Page No.
Date 29.11.20

(0580)

IGCSE_Maths

Algebra-1

Revision.

SP-20	M-20	S-20	W-19

Suresh Goel
(Former Director)
Alliance World School
Noida - Delh. N.C.R
INDIA.

Algebra Revision

1. $y = mx + c$ [SP-20/02/Q]

Find the value of y when $m = -2$, $x = -7$ and $c = -3$ --- [2]

Solution: $y = mx + c \Rightarrow y = -2 \times (-7) + (-3) = 14 - 3 = 11$ ✓

2. $y = mx + c$ [S-20/22/Q5]

Find the value of y when $m = -3$, $x = -2$ and $c = -8$

Solution: $y = mx + c \Rightarrow y = -3 \times (-2) + (-8) = 6 - 8 = -2$ ✓ --- [2]

3. Expand and simplify. $(x+3)(x-5)(3x-1)$ --- [3]

[S-20/21/Q21]

Solution:
$$\begin{aligned}(x+3)(x-5)(3x-1) &= (x+3)[x(3x-1) - 5(3x-1)]\\ &= (x+3)[3x^2 - x - 15x + 5]\\ &= (x+3)[3x^2 - 16x + 5]\\ &= x(3x^2 - 16x + 5) + 3(3x^2 - 16x + 5)\\ &= 3x^3 - 16x^2 + 5x + 9x^2 - 48x + 15\\ &= \underline{3x^3 - 7x^2 - 43x + 15}\ ✓\end{aligned}$$

4. Simplify $5c - d - 3d - 2c$ --- [2]

[W-19/21/Q5]

Solution:
$$\begin{aligned}&5c - d - 3d - 2c\\ &= 5c - 2c - d - 3d\\ &= \underline{3c - 4d}\ ✓\end{aligned}$$

5. Expand $a(a^3 + 3)$ [W-19/22/Q3] --- [1]

Solution:
$$\begin{aligned}a(a^3+3) &= a^{3+1} + 3a\\ &= a^4 + 3a\ ✓\end{aligned}$$

6. Expand and simplify; $(x+3)(x+5)$ --- [2]

[W-19/23/Q6]

Solution:
$$\begin{aligned}(x+3)(x+5) &= x(x+5) + 3(x+5)\\ &= x^2 + 5x + 3x + 15\\ &= \underline{x^2 + 8x + 15}\ ✓\end{aligned}$$

7. Solve the equation: $\frac{1-x}{3} = 5$ [S-20/22/Q14] ---[2]

Solution: $\frac{1-x}{3} = \frac{5}{1} \Rightarrow (1-x)\times 1 = 5\times 3$

$\Rightarrow 1-x = 15$

$\Rightarrow -x = 15-1 \Rightarrow -x = 14$

$\Rightarrow x = -14$ ✓

8. Solve $\frac{x-2}{3} = 3$ ---[2]

[W-19/21/Q6]

Solution: $\frac{x-2}{3} = 3 \Rightarrow x-2 = 3\times 3$

$\Rightarrow x = 9+2 = 11$

$\therefore x = 11$ ✓

9. Write $\frac{x}{2} - \frac{2x+4}{x+1}$ as a single fraction, in its simplest form.

[W-19/21/Q18] ---[3]

Solution: $\frac{x}{2} - \frac{2x+4}{x+1} = \frac{x(x+1) - 2(2x+4)}{2(x+1)}$

$= \frac{x^2 + x - 4x - 8}{2(x+1)}$

$= \frac{x^2 - 3x - 8}{2(x+1)}$ ✓

10. Write as a single fraction in its simplest form:

$\frac{x+3}{x-3} - \frac{x-2}{x+2}$ --[4]

[M-20/42/Q5(a)]

Solution: $\frac{x+3}{x-3} - \frac{x-2}{x+2}$

$= \frac{(x+3)(x+2) - (x-2)(x-3)}{(x-3)(x+2)}$

$= \frac{x^2 + 2x + 3x + 6 - (x^2 - 3x - 2x + 6)}{(x-3)(x+2)}$

$= \frac{\cancel{x^2} + 5x + \cancel{6} - \cancel{x^2} + 5x - \cancel{6}}{(x-3)(x+2)}$

$= \frac{10x}{(x-3)(x+2}$ ✓

11. Expand and simplify: $(y+3)(y-4)(2y-1)$ ---[3]

[M-20/42/Q5(c)]

Solution: $(y+3)(y-4)(2y-1) = (y+3)[y(2y-1)-4(2y-1)]$

$= (y+3)[2y^2-y-8y+4]$

$= (y+3)(2y^2-9y+4)$

$= y(2y^2-9y+4)+3(2y^2-9y+4)$

$= 2y^3-9y^2+4y+6y^2-27y+12$

$= \underline{2y^3-3y^2-23y+12}$ ✓

12.(a) $S = ut + \frac{1}{2}at^2$ ---[2]

Find the value of S when $u=5.2$, $t=7$ and $a=1.6$

Solution: $S = ut + \frac{1}{2}at^2$

$= 5.2\times 7 + \frac{1}{2}\times 1.6\times 7^2$

$= 36.4 + 0.8\times 49$

$= 36.4 + 39.2 \quad = \underline{75.6}$ ✓

(c) Solve (i) $\frac{15}{x} = -3$

(ii) $4(5-3x) = 23$ ---[3]

Solution: (i) $\frac{15}{x} = -3 \Rightarrow -3x = 15$

$\Rightarrow x = \frac{15}{-3} = -5 \Rightarrow \underline{x=-5}$ ✓

(ii) $4(5-3x) = 23$

$\Rightarrow 20-12x = 23$

$\Rightarrow -12x = 23-20$

$\Rightarrow -12x = 3$

$\Rightarrow x = \frac{-3}{12} = -\frac{1}{4} \quad \therefore \underline{x = -\frac{1}{4}\ (\text{or } -0.25)}$ ✓

(e) Expand and simplify: $(3x-5y)(2x+y)$ ---[2]

[S-20/43/Q3]

Solution: $(3x-5y)(2x+y)$

$= 3x(2x+y) - 5y(2x+y)$

$= 6x^2+3xy-10xy-5y^2$

$= \underline{6x^2-7xy-5y^2}$ ✓

12. Solve the equation $6x-3=-12$ --[2]

[W-19/43/Q2(b)]

Solution: $6x-3=-12$

$\Rightarrow 6x=-12+3$

$\Rightarrow 6x=-9$

$\Rightarrow x=-\frac{9}{6}=-\frac{3}{2}$

$\therefore x=-\frac{3}{2}$ (or -1.5) ✓

13. Solve the equation:

$$\frac{2x+5}{3-x}=\frac{14}{15}$$ ---[3]

[S-20/43/Q4(b)]

Solution: $\frac{2x+5}{3-x}=\frac{14}{15} \Rightarrow 15(2x+5)=14(3-x)$

$\Rightarrow 30x+75=42-14x$

$\Rightarrow 30x+14x=42-75$

$\Rightarrow 44x=-33$

$\Rightarrow x=-\frac{33}{44}=-\frac{3}{4}=-0.75$

$\therefore x=-0.75$ ✓

14. Oranges cost 21 cents each.
Alex buys x oranges and Bobbie buys $(x+2)$ oranges.
The total cost of these oranges is \$4.20
Find the value of x. ---[3]

[W-19/41/Q7(a)]

Solution: $21x+21(x+2)=420$ ($\because$ \$4.20 = 420 cents)

$\Rightarrow 21x+21x+42=420$

$\Rightarrow 42x=420-42=378$

$x=\frac{378}{42}=9$

$\therefore x=9$ ✓

15. Solve the inequality $n+7 < 5n-8$ ---[2]

[SP-20/02/Q13]

Solution: $n+7 < 5n-8$

$\Rightarrow n-5n < -8-7$

$\Rightarrow -4n < -15$

$\Rightarrow n > \frac{-15}{-4}$ (Dividing both sides by a -ve number will reverse the inequality)

$\Rightarrow n > \frac{15}{4}$

$\therefore n > 3.75$ ✓

Alternate:

$n+7 < 5n-8$ $\left[\because a<b \Rightarrow b>a\right]$

$\Rightarrow 5n-8 > n+7$

$\Rightarrow 5n-n > 7+8$

$\Rightarrow 4n > 15 \Rightarrow n > 3.75$ ✓

16. Solve $\frac{x}{2} - 13 > 12+3x$ ---[2]

Solution: $\frac{x}{2} - 13 > 12+3x$

$\Rightarrow \frac{x}{2} - 3x > 12+13$

$\Rightarrow -\frac{5}{2}x > 25$

Diving both sides by $-\frac{5}{2}$, will reverse the inequality

$\Rightarrow x < 25 \times \left(-\frac{2}{5}\right)$

$\Rightarrow x < -10$ ✓

[W-19/22/Q9]

Alternate Method:

$\frac{x}{2} - 13 > 12+3x$ $\left[\because a>b \Rightarrow b<a\right]$

$\Rightarrow 12+3x < \frac{x}{2} - 13$

$\Rightarrow 3x - \frac{x}{2} < -13-12$

$\Rightarrow \frac{5}{2}x < -25$

$\Rightarrow x < \frac{-25}{5/2}$

$x < -25 \times \frac{2}{5}$

$x < -10$ ✓

17. Naga has n marbles. Panav has three times as many marbles as Naga. Naga loses 5 marbles and Panav buys 10 marbles. Together they now have more than 105 marbles. Write down and solve an inequality in n. ---[3]

[M-20/42/Q7(a)]

Solution: Naga has marbles $= n$

$\therefore$ Panav has marbles $= 3n$

In new situation:

Naga has left with $= (n-5)$

and Panav will have $= (3n+10)$

Given total no. $(n-5)+(3n+10) > 105$

$\Rightarrow 4n+5 > 105$

$\Rightarrow 4n > 105-5$

$n > \frac{100}{4} \Rightarrow n > 25$ ✓

18. Solve the inequality: $3m + 12 \leqslant 8m - 5$ ---[2]

[S-20/43/Q2(a)]

Solve: $3m + 12 \leqslant 8m - 5$

$\Rightarrow 3m - 8m \leqslant -5 - 12$

$\Rightarrow -5m \leqslant -17$

dividing both sides by -5 will reverse the inequality

$\Rightarrow m \geqslant -\frac{17}{-5} \Rightarrow m \geqslant \frac{17}{5}$

$\Rightarrow \underline{m \geqslant 3.4}$

Alternatively:

$3m + 12 \leqslant 8m - 5$

$\Rightarrow 8m - 5 \geqslant 3m + 12$ [$\because a < b \Rightarrow b > a$]

$\Rightarrow 8m - 3m \geqslant 12 + 5$

$5m \geqslant 17$

$m \geqslant \frac{17}{5}$

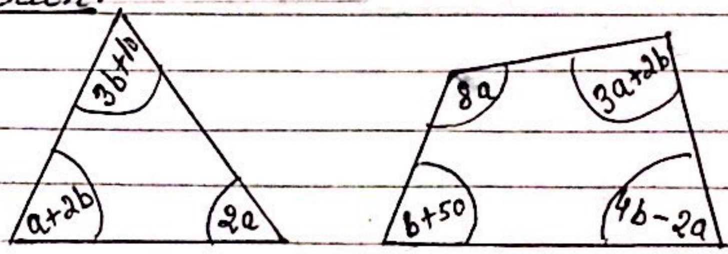
$\Rightarrow \underline{m \geqslant 3.4}$

---

§ Simultaneous Equation:

Two Linear Equation:

19(a) The diagram shows a triangle and a quadrilateral.



All angles are in degrees.

(i) For triangle, show that $3a + 5b = 170°$ ---[1]

(ii) For quadrilateral show that $9a + 7b = 310°$ ---[1]

(iii) Solve these simultaneous equations. ---[3]

(iv) Find the size of the smallest angle in the triangle. ---[1]

[W-19/43/Q2(a)]

Solution (a) $3b + 10 + a + 2b + 2a = 180°$

(i) $\Rightarrow 3a + 5b = 180 - 10 = 170°$

$\therefore 3a + 5b = 170°$ —— ①

(ii) $b + 50 + 4b - 2a + 3a + 3b + 8a = 360°$

$\Rightarrow 9a + 7b = 360 - 50$

$\Rightarrow 9a + 7b = 310°$ —— ②

(iii) To Solve: $3a + 5b = 170$ —— ①

$9a + 7b = 310°$ —— ②

multiplying eqn ① by ③

$\underline{9a + 15b = 510}$ —— ③

Subtract ③ from ① $-8b = -200$

$\Rightarrow b = \frac{-200}{-8} = 25$

Put $b = 25$ in eqn ①

$3a + 5 \times 25 = 170$

$\Rightarrow 3a = 170 - 125 = 45$

$a = 45/3 = 15°$

$\therefore a = 15°$ and $b = 25°$

(iv) Minimum angle in the triangle $= 2a$

$= 2 \times 15 = \underline{30°}$

Scanned with CamScanner

Change of Subject of a formula:

20. Make y the subject of the formula: $h^2 = x^2 + 2y^2$ ---[3]

[S-20/22/Q19]

Solution: $h^2 = x^2 + 2y^2 \Rightarrow 2y^2 = h^2 - x^2$

$\Rightarrow y^2 = \frac{h^2 - x^2}{2}$

$\Rightarrow y = \pm\sqrt{\frac{h^2 - x^2}{2}}$ ✓

21. $P = 2r + \pi r$ ---[2]

Rearrange the formula to write r in terms of P and π.

[W-19/21/Q11]

Solution: $P = 2r + \pi r \Rightarrow r(2+\pi) = P$

$\Rightarrow r = \frac{P}{(2+\pi)}$ ✓

22. Make x the subject of the formula: $x = \frac{3+x}{y}$ ---[3]

[M-20/42/Q5(d)]

Solution: $x = \frac{3+x}{y} \Rightarrow xy = 3 + x$

$\Rightarrow xy - x = 3$

$\Rightarrow x(y-1) = 3$

$\Rightarrow x = \frac{3}{(y-1)}$ ✓

23. Rearrange $2(4x - y) = 5x - 3$ to make y the subject. ---[3]

[W-19/43/Q2(c)]

Solution: $2(4x-y) = 5x-3 \Rightarrow 8x - 2y = 5x - 3$

$\Rightarrow -2y = 5x - 3 - 8x$

$\Rightarrow -2y = -3 - 3x$

$\Rightarrow 2y = 3 + 3x \Rightarrow y = \frac{3x+3}{2}$ ✓

24. Make p the subject of: (i) $5p + 7 = m$ --[2]

(ii) $y^2 - 2p^2 = h$ [W-19/42/8(a)] --[3

Solution (i) $5p + 7 = m$

$\Rightarrow 5p = m - 7$

$\Rightarrow p = \frac{m-7}{5}$ ✓

(ii) $y^2 - 2p^2 = h$

$\Rightarrow -2p^2 = -y^2 + h \Rightarrow 2p^2 = y^2 - h$

$\Rightarrow p^2 = \frac{y^2 - h}{2}$

$\Rightarrow p = \pm\sqrt{\frac{y^2 - h}{2}}$ ✓

Direct and inverse variation.

25 p is directly proportional to $(q+2)^2$ ---[3]
when $q=1$, $p=1$. Find p when $q=10$. S-20/22/Q22

Solution: Given $p \propto (q+2)^2$

$$\Rightarrow p = k(q+2)^2 \text{ ——— (1)}$$

when $q=1, p=1 \Rightarrow 1 = k(1+2)^2 \Rightarrow 1 = 9k \Rightarrow k = \frac{1}{9}$

$$\therefore \text{ from (1) } p = \frac{1}{9}(q+2)^2 \text{ ——— (2)}$$

for $q=10 \Rightarrow p = \frac{1}{9}(10+2)^2 = \frac{1}{9} \times 144 = 16$ ✓

$$\therefore p = 16 ✓$$

26. y is directly proportional to the cube root of $(x+3)$. ---[3]
when $x=5$, $y = 2/3$; Find y when $x=24$. S-20/23/Q11

Solution: Given, $y \propto (x+3)^{1/3}$

$$\Rightarrow y = k(x+3)^{1/3} \text{ ——— (1)}$$

when $x=5, y=2/3 \Rightarrow \frac{2}{3} = k(5+3)^{1/3}$

$$\Rightarrow \frac{2}{3} = k \cdot 8^{1/3} \Rightarrow 2k = 2/3$$

$$\Rightarrow k = \frac{1}{3} ✓$$

$$\therefore \text{ from (1) } y = \frac{1}{3}(x+3)^{1/3}$$

Now when $x=24 \Rightarrow y = \frac{1}{3}(24+3)^{1/3} = \frac{1}{3} \times 27^{1/3}$

$$\Rightarrow y = \frac{1}{3} \times 3 = 1$$

$$\therefore y = 1 ✓$$

27. y is inversely proportional to x^2 ---[3]
when $x=4$, $y=2$; Find y when $x=\frac{1}{2}$ W-19/21/Q15

Solution: Given, $y \propto \frac{1}{x^2} \Rightarrow y = \frac{k}{x^2}$ ——— (1)

when $x=4, y=2 \Rightarrow 2 = \frac{k}{4^2} \Rightarrow k = 16 \times 2 = 32$

$$\therefore \text{ from (1) } y = \frac{32}{x^2} \text{ ——— (2)}$$

Now when $x = \frac{1}{2} \Rightarrow y = \frac{32}{(\frac{1}{2})^2} = 32 \times 4 = 128$

$$\therefore y = 128 ✓$$

Direct and inverse variation.

28. t is inversely proportional to the square of $(x+1)$
when $x=2$, $t=5$ [W-19/23/Q20]
(a) Write t in terms of x. ---[2]
(b) When $t=1.8$, find the positive value of x. ---[2]

Solution (a) Given $t \propto \frac{1}{(x+1)^2} \Rightarrow t = \frac{k}{(x+1)^2}$ ——— ①

when $x=2, t=5 \Rightarrow 5 = \frac{k}{(2+1)^2} \Rightarrow k = 5\times 9 = 45$

$\therefore$ from ① $t = \frac{45}{(x+1)^2}$ ——— ②

(b) Now when $t=1.8$

from ② $1.8 = \frac{45}{(x+1)^2} \Rightarrow (x+1)^2 = \frac{45}{1.8} = 25$

$\Rightarrow x+1 = \pm\sqrt{25}$

$x = 5-1$ [Only + value]

$\therefore x = 4$ ✓

29. y is inversely proportional to x^2.
When $x=4$, $y=7.5$; Find y when $x=5$. ---[3]
[M-20/42/Q7(b)]

Solution: Given $y \propto \frac{1}{x^2} \Rightarrow y = \frac{k}{x^2}$ ——— ①

for $x=4$, $y=7.5 \Rightarrow 7.5 = \frac{k}{4^2} \Rightarrow k = 16\times 7.5 = 120$

$\therefore$ from ① $y = \frac{120}{x^2}$ ——— ②

Now for $x=5 \Rightarrow y = \frac{120}{5^2} = \frac{120}{25} = 4.8$

$\therefore y = 4.8$ ✓

Algebraic Indices

30. (a) Simplify $(27x^6)^{1/3}$ ---[2]
(b) Find the value of $(64x^4)^{0.5} \times 4x^{-2}$ ---[3]
[SP-20/02/Q27]

Solution (a) Simplify $(27x^6)^{1/3} = [3^3 \cdot (x^2)^3]^{1/3}$
$= ((3x^2)^3)^{1/3}$
$= (3x^2)^{3\times\frac{1}{3}} = 3x^2$ ✓

(b) $(64x^4)^{0.5} \times 4x^{-2} = (8^2(x^2)^2)^{1/2} \times 4x^{-2}$
$= [(8x^2)^2]^{1/2} \times 4x^{-2}$
$= (8x^2)^{1/2\times 2} \times 4x^{-2}$
$= 8x^2 \cdot 4x^{-2} = 32x^{2-2} = 32x^0 = 32\times 1$
$= 32$ ✓

---

31. Simplify: $\frac{p}{2q} \times \frac{4pq}{t}$ --- [2]
[S-20/22/Q10]
$= \frac{p \times 2p \times \cancel{2q}}{\cancel{2q} \times t} = \frac{2p^2}{t}$ ✓

32. Simplify $8t^8 \div 4t^4 = \frac{8t^8}{4t^4} = 2t^{8-4} = 2t^4$ ✓ ---[2]
[S-20/22/Q13]

33. Simplify (a) $(5x^4)^3$ --[2]
$= 5^3 \times (x^4)^3 = 125 \times x^{12}$ ✓ [S-20/22/Q21]
(b) $(256\, x^{256})^{3/8}$ --[2]
$= [2^8 \times (x^{32})^8]^{3/8}$
$[(2x^{32})^8]^{3/8} = (2x^{32})^{8\times\frac{3}{8}}$
$= (2x^{32})^3 = 2^3 \cdot (x^{32})^3$
$= 8\, x^{96}$ ✓

34. $\sqrt[3]{y^2} = \sqrt[6]{x}$ and $y = \sqrt[n]{x}$ find the value of n. ---[2]
[S-20/22/Q26]

Solution: Given $\sqrt[3]{y^2} = \sqrt[6]{x} \Rightarrow y^{2/3} = x^{1/6} \Rightarrow x = (y^{2/3})^6$
or $x = y^{2/3\times 6} = y^4$

Now $y^4 = x \Rightarrow y = \sqrt[4]{x}$ $(y = \sqrt[n]{x})$
$\Rightarrow n = 4$ ✓

Algebic Indices

35. Simplify: (a) $p^2 \times p^4$ --[1]

$= p^{2+4} = p^6$ ✓

(b) $m^{15} \div m^5$ --[1]

$= m^{15-5} = m^{10}$ ✓

(c) $(k^3)^5 = k^{3\times5} = k^{15}$ ✓ $\boxed{S\text{-}20/23/Q6}$ --[1]

---

36. Simplify: $2x^3 \times 3x^2$ --[2]

$= 6x^{3+2} = 6x^5$ ✓ $\boxed{W\text{-}19/21/Q7}$

37. Simplify: $\left(\frac{x^3}{8}\right)^{-\frac{4}{3}}$ $\boxed{W\text{-}19/21/Q10}$ ---[2]

$$= \left(\frac{x^3}{2^3}\right)^{-4/3} = \left(\left(\frac{x}{2}\right)^3\right)^{-\frac{4}{3}} = \left(\frac{x}{2}\right)^{3\times-\frac{4}{3}} = \left(\frac{x}{2}\right)^{-4}$$

$$= \frac{1}{\left(\frac{x}{2}\right)^4} = \frac{1}{\frac{x^4}{16}} = \frac{16}{x^4}$$ ✓

---

38. (a) $3^{-2} \times 3^x = 81$ find the value of x. ---[2]

$\Rightarrow 3^{x-2} = 3^4 \Rightarrow x-2=4 \Rightarrow x=6$ ✓

(b)

$x^{-\frac{1}{3}} = 32x^{-2}$ find the value of x. --[3]

$\Rightarrow \frac{x^{-1/3}}{x^{-2}} = 32 \Rightarrow x^{2-\frac{1}{3}} = 32$ $\boxed{W\text{-}19/22/Q21}$

$\Rightarrow x^{5/3} = 2^5$

$\Rightarrow x = (2^5)^{3/5} = 2^{5\times\frac{3}{5}} = 2^3 = 8$ ✓

$\therefore x = 8$ ✓

---

39. $2^{12} \div 2^{\frac{k}{2}} = 32$ find the value of k. --[2]

$\boxed{M\text{-}20/42/Q5(b)}$

Solution: $2^{12} \div 2^{k/2} = 32 \Rightarrow 2^{12-k/2} = 2^5$

$\Rightarrow 12 - k/2 = 5$

$\Rightarrow \frac{k}{2} = 12-5 = 7$

$\Rightarrow k = 7\times2 = 14$

$\therefore k = 14$ ✓

40. Simplify: $(27x^9)^{2/3}$ [S-20/41/Q3(d)] ---[2]

Solution: $(27x^9)^{2/3} = (3^3.(x^3)^3)^{2/3} = (3x^3)^{3\times 2/3}$
$= (3x^3)^2 = 3^2.(x^3)^2$
$= 9x^6$ ✓

Solving two simultaneous linear equations.

41. The cost of one ruler is r cents. The cost of one protractor is p cents. The total cost of 5 rulers and 1 protractor is 245 cents. The total cost of 2 rulers and 3 protractors is 215 cents. Write down two equations in terms of r and p, and solve those equations to find the cost of one protractor. ---[5]
[W-19/41/Q7(b)]

Solution: $5r + p = 245$ — (1)
$2r + 3p = 215$ — (2)
Multiplying equation (1) by 3.
$15r + 3p = 735$ — (3)
from Eqn (2) $-2r \pm 3p = 215$ — (2)
(3) − (2) ⇒ $13r = 520 \Rightarrow r = \frac{520}{13} = 40$ ✓
Put $r = 40$ in (1)
$5\times 40 + p = 245 \Rightarrow p = 245 - 200 = 45$
∴ Cost of one protractor = 45 cents.

Factorise

Page No. P-13
Date

42. Factorise Completely: $3x^2 - 12xy$ [M-20/22/Q9(a)] ---[2]

Solution: $3x^2 - 12xy = \underline{3x(x-4y)}$ ✓

43. Factorise Completely: (a) $21a^2 + 28ab$ ---[2]

$= \underline{7a(3a+4b)}$ ✓

(b) $20x^2 - 45y^2$ ---[3]

$= 5(4x^2 - 9y^2) = 5[(2x)^2 - (3y)^2]$ $[\because a^2 - b^2 = (a+b)(a-b)]$

$= \underline{5(2x+3y)(2x-3y)}$ ✓ [S-20/21/Q9]

44. Simplify: $\dfrac{2x^2 + x - 15}{ax + 3a - 2bx - 6b}$ — ① ---[5]

[S-20/22/Q25]

Numerator: $2x^2 + x - 15$

$= 2x^2 + 6x - 5x - 15$

$= 2x(x+3) - 5(x+3)$

$= (x+3)(2x-5)$ — ②

Denominator: $ax + 3a - 2bx - 6b$

$= a(x+3) - 2b(x+3)$

$= (a-2b)(x+3)$ — ③

from ② and ③ in ① $= \dfrac{\cancel{(x+3)}(2x-5)}{\cancel{(x+3)}(a-2b)} = \underline{\dfrac{2x-5}{a-2b}}$ ✓

45. (a) Factorise: $18y - 3ay + 12x - 2ax$ ---[2]

$= 3y(6-a) + 2x(6-a)$ [W-19/22/Q20]

$= \underline{(6-a)(3y+2x)}$ ✓

(b) Factorise: $3x^2 - 48y^2$ ---[3]

$= 3(x^2 - 16y^2)$

$= 3(x^2 - (4y)^2)$

$= \underline{3(x+4y)(x-4y)}$ ✓

46. Factorise: $5p + pt$ ---[1]

$= p(5+t)$ ✓ [W-19/21/Q2]

47. Factorise: (a) $12x+15$

$= \underline{3(4x+5)}$ ✓ --[1]

(b) $xy-2x+3y-6$ --[2]

$= x(y-2)+3(y-2)$

$= \underline{(x+3)(y-2)}$ ✓

[W-19/23/Q11]

48. Simplify: $\dfrac{x^2+5x}{x^2-25}$ --[3]

[W-19/43/Q2]

Solution: $\dfrac{x^2+5x}{x^2-25} = \dfrac{x(x+5)}{x^2-5^2}$

$= \dfrac{x(x+5)}{(x-5)(x+5)} = \underline{\dfrac{x}{x-5}}$ ✓

§ Quadratic Equation: $ax^2+bx+c=0, \quad a\neq 0$

(1) Solving a quad. equ^n. using factor method:

49. Solve the simultaneous equations: $y=4-x$
and $x^2+2y^2=67$ --[6]

Solution: $y=4-x$ —— ①
$x^2+2y^2=67$ —— ②

Put $y=4-x$ in ② $x^2+2(4-x)^2=67$

$x^2+2(16-8x+x^2)=67$

$x^2+32-16x+2x^2-67=0$

$3x^2-16x-35=0$ $\because$ [$35\times3=105 = 21\times5$]

$\Rightarrow 3x^2-21x+5x-35=0$

$3x(x-7)+5(x-7)=0$

$(x-7)(3x+5)=0$

$\Rightarrow x=7$ or $x=-5/3$

fm ① $y=-3$, $y=\frac{17}{3}=5\frac{2}{3}$

$\therefore x=2, y=-3$

or $x=-5/3, y=5\,2/3$ ✓

50. The perimeter of a rectangle is 80 cm. The area of the rectangle is $A\ cm^2$.

x cm.

(i) Show that $x^2 - 40x + A = 0$ --- [3]

(ii) When A is 300, solve the equation $x^2 - 40x + A = 0$ by factorisation. --- [3]

(iii) When $A = 200$, solve the equation $x^2 - 40x + A = 0$ using quadratic formula, giving your answers to 2 d.p. --- [4]

[SP-20/04/Q5(a)]

Solution: Let the length of the rectangle $= y$ cm.

Given breadth $= x$ cm.

(i) Hence perimeter $= 2(x+y) = 80$

$\Rightarrow x + y = 40 \Rightarrow y = 40 - x$ —— (1)

Area of the rectangle $A = xy$

$A = x(40 - x)$

$\Rightarrow A = 40x - x^2$

$\Rightarrow \underline{x^2 - 4x + A = 0}$ —— (2) ✓

(ii) for $A = 300$, from (2)

$x^2 - 40x + 300 = 0$

$x^2 - 30x - 10x + 300 = 0$

$x(x-30) - 10(x-30) = 0$

$(x-30)(x-10) = 0$

$x - 10 = 0$ or $x - 30 = 0 \Rightarrow \underline{x = 10\text{ cm} \text{ or } x = 30\text{ cm}}$ ✓

(iii) for $A = 200$ from (2) $x^2 - 40x + 200 = 0$

Using quad. formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

{ for quadratic eqn $ax^2 + bx + c = 0$

$a = 1,\ b = -40,\ c = 200$

$b^2 - 4ac = (-40)^2 - 4 \times 1 \times 200$

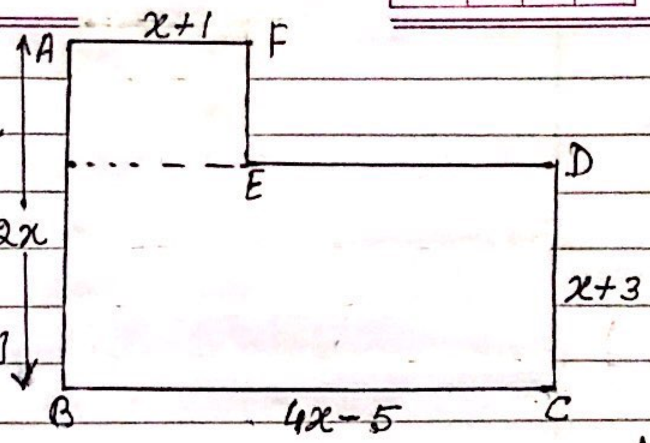
$= 1600 - 800 = 800$

$$x = \frac{+40 \pm \sqrt{800}}{2 \times 1}$$

$$= \frac{40 \pm 28.2842}{2} = \frac{68.2842}{2} \text{ or } \frac{11.7158}{2}$$

$\therefore x = \underline{34.14}$ or $x = \underline{5.86}$ ✓

51. All the lengths in this question are in centimeters. The diagram shows a shape ABCDEF. The total area of the shape is 342cm.



(a) Show that $x^2+x-72=0$ ---[5]

(b) Solve by factorisation:
$$x^2+x-72=0$$ ---[3]

(c) Work out the perimeter of the shape ABCDEF. ---[2]

(d) Calculate angle DBC. [S-20 / 43 / Q5] --[2]

Solution: Height of the small rectangle $= 2x-(x+3) = (x-3)$

(a) Area of the shape $= (x-3)(x+1) + (x+3)(4x-5) = 342$

$$\Rightarrow x^2+x-3x-3+4x^2-5x+12x-15 = 342$$
$$\Rightarrow 5x^2+5x-360=0$$
$$\Rightarrow \underline{x^2+x-72=0} \checkmark$$

(b) To Solve: $x^2+x-72=0$

$$x^2+9x-8x-72=0$$
$$x(x+9)-8(x+9)=0$$
$$(x-8)(x+9)=0$$
$$\underline{x=8} \text{ or } \underline{x=-9} \checkmark$$

(c) Perimeter of the shape $= 2x+4x-5+x+3+4x-5+x-3$

$$= 12x-10$$
$$= 12\times 8-10 \quad \left[\text{Put } x=8,\ x=-9^{\times}\right]$$
$$= \underline{86} \checkmark$$

(d) $\tan DBC = \dfrac{x+3}{4x-5} = \dfrac{8+3}{4\times 8-5} = \dfrac{11}{27}$

angle $DBC = \tan^{-1}\left(\dfrac{11}{27}\right) = 22.16$

$\therefore$ angle $DBC = \underline{22.2^\circ} \checkmark$

52. Carol walks 12 Km at x Km/h and then a further 6 Km at $(x-1)$ Km/h. The total time taken is 5 hours.

(i) Write an equation, in terms of x and show that it simplifies to $5x^2-23x+12=0$ --[3]

(ii) Factorise: $5x^2-23x+12$ --[2]

(iii) Solve the equation $5x^2-23x+12=0$ --[1]

(iv) Write down Carol's walking speed during the final 6 Km --[1]

[W-19|41|Q7]

Solution:

(i) $\frac{12}{x}+\frac{6}{x-1}=5$

$\Rightarrow \frac{12(x-1)+6x}{x(x-1)}=5 \Rightarrow 12x-12+6x=5x(x-1)$

$\Rightarrow 18x-12=5x^2-5x$

$\Rightarrow 5x^2-5x-18x+12=0$

$\Rightarrow \underline{5x^2-23x+12=0}$ ✓

(ii) $5x^2-23x+12$

$5x^2-20x-3x+12$ $\quad [\because 5\times 12=60=20\times 3]$

$5x(x-4)-3(x-4)$

$\underline{(x-4)(5x-3)}$ ✓

(iii) To solve $5x^2-23x+12=0$

or $(x-4)(5x-3)=0$ (from Part (ii))

$x-4=0$ or $5x-3=0$

$\underline{x=4 \text{ or } x=3/5}$ ✓

(iv) speed during final 6 Km $=(x-1)$ [for $x=4$

$=4-1$

$=\underline{3 \text{ Km/H}}$ ✓

53. Solve: $\frac{1}{x} - \frac{2}{x+1} = 3$

Show all your working and give your answers correct to 2 decimal places. ---[7]

[W-19/43/Q10]

Solution: $\frac{1}{x} - \frac{2}{x+1} = 3$

$\Rightarrow \frac{x+1-2x}{x(x+1)} = 3$

$\Rightarrow 1-x = 3x(x+1)$

$\Rightarrow 1-x = 3x^2+3x$

$\Rightarrow 3x^2+4x-1=0$

Quad. formula:
$ax^2+bx+c=0$
$a=3,\ b=4,\ c=-1$
$b^2-4ac = 4^2-4\times3(-1)$
$= 16+12 = 28$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-4 \pm \sqrt{28}}{2\times3}$

$= \frac{-4 \pm 5.291}{6}$

$= \frac{1.291}{6}$ or $\frac{-9.291}{6}$

$x = 0.215$ or -1.548

$\therefore x = 0.22$ or $x = -1.55$

---

54. The solution of the equation $x^2+bx+c=0$ are $\frac{-7+\sqrt{61}}{2}$ and $\frac{-7-\sqrt{61}}{2}$. ---[3]

Find the value of b and the value of c. [S-20/42/Q9(b)]

Solution: $x^2+bx+c=0$

Solution are given $x = \frac{-7 \pm \sqrt{61}}{2}$ — ①

Quadratic formula for $ax^2+bx+c=0$
$a=1,\ b=b$ and $c=c$
$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
$x = \frac{-b \pm \sqrt{b^2-4c}}{2}$ — ②

Comparing ① and ②

$-b = -7 \Rightarrow b = 7$ and $b^2-4c = 61$

$\Rightarrow 7^2-4c = 61$

$\Rightarrow c = \frac{12}{-4} = -3$

$\therefore b = 7$ and $c = -3$

Completing the Square.

55. $x^2 - 12x + a = (x+b)^2$ ---[2]
Find the value of a and the value of b. [M-20/22/Q20]

Solution: $x^2 - 12x + a = (x+b)^2$
$\Rightarrow x^2 - 12x + a = x^2 + 2bx + b^2$ —— ①
Comparing the coeff of x and the constant terms:
$\Rightarrow 2b = -12 \Rightarrow b = -6$ ✓
and $a = b^2 = (-6)^2 = 36$ ✓
$\therefore$ $a = 36$ and $b = -6$ ✓

56. Find the turning point of $y = x^2 + 4x - 3$ by completing the square.
[SP-20/02/Q25] --- [4]

Solution: $y = x^2 + 4x - 3$
$y = x^2 + 4x + 2^2 - 2^2 - 3$
$= (x+2)^2 - 7$
Turning point $(-2, -7)$ ✓

[Note: $y = (x-h)^2 + k$
Turning point (or vertex is (h, k) ✓)]

Given $x^2 + bx + c$
To make complete square add and subtract $\left(\frac{b}{2}\right)^2$
$x^2 + bx + \left(\frac{b}{2}\right)^2 - \frac{b^2}{4} + c$
$\left(x + \frac{b}{2}\right)^2 - \left(\frac{b^2 - 4c}{4}\right)$

57. (a) $x^2 - 18x - 27$ in the form $(x+k)^2 + h$ ---[2]
(b) Use your answer to part (a) to solve the eqn $x^2 - 18x - 27 = 0$ ---[2]
[S-20/23/Q18]

Solution (a) $x^2 - 18x - 27$
$= x^2 - 18x + 9^2 - 81 - 27$
$= (x-9)^2 - 108$ ✓

(b) To solve $x^2 - 18x - 27 = 0$
Part (a) $\Rightarrow$ $(x-9)^2 - 108 = 0 \Rightarrow (x-9)^2 = 108$
$x - 9 = \pm\sqrt{108} = \pm 10.39$
$\therefore x = 10.39 + 9$ or $-10.39 + 9$
$x = 19.39$ or -1.39 ✓

55. (i) Write x^2+8x-9 in the form $(x+k)^2+h$ ---[2]
(ii) Use your answer to part (i) to solve the equation $x^2+8x-9=0$ --[2]
[S-20/42/Q9(a)]

Solution: x^2+8x-9

(i)
$$= x^2+8x+\left(\frac{8}{2}\right)^2-\left(\frac{8}{2}\right)^2-9$$
$$= x^2+8x+4^2-16-9$$
$$= (x+4)^2-25 \quad \text{——— (1)}$$

(ii) To solve $x^2+8x-9=0$

from part (i) $\Rightarrow (x+4)^2-25=0$
$$\Rightarrow (x+4)^2=25$$
$$x+4=\pm\sqrt{25}=\pm 5$$
$$x=5-4 \text{ or } x=-5-4$$
$$x=1 \text{ or } x=-9$$

56. Solve the simultaneous equations: $y=5x^2+4x-19$ ---[5]
$y=4x+1$ [SP-20/02/Q28]

Solution: $y=5x^2+4x-19$ ——— (1)
$y=4x+1$ ——— (2)

from (1) and (2) $5x^2+4x-19=4x+1$
$$5x^2=20 \Rightarrow x^2=4 \Rightarrow x=\pm\sqrt{4}$$
$$\Rightarrow x=2,\ x=-2$$

from (1) $x=2 \Rightarrow y=9$
$x=-2 \Rightarrow y=-7$

$\therefore x=2,\ y=9$
or $x=-2,\ y=-7$

57. Solve the simultaneous equations: $x=7-3y$ ---[6]
$x^2-y^2=39$ [M-20/22/Q16]

Solution: $x=7-3y$ ——— (1)
$x^2-y^2=39$ ——— (2)

Put $x=7-3y$ in (2) $\Rightarrow (7-3y)^2-y^2=39$
$$\Rightarrow 49+9y^2-42y-y^2-39=0$$
$$\Rightarrow 8y^2-42y+10=0$$
$$\text{or } 4y^2-21y+5=0 \Rightarrow$$

$$4y^2-20y-y+5=0$$
$$4y(y-5)-1(y-5)=0$$
$$(y-5)(4y-1)=0$$
$$y=5,\quad y=\frac{1}{4}$$

for (1)
$y=5,\ x=-8$
$y=\frac{1}{4},\ x=\frac{25}{4}$

Sequences

5.8. (a) Complete the table for 5th and the nth term of each sequence:

1st term	2nd term	3rd term	4th term	5th term	nth term
9	5	1	−3		
4	9	16	25		
1	8	27	64		
8	16	32	64		

---[11]

(b) 0, 1, 1, 2, 3, 5, 8, 13, 21, ---

This is a Fibonacci sequence.

After first two terms, rule to find the next term is "add the two previous terms". For example, 5+8=13

Use this rule to complete each of the following Fibonacci sequence.

2 4 --- --- ---

1 --- --- ---- 11

---- −1 ---- --- 1 --[3]

(c) $\frac{1}{3}, \frac{3}{4}, \frac{4}{7}, \frac{7}{11}, \frac{11}{18}, ----$

(i) One term of this sequence is $\frac{p}{q}$

Find, in terms of p and q, the next term in this sequence. --[1]

(ii) Find the 6th term of this sequence. --[1]

[W-19/41/Q10]

Solution (a)

5th term	nth term
−7	$13-4n$
36	$(n+1)^2$
125	n^3
128	2^{n+2}

(b) ---, ---, 6, 10, 16.

---, 3, 4, 7, ----

2, ---, 1, 0, ---

(c) (i) $\frac{p}{p+q}$

(ii) $\frac{18}{29}$

59 Find the n_{th} term in each sequence.

(i) 4, 2, 0, −2, −4 ---[2]

(ii) 1, 17, 17, 31, 49 ---[2]

[M-20/42/7(C)]

Solution (i) 4, 2, 0, −2, −4

It is an arithmetic sequence,

with common difference $d = 2-4 = -2$

n_{th} term $= a + nd$ —① (Linear function in n)

for $n=1$, first term $4 = a + 1\times(-2)$ ($\because d = -2$)

$\Rightarrow a = 6$

$\therefore$ from ① n_{th} term $= \underline{(6-2n)}$ $\left[\begin{matrix} a=6 \\ d=-2 \end{matrix}\right]$

(ii) 1 7 17 31 49

Ist diff. 6 10 14 18

IInd difference 4 4 4 is constant ①

$\therefore$ n_{th} term $= an^2 + bn + c$ (quad in n)

$n = 1, 2, 3, 4$ with terms $\rightarrow$ $(a+b+c)$ $(4a+2b+c)$ $(9a+3b+c)$, $(16a+4b+c)$

② Ist diff. $3a+b$ $5a+b$ $7a+b$

IInd difference $2a$ $2a$ -- is constant.

from:

① & ② Comparing $2a = 4 \Rightarrow a = 2$

$3a+b = 6 \Rightarrow 3\times2+b = 6 \Rightarrow b = 0$

$a+b+c = 1 \Rightarrow 2+0+c \Rightarrow c = -1$

$\therefore$ Required n_{th} term an^2+bn+c

$= 2n^2 + 0 - 1$

$\therefore$ n_{th} term $= \underline{2n^2 - 1}$

60. Here is a sequence of numbers.

$7, 5, 3, 1, -1, \ldots$

(a) Find the next term in the sequence. --[1]

(b) Find an expression for the n_{th} term of this sequence. --[2]

[SP.20/02/Q/5]

Solution (a) $7, 5, 3, 1, -1, \ldots$

It is an arithmetic sequence

Next term $= -1-2 = \underline{-3}$ ✓ (d = Common differce $= 5-7 = -2$ ✓ (is constant)

(b) Let n_{th} term $= a+nd$ —— ①

for $n=1$, 1st term $7 = a+1\times(-2)$ as $d=-2$

$\Rightarrow a = 9$ ✓

$\therefore$ from ① n_{th} term $= \underline{9-2n}$ ✓ $\left\{\begin{array}{l} \text{as } d=-2 \\ a=9 \end{array}\right.$

---

61. (a) n_{th} term of a sequence is $60-8n$

Find the largest number in this sequence. ---[1]

Solution: the largest number will be for $n=1$

$= 60-8\times 1 = \underline{52}$ ✓

(b) Here are the first five terms of a different sequence:

12 19 26 33 40

--[2]

Find an expression for the n_{th} term. [S-20/21/Q5]

Solution: 12, 19, 26, 33, 40 is an arithmetic sequen

with common differen $d = 19-12 = 7$ ✓

and first term $= 12$

Now the n_{th} term $= a+nd$

$= a+7\times n$ —① ($\because d=12$)

for $n=1$, first term $12 = a+7\times 1 \Rightarrow a=5$

$\therefore$ from ① Required n_{th} term

$= a+nd$

$= \underline{5+7n}$ ✓ $\left(\because \begin{array}{l} a=5 \\ d=7 \end{array}\right)$

62. Find the n_{th} term of each sequence:

(a) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \ldots$ ---[1]

(b) 1, 5, 25, 125, 625, -- (1)

Solution (a) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10},$

Here the denominators form an A.P with even numbers

n_{th} term $= \frac{1}{2n}$ ✓

(b) 1, 5, 25, 125, 625

$= 5^0, 5^1, 5^2, 5^3, 5^4, \ldots$

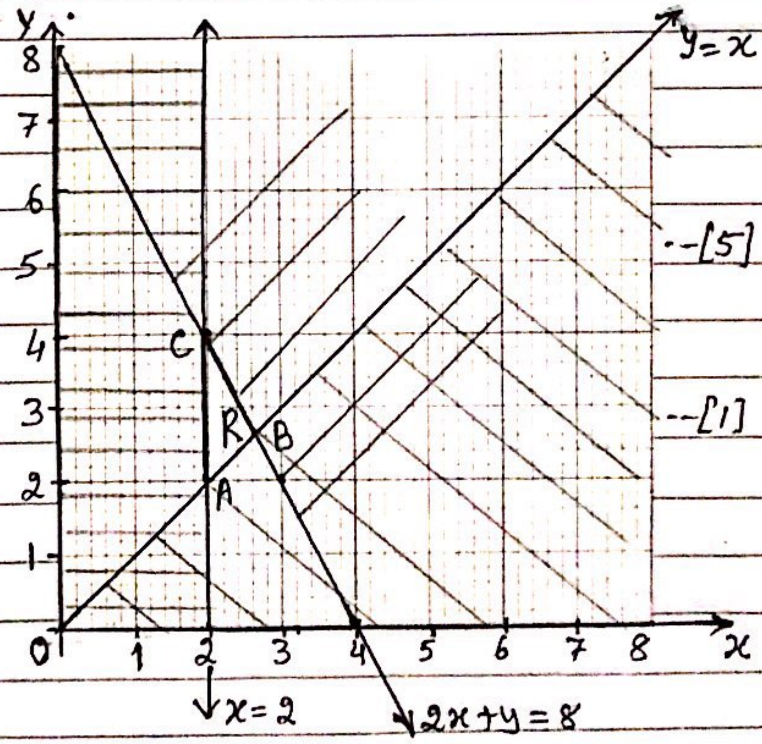
n_{th} term $= 5^{n-1}$ ✓

63 (a) By drawing suitable lines and shading unwanted region,

$x \geq 2$, $y \geq x$ and $2x + y \leq 8$. --[5]

(b) Find the largest value of $(x+y)$ in the region R. --[1]

S-20/22/Q23



Solution: $2x + y \leq 8$

for $2x + y = 8$, $(0,8), (4,0), (2,4), (3,2)$

Region R is bounded by triangle ABC. $A(2,2), B(\frac{8}{3}, \frac{8}{3}), C(2,4)$

To find the max value of $P = (x+y)$

at $A(2,2)$ $P = 2+2 = 4$

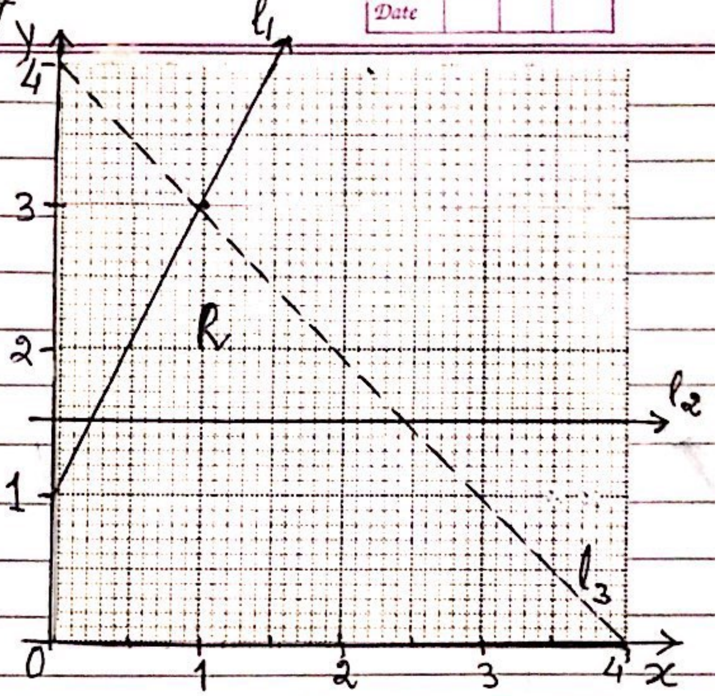
$B(\frac{8}{3}, \frac{8}{3})$ $P = \frac{8}{3} + \frac{8}{3} = \frac{16}{3} = 5\frac{1}{3}$

$C(2,4)$ $P = 2+4 = \underline{6}$ ✓

∴ Maximum value of $x+y = \underline{6}$ ✓

Linear Programming

62. Write down the three inequalities that define the region R.



S-20/23/Q13 ...[4]

Solution:

line l_1, Passes through (0,1) and (1,3), Equⁿ of l_1

$$y-1 = \frac{3-1}{1-0}(x-0)$$

$\Rightarrow$ $l_1: y = 2x+1$ ✓

line l_2; $y = 1.5$

line l_3; $x+y=4$

$\therefore$ Three inequalities defining R are;

$x+y < 4$ ✓

$y \geq 1.5$ ✓

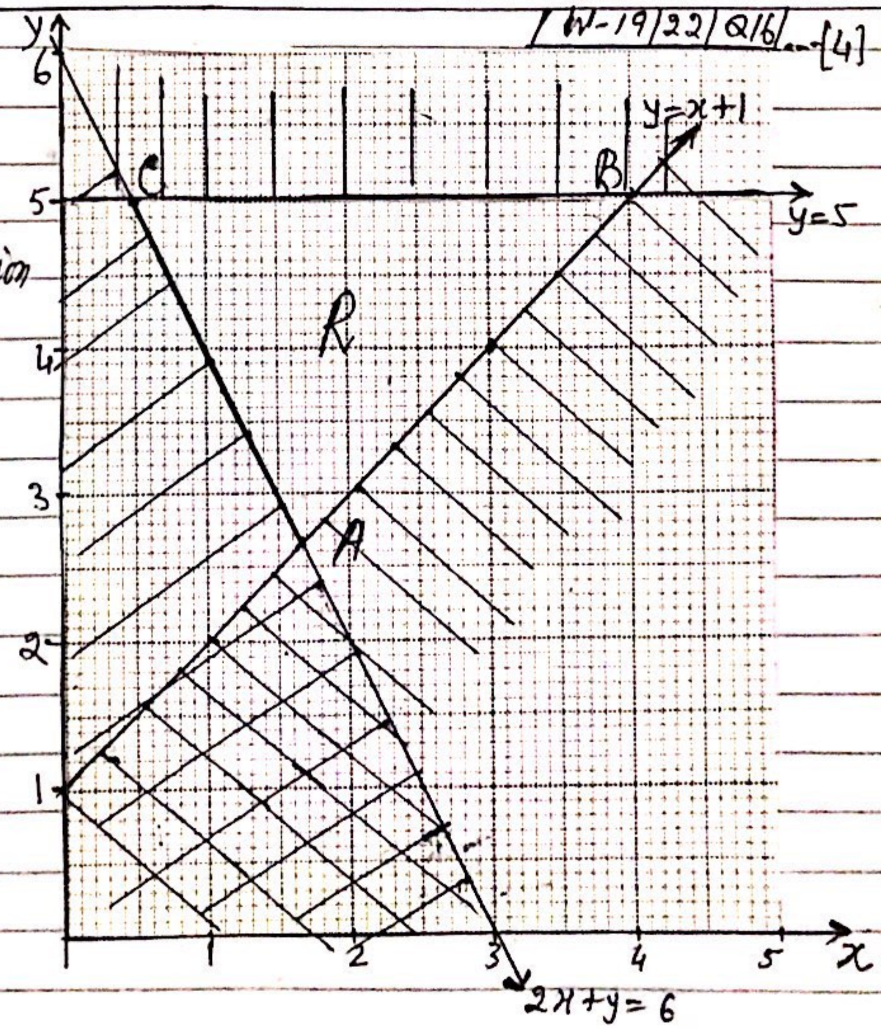
$y \leq 2x+1$ ✓

W-19/22/Q16 ...[4]

63. By shading the unwanted region of the grid, find and label the region R, that satisfies the following inequalities:

$y \leq 5$, $2x+y \geq 6$, and $y \geq x+1$

$y = x+1 \rightarrow (0,1), (3,4), (4,5)$



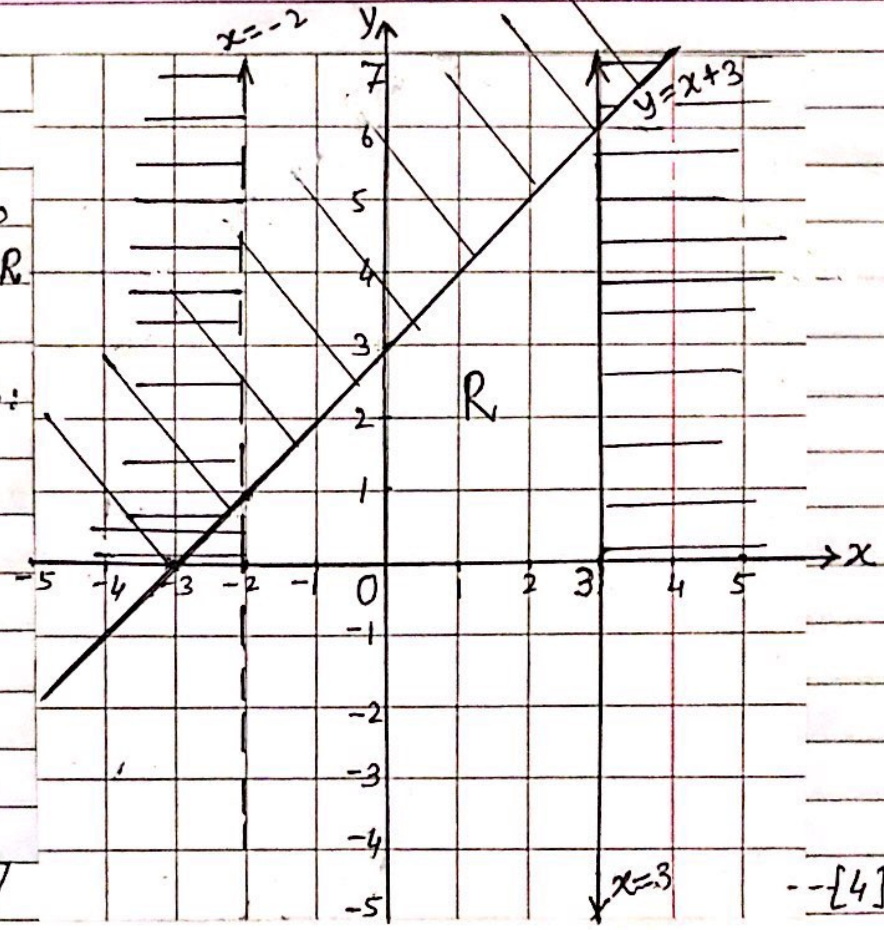
Triangle ABC and inside represents → 'R'

Scanned with CamScanner

64. By shading the unwanted regions on the grid, draw and label the region R that satisfies the following inequalities:

$-2 < x \leq 3$
and $y \leq x+3$

$y = x+3$
$(0,3), (-3,0)$



[W-19/23/Q17] --[4]

65. Raheem makes baskets and mats.
Each week he makes x baskets and y mats.
He makes fewer than 10 mats.
The number of mats he makes is greater than or equal to the number of baskets he makes.

(a) One of the inequalities that shows this information is $y < 10$.
Write down the other inequality. ---[1]

(b) He takes $2\frac{1}{4}$ hours to make a basket and $1\frac{1}{2}$ hour to make a mat. Each week he works for a maximum of 22.5 hours.
Show that $3x + 2y \leq 30$ ---[2]

(c) On the grid, draw three straight lines and shade the unwanted regions to show these inequilities. ---[5]

(d) He makes \$40 profit on each basket he sells and \$28 on each mat he sells.
Calculate the maximum profit he can make each week. ---[2]

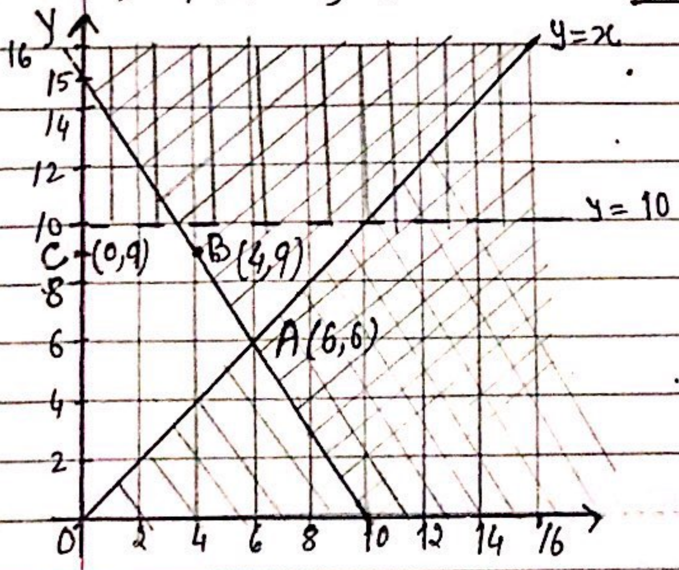
[S-20/41/Q6]

Solution: (a) $y \geq x$

(b) $\frac{9}{4}x + \frac{3}{2}y \leq 22.5$

$\Rightarrow 9x + 6y \leq 90 \Rightarrow \underline{3x + 2y \leq 30}$ ✓

(c)



(c) $3x + 2y = 30$ ——① $(0,15), (10,0)$ ✓

$y < 10$ ——②

and $y \geq x$ ——③

(d) Profit function $P = (40x + 28y)$

$O(0,0) \rightarrow P = 0$

$A(6,6) \rightarrow P = 40\times6 + 28\times6 = \$408$

$B(4,9) \rightarrow P = 40\times4 + 28\times9 = \$412$ ✓

$C(0,9) \rightarrow P = 40\times0 + 28\times9 = \$252$

Max Profit = \$412 ✓

66. A car hire company has x small cars and y large cars.
The company has at least 6 cars in total.
The number of large cars is less than or equal to the numbers of small cars,
The largest number small cars is 8.

(a) Write down three inequalities, in terms of x and/or y, to show this information. --[3]

(b) A small car can carry 4 people and a large car can carry 6 people, one day the largest number of people to be carried is 60, show that $2x+3y \leq 30$ --- [1]

(c) By shading the unwanted regions on the grid, show and label the region R that satisfy all four inequalities. --[6]

(d) (i) Find the number of small cars and the number of large cars needed to carry exactly 60 people. --[1]

(ii) When the company uses 7 cars, find the largest number of people that can be carried. --[2] W-19/41/Q9

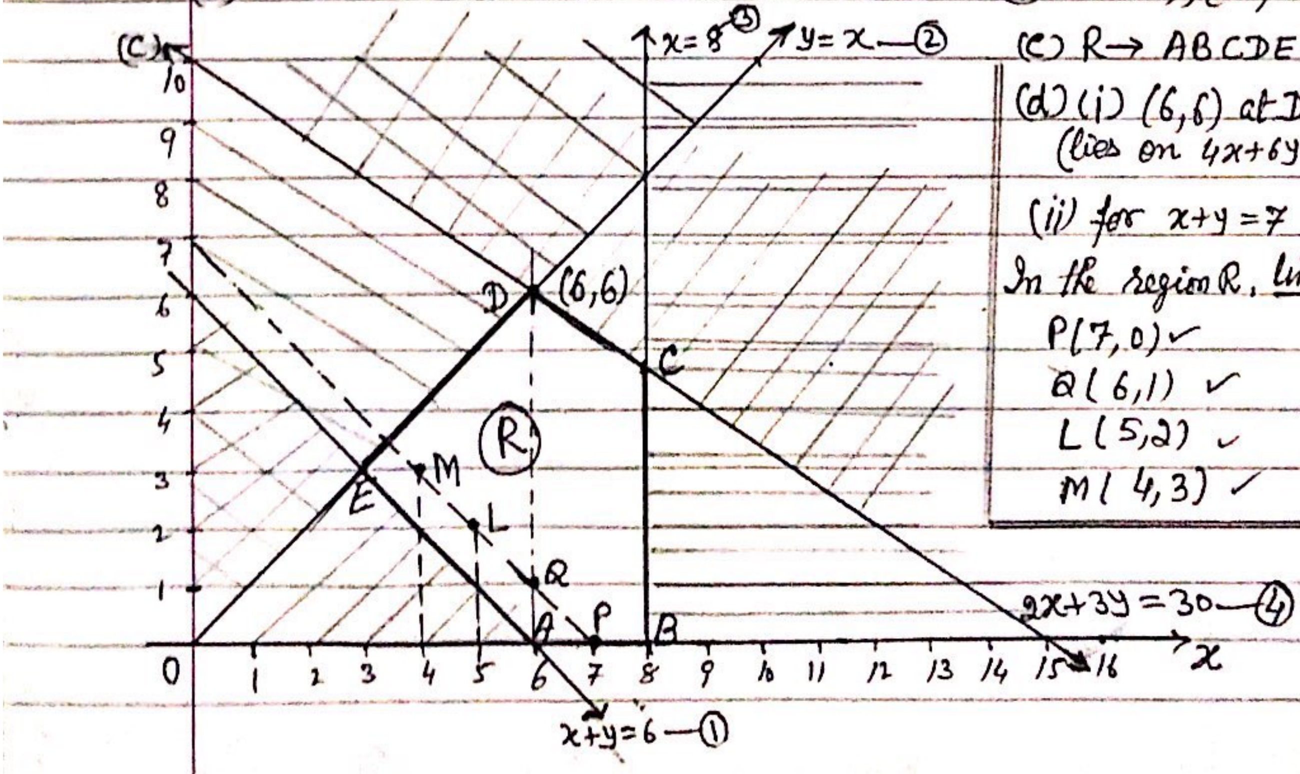
Solution (a) $x+y \geq 6$ ——— ①

$y \leq x$ ——— ②

$x \leq 8$ ——— ③

(b) $4x+6y \leq 60 \Rightarrow 2x+3y \leq 30$ ——— ④ $(0,10), (15,0)$

(c)



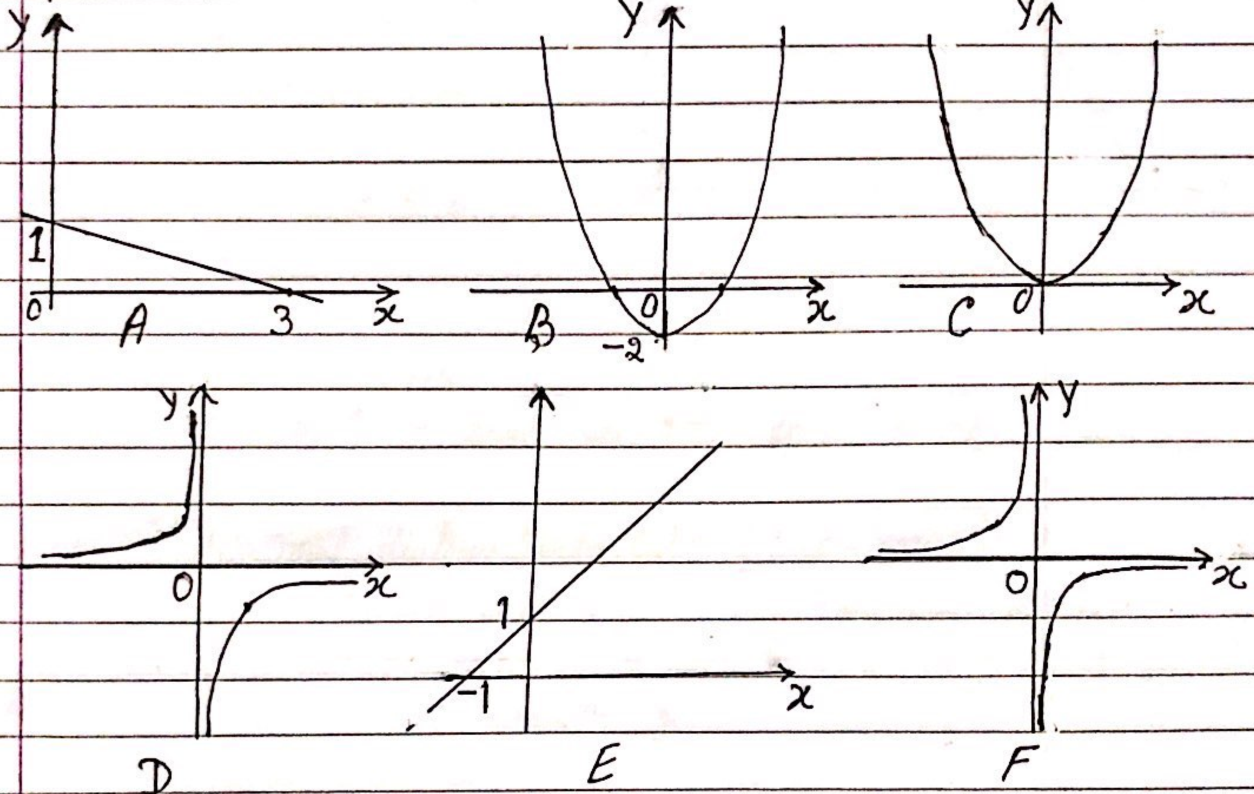
(c) $R \rightarrow ABCDE$ ✓

(d) (i) $(6,6)$ at D.
(lies on $4x+6y=60$)

(ii) for $x+y=7$
In the region R, line PM
$P(7,0)$ ✓
$Q(6,1)$ ✓
$L(5,2)$ ✓
$M(4,3)$ ✓

Graphs

67. The diagrams A, B, C, D, E and F are six graphs of different function:



Complete the table to identify the correct graph for each function. One has been done for you:

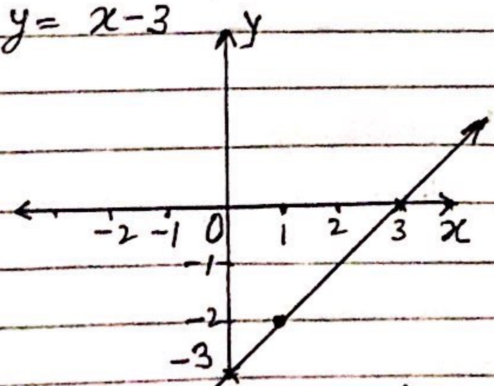
Function	$y = x+1$	$y = 1 - x/3$	$y = 2x^2$	$y = \frac{-4}{x}$
Diagram	E			

A ✓ C ✓ D ✓ ---[3]

[SP-20/02/Q21]

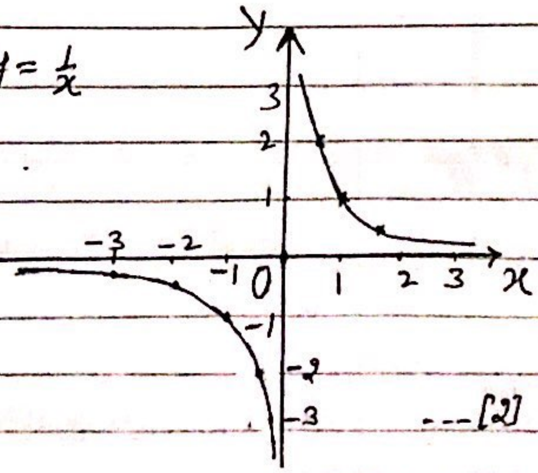
68. Sketch the graph of:

(a) $y = x-3$



(0, −3), (3, 0) (1, −2) ---[1]

(b) $y = \frac{1}{x}$



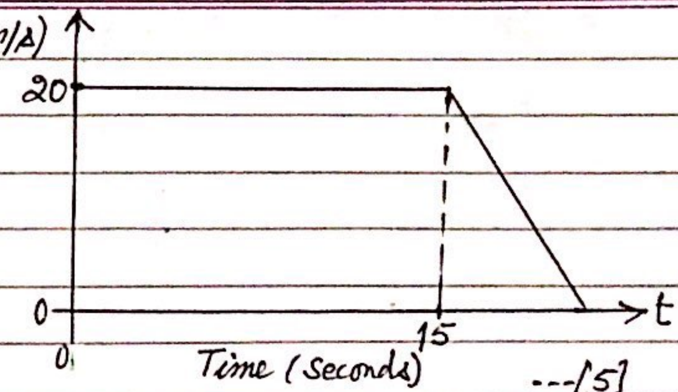
---[2]

[M-20/22/Q10]

Travel Graphs.

69.

A car travels at 20m/s for 15 seconds before it comes to rest by decelerating at $2.5\ m/s^2$.
Find the total distance travelled. ---[5]



[W-19/21/Q 26]

Solution: speed = 20 m/s

deceleration = $\frac{speed}{time} \Rightarrow 2.5 = \frac{20}{t} \Rightarrow t = \frac{20}{2.5} = 8$ sec.

$\therefore$ Total distance travelled $= \frac{1}{2}(15 + (15+8)) \times 20$

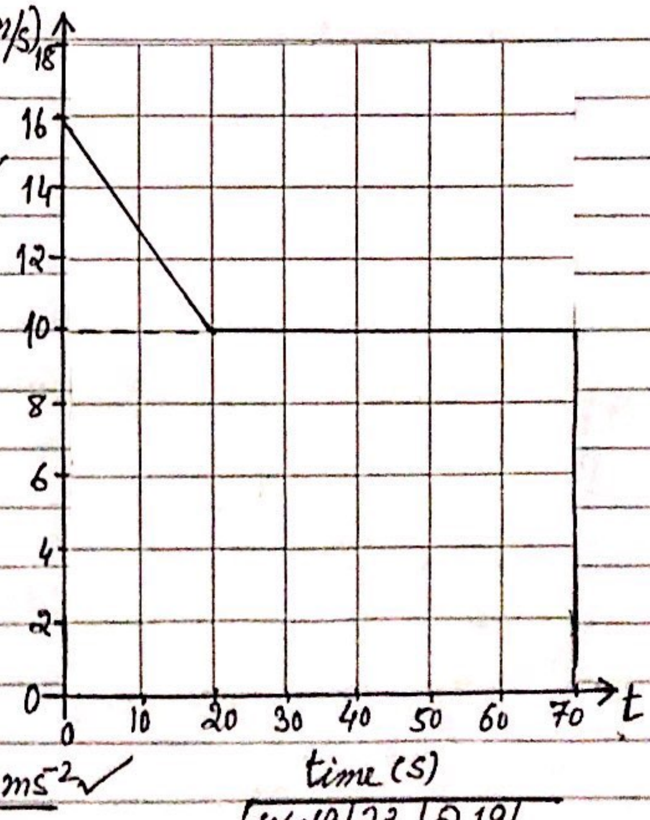
(Area of trapezium) $= \frac{1}{2} \times 38 \times 20 = \underline{380\ m}$

70.

The diagram shows the speed-time graph of 70 seconds, of a car journey.

(a) Calculate the deceleration of the car during first 20 seconds. ---[1]

(b) Calculate the total distance travelled by the car during the 70 seconds. ---[3]



[W-19/23/Q 19]

Solution:

(a) deceleration = $\frac{\text{Speed Change}}{\text{time}} = \frac{6}{20} = \underline{0.3\ ms^{-2}}$

(b) Total distance.

Area $= \frac{1}{2} \times 20 \times 6 + 70 \times 10$
$= 60 + 700 = \underline{760\ m}$

71. $f(x) = \frac{20}{x} + x, \quad x \neq 0$

(a) Complete the table:

x	-10	-8	-5	-2	-1.6	1.6	2	5	8	10
$f(x)$	-12	-10.5	-9	-12	-14.1	14.1	12			12

---[2]

(b) On the grid, draw the graph of $y = f(x)$ for $-10 \leq x \leq -1.6$ and $1.6 \leq x \leq 10$. ---[5]

(c) Using your graph solve the equation $f(x) = 11$. ---[2]

(d) K is a prime numbers and $f(x) = k$ has no solution. Find the possible value of k. ---[2]

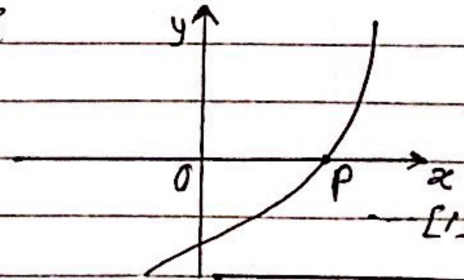
(e) The gradient of the graph of $y = f(x)$ at the point $(2, 12)$ is -4, Write down the coordinates of the other point on the graph of $y = f(x)$ whose gradient is -4. ---[1]

(f) (i) The equation $f(x) = x^2$ can be written as $x^3 + px^2 + q = 0$ show that $p = -1$ and $q = -20$ ---[2]

(ii) On the grid, draw the graph of $y = x^2$ for $-4 \leq x \leq 4$ ---[2]

(iii) Use your graphs, solve the equation $x^3 - x^2 - 20 = 0$ ---[1]

(iv) The diagram show a sketch of the graph of $y = x^3 - x^2 - 20$.
P is a point $(n, 0)$.
Write down the value of n.



---[1]

SP-20/04/Q3

Solution: (a) 9, 10.5

(b) Graph is on the next page →

Graph

71. (b)

(c) $f(x) = 11$
$\Rightarrow x = 2.6$ or $x = 9$ ✓

(d) 2, 3, 5, 7

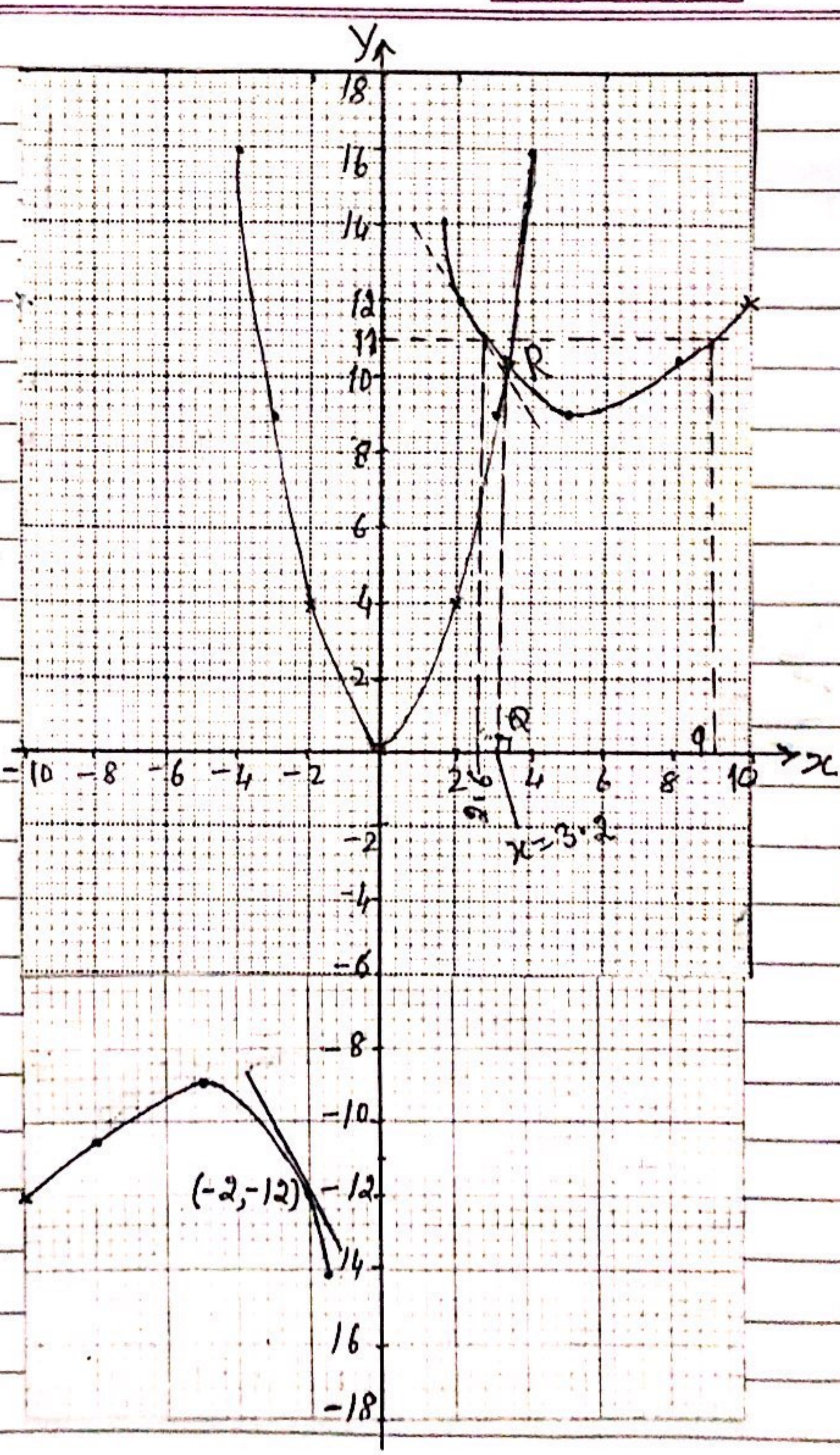
(e) $(-2, -12)$

(f) (i) $f(x) = \frac{20}{x} + x = x^2$
$\Rightarrow x^3 - x^2 - 20 = 0$
Comparing $x^3 + px^2 + q = 0$
$\Rightarrow p = -1,\ q = -20$ ✓

(ii) $y = x^2 \quad -4 \le x \le 4$

(iii) $y = x^2$ and $y = f(x)$
from Part f(i)
$\Rightarrow x^3 - x^2 - 20 = 0$
the two curves intersect at R,
Solution: $x = 3.2$ ✓

(iv) $x = 3.2$ ✓



Graphs

72(a) The table shows some values for $y = 2x^3 - 4x^2 + 3$

x	-1	-0.5	0	0.5	1	1.5	2
y	-3	1.75				0.75	3

(i) Complete the table. ---[3]

(ii) On the grid draw the graph of $y = 2x^3 - 4x^2 + 3$ for $-1 \le x \le 2$ --[4]

(iii) Use your graph to solve the equation, $2x^3 - 4x^2 + 3 = 1.5$ ---[3]

(iv) The equation $2x^3 - 4x^2 + 3 = k$ has only one solution for $-1 \le x \le 2$

Write down a possible integer value of k. ---[1]

a(i) 3, 2.25, 1,

(ii) graph

(0,3), (0.5, 2.25), (1,1)

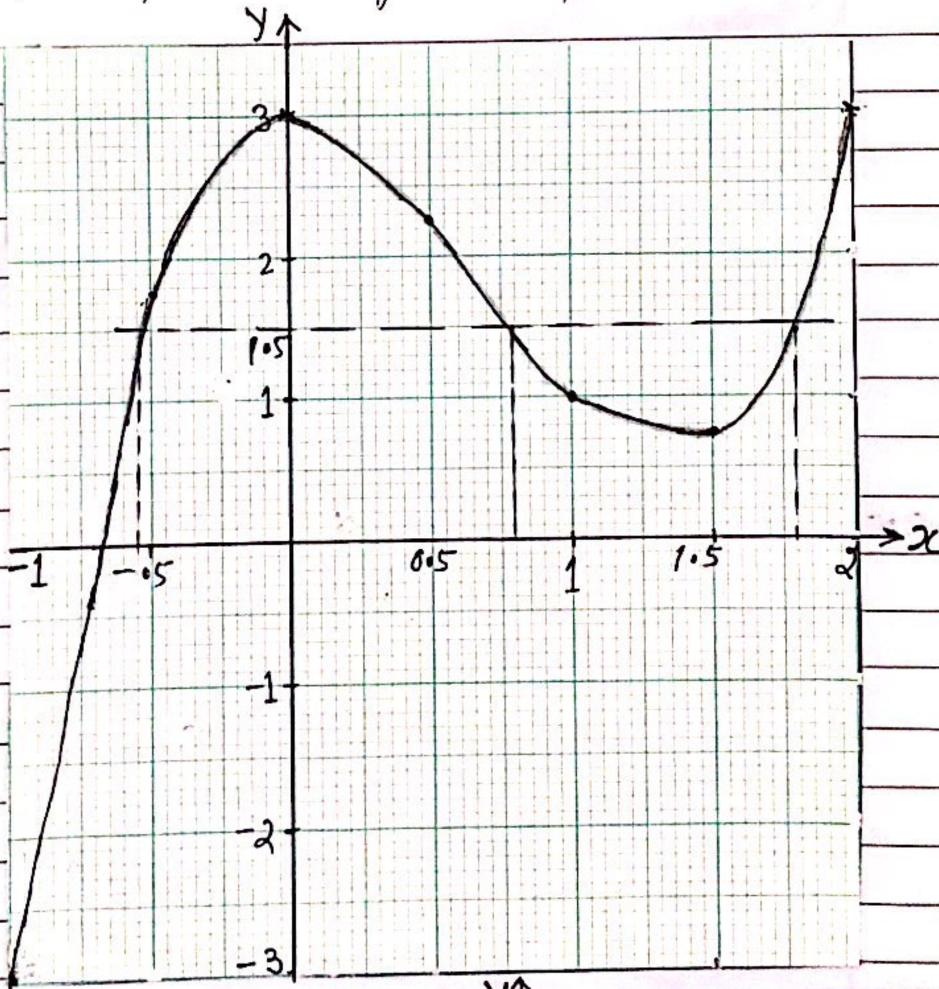
(a)(iii)

$y = 2x^3 - 4x^2 + 3 = 1.5$

$\Rightarrow x = -0.6, ✓$
$0.8, ✓$
$1.8 ✓$

a(iv)

$k = -3, -2, -1, 0$ ✓✓✓✓



(b)(i) On the grid, draw the tangent to the curve at $x=1$.

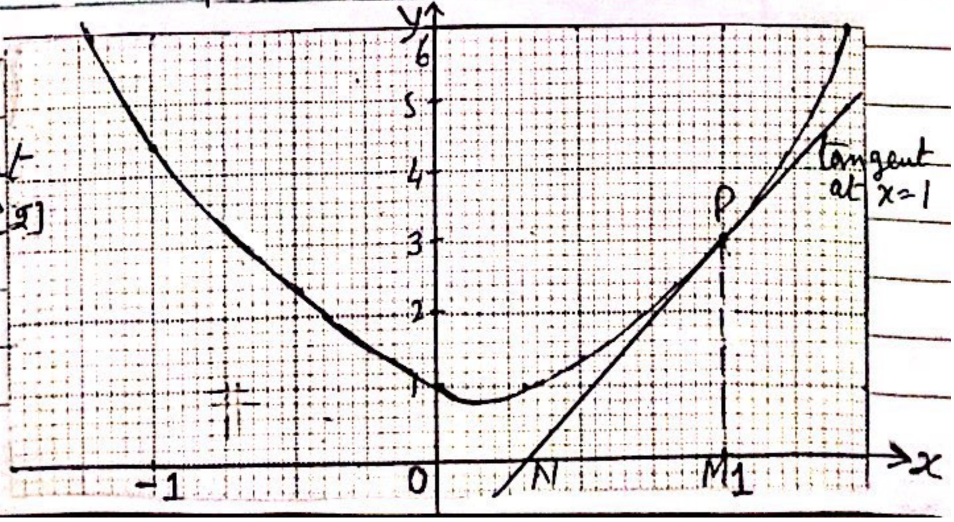
(ii) Use your tangent to estimate the gradient of the curve at $x=1$ -- [2]

(iii) Write down the equation of your tangent in the form $y = mx + c$ --[2]

Solution: (ii) $m = \frac{PM}{NM} = \frac{3}{0.7}$

$m = 4.3$ ✓

(iii) $y = 4.3x - 2$ ✓



73. (i) Write $x^2+10x+14$ in the form $(x+a)^2+b$. ---[2]

(ii) On the axes, sketch the graph of $y=x^2+10x+14$, indicating the coordinates of the turning point. ---[3]

S-20/41/Q8(C)

<u>Solution</u> (i) $x^2+10x+14 = x^2+10x+(5)^2-25+14$ $\left\{ \left(\frac{10}{2}\right)^2 \right.$ add and subtract

$= (x+5)^2-11$

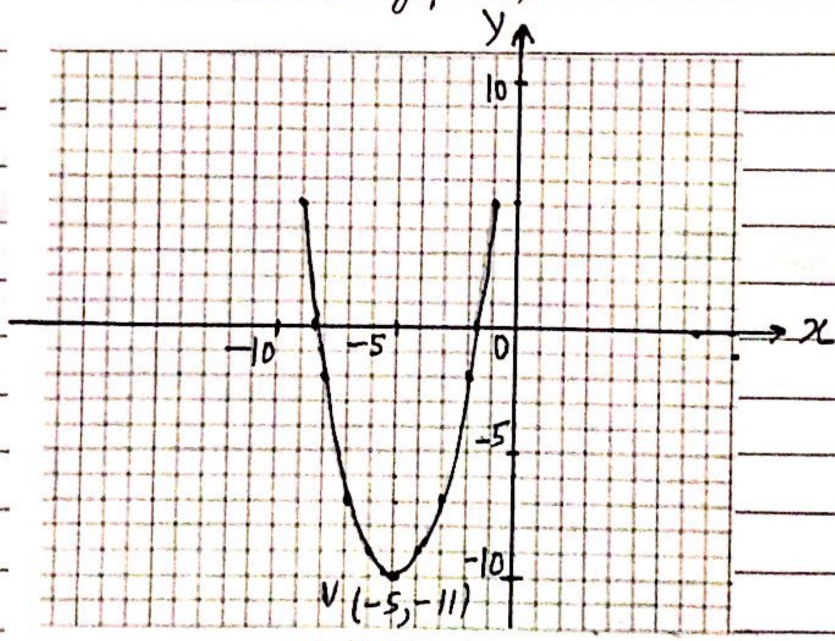
(ii) $y = x^2+10x+14$

$= (x+5)^2-11$ from Part (i)

or $= (x-(-5))^2+(-11)$

$V(-5,-11) \rightarrow$ Vertex of Parabola (or turning point).

for a quadratic function: $y=(x-a)^2+b$ Vertex at (a,b).



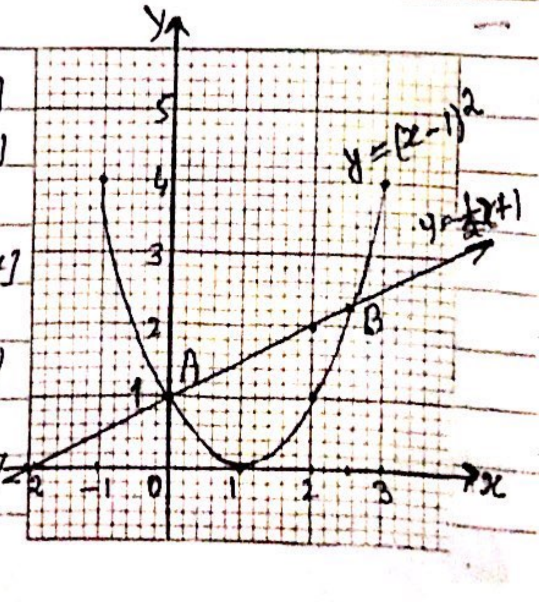
---

19 (i) on the diagram.

(a) Sketch the graph of $y=(x-1)^2$ ---[2]

(b) Sketch the graph of $y=\frac{1}{2}x+1$ --[2]

(ii) The graphs of $y=(x-1)^2$ and $y=\frac{1}{2}x+1$ intersect at A and B. Find the length AB. --[7]



<u>Solution</u>: $y=(x-1)^2$, $(1,0), (2,1), (3,4), (0,1), (-1,4)$

and $y=\frac{1}{2}x+1$, $(0,1), (2,2), (-2,0), (4,3)$

<u>AB = 2.8</u>

S-20/42/Q9C

Graph

74 The table shows some values of: $y = \frac{x^2}{2} + \frac{1}{x^2} - \frac{2}{x}$, $x \neq 0$

x	-3	-2	-1	-0.5	-0.3	0.2	0.3	0.5	1	2	3
y	5.3	3.3		8.1	17.8		4.5	0.1	−0.5	1.3	

(a) Complete the table: 3.5 ✓; 15, ✓ 3.9 ✓ --[3]

(b) On the grid draw the graph of $y = \frac{x^2}{2} + \frac{1}{x^2} - \frac{2}{x}$ for $-3 \leq x \leq -0.3$ and $0.2 \leq x \leq 3$. --[5]

(c) Use your graph to solve $\frac{x^2}{2} + \frac{1}{x^2} - \frac{2}{x} \leq 0$ --[2]

$0.6 \leq x \leq 1.5$ ✓

(d) Find the smallest integer value of k for which $\frac{x^2}{2} + \frac{1}{x^2} - \frac{2}{x} = k$ has two solutions for $-3 \leq x \leq -0.3$ and $0.2 \leq x \leq 3$

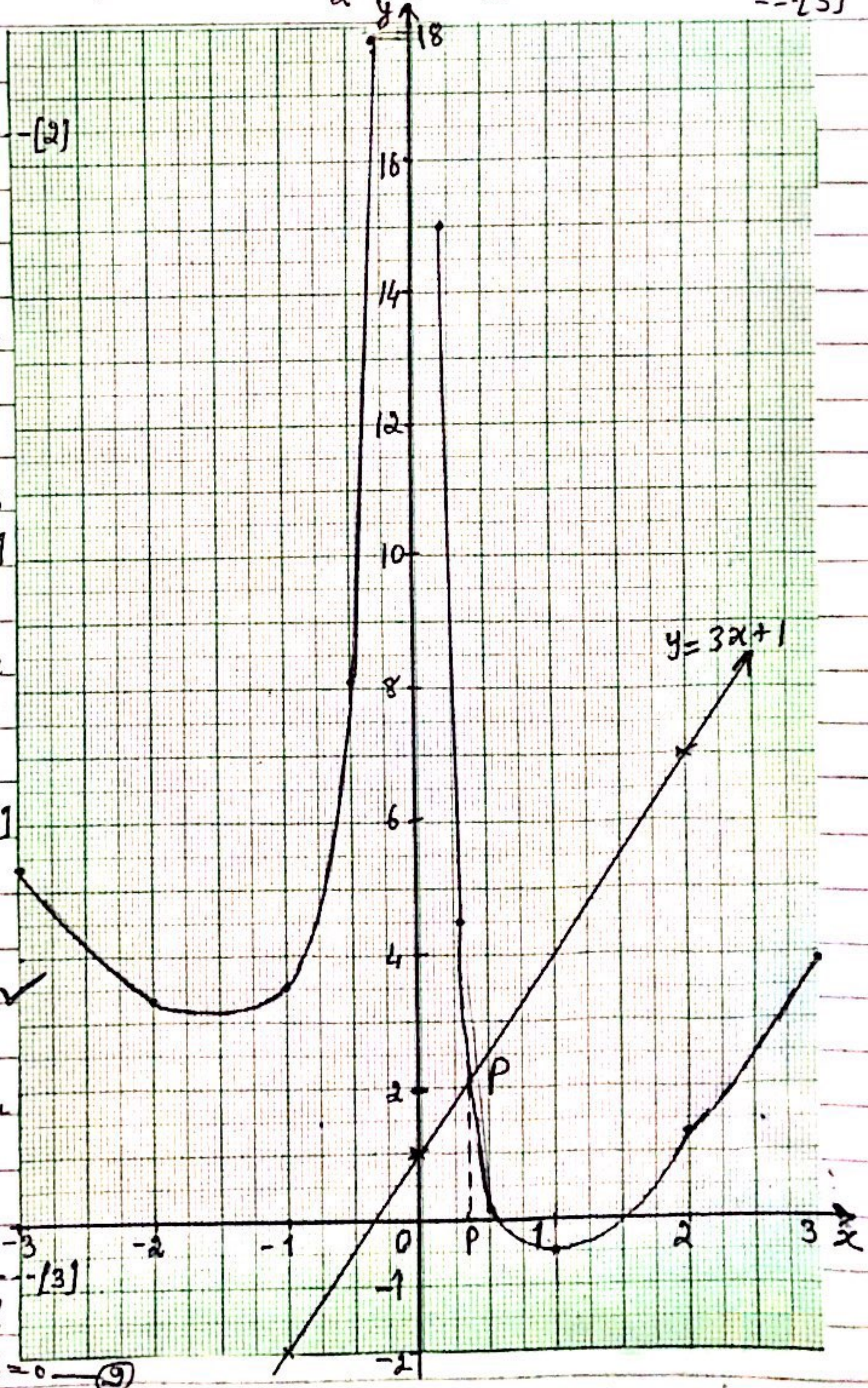
$k = 1$ ✓ ---[1]

(e) (i) By drawing suitable straight line solve, $\frac{x^2}{2} + \frac{1}{x^2} - \frac{2}{x} = 3x+1$ for $-3 \leq x \leq -0.3$ and $0.2 \leq x \leq 3$ -- [3]

Graph $y = (3x+1)$ ✓

$(0,1)$, $(2,7)$, $(-1,-2)$

Solution is at P; $x = 0.35$ ✓



e(ii) The equation,

$\frac{x^2}{2} + \frac{1}{x^2} - \frac{2}{x} = 3x+1$ — ① can be written as:

$x^4 + ax^3 + bx^2 + cx + 2 = 0$

Find the value of a, b, c. --[3]

Equation ① can be simplified to $x^4 - 6x^3 - 2x^2 - 4x + 2 = 0$ — ②

Comparing ① & ② $\Rightarrow$ $a = -6$, $b = -2$ and $c = -4$ ✓

W-19|42|Q5

75 The table shows some values for $y = x^3 + x^2 - 5x$

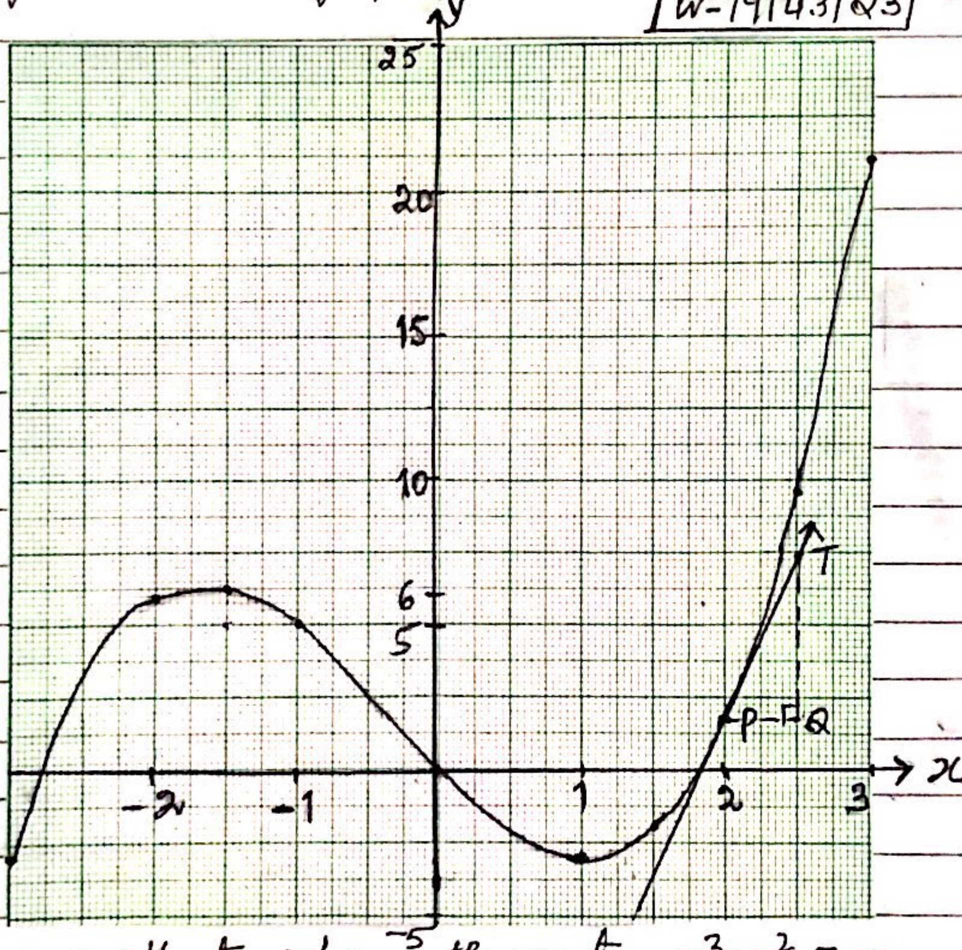
x	-3	-2	-1.5	-1	0	1	1.5	2	2.5	3
y	-3	6	6.4	....	0	....	-1.9	2	9.4	....

5 ✓ -3 ✓   21 ✓

(a) Complete the table: ---[3]

(b) On the grid, draw the graph of $y = x^3 + x^2 - 5x$ for $-3 \leq x \leq 3$

[W-19/43/Q3] ---[4]



(c) Use your graph to solve the equation $x^3 + x^2 - 5x = 0$ ---[2]

ans: -2.8, 0, 1.8 ✓

(d) By drawing suitable tangent, find an estimate of the gradient to the curve at $x = 2$. ---[3]

ans: PT is the tangent at $x = 2$, gradient $= \frac{TQ}{PQ} = \frac{6}{.5} = 12$ ✓

(e) Write down the largest value of the integer k, so that the equation $x^3 + x^2 - 5x = k$ has three solutions for $-3 \leq x \leq 3$.

ans: $k = 6$ ✓