IG- Maths 0580

Algebra - 2

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Variation

Algebra - 2.

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§ Direct Variation:

One quantity 'P' is supposed to vary directly as another quantity 'Q', such that if 'Q' changes then 'P' changes in the same ratio, we say

$$P \propto Q$$

If P_1, P_2, P_3 --- and Q_1, Q_2, Q_3 are the corresponding values of P and Q such that

$$\frac{P_1}{P_2} = \frac{Q_1}{Q_2} \text{ and } \frac{P_1}{P_3} = \frac{Q_1}{Q_3}$$

$$\text{or } \frac{P_1}{Q_1} = \frac{P_2}{Q_2} \text{ and } \frac{P_1}{Q_1} = \frac{P_3}{Q_3} = k \text{ let}$$

$\therefore$ $P \propto Q$

may be written as $P = kQ$

Consider

x: $x_1 = 1$ $x_2 = 2$ $x_3 = 3$

y: $y_1 = 5$ $y_2 = 10$ $y_3 = 15$

here $\frac{x_1}{y_1} = \frac{1}{5}$; $\frac{x_2}{y_2} = \frac{2}{10} = \frac{1}{5}$ and $\frac{x_3}{y_3} = \frac{3}{15} = \frac{1}{5}$

we can say $y \propto x$ or $y = kx$ —— (1)

when $x=1, y=5 \Rightarrow 5 = k \times 1 \Rightarrow k=5$

from $y = 5x$ ✓

Example 1. y is directly proportional to the positive square root of x. when $x=9$; $y=12$. Find y when $x = \frac{1}{4}$ --- [3]

[S-16/21/Q21]

Solution: Given $y \propto \sqrt{x}$

or $y = k\sqrt{x}$ ----(i)

for $x=9$, $y=12$ from (i) $12 = k\sqrt{9} \Rightarrow 12 = k \times 3$

or $k = 4$

from (i) $y = 4\sqrt{x}$ ---(ii) [$\because k=4$

$\therefore$ when $x = \frac{1}{4} \Rightarrow y = 4 \times \sqrt{\frac{1}{4}} = 4 \times \frac{1}{2} = 2$ ✓

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Direct Variation

Example 2: V is directly proportional to the cube of $(r+1)$; when $r=1$, $V=24$. Work out the value of V when $r=2$ [W-15/21/Q12] ---[3]

Solution: Given,

$$V \propto (r+1)^3$$
$$\Rightarrow V = k(r+1)^3 \quad ---(i)$$
$$\therefore 24 = k(1+1)^3 \quad [\because r=1, V=24$$
$$\text{or } 8k = 24 \Rightarrow k = 3$$
$$\text{from (i) } V = 3(r+1)^3 \quad —— (ii)$$

Now when $r=2$; $V = 3\cdot(2+1)^3$
$= 3\times 27 = 81$ ✓

Inverse Variation:

One quantity 'P' varies inversely to another quatity 'Q', when P varies directly to $\frac{1}{Q}$

$$\text{or } P \propto \frac{1}{Q}$$
$$\text{or } P = \frac{k}{Q}$$

Example 3. y is inversely proportial to x^2; when $x=5$, $y=16$. Find y when $x=10$. [M-17/22/Q12] ---[3]

Solution: Given $y \propto \frac{1}{x^2}$

$$\text{or } y = \frac{k}{x^2} \quad ---(i)$$

Given $x=5$, $y=16$ $\Rightarrow 16 = \frac{k}{5^2} \Rightarrow k = 16\times 25 = 400$

$\therefore$ from (i) $y = \frac{400}{x^2} \quad ---(ii)$

Now when $x=10 \Rightarrow y = \frac{400}{10^2} = 4$ ✓

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Variation

Algebra-2

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Inverse Variation:

Example 4. y is inversely proportional to $\sqrt{1+x}$, when $x=8$, $y=2$
Find y when $x=15$. S-17/22/21 ---[3]

Solution Given,

$$y \propto \frac{1}{\sqrt{1+x}}$$

$$\text{or } y = \frac{k}{\sqrt{1+x}} \quad \text{---(i)}$$

When $x=8, y=2 \Rightarrow 2 = \frac{k}{\sqrt{1+8}} \Rightarrow 2 = \frac{k}{3}$

$$\Rightarrow k = 6$$

$$\text{from (i)} \quad y = \frac{6}{\sqrt{1+x}} \quad \text{---(ii)}$$

Now when $x=15$, $y = \frac{6}{\sqrt{1+15}} = \frac{6}{4} = \frac{3}{2}$ ✓ or $1\frac{1}{2}$ ✓

Example 5. y is inversely proportion to x^3, $y=5$ when $x=2$
Find y when $x=4$. S-13/22/Q14 ---[3]

Solution:

Given, $y \propto \frac{1}{x^3}$

$$\Rightarrow y = \frac{k}{x^3} \quad \text{---(i)}$$

When $y=5, x=2 \Rightarrow 5 = \frac{k}{2^3} \Rightarrow k = 40$

$$\text{from (i)} \quad y = \frac{40}{x^3} \quad \text{----(ii)}$$

Now when $x=4$, $y = \frac{40}{4^3} = \frac{40}{64} = \frac{5}{8}$ or 0.625 ✓

—— x —— x —— x —— x ——

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§ Distance-time / Speed-time Travel Graphs

We know (i) Speed = $\frac{\text{distance}}{\text{Time}}$ or distance = Speed × time

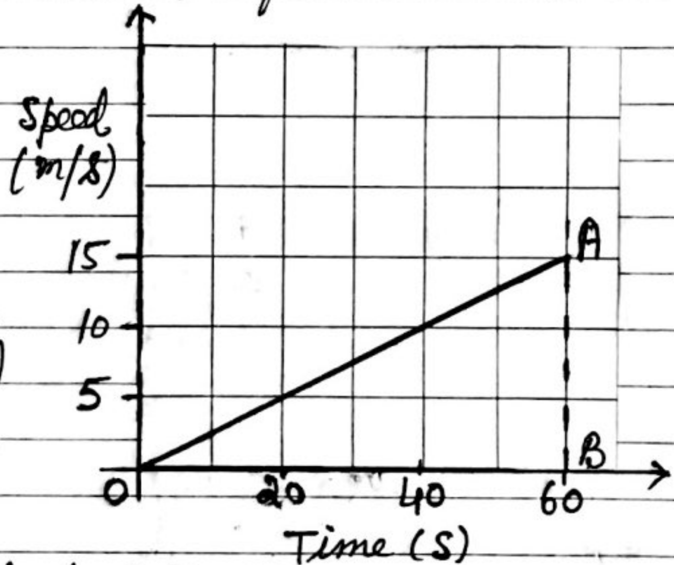
(ii) Average Speed = $\frac{\text{Total distance}}{\text{Total Time}}$

(iii) Acceleration (or deceleration) = $\frac{\text{Speed}}{\text{Time}}$.

§ On Speed-time graph the gradient gives the acceleration.

Note: Constant speed is represent by a line parallel to X-axis.

Example 1. The speed time graph shows the first 60 seconds of train journey.



(a) Find the acceleration of the train. ---[1]

(b) Calculate the distance the train has travelled in this time. Give your answer in kilometres. ---[3]

M-17/22/Q16

Solution

(a) Accelation of the train = gradiant of line OA

$= \frac{15}{60} = \frac{1}{4}$ or $0.25\ m/s^2$ ✓

(b) distance travelled = Area under the line OA.

= Area of triangle OAB

$= \frac{1}{2} \times OB \times AB$

$= \frac{1}{2} \times 60 \times 15$

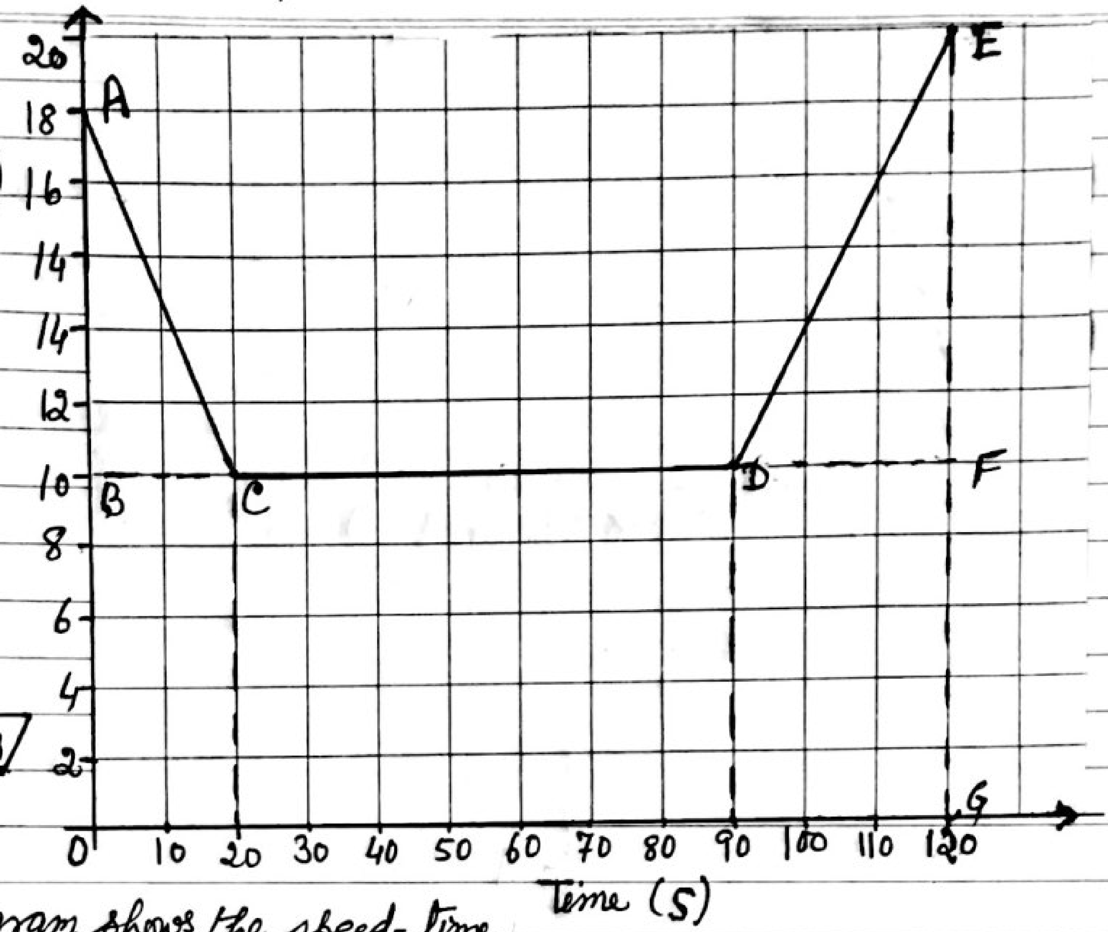
$= 450\ m$.

$= \frac{450}{1000} = 0.45\ Km$ ✓

Travel Graphs

Example 2:

[S-15/22/Q23]



The diagram shows the speed-time graph for 120 seconds of a car journey.

(a) Calculate the deceleration of the car during the first 20 seconds. ---[1]

(b) Calculate the total distance travelled by the car during 120 seconds. --[3]

(c) Calculate the average speed for this 120 second journey. --[1]

Solution:

(a) deceleration = Gradient of AC $= \frac{-8}{20} = -0.4$ or $2/5 \; m/s^2$

(b) Area under ACDE = ar Δ ABC + ar Δ EDF + ar rect OBFG

$= \frac{1}{2} \times 20 \times 8 + \frac{1}{2} \times 30 \times 10 + \;\; 120 \times 10$

$= 80 + 150 + 1200 = 1430\,m$

(c) Average speed $= \frac{\text{Total distance}}{\text{Total Time}}$

$= \frac{1430}{120} = 11.92 \; m/s$

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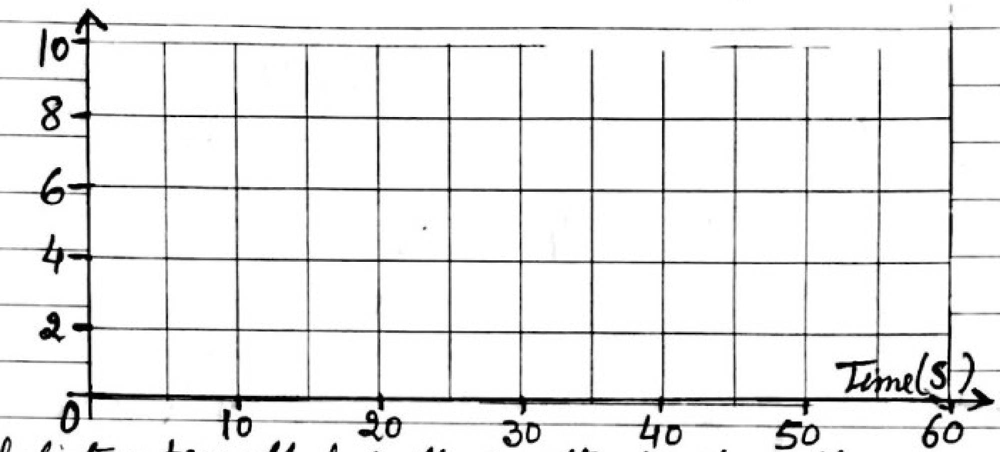
Travel Graphs

Example 3: A car passes through a checkpoint at time $t=0$, travelling at $8 m/s$. It travels at this speed for 10 seconds. [W-15/22/Q20]
The car then decelerates at a constant rate unit it stops when $t=55$ sec.

(a) On the grid, draw the speed-time graph. --- [2]



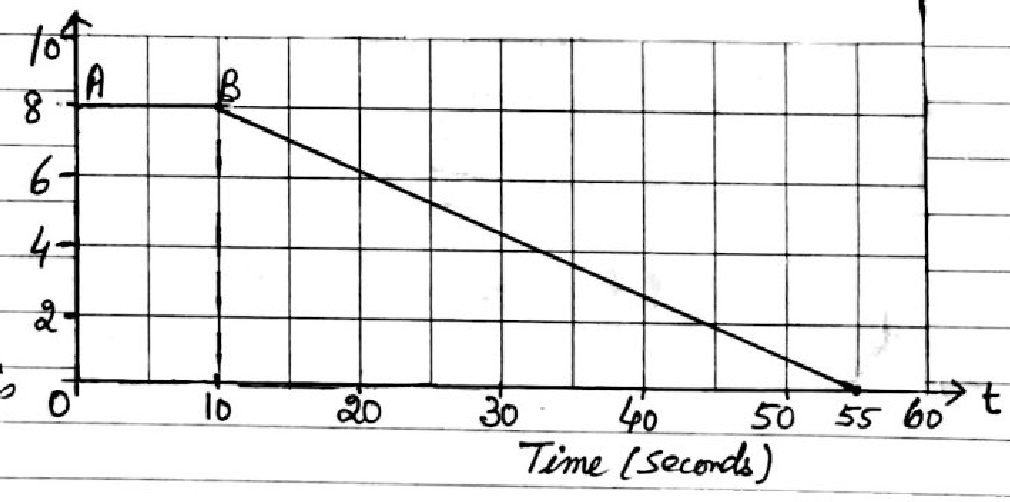
(b) Calculate the total distance travelled by the car after passing through the checkpoint. --- [3]

Solution:

(a)

Speed (m/s)

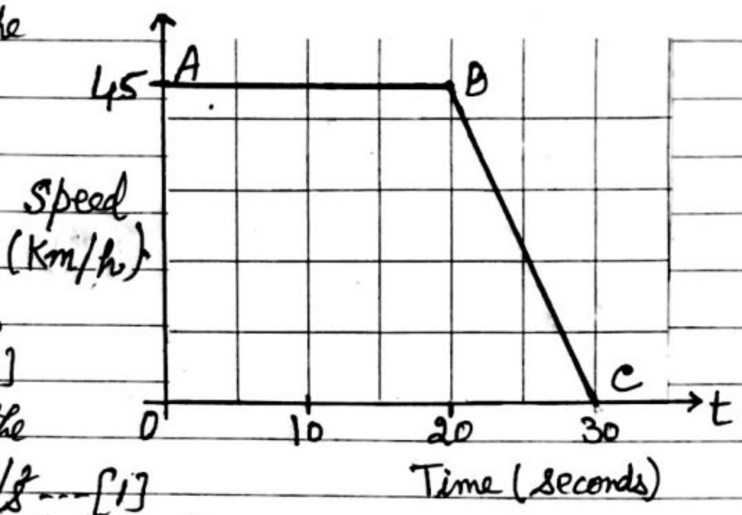
for first 10 seconds speed is constant. 8 m/s. ∴ on the graph AB // X-axis



(b)

$$\text{Distance} = \text{ar of rect} + \text{ar. of triangle}$$
$$= 8 \times 10 + \frac{1}{2} \times 8 \times (55-10)$$
$$= 80 + 4 \times 45$$
$$= 80 + 180$$
$$\therefore \text{Distance} = 260 \text{ m}$$

Example 4: The diagram shows the speed-time graph of a car. The car travels at 45 Km/h for 20 seconds. The car then decelerates for 10 seconds until it stops.



(a) Change 45 Km/h into m/s ---[2]

(b) Find the deceleration of the car, giving your answer in m/s^2 ---[1]

(c) Find the distance travelled by the car during 30 seconds, giving your answer in metres. ---[3]

W-15/23/Q26

Solution (a) $45\ Km/h = 45 \times \frac{5}{18} = 12.5\ m/s$ ✓ $\left[1\ Km/h = \frac{1000}{60 \times 60} = \frac{5}{18}\ m/s\right]$

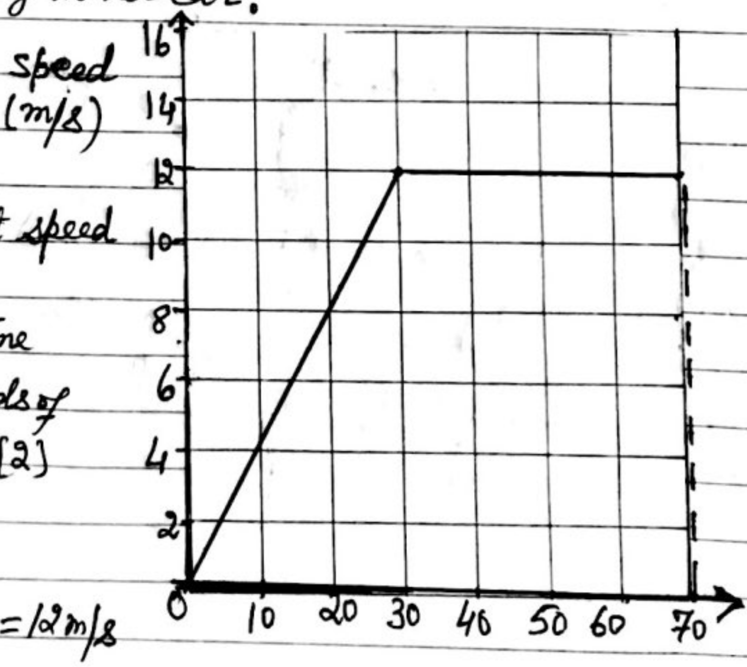
(b) Deceleration $= \frac{\text{Speed}}{\text{Time}} = \frac{12.5}{10} = 1.25\ m/s^2$ ✓ $[30 - 20 = 10\ s]$

(c) Distance = Area of Trap. $0ABC = \frac{1}{2}(AB + 0C) \times 0A$

$= \frac{1}{2}(20 + 30) \times 12.5$

$= 312.5\ m$ ✓

Example 5: Petra begins a journey in her car. She accelerates from rest at a constant rate of $0.4\ m/s^2$ for 30 seconds. She then travels at a constant speed for 40 seconds. On the grid draw the speed time graph for the first 70 seconds of Petra's journey. ---[2]



W-17/21/Q7

Solution: speed at $t = 30$ sec $= .4 \times 30 = 12\ m/s$

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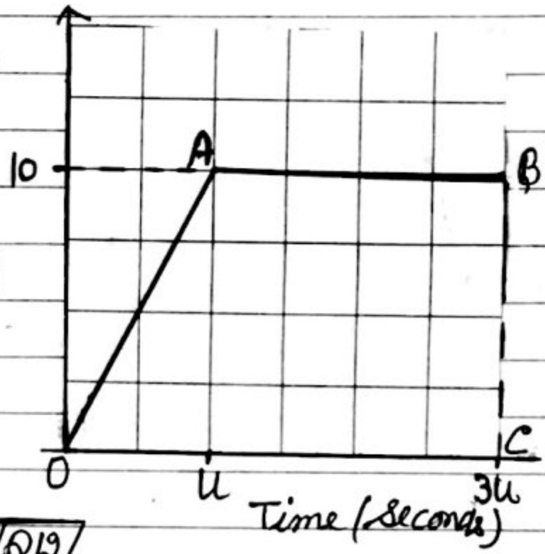
Travel Graphs

Example 6. A car starts from rest and accelerates for u seconds until it reaches a speed of 10 m/s.
The car then travels at 10 m/s for $2u$ seconds.
The diagram shows the speed-time graph for this journey.
The distance travelled by the car in the first $3u$ seconds is 125 m.



(a) Find the value of u. ----- [3]

(b) Find the acceleration in first u seconds. --- [1]

S-15/23/Q12

Solution:

(a) Distance travelled in $3u$ seconds = Area of Trapezium OABC

$= \frac{1}{2}(3u + 2u) \times 10$

$= 25u = 125$ given

$\Rightarrow u = \frac{125}{25} = 5$ m/s

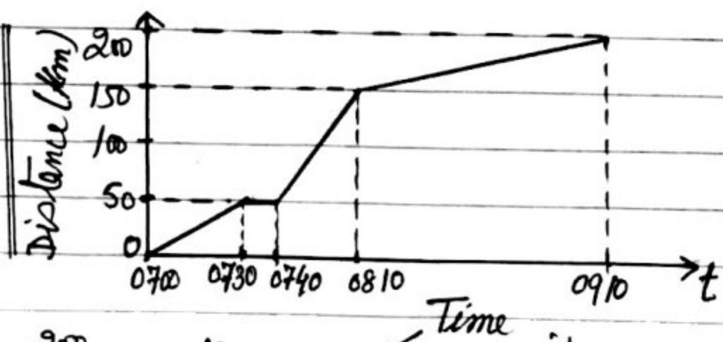
(b) Acceleration in first sec. $= \frac{\text{Speed}}{\text{Time}}$

$= \frac{10}{u} = \frac{10}{5} = 2\,ms^{-2}$ $(\because u = 5)$

Example 7 The distance-time graph shows the journey of a train.

(i) Find the speed of the train between 0700 and 0730. ---- [1] S-17/42/Q9(a)

(ii) Find the average speed for the whole journey. --- [3]



Solution: (i) Speed $= \frac{\text{distance}}{\text{Time}} = \frac{50}{1/2}$

$= 100$ km/h

(ii) Average Speed $= \frac{\text{Total distance}}{\text{Total time}} = \frac{200}{\frac{13}{6}} = \frac{1200}{13} = 92.3$ km/h $\left[2h\ 10m = \frac{13}{6} h\right]$

Travel Graphs.

Example 8. Ali leaves home at 10 00 to cycle to his grand mother house. He arrives at 1300. The distance time graph represents his journey.

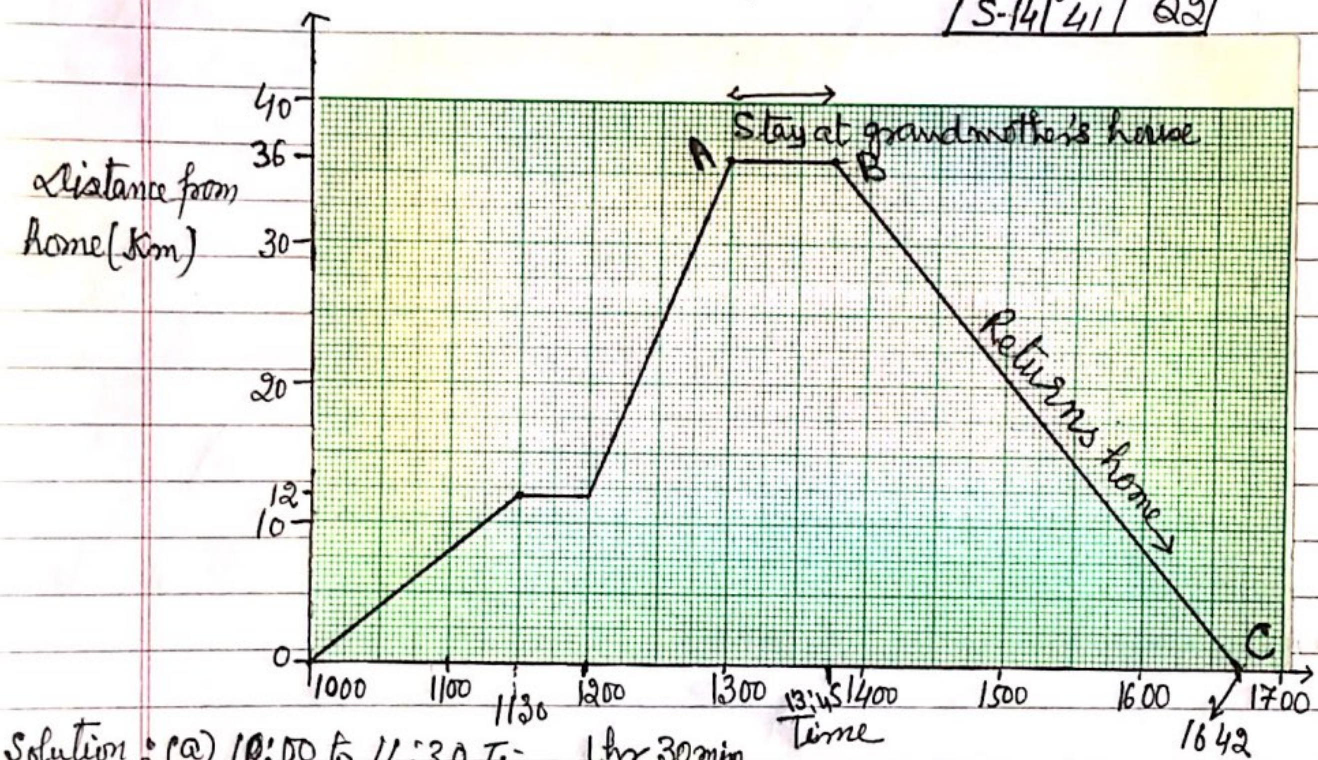
(a) Calculate Ali's speed between 1000 and 1130. Give your answer in Km/h --[2]

(b) Show that Ali's average speed for the whole journey to his grandmother's house is 12 Km/h. --[2]

(c) Change 12 Km/h into metres per minute. --[2]

(d) Ali stays for 45 minutes at his grandmother's house and he returns home. He arrives home at 1642. Complete the distance-time graph. --[2]

[S-14/ 41/ Q2]



Solution: (a) 10:00 to 11:30 Time = 1 hr 30 min.

$= \frac{3}{2}$ Hr

$\therefore$ Speed $= \frac{\text{distance}}{\text{Time}} = \frac{12}{3/2} = 8$ Km/h ✓

(b) Time to reach grandmother's house = 3 Hr

and distance = 36 Km

$\therefore$ Average speed $= \frac{\text{Total dis}}{\text{Total Time}}$

$= \frac{36}{3} = 12$ Km/h ✓

(c) $12 \text{ Km/h} = \frac{12 \times 1000}{60} = 200$ m/min.

(d) He stays 45 min at grandmother's house. graph is line parallel to x-axis 13:00 to 13:45. AB ✓

Now Returns home from B to C. Join (13:45, 36) to (16:42, 0)

$\overline{BC}$ ✓

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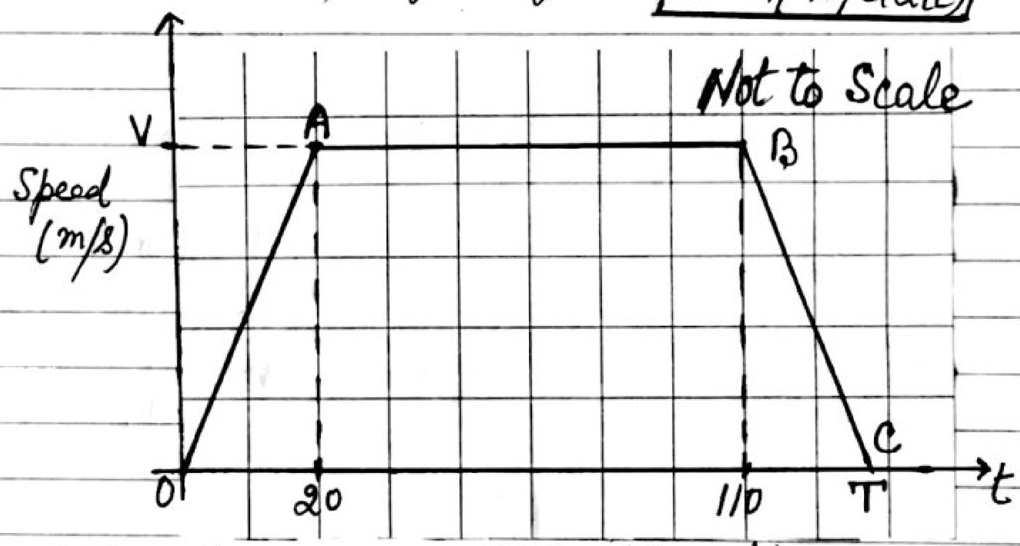
Travel Graph.

Example 9. The diagram shows the speed-Time graph for a car travelling along a road for T seconds.
To begin with the car accelerated at $0.75\ m/s^2$ for 20 seconds, to reach a speed of v m/s.

(i) Show that the speed, v, of the car is 15 m/s. ---[1]

(ii) The total distance travelled is 1.8 Kilometres.
Calculate the total time, T, of the journey. ---[4]

W-14/41/Q2(c)



(i) acceleration $= 0.75\ m/s^2$ for 20 seconds

$\therefore V = acc \times time$

or $V = 0.75 \times 20 = 15\ m/s$ ✓

(ii) Total distance $= 1.8\ Km = 1800\ m$ --- (i)

Also the total is area of Trapezium OABT

$= \frac{1}{2}(OT + AB) \times V$

$= \frac{1}{2}(T + 90) \times 15$ --- (ii)

$\left\{ \begin{array}{l} AB = 110 - 20 \\ \quad = 90\ s \\ V = 15\ m/s \end{array} \right.$

from (i) and (ii) $\frac{15}{2}(T+90) = 1800$

or $T + 90 = 1800 \times \frac{2}{15} = 240$

$\therefore T = 240 - 90 = 150$ seconds ✓

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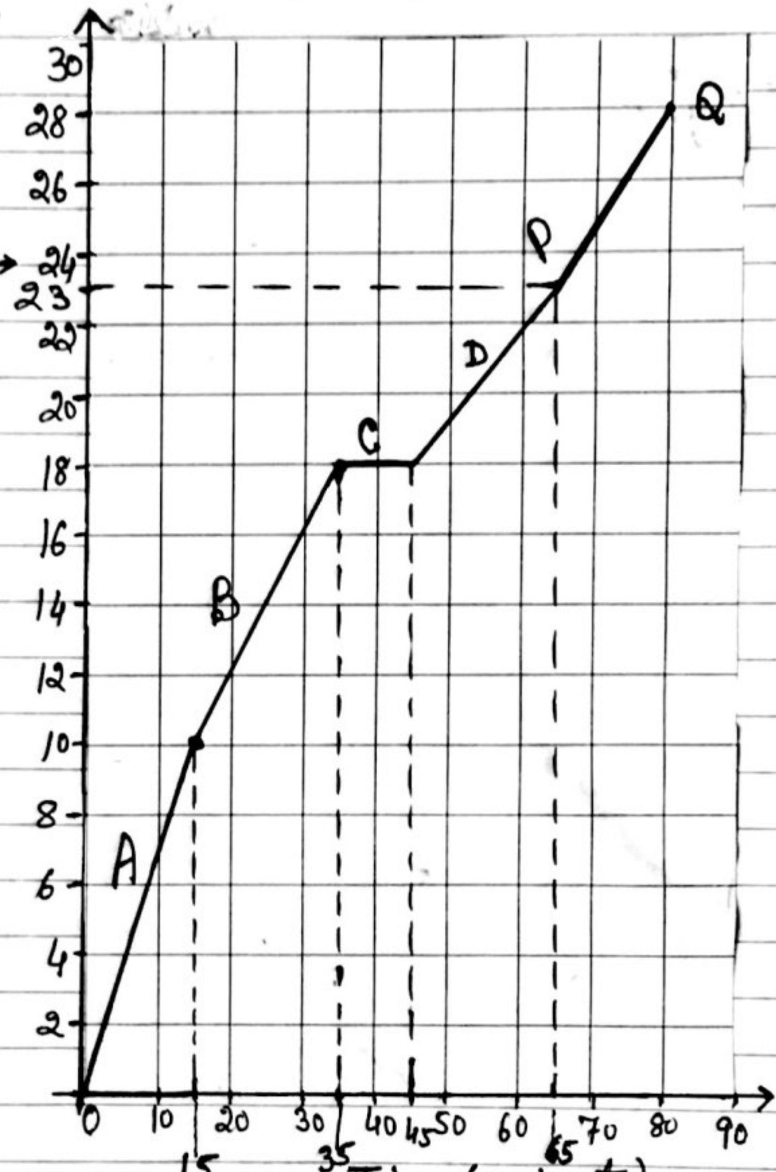
Travel Graphs

Example 10. The diagram shows the distance-time graph for the first 65 minutes of a bicycle journey.

(a) There are four different parts to the journey labelled A, B, C and D. Write down the part of the journey with fastest speed. ----[1]



(b) After the first 65 minutes the bicycle travels at a constant speed of 20km/h for 15 minutes. Draw this part of the journey on the diagram. ----[1]

W-16/23/Q9

Solution:

(a) The gradient of line shows the speed. Here we find "Part-A" has greatest gradient, hence fastest speed.

(b) speed 20 Km/h for '15' minutes (or $\frac{1}{4}$ hrs) (from 65 min to 80 mins)

distance travelled = Speed × Time

$= 20 \times \frac{1}{4} = 5$ Km. (from 65 min to 80 min) on the graph.

(23 km to 28 km) ⟶

Join P(65, 23) to Q(80, 28) ✓

Functions.

"An Introduction"

§ Allotment of **rooms**, to the **students**, in a hostel of a residential school.

Set of Students $S = \{A, B, C, D, E\}$

Set of Rooms $R = \{1, 2, 3, 4, 5, 6\}$

Consider:

(i) each student must get a room. (Unique - one but not more than one room)

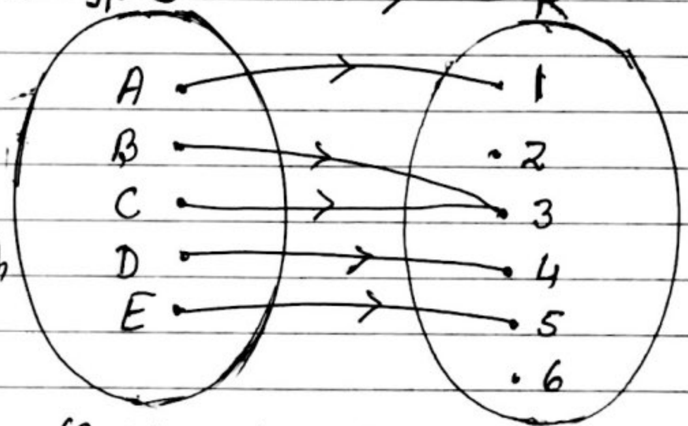
(ii) Some rooms may remain unoccupied.

(iii) two or more students may be alloted a single room. $f_1 : S \longrightarrow R$

Case I

f_1 :

(Many-one function)



$f_1 = \{(A,1), (B,3), (C,3), (D,4), (E,5)\}$

§ A function $f : S \to R$ is a mapping (or association) from set S to R such that each element of set S is associated to a unique (one and only one) element of set R.

We say that f-image of A is 1 or $f(A) = 1$

and conversely Pre-image of 1 is A or $f^{-1}(1) = A$

Domain of $f_1 = \{A, B, C, D, E\}$

and Codomain of $f_1 = \{1, 2, 3, 4, 5, 6\}$

Range of $f_1 = \{1, 3, 4, 5\}$ (Rooms occupied)

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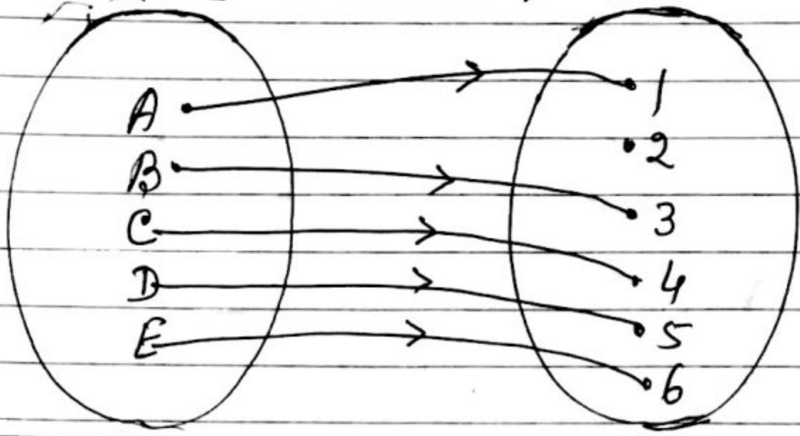
Functions

Case II

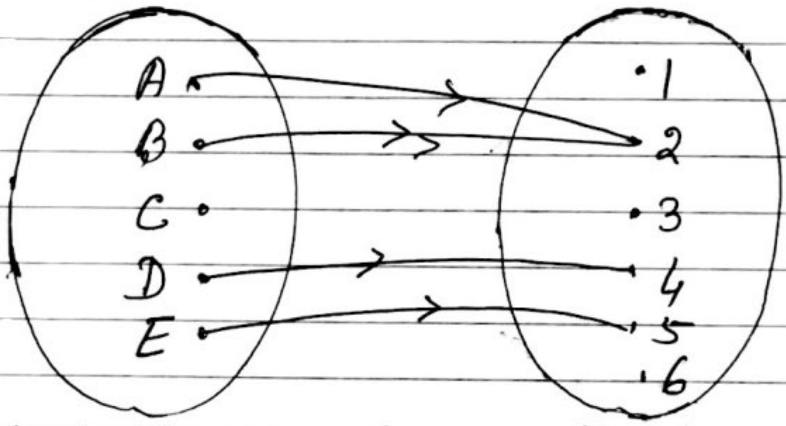
$f_2 : S \longrightarrow R$



f_2 is a **One-one** function.

Case III

$f_3 : S \longrightarrow R$

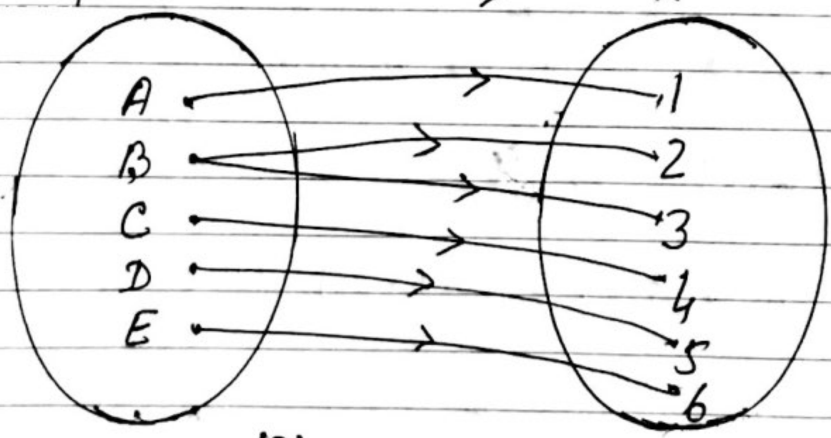


$f_3 : S \to R$ is not a fu^n ?

$C \in S$
Because student 'C' has not been alloted any room.

Case (iv)

$f_4 : S \longrightarrow R$



f_4 is not a function. ?

$B \in S$; Student 'B' has occupied two rooms '2' and '3' which is against the rule of function.
(Can a student occupy two or more rooms ?)

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Algebra-2

Functions

Example 1 $f: x \to 5-3x$ or $f(x) = 5-3x$

(a) Find $f(6)$ ---[1]

(b) Find $f(x+2)$ [S-15/21/Q23(a),(b)] ---[1]

Solution Given $f(x) = 5-3x$

(i) $f(6) = 5 - 3\times 6 = -13$ ✓

(ii) $f(x+2) = 5-3(x+2)$

$= 5-3x-6$

$= -3x-1$

or $= -(3x+1)$ ✓

Example 2. $g(x) = \frac{2}{(x+1)}$ ---(i)

[W-14/21/Q20(b)]

Solve $g(x) = 10$ ---[2]

or $\frac{2}{x+1} = 10$ from (i)

or $10(x+1) = 2$

or $x+1 = \frac{1}{5} \Rightarrow x = \frac{1}{5} - 1 = -\frac{4}{5}$

$\therefore x = -\frac{4}{5}$ ✓

Example 3. $f(x) = 1-2x$; $g(x) = x+4$ [W-17/42/Q9(a)(b)]

(a) Find $f(-1)$ --- [1]

(b) Solve the equation $2f(x) = g(x)$ --- [2]

Solution: $f(x) = 1-2x$

(a) $f(-1) = 1-2(-1)$

$= 3$ ✓

(b) $2\cdot f(x) = g(x)$

or $2(1-2x) = x+4$

$2-4x = x+4$

$\Rightarrow -5x = 2 \Rightarrow x = -\frac{2}{5}$ ✓

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Functions

§ **Composite function:** Given two functions, $f(x)$ and $g(x)$

Then (i) $gf(x) = g[f(x)]$

(ii) $fg(x) = f[g(x)]$

(iii) $ff(x) = f[f(x)]$

Example 1. $f(x) = (x-3)^2$ $\quad g(x) = \frac{x-1}{4}$ $\quad h(x) = x^3$

Find (a) $hf(1)$ --- [2]

(c) $gh(x)$ --- [1] $\boxed{W-14/23/Q16}$

(d) the solution to the eqn $f(x) = 0$ --- [1]

Solution (a) $hf(1) = h[f(1)]$

$= h[(1-3)^2]$ $\quad \because f(x) = (x-3)^2$

$= h(4)$

$= 4^3$ $\quad \because h(x) = x^3$

$= 64$ ✓

(c) $gh(x) = g[h(x)]$

$= g(x^3)$ $\quad \because h(x) = x^3$

$= \frac{x^3-1}{4}$ ✓ $\quad \because g(x) = \frac{x-1}{4}$

(d) $f(x) = 0$

$\Rightarrow (x-3)^2 = 0$

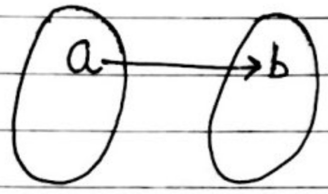
$\Rightarrow x - 3 = 0$

$\Rightarrow x = 3$ ✓

Function

§ **Inverse functions:** Given $y = f(x)$

$$f : x \longrightarrow y$$



or $f(a) = b$

Then $f^{-1}(x)$ is such that $f^{-1}(b) = a$

Example 1. $f(x) = 5x - 3$ find $f^{-1}(x)$ ... [2] | M-15/22/Q22(c) |

Let $y = 5x - 3$

§
(i) get x in terms of y
(ii) Interchange x & y
(iii) Replace y by $f^{-1}(x)$

$\Rightarrow 5x = y + 3$

$\Rightarrow x = (y+3)/5$

Interchange x & y

$y = (x+3)/5$

or $f^{-1}(x) = \dfrac{x+3}{5}$

Observe

$f(2) = 5 \times 2 - 3$
$= 7$

$f^{-1}(7) = \dfrac{7+3}{5}$
$= 2$ ✓

Example 2: $g(x) = \dfrac{x-1}{4}$; find $g^{-1}(x)$ ---- [2]

| W-14/23/Q16(b) |

Solution Let $y = \dfrac{x-1}{4}$

or $4y = x - 1$

or $x = 4y + 1$

Interchange x & y

$y = 4x + 1$

or $f^{-1}(x) = 4x + 1$ ✓

Note: 1 $f f^{-1}(x) = f^{-1} f(x) = x$

Note: 2: $f^{-1}(x) = f(x)$ — for $f(x) = \begin{cases} \text{(i) } x \text{ or} \\ \text{(ii) } 1/x \text{ or} \\ \text{(iii) } 1-x \end{cases}$

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Functions

Example 2. $f(x) = 2x-1$ $\quad g(x) = \frac{1}{x},\ x \neq 0$ $\quad h(x) = 2^x$

W-15/43/Q9

(a) Find $h(3)$ --- [1]

(b) Find $fg(0\cdot5)$ --- [2]

(c) Find $f^{-1}(x)$ --- [2]

(d) Find $ff(x)$, giving your answer in its simplest from --- [2]

(e) Find $(f(x))^2 + 6$, giving your answer in its simplest form --- [2]

(f) Simplify $hh^{-1}(x)$. --- [1]

(g) Which of the following statements is true?

$f^{-1}(x) = f(x)$

$g^{-1}(x) = g(a)$

$h^{-1}(x) = h(x)$ --- [1]

(h) Use two of the functions $f(x)$, $g(x)$ and $h(x)$ to find the composite function which is equal to $2^{x+1} - 1$ --- [1]

Solution:

(a) $h(3) = 2^3$ $\quad \because h(x) = 2^x$

$= 8$ ✓

(b) $fg(0\cdot5) = f[g(0\cdot5)]$

$= f\left(\frac{1}{0\cdot5}\right)$ $\quad \because g(x) = \frac{1}{x}$

$= f(2)$

$= 2\times2-1$ $\quad \because f(x) = 2x-1$

$= 3$ ✓

(c) To find $f^{-1}(x)$, $\quad f(x) = 2x-1$

Let $y = f(x)$ or $y = 2x-1$

or $x = \frac{y+1}{2}$

Interchange x and y

$\therefore y = \frac{x+1}{2}$

or $f^{-1}(x) = \frac{y+1}{2}$ ✓

(d) $ff(x) = f[f(x)]$

$= f(2x-1)$ $\quad \because f(x) = 2x-1$

$= 2(2x-1)-1$

$= 4x-2-1$

$= 4x-3$ ✓

(e) $(f(x))^2 + 6 = (2x-1)^2 + 6$

$= 4x^2 - 4x + 1 + 6$

$= 4x^2 - 4x + 7$ ✓

(f) $h(h^{-1}(x))$

$= h(\log_2)$

$= 2^{\log_2 x}$

$= x$ ✓

$\boxed{hh^{-1}(x) = x}$ ✓

Let $z = h(x)$

$z = 2^x$

$\Rightarrow x = \log_2 z$

$\therefore z = \log_2 x$

$h^{-1}(x) = \log_2 x$

(g) $g^{-1}(x) = g(x)$

State the result, Proof is not required

(h) Consider $fh(x) = f(h(x)) = f(2^x)$ $\quad (\because f(x) = 2x-1)$

$= 2\times2^x - 1 = 2^{x+1} - 1$ ✓

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Example 3. $f(x) = 3x - 2 \quad g(x) = x^2 \quad h(x) = 3^x$

(a) Find $f(-3)$ --- [1] $\boxed{\text{S-17}}\boxed{42}\boxed{Q7}$

(b) Find the value of x when $f(x) = 19$ --- [2]

(c) Find $fh(2)$ --- [2]

(d) Find $gf(x) + f(x) + x$

give your answer in its simplest form. --- [3]

(e) Find $f^{-1}(x)$ --- [2]

Solution :

(a) $f(-3) = 3(-3) - 2$ $\quad \because f(x) = 3x - 2$

$= -9 - 2$

$= -11$ ✓

(b) To solve. $f(x) = 19$

or $3x - 2 = 19$ $\quad \because f(x) = 3x - 2$

or $3x = 21$

$\therefore x = 7$ ✓

(c) $fh(2) = f[h(2)]$

$= f(3^2)$ $\quad \because h(x) = 3^x$

$= f(9)$

$= 3 \times 9 - 2 = 25$ ✓ $\quad \because f(x) = 3x - 2$

(d) $gf(x) + f(x) + x$

$= g(f(x)) + f(x) + x$

$= g(3x-2) + (3x-2) + x$

$= (3x-2)^2 + 3x - 2 + x$ $\quad \because g(x) = x^2$

$= 9x^2 - 12x + 4 + 3x - 2 + x$

$= 9x^2 - 8x + 2$ ✓

(e) To find $f^{-1}(x)$ let $y = f(x)$

or $y = 3x - 2$ $\quad f(x) = 3x - 2$

or $3x = y + 2$

or $x = \frac{y+2}{3}$

Interchange x & y $\Rightarrow$ $y = \frac{x+2}{3}$

$\therefore f^{-1}(x) = \frac{x+2}{3}$ ✓

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Example 4. $f(x) = 2x+1$ $g(x) = x^2+4$ $h(x) = 2^x$

(a) Solve the equation $f(x) = g(1)$ ---[2] $\boxed{SP\text{-}2020/04/Q7}$

(b) Find $f^{-1}(x)$ ---[2]

(c) Find $gf(x)$ in its simplest form. ---[3]

(d) Solve the equation $h^{-1}(x) = 0.5$ ---[1]

(e) $\frac{1}{h(x)} = 2^{kx}$

write down the value of k ---[1]

Solution:

(a) $f(x) = g(1)$ $\because f(x) = 2x+1$

$2x+1 = 1^2+4$ and $g(x) = x^2+4$

or $2x = 4$

$\therefore x = 2$ ✓

(b) Find $f^{-1}(x)$,

Let $y = f(x)$

or $y = 2x+1$

or $x = \frac{y-1}{2}$

Interchange x and y

$\therefore y = \frac{x-1}{2}$

$\therefore f^{-1}(x) = \frac{x-1}{2}$ ✓

(c) $gf(x) = g(f(x))$

$= g(2x+1)$

$= (2x+1)^2+4$ $\because g(x) = x^2+4$

$= 4x^2+4x+1+4$

$\therefore gf(x) = 4x^2+4x+5$ ✓

(d) To solve $h^{-1}(x) = 0.5$

$\Rightarrow x = h(0.5)$

$= 2^{0.5}$ $\because h(x) = 2^x$

$= 2^{1/2}$

or $x = \sqrt{2}$ or 1.41

(e) $\frac{1}{h(x)} = 2^{kx}$

or $\frac{1}{2^x} = 2^{kx}$ $(\because h(x) = 2^x)$

or $2^{-x} = 2^{kx}$

or $-\cancel{x} = k\cancel{x}$

$\therefore k = -1$ ✓

Graphs of some Particular Functions:

§ 1. **Linear function:** $f(x) = ax+b$

or $y = ax+b$

The graph of a linear function is a straight line, here 'a' is the gradient and 'b' the y-int of the line.

To draw the graph we need to take any two point (Value of x and y) satisfying the function.

Example 1. Draw the graph of $f(x) = 2x+3$

or $y = 2x+3$ ----(i)

In (i)

$x=0$, $y=3$,

and $x=-2$, $y=-1$

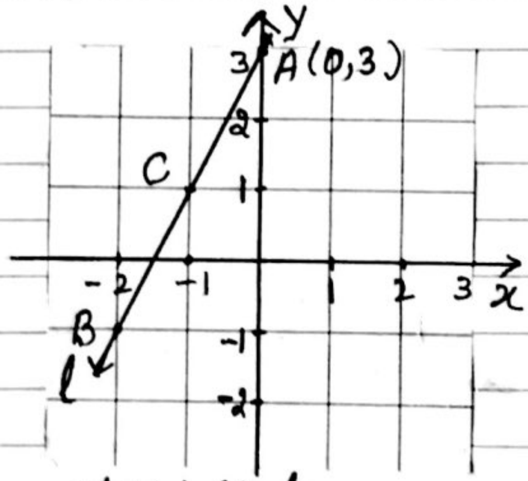
$\therefore$ Points $A(0,3)$ and $(-2,-1)$

we find that the line 'l' passes through a point $C(-1,1)$

Put $x=-1$ and $y=1$ in (1)

$1 = 2(-1)+3$

or $1 = -2+3$ True



$\therefore$ Graph of a linear function is a straight line.

§ 1(a) Special Cases:

(i) Draw the graph of $x = a$ (or $x=3$)

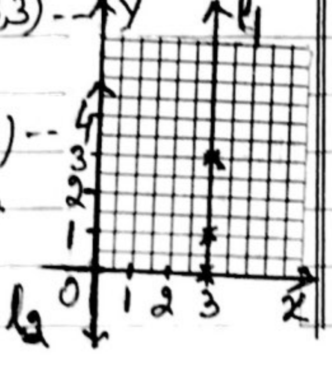
(ii) Draw the graph of $x=0$

(i) Points $(3,0)$, $(3,1)$, $(3,3)$ --

Graph is $l_1 \parallel$ Y-axis

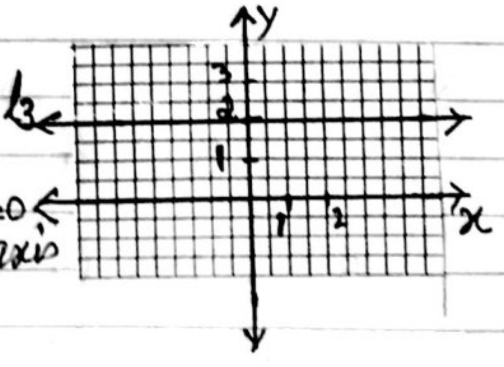
(ii) Points $(0,1)$, $(0,3)$ --

l_2 is Y-axis Only



1(b)(i) Graph of $y=2$, $l_3 \parallel$ X-axis

(ii) Graph of $y=0$ is X-axis.



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I G-Maths 0580 Graphs of Functions Algebra-2 P-22

§2. Quadratic function: $f(x) = ax^2 + bx + c \; ; \; a \neq 0$

Note 1. If $a > 0$ the graph of $f(x)$ is parabolic curve opening upwards and has a min. at Vertex.

2. If $a < 0$ the graph of $f(x)$ is parabolic curve opening downwards, and the curve has a max. at vertex.

3. If the curve intersects the x-axis at $x = \alpha$ and $x = \beta$ (which are the solutions of $ax^2+bx+c=0$) then vertex lies at $x = \frac{\alpha+\beta}{2} = p$ (let)

Then Vertex at (p, q) when $q = f(p)$

Example 1. Draw the graph of $f(x) = x^2 - 5x + 6$ ---(i)

Solve $x^2 - 5x + 6 = 0$

or $(x-3)(x-2) = 0$

$x = 3 \; ; \; 2$

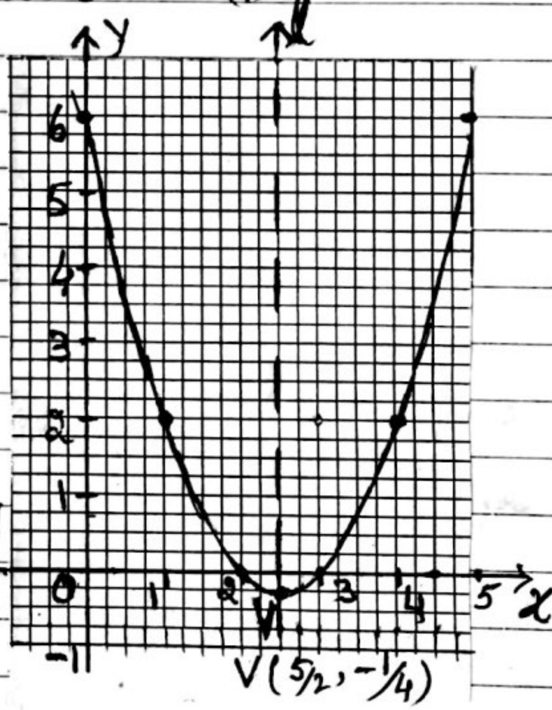
Curve cuts X-axis $(3,0)$ and $B(2,0)$

Vertex is at $\frac{3+2}{2} = 5/2$

$f(\frac{5}{2}) = (\frac{5}{2})^2 - 5(\frac{5}{2}) + 6 = -\frac{1}{4}$

Vertex at $V(\frac{5}{2}, -\frac{1}{4})$.

x	0	1	2	5/2	3	4	5
$f(x)$	6	2	0	$-1/4$	0	2	6



Note: The curve is symmetrical about line $l \parallel$ Y-axis passing through Vertex.

Alternate Method 1. $f(x) = ax^2 + bx + c$

Then Vertex at $x = -\frac{b}{2a}$

Example. $f(x) = x^2 - 5x + 6$

Vertex at $x = -\frac{b}{2a}$

$x = \frac{-(-5)}{2 \times 1} = \frac{5}{2}$ ✓

$\left\{ \begin{array}{l} a = 1 \\ b = -5 \\ c = 6 \end{array} \right.$

Alternate 2:

Express: $ax^2 + bx + c = a(x-p)^2 + q$

$V(p, q)$

Example:

$x^2 - 5x + 6 = (x - \frac{5}{2})^2 - \frac{1}{4}$

$\therefore V = (5/2, -\frac{1}{4})$

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§ Quadratic functions: $y = 2x^2 + 5x - 3$ for $-4 \leqslant x \leqslant 1.5$

Example 2.

(a) Complete the table:

x	-4	-3	-2	-1	0	1	1.5
y	----	0	-5	----	-3	4	----

---[3]

$-4 \to 9$

$-1 \to -6$

$1.5 \to 9$

(b) Draw the graph. $-4 \leqslant x \leqslant 1.5$

[4]

Vertex at

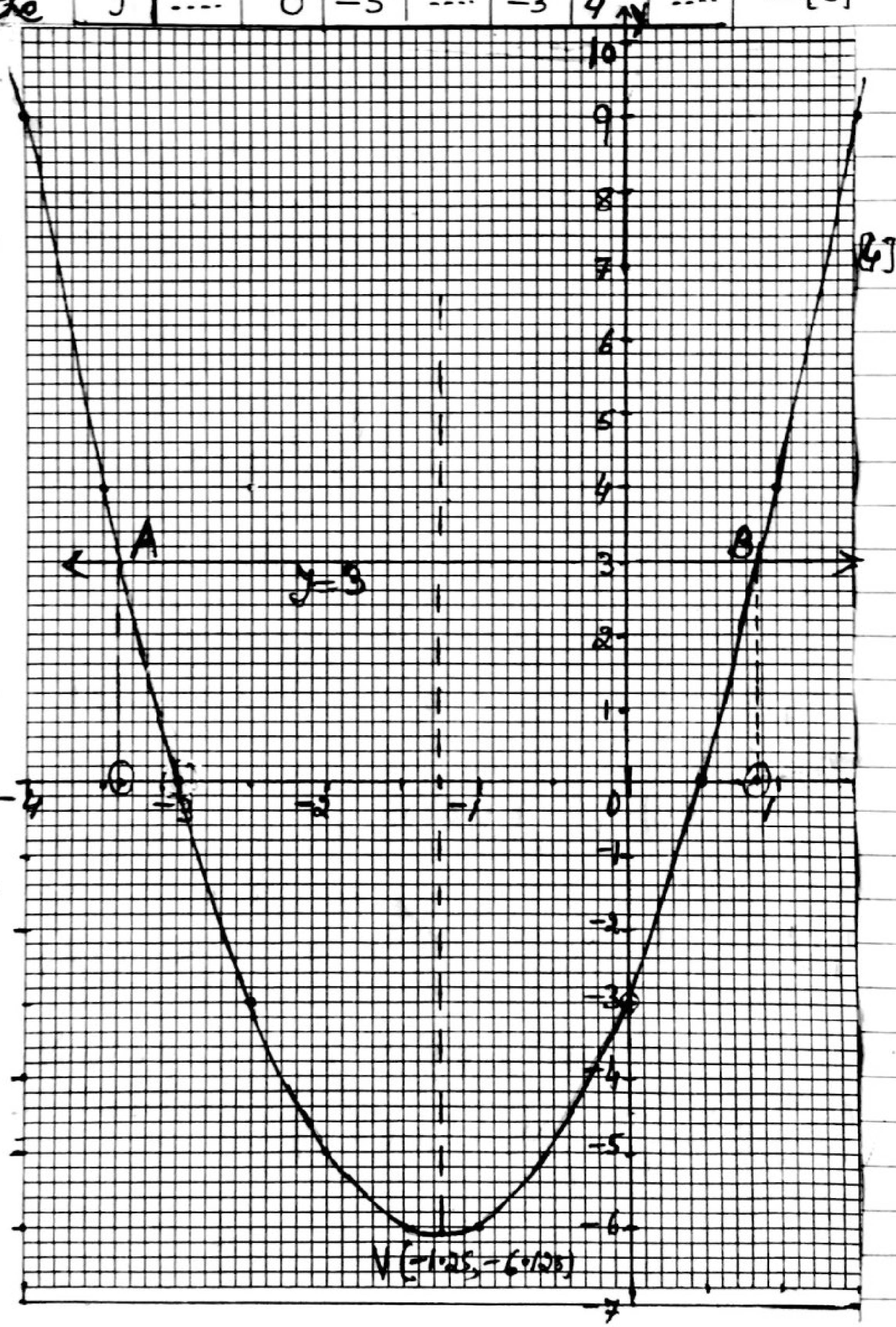
$x = -\frac{b}{2a}$

$= -\frac{5}{2\times 1}$

$= -\frac{5}{4}$

$\left(-\frac{5}{4}, -\frac{49}{8}\right)$

$(-1.25, -6.125)$



W-17/43/Q7

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Quadratic functions

(Continued →)

Example 2 (C). Use your graph to solve the equation.

$2x^2+5x-3=3$ ---[2]

Solution: Draw a line $y=3$ to intersect the curve at A and B, where $x=-3.4$ ✓ and $x=0.85$ to 0.9 ✓

(d) $y=2x^2+5x-3$ can be written in the form,

$y=2(x+a)^2+b$, find the value of a and b. ---[3]

Solution

Consider $y=2x^2+5x-3$

$=2\left[x^2+\frac{5}{2}x\right]-3$ add and sub the $\left(\frac{\text{Coeff } x}{2}\right)^2$

$=2\left[x^2+\frac{5}{2}x+\left(\frac{5}{4}\right)^2-\left(\frac{5}{4}\right)^2\right]-3$

$=2\left[\left(x+\frac{5}{4}\right)^2-\frac{25}{16}\right]-3$

$=2\left(x+\frac{5}{4}\right)^2-\cancel{2}\times\frac{25}{\cancel{16}\,8}-3$

$=2\left(x+\frac{5}{4}\right)^2-\frac{49}{8}$ ✓

$\therefore\ a=5/4$ ✓ and $b=-\frac{49}{8}$ ✓

---

§ Given a quad function,

$y=ax^2+bx+c$ $=a\left(x+\frac{b}{2a}\right)^2+\frac{(4ac-b^2)}{4a}$

or can be expressed as

$y=a(x-p)^2+q$ [using completing the square method]

Then **Vertex** of graph of this quad. function is

$V(p,q)$ or $\left(\frac{-b}{2a},\ \frac{4ac-b^2}{4a}\right)$

$\boxed{\text{Note}}$ 1. If $a>0$, Then the it has a Min value $=q$

2. If $a<0$, Then a Max. Value $=q$ (at $x=p$) at Vertex

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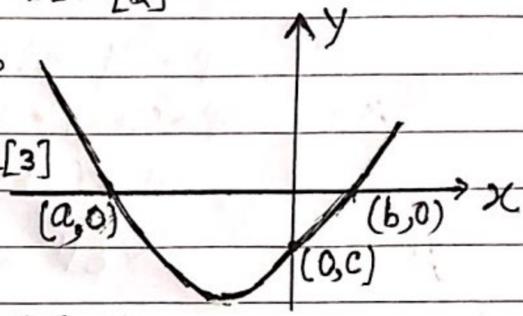
Quadratic function.

Example 3. (b)(i) Factorise $x^2+3x-10$ ---[2]

(ii) The graph of $y=x^2+3x-10$ is sketched.

Write down the value of a, b and c ---[3]



(iii) Write down the equation of the line of symmetry of the graph of $y=x^2+3x-10$ ---[1]

(c) Sketch the graph of $y=18+7x-x^2$ on the axes below. Indicate clearly the values where the graph crosses x and y axis. ---[4]

(d) (i) $x^2+12x-7=(x+p)^2-q$, Find the value of p and q. ---[3]

(ii) Write down the minimum value of y for the graph of $y=x^2+12x-7$. ---[1]

W-14/43/Q8

Solution:

(b)(i) $x^2+3x-10$

$=x^2+5x-2x-10$

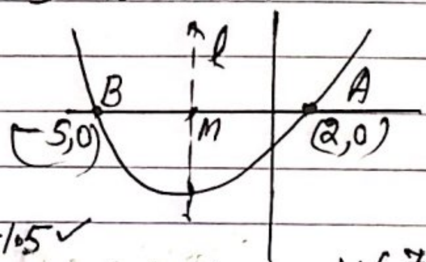
$=x(x+5)-2(x+5)$

$=(x-2)(x+5)$ ✓

(ii) $y=(x-2)(x+5)=0 \Rightarrow x=2$ or -5

$\therefore$ $b=2$ and $a=-5$ ✓

(iii) Line of symmetry 'l' passes the mid point of AB. $\frac{-5+2}{2}=\frac{-3}{2}$



$\therefore$ l // Y-axis passing through M is

$\therefore$ Equation of line of Symm $x=-3/2$ or -1.5 ✓

(c) $y=18+7x-x^2$

$=18+9x-2x-x^2$

$=9(2+x)-x(2+x)$

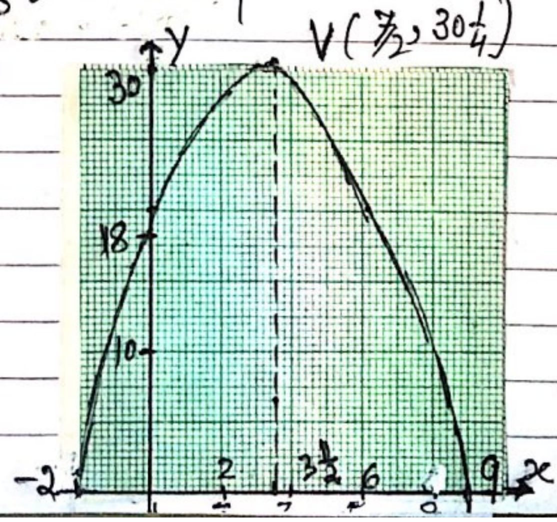
$=(2+x)(9-x)=0$

$x=-2,\ x=9$

Curve intersects the X-axis at $(-2,0)$, $(9,0)$

Vertex at $x=\frac{-2+9}{2}=\frac{7}{2}$, $V\left(\frac{7}{2},\frac{121}{4}\right)$

Y-Int $=18$ $(x=0)$



(Continued→)

Quadratic functions

(Continued→)

Example 3(d) (i) $x^2+12x-7 = (x+p)^2-q$

or $x^2+12x+6^2-36-7 = (x+p)^2-q$

or $(x+6)^2-43 = (x+p)^2-q$

$\therefore$ $p=6$ and $q=43$ ✓

(ii) Minimum Value of $y = -43$ ✓

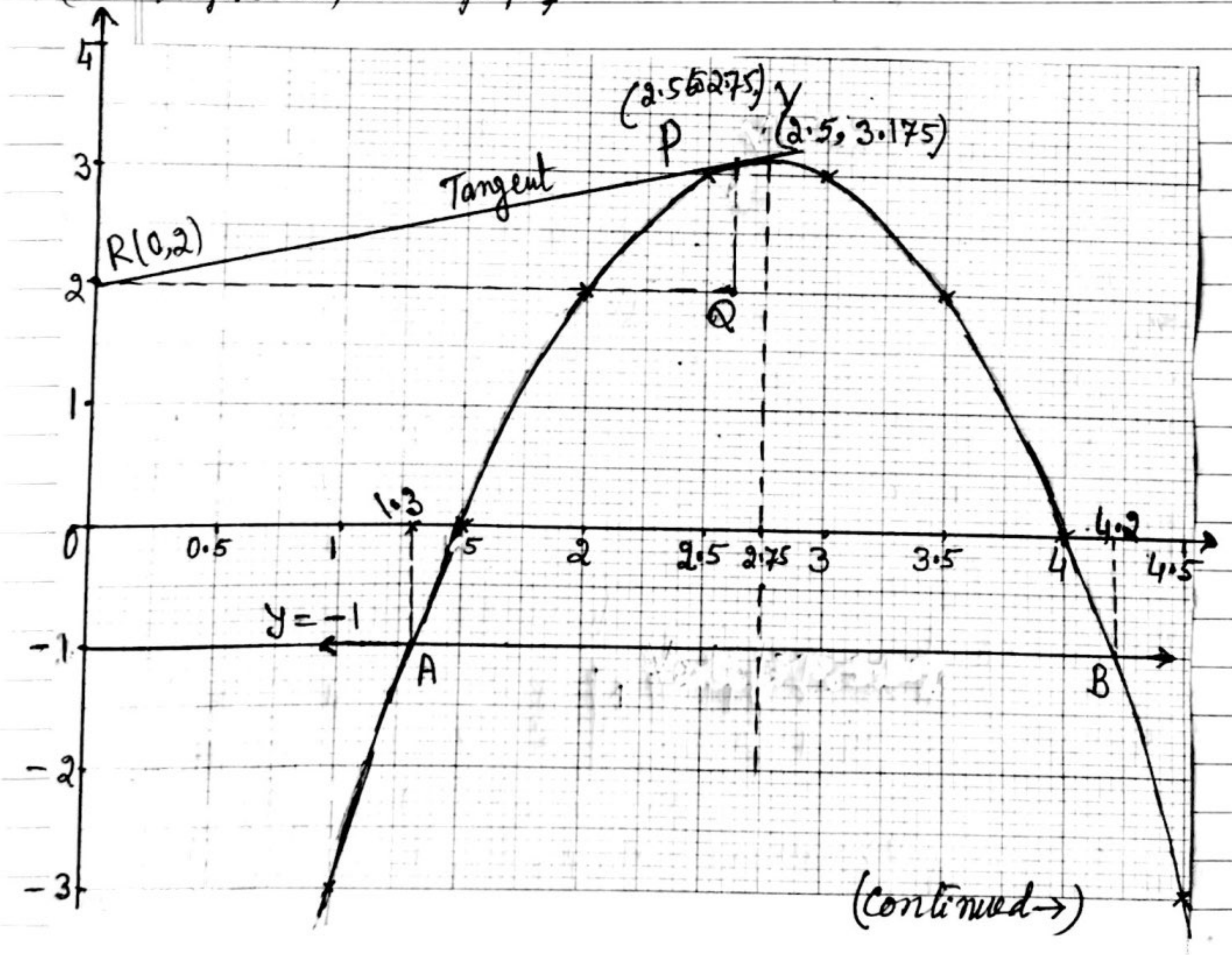
Example 4. The table shows some values for the function. [S-13 | 42 | Q3]

$y = 11x-2x^2-12$ for $1 \le x \le 4.5$

x	1	1.5	2	2.5	3	3.5	4	4.5
y	−3	---- 0	2	3	3	--- 2	--- 0	--- −3

(a) Complete the table of values. ---[3]

(b) On the grid below, draw the graph of $y=11x-2x^2-12$ for $1 \le x \le 4.5$ ---[4]



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Quadratic functions

(Continued →)

Example 4(c) By drawing a suitable line, use your graph to solve the equation: $11x - 2x^2 = 11$ ---[2]

Solution: $11x - 2x^2 = 11$

$\Rightarrow 11x - 2x^2 - 12 = 11 - 12$

or $y = -1$ intersection the curve

(see graph) at $x = 1.3$ and 4.2 ✓

(d) The line $y = mx + 2$ is a tangent to the curve $y = 11x - 2x^2 - 12$ at point P. By drawing this tangent:

(i) Find the coordinates of point P... ---- [2]

(ii) Work out the value of m. ---- [2]

Solution (i) $y = mx + 2$ passes through R(0,2) meets curve at point P(2.5 to 2.75, 3 to 3.1) ✓ (see graph)

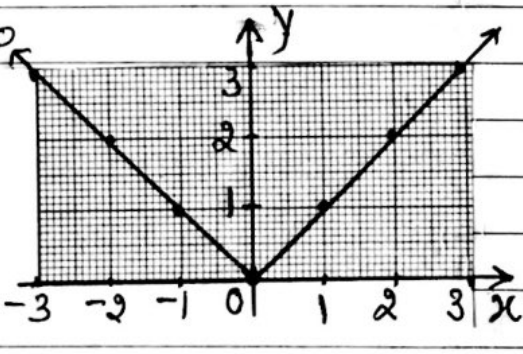
(ii) Gradient of tangent P $= m = \frac{PQ}{RQ} = \frac{\text{rise}}{\text{run}} = \frac{1.1}{2.6} = 0.42$ ✓

---

§ Modulus function (or Absolute value function):

$$y = |x| = \begin{cases} x, & \text{if } x \geqslant 0 \\ -x, & \text{if } x < 0 \end{cases}$$

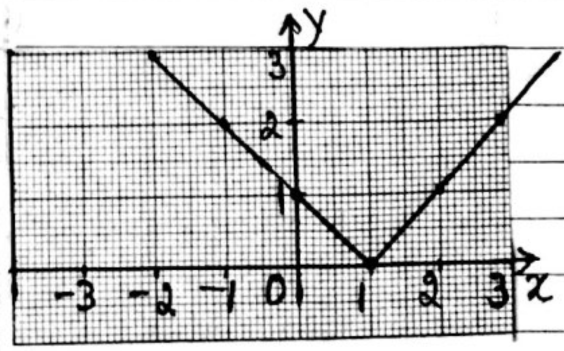
x	-2	-1	0	1	2	3
y	2	1	0	1	2	3



Example. Draw the graph of $y = |x-1|$

$$y = |x-1| = \begin{cases} x-1 & \text{if } x \geqslant 1 \\ 1-x & \text{if } x < 1 \end{cases}$$

x	-3	-2	-1	0	1	2	3
y	4	3	2	1	0	1	2



§ Graph of Rational functions (and Asymptotes)

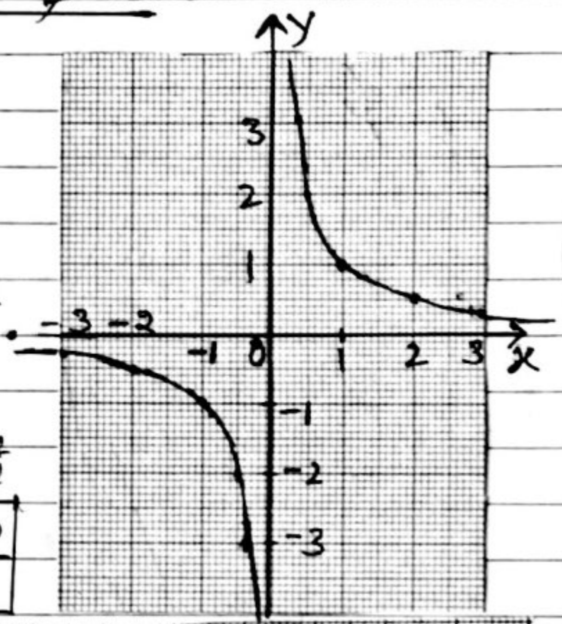
Case I Degree Numerator < Degree of Denominator.

Example 1, Draw the graph of,

$$y = \frac{1}{x} \quad \text{(Reciprocal function)}$$

(i) here as $x \to 0$ (tends to) ; $y \to \infty$

[y - axis (or $x = 0$) is an asym



(ii) as $x \to \infty$, $y \to 0$

[x - axis or $y = 0$ is an asymptote

x	-3	-2	-1	0	1	2	3	$\to\infty$
y	-0.33	-0.5	-1	Not def	1	$1/2$	$1/3$	0

Example 2: Draw the graph of.

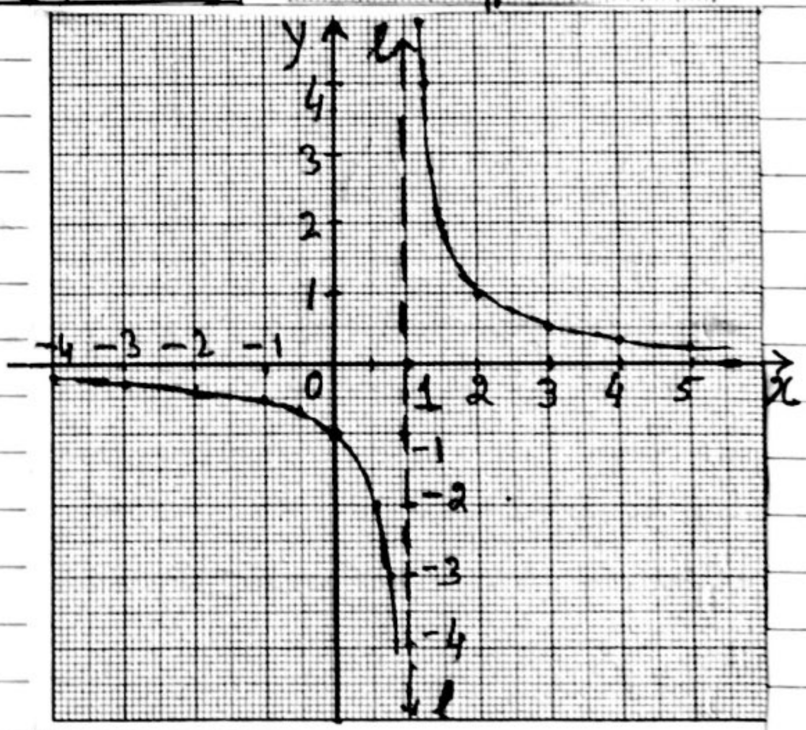
$$y = \frac{1}{x-1}$$

(i) $x \to 1$, $y \to \infty$

(line $l \parallel$ y-axis through $x=1$ is asymptote)

(ii) $x \to \infty$, $y \to \infty$

x - axis is asymptote



x	$-\infty$	--	-4	-3	-1	0	0.5	1	1.001	1.5	2	3	4	5	--	$x \to \infty$
y	$y \to 0$	--	$-.2$	$-.25$	$-.5$	-1	-2	not def	1000	2	1	$.5$	$.33$	$.25$	$\to$	$y \to 0$

x - axis is as asymptote

line $l \parallel$ y-axis through $x = 1$ as also an asymptote.

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§ Graph of Rational functions (and Asymptote)

Case II Deg of Numerator = deg of Denominator.

Example 3: Draw the graph of: [S-16/43/Q3]

$g(x) = \frac{2}{x} + 3$ $\left[\text{or } y = \frac{2+3x}{x}\right]$

or $y = 3 + \frac{2}{x}$ for $-3.5 \leq x \leq -0.5$ and $0.5 \leq x \leq 2.5$

tends to

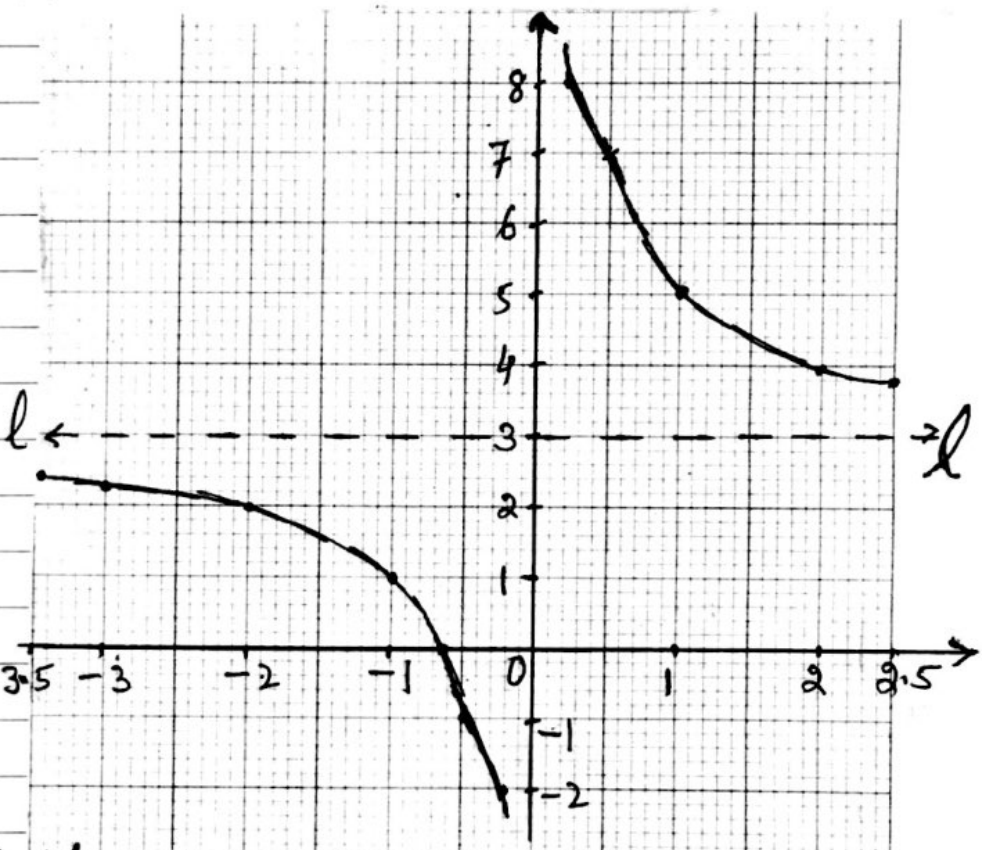
as $x \to \infty$, $y \to 3$

line $l \parallel$ X axis through $y = 3$ is an Asymptote

and

$x \to 0$, $y \to \infty$

y-axis is another Asymptote.



(i) Complete the table: ---- [3]

x	-3.5	-3	-2	-1	-0.5	0.5	1	2	2.5
$g(x) = y$	2.4	2.3	--- ↓ 2	1	--- ↓ -1	7	5	--- ↓ 4	3.8

(ii) Draw the graph of $y = g(x)$ ---- [4]

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§ Graph of Rational functions (and Asymptotes)

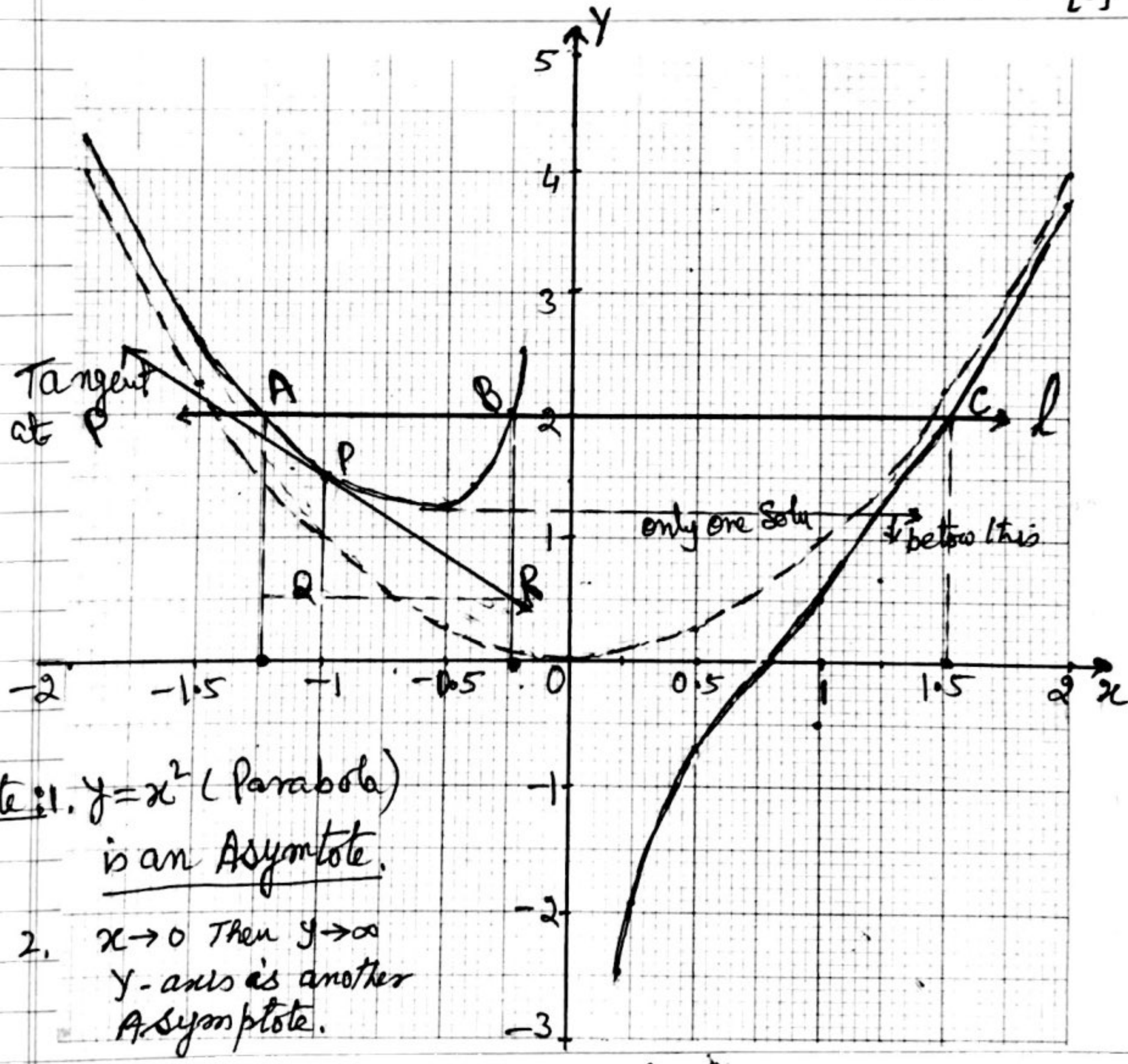
Case 3: Deg. of Numerator > deg. of Denominator ⊗

Example 4. $y = x^2 - \frac{1}{2x}$, $x \neq 0$ $\left(\text{or } y = \frac{2x^3 - 1}{2x}\right)$ ⊗ S-15/41/Q2

x	-2	-1.5	-1	-0.5	-0.25	-0.2	0.2	0.25	0.5	1	1.5	2
y	4.25	2.58	--- 1.5	--- 1.25	2.06	2.54	-2.46	-1.94	--- -0.75	--- 0.5	1.92	3.75

(a) Complete the table of values. ---[4]

(b) On the grid, draw the graph of $y = x - \frac{1}{2x}$ for $-2 \leq x \leq 0.2$ and $0.2 \leq x \leq 2$ ---[5]



Note: 1. $y = x^2$ (Parabola) is an Asymtote.

2. $x \to 0$ Then $y \to \infty$
Y-axis is another Asymptote.

(Continued →)

Graph of functions — Algebra-2

§ Graph of Rational functions:

(Continued→) Example 4 (c) By drawing a suitable line, use your graph to solve the equation $x^2 - \frac{1}{2x} = 2$ - - - [3]

Soln- Draw a line $y = 2$ on the graph ($l \parallel$ x-axis through (0,2)) intersects the curve at the point A, B and C,
for $x = -1.25$ ✓; -0.25 ✓ and 1.55 ✓

(d) The equation: $x^2 - \frac{1}{2x} = k$ has only one solution, write down the range ______ of values of k.

Solution: $k < 1.2$

(e) By drawing a suitable tangent, find an estimate of the gradient of the curve at the point where $x = -1$.

Solution: gradient of tangent at ($x = -1$) P $= \frac{\text{rise}}{\text{run}} = \frac{-PQ}{QR}$

$= \frac{-1}{0.75} = -1.33$ ✓

gradient $= -1.33$

Example 5: Draw the graph of: $y = \frac{x^2+2}{x}$; $- \leq x < 0$ and $0 < x \leq$

x	-4	-3	-2	-1	-0.5	-0.4	0.4	$.5$	1	1.5	2	3	4
y	-4.5	-3.66	-3	-3	-4.5	-5.4	5.4	4.5	3	2.83	3	3.66	4.5

consider $y = \frac{x^2+2}{x}$

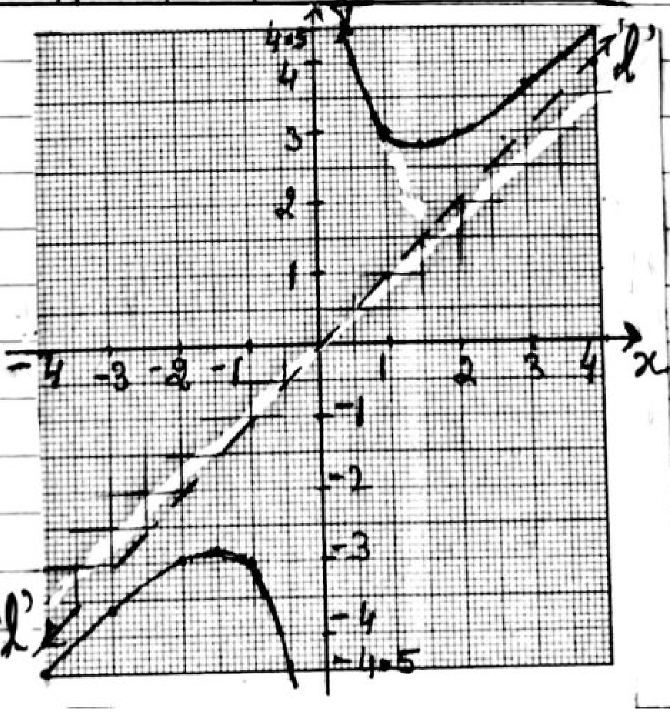
or $y = x + \frac{2}{x}$

as $x \to 0$, $y \to \infty$

$\therefore$ y-axis is an Asymptote.

(*) as $x \to \infty$, $y \to x$

$\therefore$ line 'l' $y = x$ is an Asymptote (Dotted line l)



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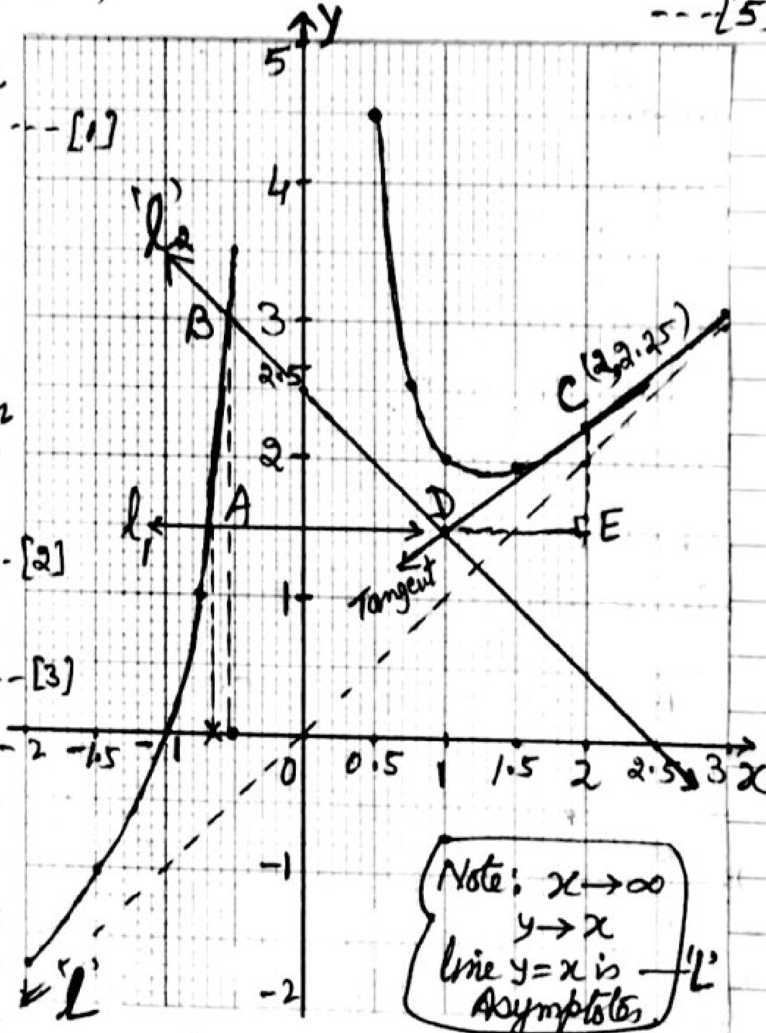
Graph of Rational Functions:

Example 6. The table shows some values of $y = x + \frac{1}{x^2}$, $x \neq 0$ [M-16/42/Q7]

x	-2	-1.5	-1	-0.75	-0.5	0.5	0.75	1	1.5	2	3
y	-1.75	-1.06	0	1.03	--- ↓ 3.50	4.50	2.53	2	--- ↓ 1.94	2.25	--- ↓ 3.11

(a) Complete the table of values. ---[3]

(b) On the grid, draw the graph of $y = x + \frac{1}{x^2}$ for $-2 \leq x \leq -0.5$ and $0.5 \leq x \leq 3$. ---[5]



(c) Use your graph to solve the equation $x + \frac{1}{x^2} = 1.5$ ---[1]

Solution: or $y = 1.5$ for which draw line $l_1 \parallel$ x-axis which intersects the curve at A, $x = -0.65$ ✓

(d) The line $y = ax + b$ can be drawn on the grid to solve $\frac{1}{x^2} = 2.5 - 2x$

(i) Find the value of a & b. ---[2]

(ii) Draw the line $y = ax + b$ to solve $\frac{1}{x^2} = 2.5 - 2x$ ---[3]

Solution:

(i) $\frac{1}{x^2} = 2.5 - 2x$

or $\frac{1}{x^2} + x = -x + 2.5$ (or $ax + b$)

$\therefore a = -1$ & $b = 2.5$ ✓

(ii) Draw the line 'l₂' $y = -x + 2.5$

x	0	2.5	1
y	2.5	0	1.5

line 'l₂'

line 'l₂' intersects curve at B, $x = -0.55$ ✓

(e) By drawing a suitable tangent, find an estimate of the gradient of the curve at the point where $x = 2$ C(2, 2.25) ---[3]

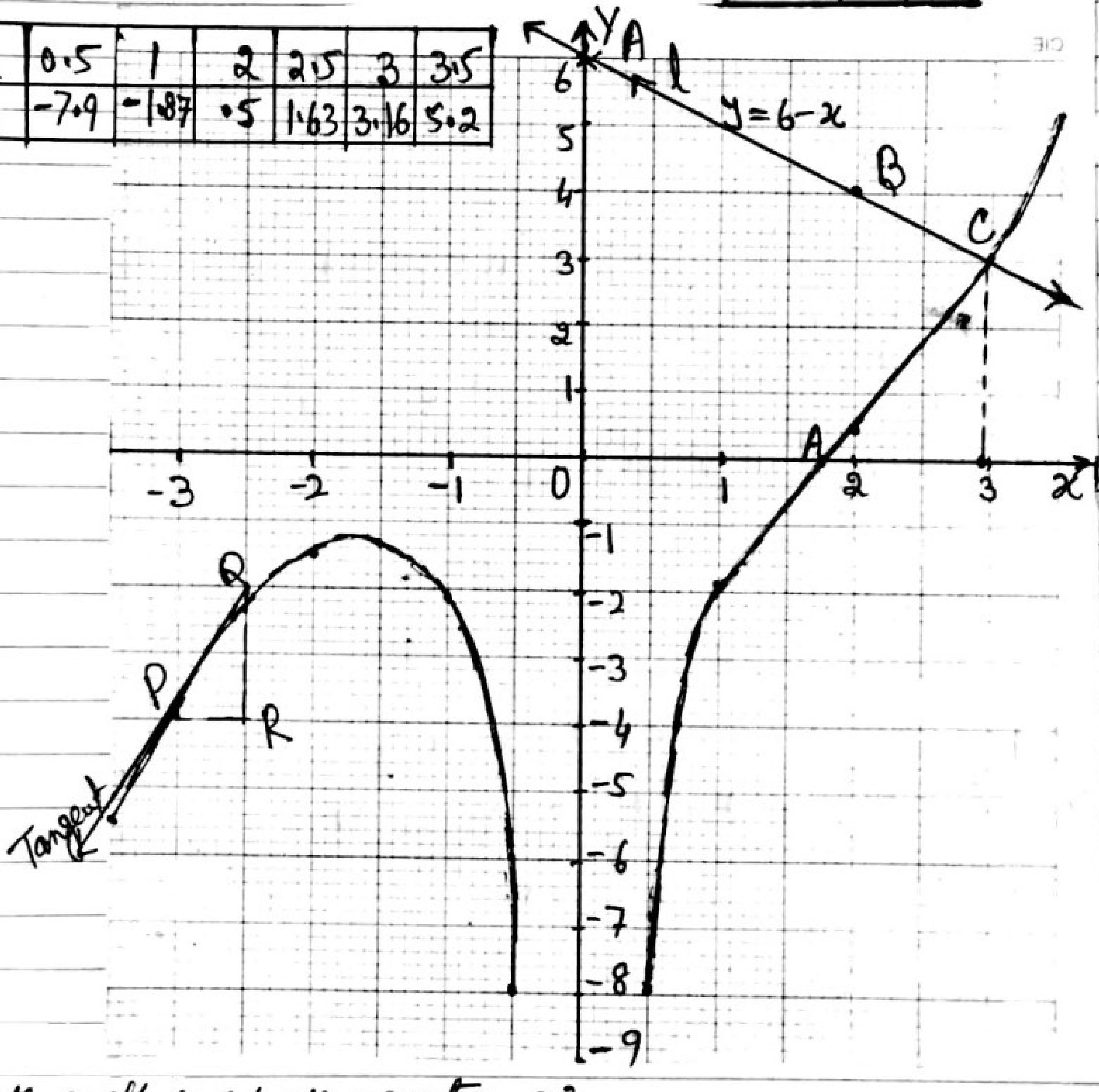
Solution: CD is tangent at C, gradient $= \frac{CE}{DE} = \frac{0.75}{1} = 0.75$ ✓

Graph of Rational Functions.

Example 7(b) The graph of $y = \frac{x^3}{8} - \frac{2}{x^2}$ for $-3.5 \leq x \leq -0.5$ has already been drawn on the grid, draw the graph of $y = \frac{x^3}{8} - \frac{2}{x^2}$ for $0.5 \leq x \leq 3.5$ --- [4]

W-17/42/Q5

x	0.5	1	2	2.5	3	3.5
y	-7.9	-1.87	.5	1.63	3.16	5.2



(C) use the graph to solve the equation $\frac{x^3}{8} - \frac{2}{x^2} = 0$ --- [1]
or $y = 0$;

Curve cuts the x-axis at 1.75 ✓ is the required soln.

(d) $\frac{x^3}{8} - \frac{2}{x^2} = k$, where k is an integer. Write down a value of k, when the equation $\frac{x^3}{8} - \frac{x^2}{2} = k$ has

(i) One answer. ⟶ $k \geq -1$ --- [1]

(ii) Three answer. ⟶ $k < -1$ -- [1]

(Continued ⟶)

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Example 7(e) By drawing a suitable tangent, estimate the gradient of the curve where $x = -3$. ---[3]

Soln: at $x=-3$ at point P → PQ is tangent.

gradient of tangent $= \frac{QR}{PR} = \frac{2}{0.5} = 4$ ✓

(f)(i) By drawing a suitable line on the grid, find x, when $\frac{x^3}{8} - \frac{2}{x^2} = 6 - x$ ---[3]

Solution Draw a line 'l' $y = 6 - x$

Consider point A(0,6), B(2,4) draw line 'l' which intersects the curve at C... or $x = 2.95$ ✓

(ii) The equation $\frac{x^3}{8} - \frac{2}{x^2} = 6 - x$ can be written as ---[4]

$x^5 + ax^3 + bx^2 + c = 0$, find the value of a, b and c.

Solution:

$$\frac{x^3}{8} - \frac{2}{x^2} = 6 - x$$

multiply by x^2, $\frac{1}{8}x^5 - 2 = 6x^2 - x^3$

or $\frac{1}{8}x^5 + x^3 - 6x^2 - 2 = 0$

or $x^5 + 8x^3 - 48x^2 - 16 = 0$

$\therefore$ $a = 8$, $b = -48$ and $c = -16$ ✓

Example 8. Draw the graph of,

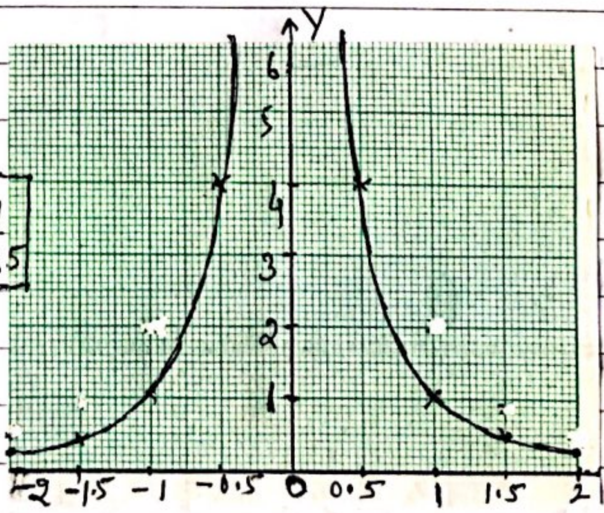
$f(x) = \frac{1}{x^2}$, $-2 \le x \le 2, x \ne 0$

x	-2	-1.5	-1	-0.5	0.5	1	1.5	2	0.4
y	0.25	0.44	1	4	4	1	0.44	0.25	6.25

(i) $x \to 0, y \to \infty$

$\therefore$ y-axis is Asymptote.

(ii) $x \to \infty, y \to 0$, x-axis is Asymptote.



IG-Maths 0580

Graph of functions Algebra-2

"Rational Function"

Example 9: $f(x) = \frac{20}{x} + x$, $x \neq 0$ SP-2020/04/Q3/S-16/41/Q5

(a) Complete the table: ---[2]

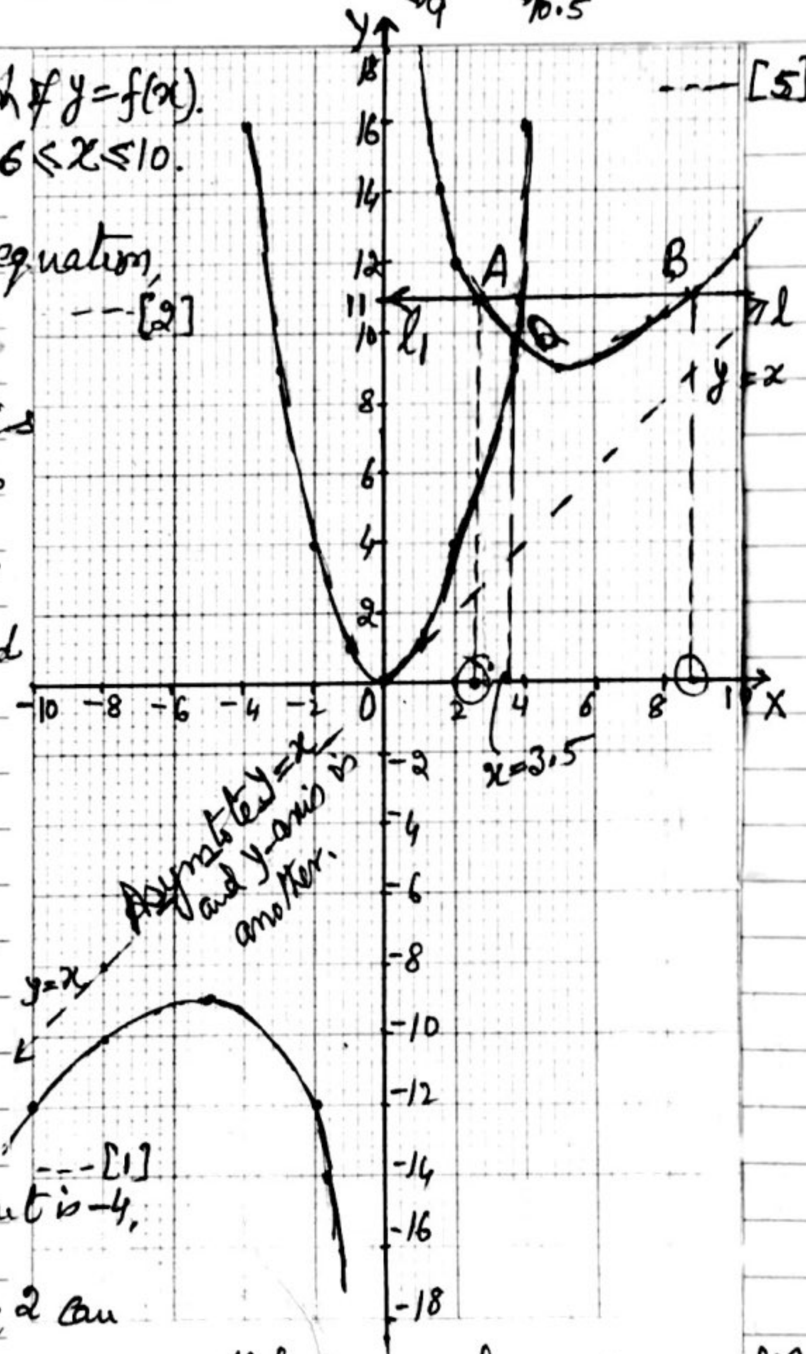
x	-10	-8	-5	-2	-1.6	1.6	2	5	8	10
y	-12	-10.5	-9	-12	-14.1	14.1	12	--- 9	--- 10.5	12

(b) On the grid, draw the graph of $y = f(x)$. for $-10 \leq x \leq -1.6$ and $1.6 \leq x \leq 10$. ---[5]

(c) Using graph solve the equation, $f(x) = 11$. ---[2]

Solution Draw a line l_1, $y = 11$, line $l_1 \parallel x$-axis intersects the the curve at A, (2.6, 11), B(8.8, 11)

$\therefore$ Rep Solu. 2.6 and 8.8



(d) K is a prime number and $f(x) = K$ has no solution. find the value of K.

Solution: $K = 2, 3, 5, 7$ [since $f(x) > 9$] graph.

(e) The gradient of the graph of $y = f(x)$ at point (2, 12) is -4. Write down the coordinate of other point on the graph $y = f(x)$. where the gradiant is -4. ---[1]

Solu. $(-2, -12)$.

(f) (i) The equation $f(x) = x^2$ can be written as $x^3 + px^2 + q = 0$ show that $p = -1$ and $q = -20$ ---[2]

Solution: $f(x) = x^2 \Rightarrow \frac{20}{x} + x = x^2 \Rightarrow 20 + x^2 = x^3$

or $x^3 - x^2 - 20 = 0$

$\therefore p = -1$ and $q = -20$

(Continued →)

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IG-Maths 0580 Graph of functions Algebra-2

(Continued→)

Example 9(f)(ii) on the grid, draw the graph of $y=x^2$ $-4\le x\le 4$ --[2]

Draw the correct graph, a parabola.

(iii) Using graph solve the the equation,

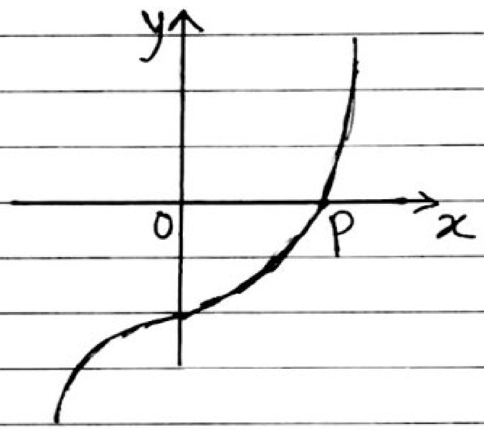
$x^3-x^2-20=0$ or $f(x)=x^2$ ---[1]

Curves $y=f(x)$ and $y=x^2$ intersect at the point Q

where $x=3.5$ ✓ is the solution.

(iv) The diagram show a sketch of the graph of $y=x^3-x^2-20$, P is the point $(n,0)$, write down the value of n. --[1]

Solution $x=3.5$



Graph of cubic function

Example 10: (a) Complete the table for the function, $f(x) = \frac{x^3}{2} - 3x - 1$ --- [3]

x	-3	-2	-1.5	-1	0	1	1.5	2	3	3.5
$f(x)$	-5.5	--- ↓ 1	1.8	1.5	--- ↓ -1	-3.5	-3.8	-3	--- ↓ 3.5	9.9

(b) On the grid draw the graph of $y = f(x)$ for $-3 \leq x \leq 3.5$ --- [4]

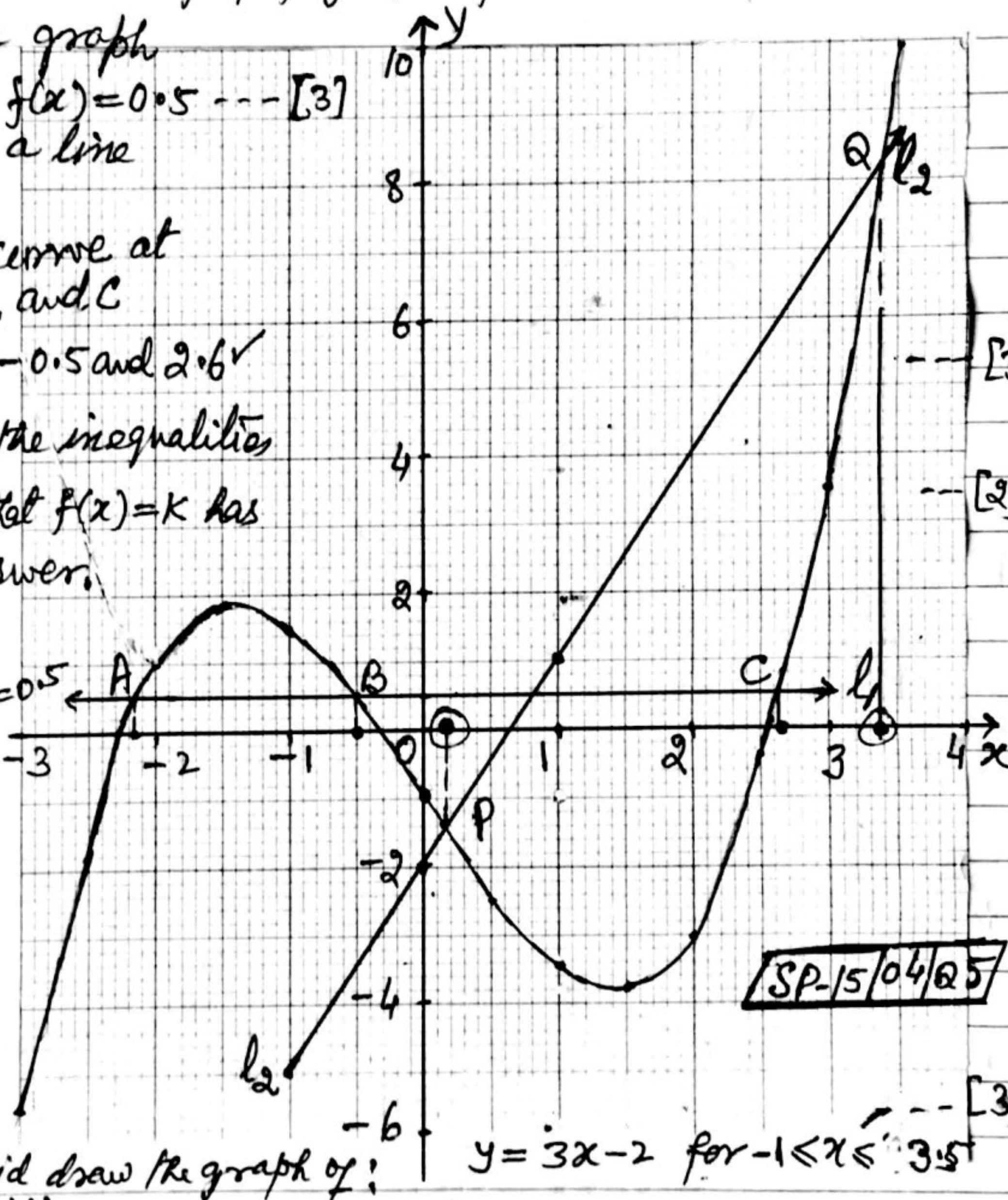
(c)(i) Use your graph to solve $f(x) = 0.5$ --- [3]

Solution: Draw a line 'l_1' $y = 0.5$ intersects the curve at the points A, B, and C for $x = -2.2, -0.5$ and 2.6 ✓

--- [3]

(C)(ii) Find the inequalities for K, so that $f(x) = K$ has only one answer.

$K < -3.8$ or $K > 1.8$ --- [2]



--- [3]

(d)(i) On the grid draw the graph of: $y = 3x - 2$ for $-1 \leq x \leq 3.5$

Draw line 'l_2'

(ii) The equation $\frac{x^3}{2} - 3x - 1 = 3x - 2$ can be written in the form $x^3 + ax + b = 0$ find the value of a & b.

$\Rightarrow x^3 - 12x + 2 = 0 \Rightarrow a = -12$ and $b = 2$ ✓

(iii) Use your answer to find the positive answer to $\frac{x^3}{2} - 3x - 1 = 3x - 2$ at P and Q, at 1.55 and 3.55 ✓

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IG-Maths 0580 **Graph of functions** **Algebra-2** Date Page P-38

Graph of Cubic function.

Example 11 (a) The table shows some value for $y=2x^3+4x^2$ [S-17/43/Q3]

x	-2.2	-2	-1.5	-1	-0.5	0	0.5	0.8
y	-1.94	--- 0	--- 2.25	--- 2	0.75	0	--- 1.25	3.58

---[4]

(b) Draw the graph of $y=2x^3+4x^2$ for $-2.2 \leq x \leq 0.8$ ---[4]

(c) Find the number of solution of the equation $2x^3+4x^2=3$ ---[1]

(d)(i) The equation $2x^3+4x^2-x=1$ can be solved by drawing a straight line on the grid. Write down the equation of the straight line. ---[1]

d(ii) Use your graph to solve the equation $2x^3+4x^2-x=1$ ---[3]

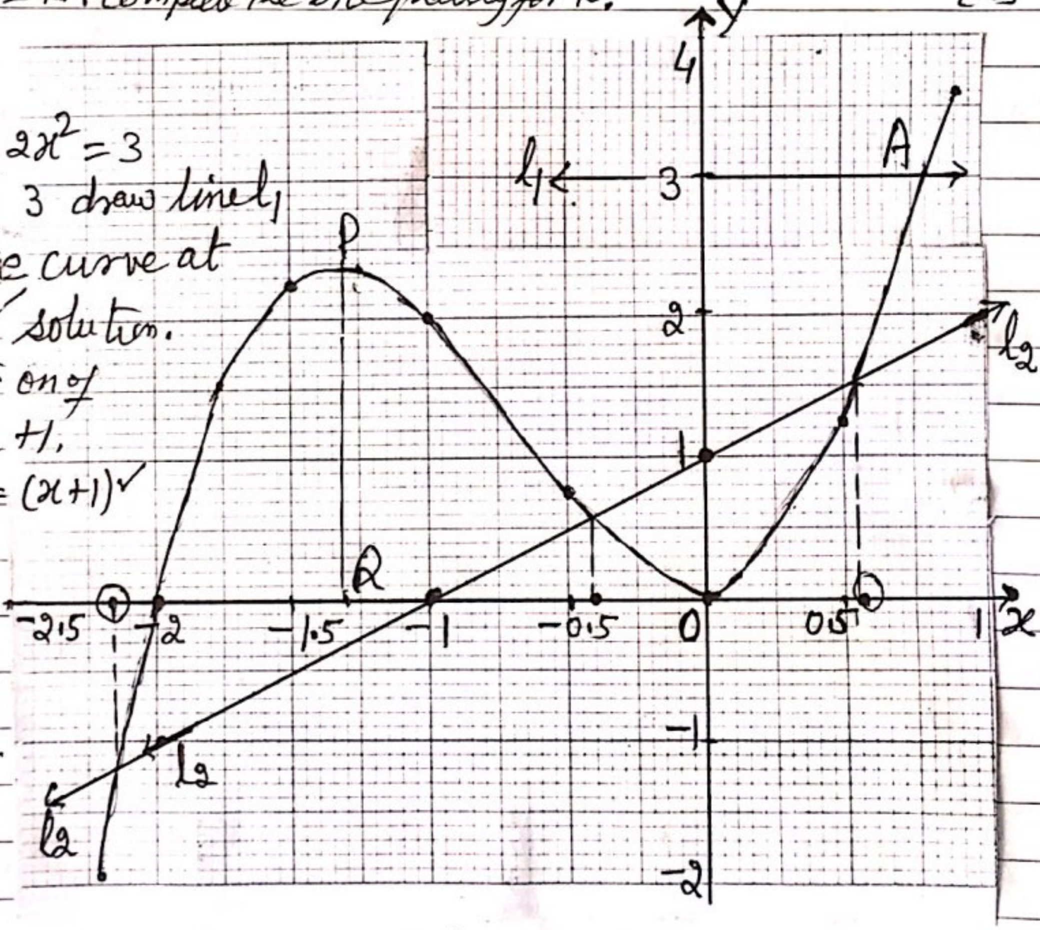
(e) The tangent to the graph of $y=2x^3+4x^2$ has a negative gradient when $x=k$. Complete the inequality for k. ---[2]

Solution:

c(i) $2x^3+2x^2=3$
or $y=3$ draw line l_1
intersects the curve at A. Only 1 solution.

(d) (i) Equation of
$2x^3+4x^2=x+1$.
$\therefore$ Eqn of line $y=(x+1)$

(ii) Draw the line l_2, $y=x+1$
solution
-2.15, -0.45 and 0.55



(e)

$-1.33 < k < 0$

[Q < Point < 0]

(gradient < 0)
all the points on $\widehat{OP}$.

$\frac{dy}{dx} = 6x^2+8x$
$= 2x(3x+4) < 0$
$-\frac{4}{3} < x < 0$
or $-1.33 \leq x < 0$

§ Exponential Functions:

$f(x) = b^x$; $b \neq 1$ and $b > 0$ [$f(x) = (\text{Constant})^{\text{Variable}}$

Example 12: (i) Complete this table of values for 2^x. --- [2]

x	-3	-2	-1	0	1	2	3
y	0.125	--- (0.25)	0.5	--- (1)	2	4	8

W-15/41/Q2(c)

(ii) On the grid draw, draw the graph of $y = 2^x$ for $-3 \leq x \leq 3$ -- [4]

(iii) Use graph to solve $2^x = 5$ --- [1]

(iv) Find the equation of the line joing (1,2) and (3,8) --- [3]

(v) By drawing a suitable line on your graph, solve $2^x - 2 - x = 0$ -- [2]

Solution:

(i) 0.25, 1

(iii) To solve $2^x = 5$

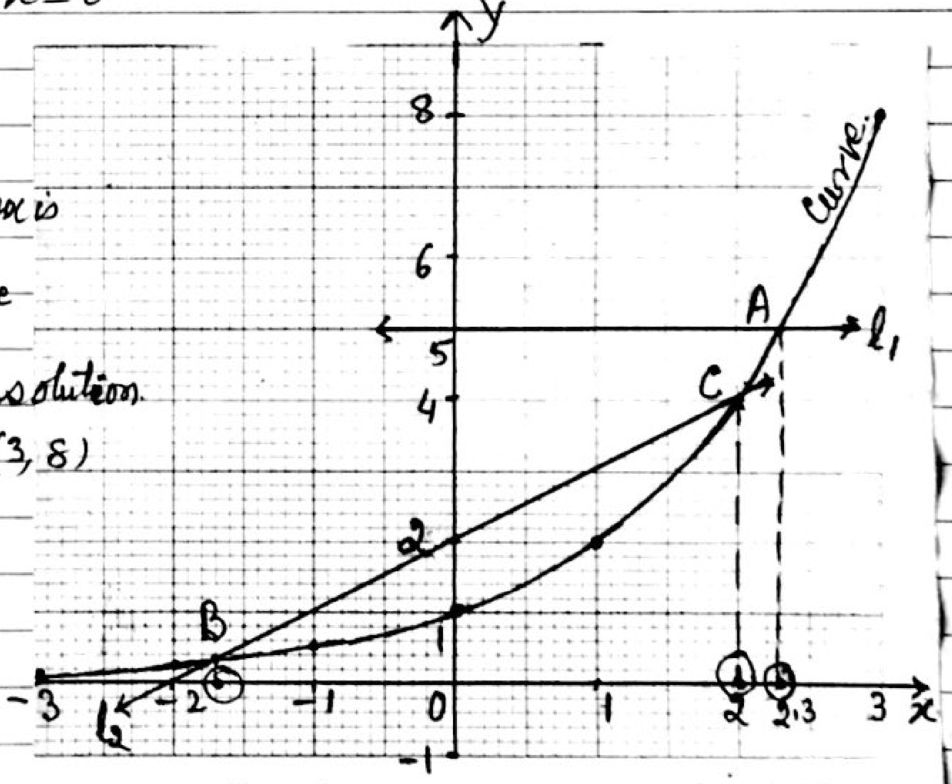
draw a line $l_1 \parallel$ X-axis $y = 5$

l_1 intersects the curve at a point A for $x = 2.3$ ✓ is the solution.

(iv) Line joing (1,2), (3,8)

$y - 2 = \frac{8-2}{3-1}(x-1)$

or $y = 3x - 1$ ✓



(V) To Solve

$2^x - 2 - x = 0$

or $2^x = x + 2$, draw a line l_2 $y = x + 2$

l_2 intersects the curve at the points B ($x = -1.7$) and C ($x = 2$)

∴ Required Solu. -1.7 and 2

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Exponential function

Example 13 The table shows some values for $y = 1.5^x - 1$ [M-17/42/Q3]

x	-2	-1	0	1	2	3	4	5
y	-0.56	-0.33	---- 0	---- 0.5	---- 1.25	2.38	4.06	6.59

(a) Complete the table. ---[3]

(b) Draw the graph of $y = 1.5^x - 1$ for $-2 \le x \le 5$ ---[4]

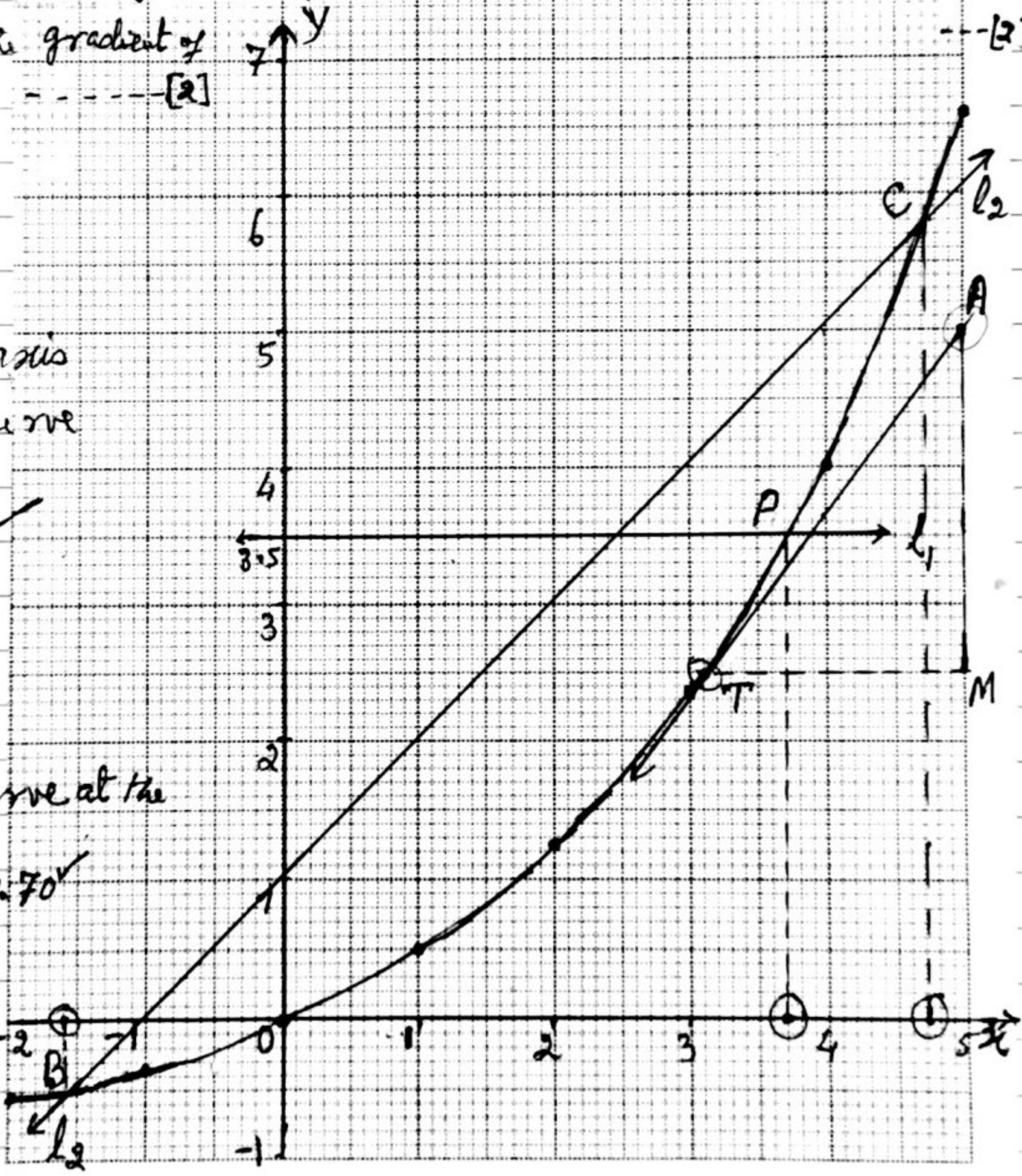
(c) Use your graph to solve the equation, $1.5^x - 1 = 3.5$ --[2]

(d) By drawing a suitable straight line, solve $1.5^x - x - 2 = 0$ --[3]

(e) (i) On the grid plot the point $A(5,5)$ --[1]

(ii) Draw the tangent to the graph of $y = 1.5^x - 1$, that passes through A. --[1]

(iii) Work out the gradient of this tangent. ------[2]



Solution

(c) To solve $1.5^x - 1 = 3.5$

Draw a line 'l_1' $y = 3.5$ // x-axis

Intersect the curve at $P(3.7, 3.5)$

Soln is $x = 3.7$ ✓

(d) $1.5^x - x - 2 = 0$

or $1.5^x - 1 = x + 1$

Draw a line 'l_2' $y = x + 1$ ✓

Intersects the curve at the points B and C

$x = -1.6$ ✓ and 4.70 ✓

(e)(i) $A(5,5)$ ✓

(ii) Draw tangent AT. ✓

(iii) gradient of $AT = \frac{\text{rise}}{\text{run}}$

$= \frac{AM}{TM} = \frac{2.5}{1.9} = 1.32$ ✓ $\therefore$ grad of tangent $= 1.32$ ✓

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