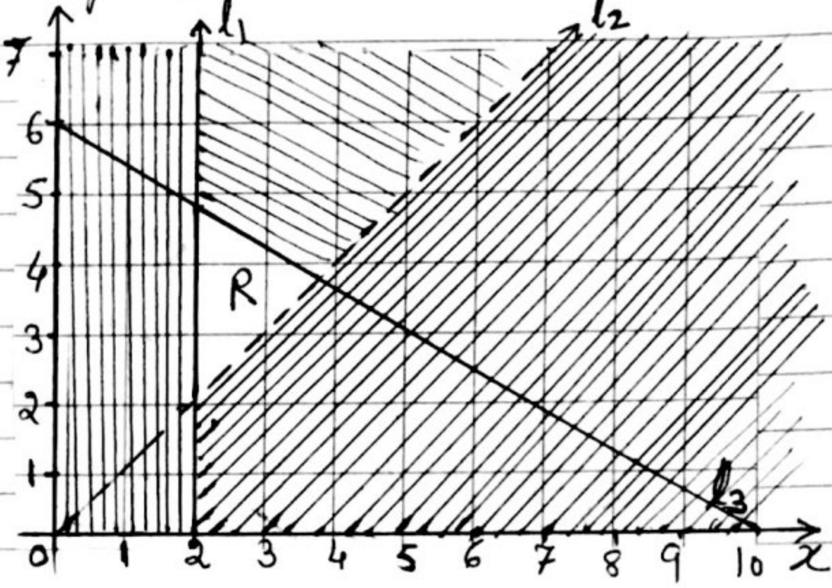
Algebra.

§ Graph of Linear Inequalities:

Example 6: Find the three inequalities that define the unshaded region R. --- [5]

W-16/23/Q24



Line 'l_1' equ. is $x = 2$

∴ for l_1 the unshaded region is $x \geq 2$ — (i)

line 'l_2' Join (0,0) and (7,7)

Equ. of l_2 is $y = x$

the unshaded region is

$y > x$ —— (ii)

as line l_2 is

and line 'l_3' passes through points (0,6) and (10,0)

Equ. of l_3 is $3x + 5y = 30$

∴ for unwanted region being on 0-side, ∴ $3x + 5y \leq 30$

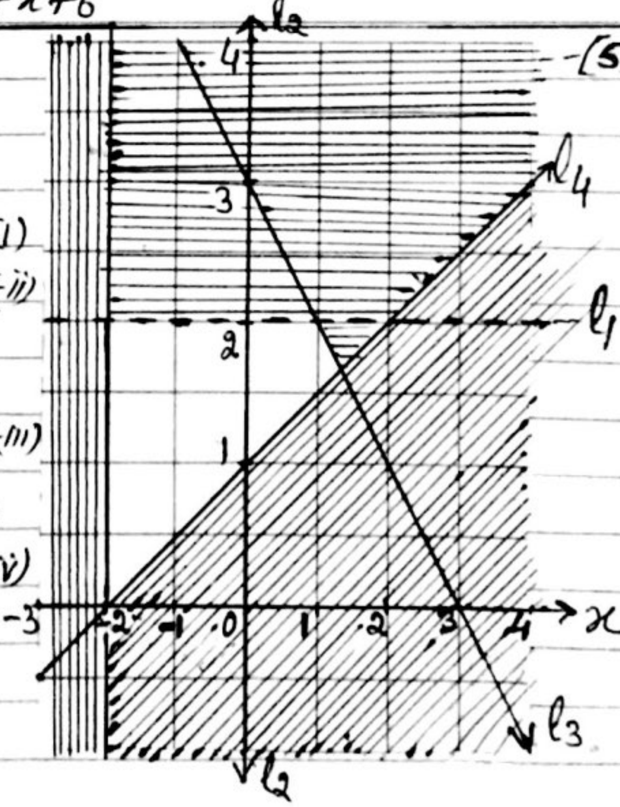
or $\frac{3}{5}x + y \leq 6$

or $y \leq -\frac{3}{5}x + 6$ -- (iii)

∴ Required: $x \geq 2$, $y > x$

and $y \leq -3/5\, x + 6$

Example 6: Find the four inequalities that define the region that is **not** shaded. --- [5]



Solution 'l_1' equ. $y = 2$, inequal. $y < 2$ — (i)

l_2 equ. $x = -2$, ineq. $x \geq -2$ — (ii)

l_3 equ. Passes through (3,0) and (0,3)

l_3 is $x + y = 3$ or $y = -x + 3$

Inequality $y \leq -x + 3$ - (iii)

l_4 joins (0,1) and (2,2), $y = \frac{x}{2} + 1$

Inequality is $y \geq \frac{1}{2}x + 1$ — (iv)

M-16/22/Q19

§ Graph of Inequalities and Linear Programming Problems:

SP-15/04/Q8

Example 1. Mr Cheng hires x large coaches and y small coaches to take 300 students on a school trip, Large coaches can carry 50 students and small coaches 30 students.
There is maximum of 5 large coaches.

(a) Explain clearly how the following two inequalities satisfy these conditions.

(i) $x \leq 5$ --- [1]

(ii) $5x + 3y \geq 30$ --- [2]

Mr Chang also knows that $x + y \leq 10$

(b) On the grid show the information above by drawing three straight lines and shading the **unwanted** regions. --- [5]

(C) A large coach costs $450 to hire and a small coach costs $350. (i) Find the number of large coaches and the number of small coaches that would give the minimum hire cost for the school trip. --- [2]

(ii) Calculate the hire cost. --- [1]

Solution (a) (i) there are upto 5 large coches $\therefore x \leq 5$ --- (i)

(ii) x large coaches can have $50x$ students and y small coaches can have $30y$ student this total has to be 300 or more,

$\therefore 50x + 30y \geq 300$

divide by 10, or $5x + 3y \geq 30$ --- (ii)

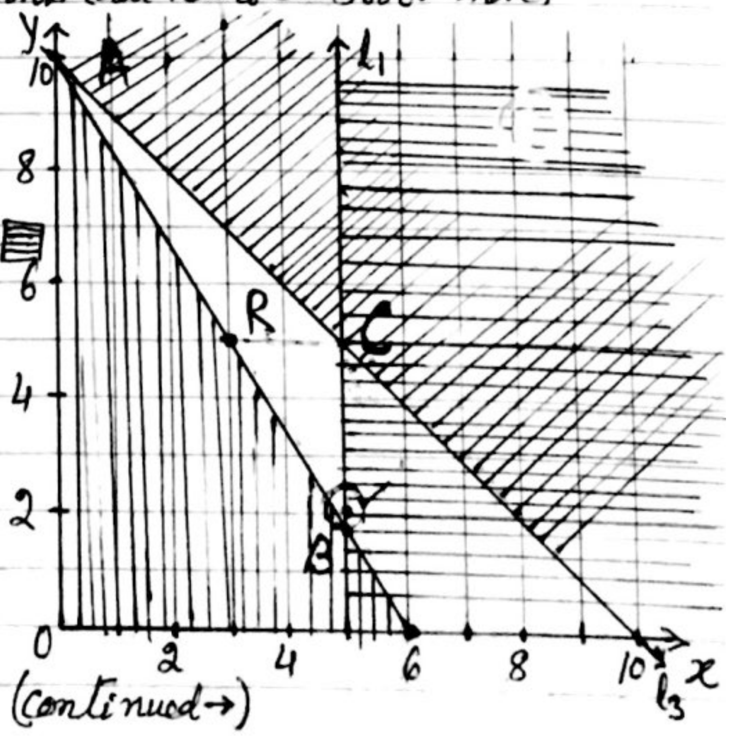
also given $x + y \leq 10$ -- (iii)

(b) for $x \leq 5$, draw line l_1, $x = 5$ and shade

for $5x + 3y \geq 30$ draw line l_2 consider point $(0,10)$, $(6,0)$, $(3,5)$ and shade

for $x + y \leq 10$, draw $x + y = 10$ line l_3 $(0,10)$, $(10,0)$. and shade

Now Region 'R' (Triangle ABC-inside) and the points on its sides all satisfy.



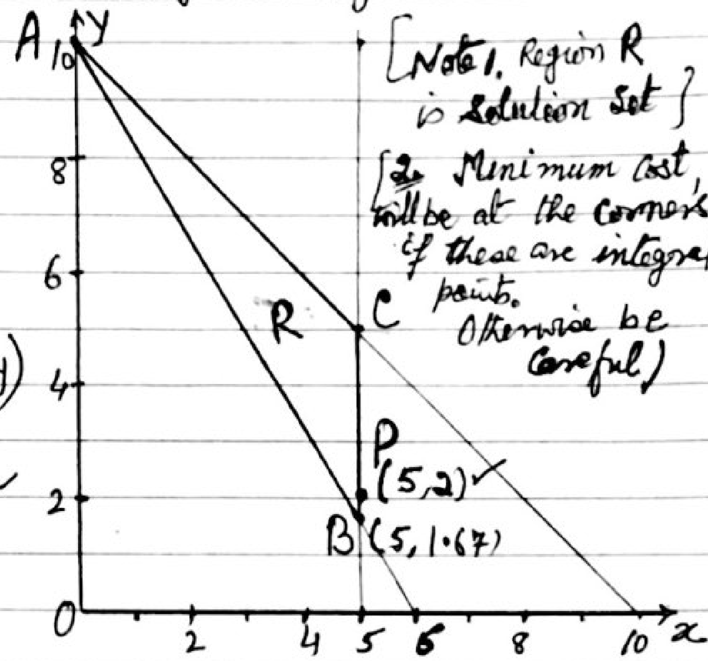
(Continued →)

§ Graph of Inequalities and Linear Programming Problems:
(Continued→)

Example 1: Now cost of a large
C (i) Coach is \$450 (x-coaches)
and the cost of a small coach
is \$ 350 (y-Coaches)

∴ Cost Function $C = (450x + 350y)$

from the graph it is clear
that the cost will be
minimum at $B(5, 1.67)$
∴ $x = 5$ ✓
and $y = 1.67$ ✗ but y is a whole number.
∴ $y = 2$ ✓



[Note 1. Region R is Solution Set]
[2. Minimum cost will be at the corners if these are integral points. Otherwise be Careful]

∴ Number of large coaches = 5
" Small Coaches = 2 } ✓

C(ii) ∴ Here Cost $= 450x + 350y$
$= 450 \times 5 + 350 \times 2$ [for $x = 5$ and $y = 2$]
$=$ \$ 2950

Example 2: Bernie buys x packets of seeds and y plants for his garden.
He wants to buy more packets of seeds than plants.
The inequality $x > y$ shows this information.
He also wants to buy,
- less than 10 packets of seeds
- atleast 2 plants

(a) Write 2 more inequalities in x or y to shows this information. --- [2]
(b) Each packet of seeds costs \$1 and each packet of plant cost \$3.
The maximum amount Bernie can spend is \$ 21.
write down another inequality in x and y to show this information. --- [1]
(C) The line $x = y$ is drawn on the grid, draw three lines to show your inequalities and shade the Unwanted regions. --- [5]

(Continued→)

§ Linear Programming Problems:

(Continued→)

Example 2(d): Bernie buys 8 packet of seeds. [M-17/42/Q9]

(i) Find the maximum number of plants he can buy. ---[1]

(ii) Find the total cost of these packets of seeds and plants. ---[1]

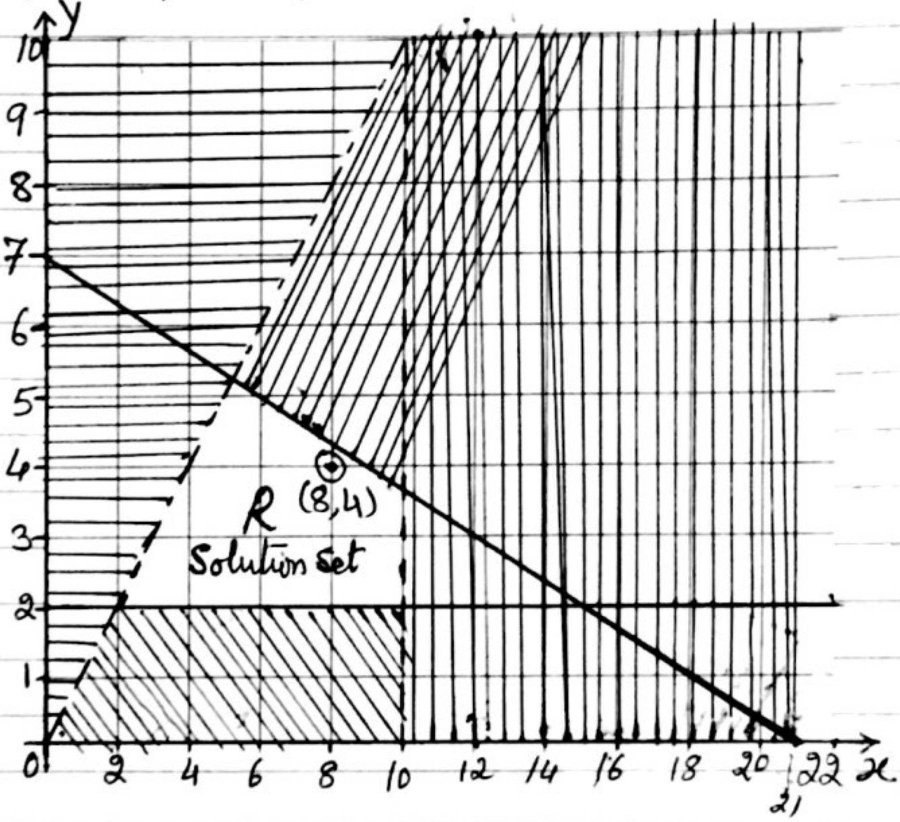
Solution: $x > y$ --(i)

$x < 10$ --(ii)

$y \geq 2$ --(iii)

$x + 3y \leq 21$ --(iv)

on $l_4 \rightarrow (0,7); (0,21)$



(d)(i) $x = 8$ from (iv)

$8 + 3y = 21$

$y = \frac{13}{3} = 4.3$

In region R for $x = 8$

Max $y = 4$ ✓

Max No. of Plants $y = 4$ ✓

(ii) $x = 8, y = 4$

Cost $C = x + 3y$

$= 8 + 3 \times 4 = \$20$

[Note: 1. Region R is called "Solution set"]

[Max/Minimum value lies at the corner points - If these are integral points (otherwise we have to be careful.)]

Example 3: A bag of sweets contains x orange sweets and y lemon sweets. Each orange sweet costs 2 cents and each lemon sweet costs 3 cents. The cost of a bag of sweets is less than 24 cents. There are atleast 9 sweets in each bag. [W-16/42/Q6(b)]
There are at least 2 lemon sweets in each bag.

(i) One of the inequalities that shows the information is $2x + 3y < 24$. Write down the other two other two inequalities. ---[2]

(Continued→)

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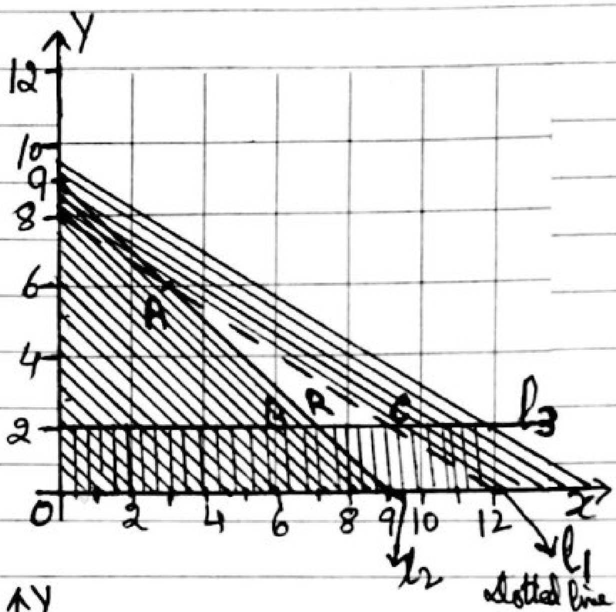
§ Graph of Inequalities and Linear Programming Problems: (Continued →)

Example 3 (ii) On the grid, by shading the unwanted region, show that the region which satisfy the three inequalities. ---[4]

(iii) Find the lowest cost of a bag of sweets.
Write down the value of x and the value of y that give this cost --[3]



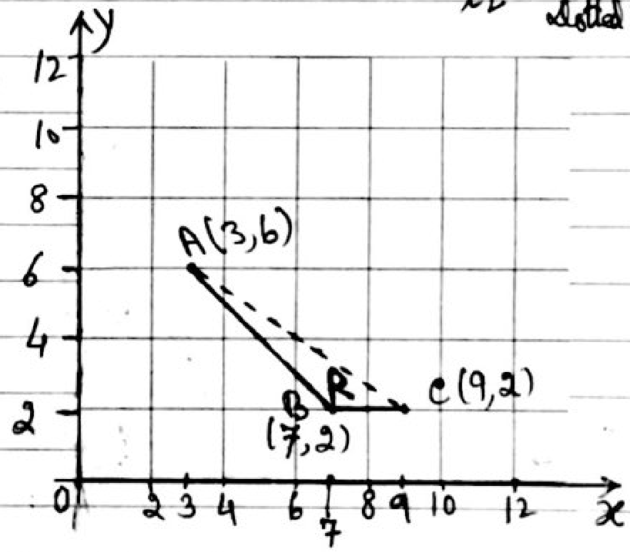
Solution: No of oranges sweets $= x$
No of lemon sweets $= y$
Each orange sweet cost 2 cents
and each lemon sweet cost 3 cents
Given that cost of bag is less than 24 cents
$\therefore \quad 2x + 3y < 24$ ---(i)
and given $x + y \geq 9$ ----(ii)
also $y \geq 2$ ---(iii)

To draw the graph of (i) $2x + 3y < 24$
Consider $2x + 3y = 24$
$(0, 8), (12, 0)$ —— l_1 dotted line.

for $x + y \geq 9$
consider $x + y = 9$, $(9,0), (0,9)$ — l_2

for $y \geq 2$, Consider $y = 2$, $l_3 \parallel$ X-axis

$\therefore$ the solution set 'R' is Δ ABC, and inside
(But point on AC not included)



Cost Function $C = 2x + 3y$ (iv)

Value of C at corner A (3,6)
fn (iv) $= 2\times3 + 3\times6$
$= 24$

and the Value of Cost C at Vertex B(7,2)
$= 7\times2 + 2\times3 = 20$

$\therefore$ Minimum Cost of bag $= 20$ Cents
for $x = 7$
and $y = 2$

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Algebra.

§ Sequences

Consider the following collection of

(i) 1, 4, 9, 16, 25, - - - -
(ii) 2, 5, 8, 11, 14, - - - -
(iii) 3, 6, 12, 24, 48, - - - -
(iv) 7, 5, 3, 1, −1, - - - -
(v) $\frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \frac{4}{6}, \frac{5}{7}, \ldots$

In each of the above collection numbers are arranged in a definite order with **some rule**.
These are called **sequences**.
The terms of a sequence are denoted by:
$t_1, t_2, t_3, \ldots t_n, t_{n+1}, \ldots$

First term $= t_1$
Second term $= t_2$
- - -

General term or n_{th} term $= t_n$
next to it $(n+1)_{th}$ term $= t_{n+1}$.

Now let us consider the above 5 sequences one by one and **observe** <u>term to term rule.</u>

(i) 1, 4, 9, 16, 25 or may be written as:
$1^2, 2^2, 3^2, 4^5, 5^{-2}$ are squred numbers.
$\therefore$ next term (6_{th} term) $= 6^2 = 36$ ✓
and the n_{th} term $= n^2$ ✓

(ii) 2, 5, 8, 11, 14, - - in this sequence
$(5-2) = 3$, $(8-5) = 3$, $(11-8) = 3$ i,e the difference between any two **consecutive** term remains constant (<u>Common difference</u> <u>is constant</u>).
term to term rule $\boxed{t_{n+1} = t_n + 3}$
such kind of sequence is defined as:
<u>Arithmetic Sequence.</u>

(Continued →)

Sequences

(iii) 3, 6, 12, 24, 48, ---

In this sequence ratio of any term to the previous term

$\frac{6}{3} = \frac{12}{6} = \frac{24}{12} = \frac{48}{24} = 2$ is constant, called common ratio.

and the term to term rule - $\frac{t_{n+1}}{t_n} = 2$ (Constant)

such a sequence is called Geometric Sequence.

(iv) 7, 5, 3, 1, -1, --

It is also an arithmetic sequence as the common difference $5-7 = 3-5 = 1-3 = -1-1 = -2$ Constant

and the term to term rule: $t_{n+1} = t_n + (-2)$ ✓

(v) $\frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \frac{4}{6}, \frac{5}{7}$ ---

terms in the numerator 1, 2, 3, 4, 5, --

terms in the denominator 3, 4, 5, 6, 7 [$D^r = N^r + 2$]

n_{th} term $= \frac{n}{(n+1)}$ ✓

Term to Term Rule: $t_{n+1} = t_n + d$

or $t_{n+1} - t_n = d$ (Constant)

§ Arithmetic Sequence:

Consider a sequence: 7, 10, 13, 16, 19, ---

The differences $10-7 = 13-10 = 16-13 = 19-16 = 3$ ✓ Common difference.

Now just observe the terms;

First term $t_1 = 7 = 4 + 1 \times 3$

Second term $t_2 = 10 = 4 + 2 \times 3$

Third term $t_3 = 13 = 4 + 3 \times 3$

Fourth term $t_4 = 16 = 4 + 4 \times 3$

{ here $d = 3$ and $4 = t_1 - d$, $a = 7 - 3 = 4$ ✓ }

$\boxed{\therefore \ n_{th} \text{ term of the Sequence} = a + nd \ (\text{linear function})}$

$\therefore$ n_{th} term of an Arithmetic Sequence $= 4 + n \times 3$

$= (4 + 3n)$ ✓

[$a = t_1 - d = 7 - 3 = 4$

d = Common diff

n = No. of term.

✓

Sequences

Arithmetic Sequence:

Example 2. These are the first five terms of a sequence,

32, 25, 18, 11, 4, ... [S-14/22/Q20]

find (a) the 6th term. ---[1]

(b) the nth term. ---[2]

(c) which term is equal to -332.

Solution: Given sequence 32, 25, 18, 11, 4 is an arithmetic sequence

as $25-32 = 18-25 = 11-18 = 4-11 = -7$ (constant)

$\therefore$ common difference $d = -7$ ---(i)

(a) $\therefore$ 6th term = 5th term + d

$= 4 + (-7) = -3$ ✓

(b) nth term $= a + nd$

$= a + n(-7)$ [from (i) $d = -7$

$= a - 7n$ ---(ii)

for $n = 1$, first term $= a - 7\times1 = 32$ given

$\Rightarrow a = 32 + 7$

$\Rightarrow a = 39$

$\therefore$ from (ii) nth term $= (39 - 7n)$ ✓

(c) Let nth term

$(39 - 7n) = -332$

$\Rightarrow -7n = -332 - 39$

$= -371$

or $n = \frac{+371}{+7} = 53$

$\therefore$ Required term = 53 ✓

(b) **Alternate method:** nth term $= a + nd$

$= 39 + n\times(-7)$

$= (39 - 7n)$ ✓

here $d = -7$ ✓

$a = t_1 - d$

$= 32 - (-7)$

or $a = 39$ ✓

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Algebra

Sequences

§ Arithmetic Sequences (Linear function)

Example 3. Complete the table for the following sequences. The first has been completed for you.

W-13/42/Q10

	Sequence				Next two terms	n_{th} term.	
	1	5	9	13	17 21	$4n-3$	
(a)	12	21	30	39			--- [3]
(b)	80	74	68	62			--- [3]
(c)	1	8	27	64			--- [2]
(d)	2	10	30	68			--- [2]

Solution:

	Sequence				Next two terms	n_{th} term
(a)	12	21	30	39	48 ; 57	$9n+3$
(b)	80	74	68	62	56 ; 50	$86-6n$
(c)	1	8	27	64	125 ; 216	n^3
(d)	2	10	30	68	130 , 222	n^3+n

(a) 12, 21, 30, 39 is an arithmetic sequence with common diff $d=9$

$\therefore$ next two terms are $(39+9), (48+9) = 48 \ \& \ 57$ ✓

n_{th} term $= a + nd$

$= 3 + 9n$ ✓

$\left\{ \begin{array}{l} a = t_1 - d \\ a = 12 - 9 = 3 ✓ \\ d = 9 ✓ \end{array} \right.$

(b) 80, 74, 68, 62 is an arithmetic seq. $d = 74 - 80 = -6$

next terms $62 - 6 = 56$ ✓ & $56 - 6 = 50$ ✓

n_{th} term $= a + nd = 86 + n(-6) = 86 - 6n$ ✓

$\left[\begin{array}{l} a = t_1 - d \\ = 80 - (-6) = 86 \end{array} \right.$

(c) 1, 8, 27, 64, -- or $1^3, 2^3, 3^3, 4^3$, --

$\therefore$ next two terms are $5^3 \ \& \ 6^3 \Rightarrow 125 \ \& \ 216$ ✓

n_{th} term $= n^3$ ✓

(d) 2, 10, 30, 68

or $1+1^3,\ 2+2^3,\ 3+3^3,\ 4+4^3$ $\therefore$ next two terms are $5+5^3 \ \& \ 6+6^3$

or 130 & 222 ✓

by observation:

and n_{th} term $= (n + n^3)$ ✓

Sequences

Note 1: The 1st difference $(t_{n+1} - t_n)$ is contant for Arithmetic sequence and the n_{th} term = Linear. "$a + nd$"

Note 2: If the 2nd difference is constant then the n_{th} term is quadratic function of n. n_{th} term $= t_n = an^2 + bn + c$

Using Difference Method:

Example 4: Deduce the rule for the n_{th} term for the sequence:

4, 8, 14, 22, 32, - - - [S-17/41/Q9(b)(ii)]

Position	1	2	3	4	5
Term	4	8	14	22	32
1st Difference		4	6	8	10
2nd Difference →		2	2	2	constant

→ Table 1

The second difference is constant.

∴ n_{th} term is a Quadratic $\Rightarrow t_n = an^2 + bn + c$ — (1)

Now see another table - 2.

Position	1	2	3	4	5
Term	$a+b+c$	$4a+2b+c$	$9a+3b+c$	$16a+4b+c$	$25a+5b+c$
1st difference		$3a+b$	$5a+b$	$7a+b$	$9a+b$
2nd difference →		$2a$	$2a$	$2a$	is constant

Table 2

Now Comparing the two tables:

2nd diff $\quad 2a = 2 \Rightarrow a = 1$ ✓

1st diff. $\quad 3a + b = 4 \Rightarrow 3 + b = 4 \Rightarrow b = 1$ ✓ [∵ a = 1

1st term $\quad a + b + c = 4 \Rightarrow 1 + 1 + c = 4$ [a = 1, b = 1

$\Rightarrow c = 2$ ✓

∴ from (1) n_{th} term $= an^2 + bn + c$

∴ Required n_{th} term $= 1 \times n^2 + 1 \times + 2$ $\quad$ | a = 1, b = 1, c = 2

n_{th} term $= n^2 + n + 2$ ✓

§ **Note:** If in a sequence the 2nd diff is constant. $t_n = an^2 + bn + c$

Please Note: First term $= a + b + c$

First difference $= 3a + b$

Second diff $= 2a$.

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Sequences:

Note 3. If the 3rd difference is contant. Then the n_{th} term = Cubic function = an^3+bn^2+cn+d

Example 5: Deduce the rule for the n_{th} term for the sequence;
5, 15, 41, 89, 165, --- (use difference method).

Solution:

Position	1	2	3	4	5
Term	5	15	41	89	165
Ist difference		10	26	48	76
IInd difference			16	22	28
IIIrd difference				6	6

⟵ Table 1

→ is constant.

$\therefore$ n_{th} term is cubic $= an^3+bn^2+cn+d$ — ①

Position	1	2	3	4	5
Term	$a+b+c+d$	$8a+4b+2c+d$	$27a+9b+3c+d$	$64a+16b+4c+d$	$125a+25b+5c+d$
Ist difference		$7a+3b+c$	$19a+5b+c$	$37a+7b+c$	$61a+9b+c$
IInd difference			$12a+2b$	$18a+2b$	$24a+2b$
IIIrd difference				$6a$	$6a$

→ Constant

Table-2

Now comparing the two tables.

3rd diff $6a=6 \Rightarrow a=1$ ✓

2nd diff. $12a+2b=16 \Rightarrow 12+2b=16 \Rightarrow b=2$ ✓ $(\because a=1)$

Ist diff $7a+3b+c=10 \Rightarrow 7+6+c=10 \Rightarrow c=-3$ ✓ $\left(\begin{smallmatrix}a=1\\b=2\end{smallmatrix}\right)$

Ist term, $a+b+c+d=5 \Rightarrow 1+2-3+d=5 \Rightarrow d=5$ ✓

$\therefore$ for ① n_{th} term $= an^3+bn^2+cn+d$

$\therefore$ n_{th} term $= n^3+2n^2-3n+5$

$\because$ $a=1$, $b=2$, $c=-3$, $d=5$

Note: If the third diff. is constant then n_{th} term $= an^3+bn^2+cn+d$

First term $= a+b+c+d$
Ist diff $= 7a+3b+c$
IInd diff $= 12a+2b$
IIIrd diff $= 6a$

Sequences

§ Difference method:

(Alternate method)

Example 6. Find the n_{th} term of the sequence: --- [2]

S-15/21/Q11(b)

11 , 20 , 35 , 56 , 83 , ---

Ist difference: 9 15 21 27

2nd difference: 6 6 6 → is constant.

∵ the 2nd difference is constant, hence n_{th} terms is Quadratic.

$$n_{th}\ term = an^2 + bn + c \quad \text{--- (i)}$$

for $n = 1$, Ist term, $a + b + c = 11$ --- (ii)

for $n = 2$, IInd term, $4a + 2b + c = 20$ --- (iii)

for $n = 3$, IIIrd Term, $9a + 3b + c = 35$ --- (iv)

Now equation (iv) − (i) ⇒ $8a + 2b = 24$

or $4a + b = 12$ --- (v)

also (iii) − (i) ⇒ $3a + b = 9$ --- (vi)

(v) − (vi) ⇒ $a = 3$ ✓

Put $a = 3$ in (vi) ⇒ $b = 0$ ✓

$a = 3$ and $b = 0$ in (ii) ⇒ $c = 8$ ✓

∴ from (i) n_{th} term $= 3n^2 + 8$ ✔

Sequences

§ Geometric Sequence:

term to term Rule

$\frac{U_{n+1}}{U_n} = r$ constant (Common-ratio)

Consider a sequence of numbers:

2, 6, 18, 54, ---

observe: $\frac{6}{2} = \frac{18}{6} = \frac{54}{18} = 3$ Constant Ratio denoted by 'r' in general.

General form of a Geometric sequence:

$a, ar, ar^2, ar^3, ---$

First term $t_1 = a$

Second term $t_2 = ar$

third term $t_3 = ar^2$

$\vdots$

nth term of a geometric sequence:

$$t_n = a \cdot r^{n-1}$$

Example 1. Find the next two terms and the nth term of the sequence, 2, 6, 18, 54, --- [3] (W-15/43/Q10(D))

Solution: we note $\frac{6}{2} = \frac{18}{6} = \frac{54}{18} = 3$ constant or common ratio or $r = 3$.

$\therefore$ 5th term $= 54 \times 3 = 162$ ✓

and 6th term $= 162 \times 3 = 486$ ✓

nth term $= ar^{n-1}$ [first term $a = 2$ and $r = 3$]

$= 2 \cdot 3^{n-1}$

$= 2 \cdot 3^{n-1}$ ✓

Example 2. Which term of the sequence $12, 6, 3, \frac{3}{2}, ---$ is $\frac{3}{32}$.

Solution: The given sequence $12, 6, 3, \frac{3}{2}$ is a geometric seq, with common ratio $r = \frac{1}{2}$

$\therefore$ Let nth term $= \frac{3}{32}$

$\Rightarrow 12 \cdot \left(\frac{1}{2}\right)^{n-1} = \frac{3}{32}$

$\Rightarrow \left(\frac{1}{2}\right)^{n-1} = \frac{3}{32 \times 12}$

$\Rightarrow \left(\frac{1}{2}\right)^{n-1} = \frac{1}{128} = \left(\frac{1}{2}\right)^7$

$\Rightarrow n - 1 = 7$

$\Rightarrow n = 8$ ✓

Sequences:
Applications:

Compound Interest: $A = P\left(1 + \frac{r}{100}\right)^n$

Example 1. $A = 5000(1.12)^5$

gives the amount for Rs 5000, invested at 12% compound interest after 5 years.

Example 2 (a) The nth term of a sequence is $8n-3$.

(i) Write down the first two terms of the sequence. --- [1]

(ii) Show that 203 is not in this sequence. --- [2]

(b) Find the nth terms of these sequences.

(i) 13, 19, 25, 31, --- [2]

(ii) 4, 8, 14, 22, --- [2]

(c) The second term of a sequence is 20 and the third term is 50.
The rule for finding the next term in this sequence is subtract y and then multiply by 5. --- [4]
Find the value of y and work out the first term of the sequence.

$\boxed{\text{S-17/41/Q9}}$

Solution: (a) nth term $= 8n-3$

$\therefore$ first term $= 8\times1-3 = 5$ ✓

and second term $= 8\times2-3 = 13$ ✓

(b)(i) 13, 19, 25, 31, --- is an arithmetic sequence with common diff. $d = 19-13 = 6$

nth term $= a + nd$ $\left[\begin{array}{l} a = t_1 - d \\ = 13-6 = 7 \end{array}\right.$

$= 7 + n\times 6$

$= 6n+7$ ✓

(ii) 4, 8, 14, 22

I st difference: 4, 6, 8

II nd diff: 2, 2 is constant $\longrightarrow$

$\therefore$ nth term is quadratic

nth term $= an^2+bn+c$ —— (i)

for $n=1$, $a+b+c=4$ —— (ii)

for $n=2$, $4a+2b+c=8$ —— (iii)

for $n=3$, $9a+3b+c=14$ —— (iv)

(iii)−(ii) and (iv)−(iii)

$\Rightarrow$ $3a+b=4$ —— (v)

$5a+b=6$ —— (vi)

Solv. (v) and (vi) $a=1, b=1$

& (ii) $c=2$

$\therefore$ Req. nth term $= n^2+n+2$ ✓

(Continued →)

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Sequences.

Algebra

(Contined→)

Example 2(C). Second term = 20

Third term = 50

according to the rule $(20-y)\times 5 = 50$

$\Rightarrow 20-y=10 \Rightarrow y=10$ ✓

Let the first term is t_1; $(t_1 - y)\times 5$ = second term

$(t_1-10)5 = 20 \Rightarrow t_1 - 10 = 4$

$\Rightarrow t_1 = 14$ ✓

Example 3. S-16/43/Q10

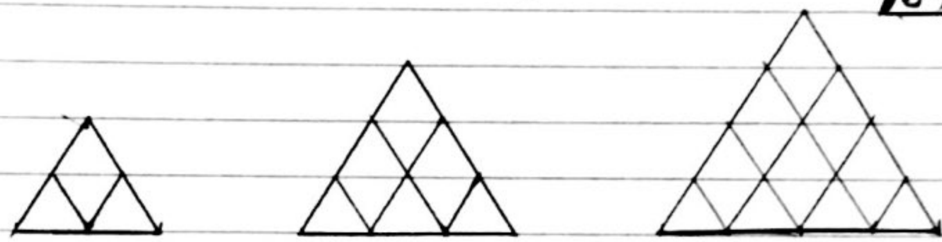

Diagram 1 Diagram 2 Diagram 3.

Each diagram is made form tiles in the shape of equilateral triangle and rhombuses. The length of a side of each tile is 1 unit.

(a) Complete the table below for this sequence of diagrams. ---[6]

Diagram	1	2	3	4	5
Number of equilateral triangle shape tiles.	2	3	4	5	6
(i) Number of rhombus shaped tiles.	1	3	6		
(ii) Total number of tiles	3	6	10		
(iii) Number of 1 unit lengths	8	15	24		

Note: See the formula in Part (b) and (c)

Solution (i) → 10 15 ✓

(ii) → 15 21 ✓

(iii) → 35 48 ✓

(b) (i) The number 1 unit length in Diagram n is n^2+4n+p

Find the value of p ---[2]

Solution: for $n=1 \Rightarrow 1^2+4\times 1+p = 8$ given $\Rightarrow p=3$ ✓

(ii) Calculate the number of 1 unit length in Diagram 10 ---[1]

Solution In Diagram $n = n^2+4n+3 \Rightarrow$ In Diagram $10 = 10^2+4\times 10+3 = 143$ ✓

(Continued→)

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Algebra

Sequences

(Continued→)

Example 3(c). The total number of tiles in diagram n is an^2+bn+1
Find the value of a and the value of b. ---[5]

Solution for $n=1$, no. of tiles $a+b+1=3$ given
$\Rightarrow a+b=2$ ---(i)

for $n=2$, $a\times2^2+b\times2+1=6$ given
$\Rightarrow 4a+2b=5$ ---(ii)

Solving (i) and (ii) $a=\frac{1}{2}$ ✓ and $b=\frac{3}{2}$ ✓

(d) Part of Louvre museum in Paris is in the shape of a square-based pyramid made from glass tiles. Each of the triangular faces of the pyramid is represented by diagram 17 in the seq.

(i) Calculate the total number tiles on one triangular face of this pyramid.

Solu. No of tiles in n_{th} diagram $=\frac{1}{2}n^2+\frac{3}{2}n+1$
$\therefore$ in 17_{th} diagram $=\frac{1}{2}\times17^2+\frac{3}{2}\times17+1$
$=171$ ✓

(ii) 11 tiles are removed from on of the triangular faces to create entrance into the pyramid.
Calculate the total number of glass tiles used to construct this pyramid.

Solu. No of Tiles $=4\times171-11=\underline{673}$ ✓

Example 4.

M-15/42/Q3

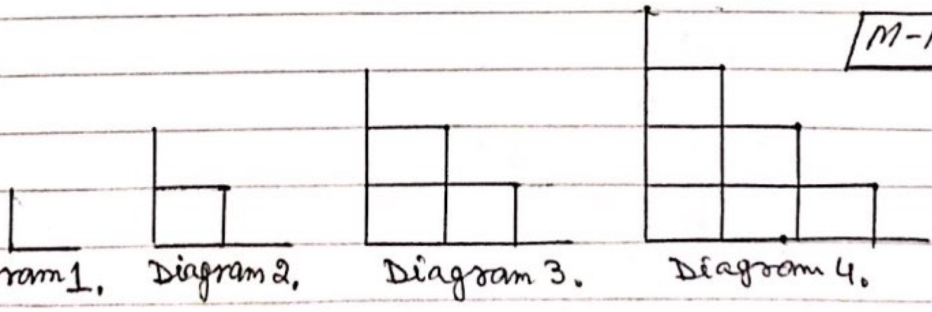


Diagram 1 shows two lines of length 1 unit at right angles forming an L.

(continued→)

Sequences

(Continued→)

Example 4. Two L's are added to diagram 1, to make diagram 2, this forms one small square.
Three L's are added to diagram 2 to make diagram 3, this forms three small squares.
The sequence of diagrams continues.

(a) Draw diagram 5. ---[1]

(b) Complete the table. ---[2]

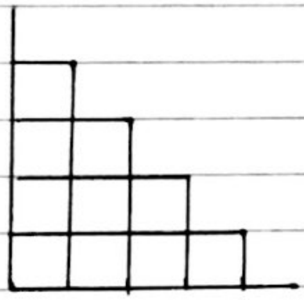
	Diagram 1	Diagm 2	Diagr 3	Diagr 4	Digm. 5
Number of lines of length 1 Unit	2	6	12	20	
Number of small squares	0	1	3	6	

(c) Find an expression, in terms of n, for number of lines. --[2]

(d) Find an expression, in terms of n, for number of small squares in Diagram n. -- [2]

Solution: (a)

diagram 5



(b) 30, 10

(c) 2, 6, 12, 20, --

Ist diff 4, 6, 8

IInd diff. 2, 2 is constant ∴ n_{th} term is Quadratic $= an^2+bn+c$ --①

for $n=1 \Rightarrow a+b+c=2$ —②

$n=2 \Rightarrow 4a+2b+c=6$ —③

$n=3 \Rightarrow 9a+3b+c=12$ —④

④-③ & ③-② $\Rightarrow \begin{cases} 5a+b=6 & —⑤ \\ 3a+b=4 & —⑥ \end{cases}$

fr ⑤ & ⑥ $a=1, b=1$

fr ② $c=0$

No of lines — from ① n_{th} term $= n^2+n$ or $n(n+1)$ ✓

(d) Proceed similarly. Small no. of squares $= \frac{1}{2}n(n-1)$ ✓