

IG - Maths

0580

Algebra

Exercise - Paper - 4

SP-20; M-19; M-18; S-19; S-18

W-18.

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Contents	Page No.
1. Linear equations, inequations, proportion, Factorise, Simplify an algebraic fraction.	1-2
2. Quadratic equations and laws of exponents.	3-7
3. Sequences.	8-10
4. Functions and Graphs	11-23
5. Derived functions.	24
<u>Answers.</u>	<u>25-29.</u>

Factorise.

1. (a) Solve the simultaneous equations.
You must show all your working. $6x + 5y = 27$ ---[4]
 $5x - 3y = 44$

(b) y is inversely proportional to $(x+3)^2$
When $x = 2$, $y = 8$. Find y when $x = 7$, ---[3]

(c) Solve the inequality.
 $3(x-2) < 7(x+2)$ ---[3]
M-19/42/Q10

2 (a) Make t the subject of the formula $s = k - t^2$ ---[2]

(b) (i) Factorise $x^2 - 25$ ---[1]

(ii) Simplify $\frac{x^2 - 25}{x^2 - 2x - 35}$ ---[3]

(c) Write as a single fraction in its simplest form: $\frac{x-8}{x} + \frac{3x}{x+1}$ ---[3]

(d) Find the integer values of n that satisfy the inequality:

$$18 - 2n \leq 6n \leq 30 + n$$

M-18/42/Q4 [3]

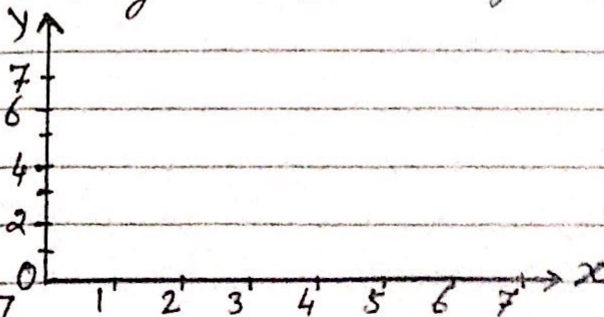
3. Klaus buys x silver balloons and y gold balloons for a party.
He buys

- more gold balloons than silver balloons.
- at least 15 silver balloons
- less than 50 gold balloons.
- a total of no more than 70 balloons.

(a) Write down four inequalities, in terms of x and/or y ,
to show this information. ---[4]

(b) On the grid, show the information from part (a) by drawing
four straight lines and shading the unwanted regions. ---[5]

(c) Silver balloons cost \$2
and gold balloons
cost \$3.
Calculate the most that
Klaus could spend. ---[2]



M-18/42/Q6



4(a) Expand and simplify. $(x+7)(x-3)$ ---[2]

(b) Factorise completely: (i) $15p^2q^2 - 25q^3$ ---[2]

(ii) $4fg + 6gh + 10fk + 15hk$ ---[2]

(iii) $8(k^2 - m^2)$ ---[2]

(c) Solve the equation. $3(x-4) + \frac{(x+2)}{5} = 6$ ---[4]

S-19/42/Q6

5(a) Solve $5x - 17 = 7x + 3$ ---[2]

(b) Find the integer values of n that satisfy this inequality.

$-7 < 4n \leq 8$ [3]

S-19/43(a)(b)

6(a) Factorise. (i) $2mn + m^2 - 6n - 3m$ ---[2]

(ii) $4y^2 - 81$ ---[1]

(iii) $t^2 - 6t + 8$ ---[2]

(b) Rearrange the formula to make x the subject.

$k = 2m - x$ ---[4]

(c) Solve the simultaneous equations:

$\frac{1}{2}x - 3y = 9$

you must show all your working.

$5x + y = 28$ ---[3]

S-18/41/Q5(a,b,c)

7(a) Factorise $5m^2 - 20p^4$ ---[3]

(b) Make P the subject of the formula. $A = P + \frac{PRT}{100}$ ---[3]

W-18/41/Q17

8(a) Solve $30 + 2x = 3(3 - 4x)$ ---[3]

(b) Factorise $12ab^3 + 18a^3b^2$ ---[2]

(c) Simplify (i) $5a^3c^2 \times 2a^2c^7$

(ii) $\left(\frac{16a^8}{c^{12}}\right)^{\frac{3}{4}}$ ---[2]

(d) y is inversely proportional to the square of $(x+2)$

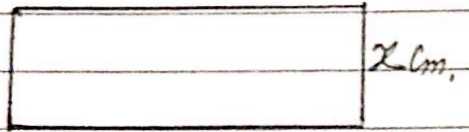
When $x=3$, $y=2$; Find y when $x=8$. ---[3]

(e) Write as a single fraction in its simplest form.

$\frac{5}{x-2} - \frac{x-5}{2}$ ---[3]

W-18/42/Q2

1(a) The perimeter of the rectangle is 80 cm. The area of the rectangle is $A \text{ cm}^2$.



(i) Show that $x^2 - 40x + A = 0 \dots [3]$

(ii) When $A = 300$, solve the equation $x^2 - 40x + A = 0$ by factorising. $\dots [3]$

(iii) When $A = 200$, solve the equation $x^2 - 40x + A = 0$ using the quadratic formula. Show all your working and give your answers correct to 2 decimal places. $\dots [4]$

(b) A car completes a 200 km journey at an average speed of $x \text{ km/h}$. The car completes the return journey of 200 km at an average speed of $(x + 10) \text{ km/h}$.

(i) Show that the difference between the time taken for each of the two journeys is $\frac{2000}{x(x+10)}$ hours. $\dots [3]$

(ii) Find the difference between the time taken for each of the two journeys when $x = 80$. Give your answer in minutes and seconds. $[SP-20/04/Q5] \dots [3]$

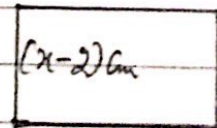
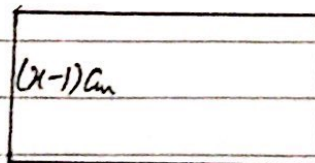
2(a) $S = ut + \frac{1}{2}at^2$

(i) Find S when $t = 26.5$, $u = 104.3$ and $a = -2.2$

Give your answer in standard form, correct to 4 significant figures. $\dots [4]$

(ii) Rearrange the formula to write a in terms of u , t and s . $\dots [3]$

(b) The difference between the areas of two rectangles is 62 cm^2 .



(i) Show that $x^2 + 2x - 63 = 0$ $\dots [3]$

(ii) Factorise $x^2 + 2x - 63$ $\dots [2]$

(iii) Solve the equation $x^2 + 2x - 63 = 0$ to find the difference between the perimeters of the two rectangles. $\dots [2]$

[S-19/41/Q7]

- 3 The probability that Andrei cycles to school is r .
- (a) Write down, in terms of r , the probability that Andrei does not cycle to school. ---[1]
- (b) The prob. that Benoit does not cycle to school is $1.3-r$.
 The prob. that both Andrei and Benoit do not cycle to school is 0.4 .
- (i) Complete the equation in terms of r .
 $(\dots) \times (\dots) = 0.4$ ---[1]
- (ii) Show that this equation simplifies to $10r^2 - 23r + 9 = 0$ ---[3]
- (iii) Solve by factorisation $10r^2 - 23r + 9 = 0$ ---[3]
- (iv) Find the prob. that Benoit does not cycle to school. ---[1]

[S-19/42/Q3]

4. Simplify,
- (i) $a^3 \times a^6$ --- [1]
- (ii) $(5xy^2)^3$ --- [2]
- (iii) $\left(\frac{27x^{12}}{64y^3}\right)^{-\frac{1}{3}}$ --- [3]

[S-19/43/Q2(c)]

5.(a) Angelo has a bag containing 3 white counters and x black counters. He takes two counters at random from the bag without replacement.

- (i) Complete the following statement.
 The probability that Angelo takes two black counters is,
 $\frac{x}{x+3} \times \frac{\dots}{\dots}$ --- [2]

- (ii) The prob. that Angelo takes two black counters is $\frac{7}{15}$.
- (a) Show that $4x^2 - 25x - 21 = 0$ --- [4]
- (b) Solve by factorisation $4x^2 - 25x - 21 = 0$ --- [3]
- (c) Write down the number of black counters in the bag. --- [1]

[S-19/43/Q8(a)]

6. $\frac{3}{m+4} - \frac{4}{m} = 6$

- (i) Show that this equation can be written as $6m^2 + 25m + 16 = 0$ --- [3]
- (ii) Solve the equation $6m^2 + 25m + 16 = 0$
 Show all your working and give your answers correct to 2 decimal places. --- [4]

[S-18/41/5(d)]

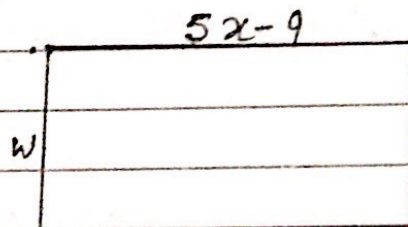
- 7 (a) Simplify (i) $(3p^2)^5$ --- [2]
 (ii) $18x^2y^6 \div 2xy^2$ --- [2]
 (iii) $\left(\frac{5}{m}\right)^{-2}$ --- [1]

(b) In this part, all measurements are in metres.

The diagram shows a rectangle.

The area of the rectangle is 310m^2 .

Work out the value of w .



S-18/42/Q4 --- [4]

8 In this question, all measurements are in metres.

The diagram shows a right-angled triangle.

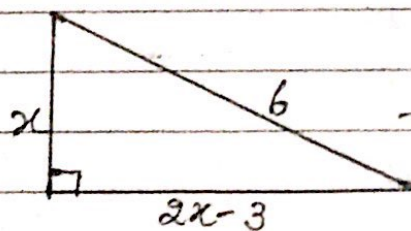
(a) Show that $5x^2 - 12x - 27 = 0$

(b) Solve $5x^2 - 12x - 27 = 0$

Show all your working and give your answers correct to 2 decimal places.

(c) Calculate the perimeter of the triangle.

(d) Calculate the smallest angle of the triangle. S-18/42/Q7 --- [2]



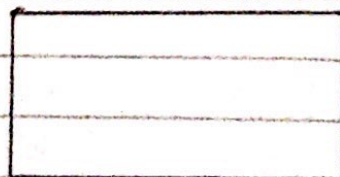
9(a) At a football match, the price of an adult ticket is \$ x and the price of a child ticket is \$ $(x - 2.50)$. There are 18500 adults and 2400 children attending the football match. The total amount paid for the tickets is \$320040.

Find the price of an adult ticket.

(b) (i) Factorise $y^2 + 5y - 84$

(ii) The area of the rectangle is 84m^2 .

Find the perimeter.



$(y+5)\text{cm}$

(Continued →)

(continued →)

9(c) In a shop, the price of a monthly magazine is \$ m and the price of a weekly magazine is \$ $(m - 0.75)$, one day the shop receives.

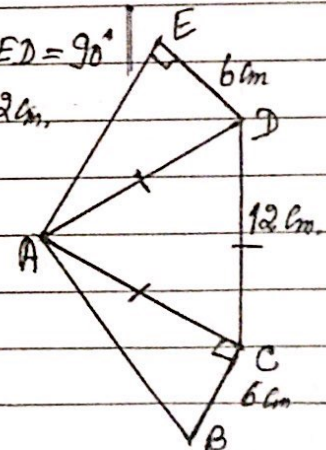
- \$ 168 from selling monthly magazines,
- \$ 207 from selling weekly magazines.

The total number of these magazines sold during this day is 100.

(i) Show that $50m^2 - 225m + 63 = 0$ --- [3]

(ii) Find the price of a monthly magazine. Show all your working. [S-18/43/Q5] --- [3]

10 (a) In the pentagon ABCDE, angle ACB = angle AED = 90°. Triangle ACD is equilateral with side length 12cm. DE = BC = 6cm.



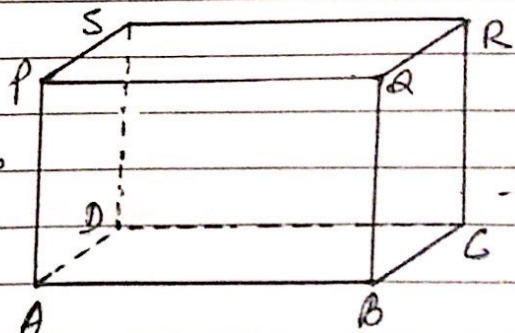
(i) Calculate angle BAE. --- [4]

(ii) Calculate AB. --- [2]

(iii) Calculate AE. --- [3]

(iv) Calculate the area of the pentagon. --- [4]

(b) The diagram shows a cuboid. AB = 8cm, BC = 4cm and CR = 5cm.



(i) Write down the number of planes of symmetry of this cuboid. --- [1]

(ii) Calculate the angle between the diagonal AR and the plane BCRQ. [S-18/43/Q6] --- [4]

11. Paulo and Jim each buy sacks of rice but from different shops. Paulo pays \$ 72 for sacks costing \$ m each. Jim pays \$ 72 for sacks costing \$ $(m + 0.9)$ each.

- (a) (i) Find an expression, in terms of m , for the number of sacks Paulo buys. --- [1]
(ii) Find an expression, in terms of m , for the number of sacks Jim buys. --- [1]
(Continued →)

(continued →)

11(b) Paulo buys 4 more sacks than Jim.

Write down an equation, in terms of m , and show that it

simplifies to $10m^2 + 9m - 162 = 0$ --- [4]

(c) (i) Solve $10m^2 + 9m - 162 = 0$ --- [3]

(ii) Find the number of sacks of rice that Paulo buys. --- [1]

[W-18/41/Q9]

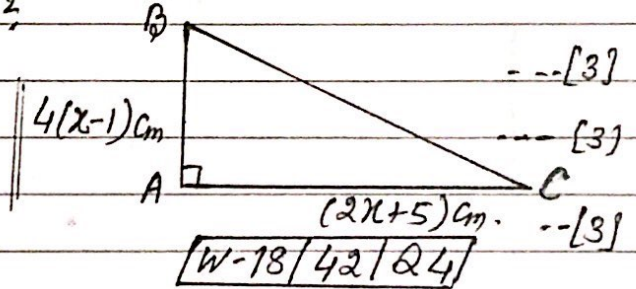
12. The diagram shows a right-angled triangle ABC.

The area of this triangle is 30 cm^2 .

(a) Show that $3x^2 + 3x - 20 = 0$ --- [3]

(b) Use factorisation to solve the equation $3x^2 + 3x - 20 = 0$ --- [3]

(c) Calculate BC. --- [3]



[W-18/42/Q4]

1. (a) The table shows the first five terms of sequence A and sequence B.

Term	1	2	3	4	5	6
Sequence A	7	13	23	37	55	
Sequence B	1	3	9	27	81	

(i) Complete the table for the 6th term of each sequence. ---[2]

(ii) Find the n th term of.

(a) sequence A. --[2]

(b) sequence B. --[2]

(b) The n th term of another sequence is $4n^2 + n + 3$

Find (i) the 2nd term. --[1]

(ii) the value of n when the n th term is 498. ---[3]

[M-19/42/Q11]

2. The table shows the first five terms of sequences A, B and C.

Sequence	1st term	2nd term	3rd term	4th term	5th term	6th term
A	0	1	4	9	16	
B	4	5	6	7	8	
C	-4	-4	-2	2	8	

(a) Complete the table. ---[3]

(b) Find an expression for the n th term of

(i) sequence A. --[2]

(ii) sequence B. --[1]

(c) Find the value of n when the n th term of sequence A is 576. --[2]

(d) (i) Find an expression for the n th term of sequence C.

Give your answer in its simplest form. --[3]

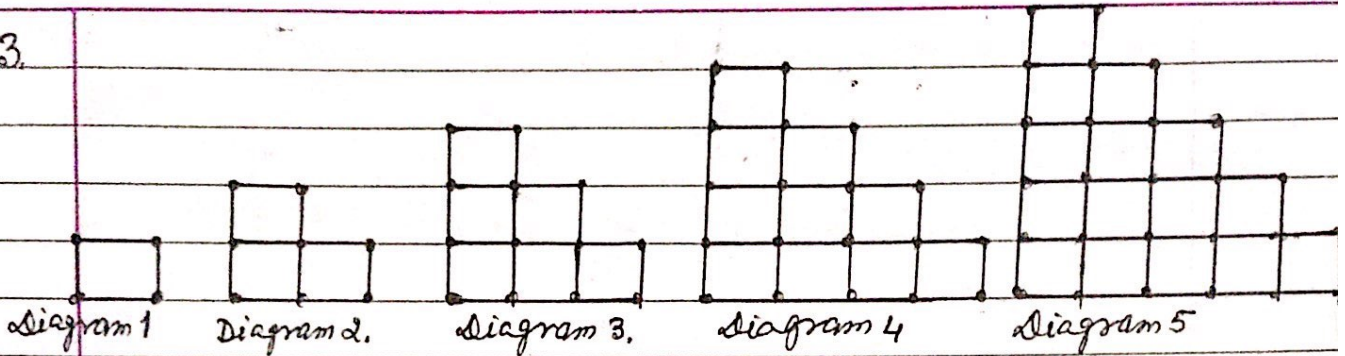
(ii) Find the value of the 30th term of sequence C. ---[2]

[M-18/42/Q11]

Sequences.



3.



The sequence of diagrams above is made up of small lines and dots.

(a) Complete the table.

---[4]

	Diagram 1	Diagram 2	Diagram 3	Diagram 4	Diagram 5	Diagram 6
Number of small lines	4	10	18	28		
Number of dots	4	8	13	19		

(b) For diagram n find an expression, in terms of n , for the number of small lines. ---[2]

(c) Diagram n has 10300 small lines.

Find the value of n .

---[2]

(d) The number of dots in diagram n is $an^2 + bn + 1$.

Find the value of a and value of b .

[5-19/42/Q11]

---[2]

4(a)

19, 15, 11, 7, ----

(i) Write down the next two terms of the sequence. ---[2]

(ii) Find the n th term of this sequence. ---[2]

(iii) Find the value of n when the n th term is -65 . ---[2]

(b) Another sequence has n th term $2n^2 + 5n - 15$.

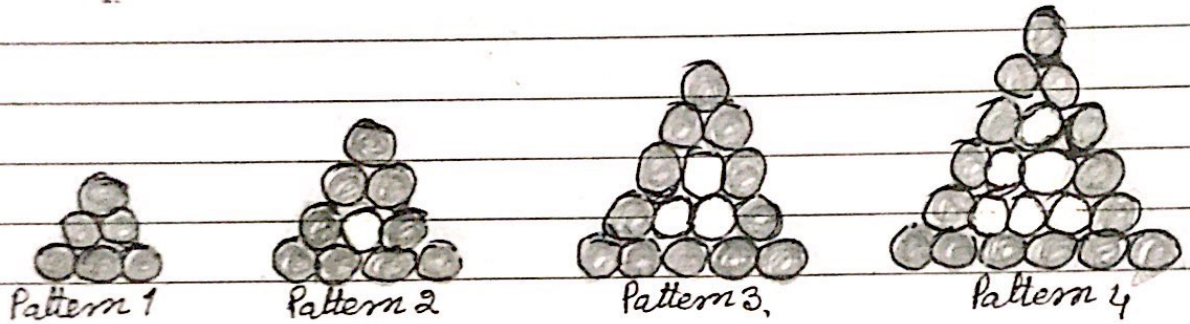
Find the difference between the 4th term and the 5th term of this sequence.

[5-19/43/Q10]

---[2]



5.



The patterns form a sequence.

Marco makes a table to show some information about the patterns.

Pattern Number	1	2	3	4	5
Number of grey mats	6	9	12	15	
Total Number of mats	6	10	15	21	

(a) Complete the table for Pattern 5. ---[2]

(b) Find an expression, in terms of n , for the number of grey mats in Pattern n . --[2]

(c) Marco makes a pattern with 24 grey mats. Find the total number of mats in this pattern. --[2]

(d) Marco needs a total of 6 mats to make the first pattern.

He needs a total of 16 mats to make the first two patterns.

He needs a total of $\frac{1}{6}n^3 + an^2 + bn$ mats to make the first n patterns.

Find the value of a and the value of b . [5-18/41/Q12] [6]

6(a) Find the next term and the n th term of this sequence. ---[3]

$$\frac{3}{5}, \frac{4}{7}, \frac{5}{9}, \frac{6}{11}, \frac{7}{13}, \dots$$

(b) Find the n th term of each sequence.

(i) $-1, -3, -5, -7, -9, \dots$ ---[2]

(ii) $2, 9, 28, 65, 126, \dots$...[2]

[W-18/43/Q10]



SP-20/04/Q3

1. $f(x) = \frac{20}{x} + x, x \neq 0$

(a) Complete the table:

x	-10	-8	-5	-2	-1.6	1.6	2	5	8	10
f(x)	-12	-10.5	-9	-12	-14.1	14.1	12			12

(b) On the grid, draw the graph of $y = f(x)$,
for $-10 \leq x \leq -1.6$ and $1.6 \leq x \leq 10$

(c) Using your graph,
Solve the equation $f(x) = 11$.

(d) k is a prime number and
 $f(x) = k$ has no solution.
Find the possible values
of k .

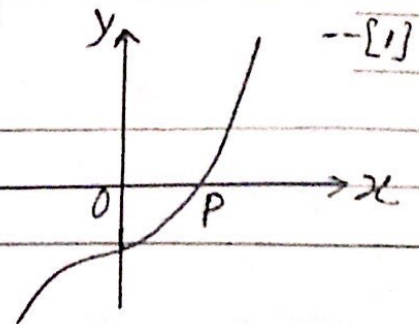
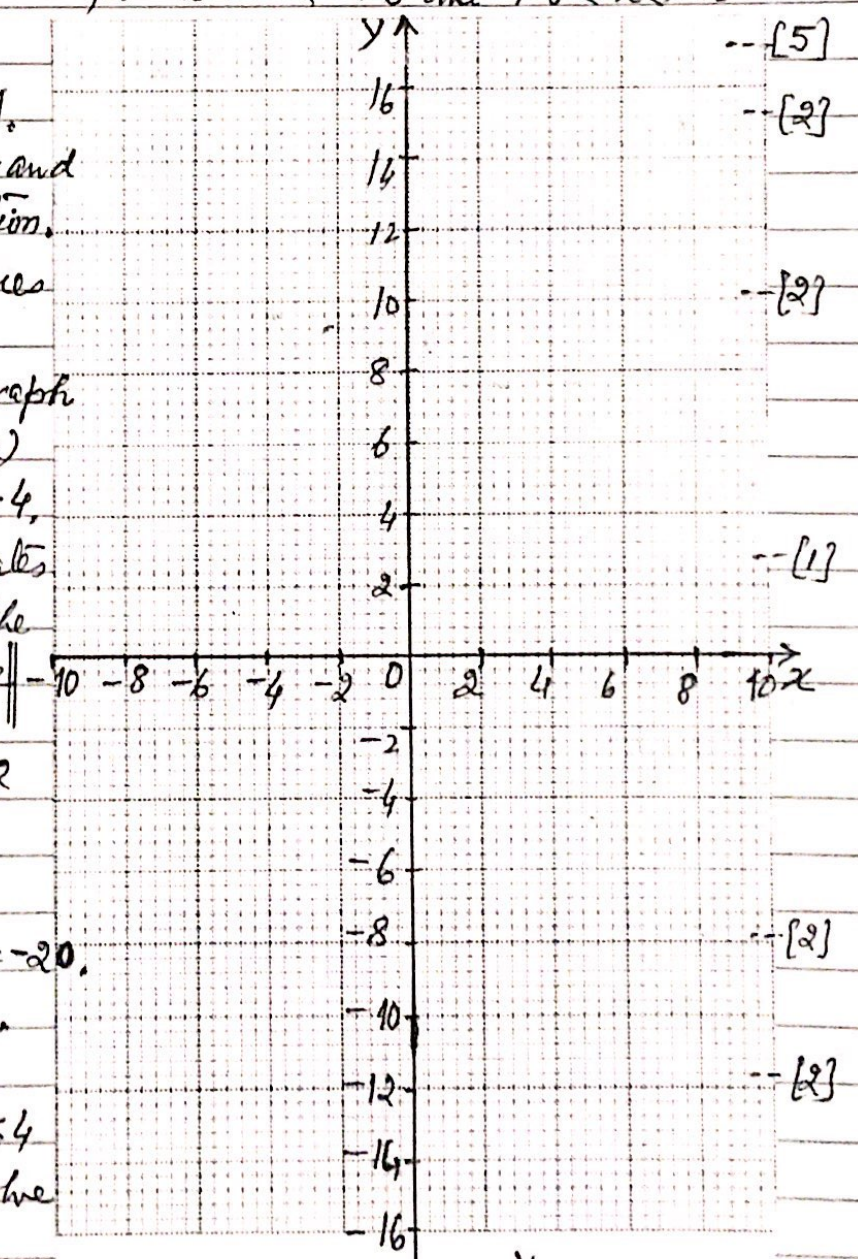
(e) The gradient of the graph
of $y = f(x)$ at the point $(2, 12)$ is -4 .
Write down the coordinates
of the other point on the
graph of $y = f(x)$ where
the gradient is -4 .

(f)(i) The equation $f(x) = x^2$
can be written as
 $x^3 + px^2 + q = 0$
Show that $p = -1$ and $q = -20$.

(ii) On the grid opposite,
draw the graph of
 $y = x^2$ for $-4 \leq x \leq 4$

(iii) Using your graphs, solve
the equation,
 $x^3 - x^2 - 20 = 0$

(iv) The diagram shows a sketch of the graph
of $y = x^3 - x^2 - 20$.
 P is the point $(n, 0)$, write down the value of n .





2. $f(x) = 2x + 1$ $g(x) = x^2 + 4$ $h(x) = 2^x$

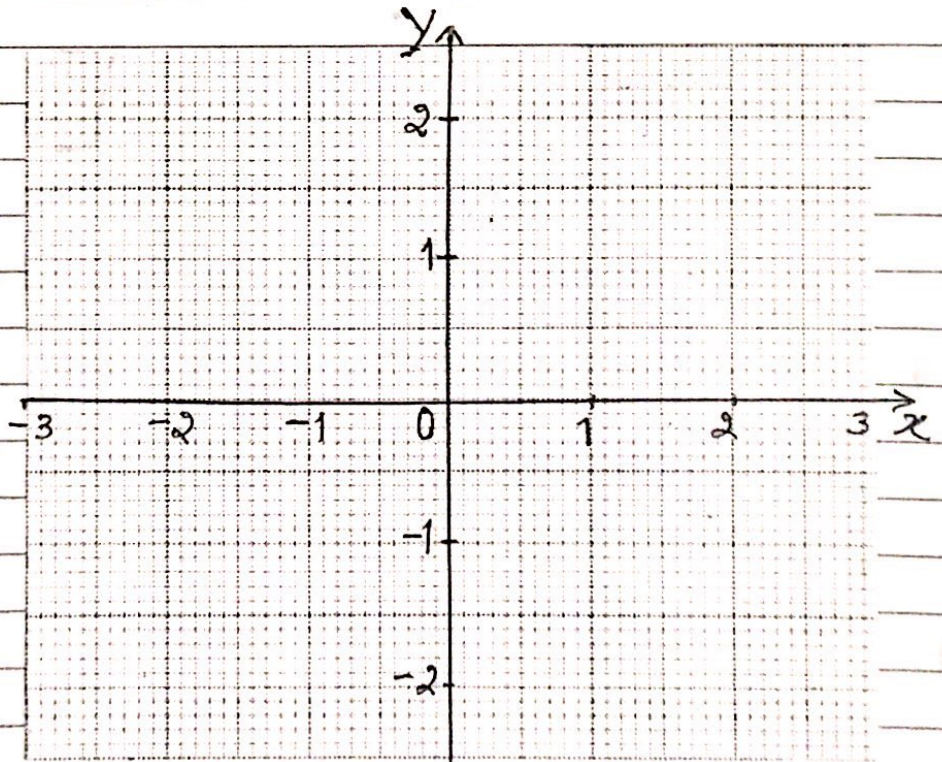
- (a) Solve the equation $f(x) = g(1)$ --- [2]
- (b) Find $f^{-1}(x)$ --- [2]
- (c) Find $gf(x)$ in its simplest form. --- [3]
- (d) $\frac{1}{h(x)} = 2^{kx}$, write down the value of k . --- [1]

[SP-20/04/Q7]

3. The table shows some values for $y = \frac{3}{10}x^3 - 2x$ for $-3 \leq x \leq 3$

x	-3	-2	-1.5	-1	0	1	1.5	2	3
y			2	1.7	0		-2	-1.6	

- (a) Complete the table. --- [3]
- (b) On the grid, draw the graph of $y = \frac{3}{10}x^3 - 2x$ for $-3 \leq x \leq 3$



- (c) On the grid opposite, draw a suitable straight line to solve the equation $\frac{3}{10}x^3 - 2x = \frac{1}{2}(1-x)$ for $-3 \leq x \leq 3$. --- [4]
- (d) For $-3 \leq x \leq 3$, the equation $\frac{3}{10}x^3 - 2x = 1$ has n solutions. Write down the value of n . --- [1]

[M-19/42/Q5]



4. $f(x) = \frac{3}{x+2}, x \neq -2$ $g(x) = 8x - 5$ $h(x) = x^2 + 6$

- (a) Work out $g(\frac{1}{4})$ --- [1]
- (b) Work out $ff(2)$ --- [2]
- (c) Find $gg(x)$, giving your answer in its simplest form. --- [2]
- (d) Find $g^{-1}(x)$ --- [2]
- (e) Write $g(x) - f(x)$ as a single fraction in its simplest form. --- [3]
- (f) (i) Show that $hg(x) = 19$ simplifies to $16x^2 - 20x + 3 = 0$ --- [3]
- (ii) Use the quadratic formula to solve $16x^2 - 20x + 3 = 0$
 show all your working and give your answers correct to
 2 decimal places. [M-19/42/Q8] --- [4]

5. The table shows some values for $y = 2x + \frac{1}{x} - 3$ for $0.125 \leq x \leq 3$

x	0.125	0.25	0.375	0.5	0.75	1	1.5	2	2.5	3
y	5.25	1.5	0.42			0	0.67	1.5		3.33

- (a) Complete the table. --- [3]
- (b) on the grid, draw the graph of $y = 2x + \frac{1}{x} - 3$ for $0.125 \leq x \leq 3$. --- [4]

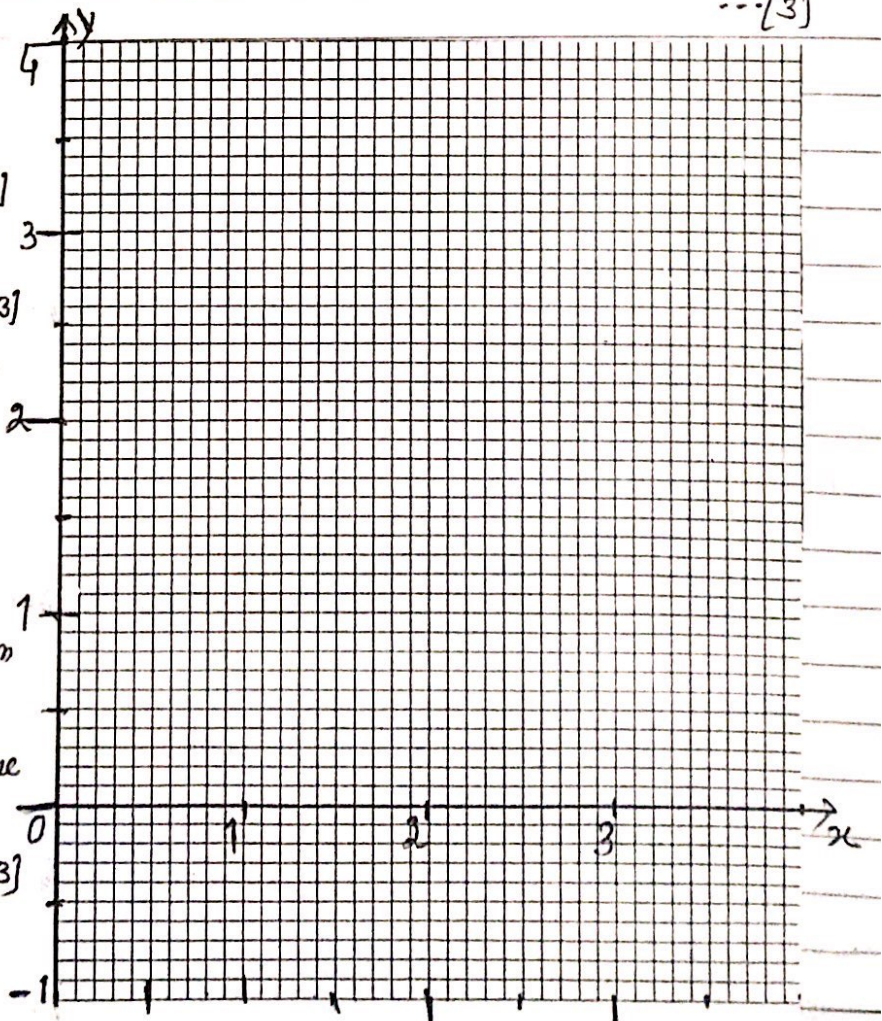
(c) use graph to solve $2x + \frac{1}{x} - 3 \geq 2$ --- [3]

(d) The equation, $\frac{1}{x} = 7 - 3x$ can be solved using your graph in part (b) and a straight line.

(i) write down the equation of this straight line.

(ii) draw this straight line and solve the equation, $\frac{1}{x} = 7 - 3x$ --- [3]

[M-18/42/Q3]





6. The table shows some values for $y = x^3 + 3x^2 + 2$

x	-3.5	-3	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5
y	-4.1		5.1	6	5.4	4	2.6		2.9		12.1

(a) Complete the table. ---[3]

(b) On the grid, draw the graph of $y = x^3 + 3x^2 + 2$ for $-3.5 \leq x \leq 1.5$. ---[4]

(c) Use your graph to solve the equation.

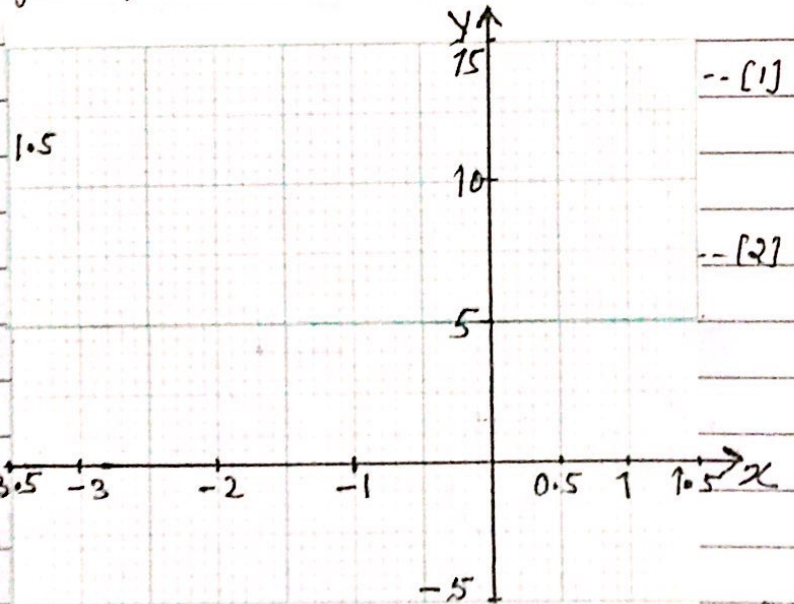
$$x^3 + 3x^2 + 2 = 0; -3.5 \leq x \leq 1.5$$

(d) By drawing a suitable line, solve the equation.

$$x^3 + 3x^2 + 2x + 2 = 0$$

for $-3.5 \leq x \leq 1.5$

(e) For $-3.5 \leq x \leq 1.5$, the equation $x^3 + 3x^2 + 2 = k$ has three solutions and k is an integer. Write down a possible value of k .



S-19/41/Q2

7. $f(x) = 7x - 2$ $g(x) = x^2 + 1$ $h(x) = 3^{2x}$

(a) Find $gh(2)$ ---[2]

(b) Find $f^{-1}(x)$ ---[2]

(c) $g(g(x)) = ax^4 + bx^2 + c$

Find the values of a, b and c . ---[3]

(d) Find x when $h(f(x)) = 81$

S-19/41/Q9 ---[3]

8. The table shows some values of $y = \frac{1}{2x} - \frac{x}{4}$ for $0.15 \leq x \leq 3.5$

x	0.15	0.2	0.5	1	1.5	2	2.5	3	3.5
y	3.30		0.88		-0.04		-0.43	-0.58	-0.73

(a) Complete the table. ---[3]

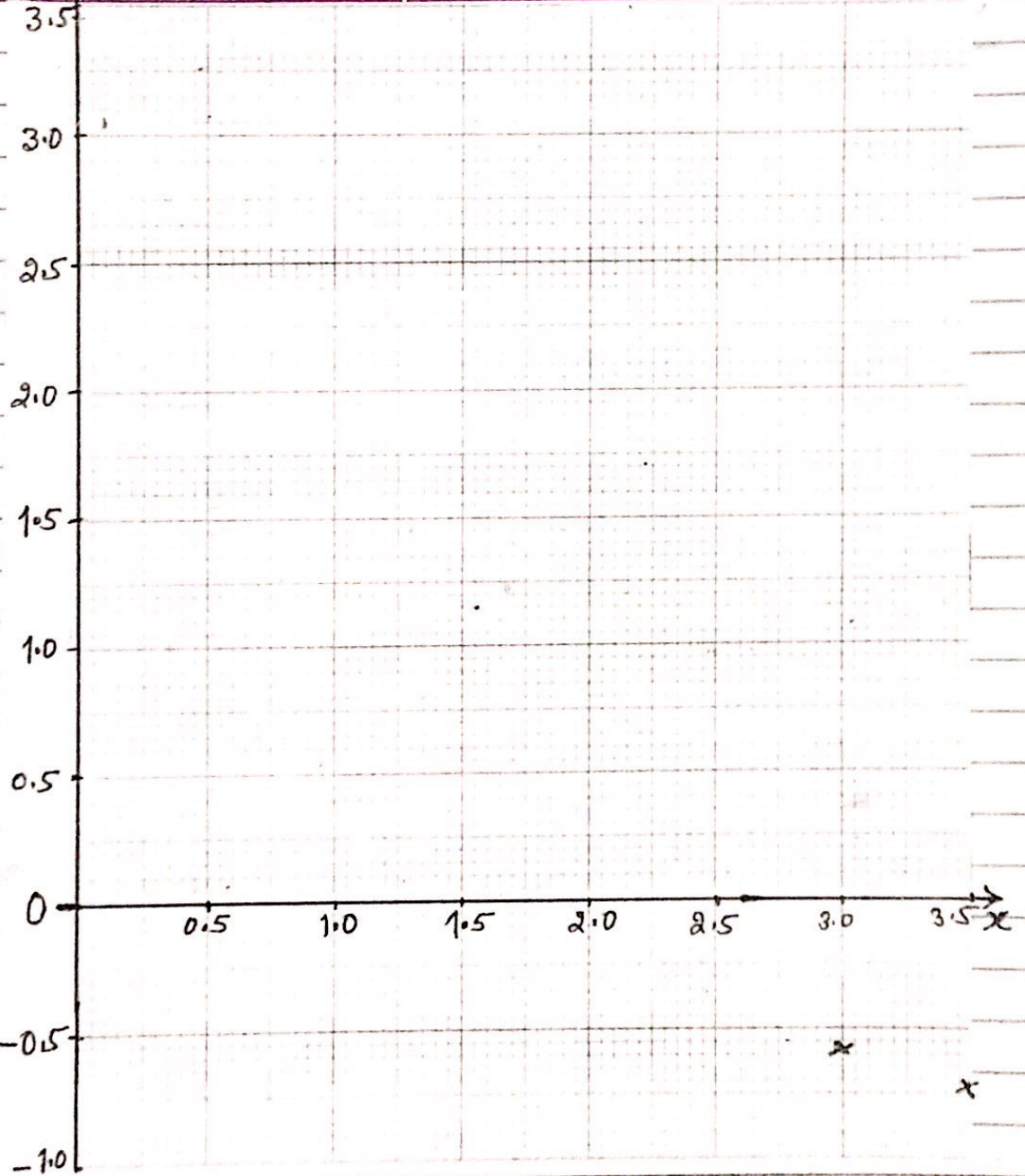
(b) On the grid, draw the graph of $y = \frac{1}{2x} - \frac{x}{4}$ for $0.15 \leq x \leq 3.5$

The last two points have been plotted for you. ---[4]

(continued →)

(continued →) y

8(b)



- (c) Use your graph to solve the equation $\frac{1}{2x} - \frac{x}{4} = \frac{1}{2}$ for $0.15 \leq x \leq 3.5$... [1]
- (d) (i) on the grid draw the line $y = 2 - x$... [2]
 (ii) Write down the x-co-ordinates of the points where the line $y = 2 - x$ crosses the graph of $y = \frac{1}{2x} - \frac{x}{4}$ for $0.15 \leq x \leq 3.5$... [2]
- (e) Show that the graph of $y = \frac{1}{2x} - \frac{x}{4}$ can be used to find the value of $\sqrt{2}$ for $0.15 \leq x \leq 3.5$, ... [2]

S-19|42|Q5



9 The diagram shows the graph of $y = f(x)$ where $f(x) = x^2 - \frac{2}{x} - 2, x \neq 0$

(a) Use the graph to find:

(i) $f(1)$

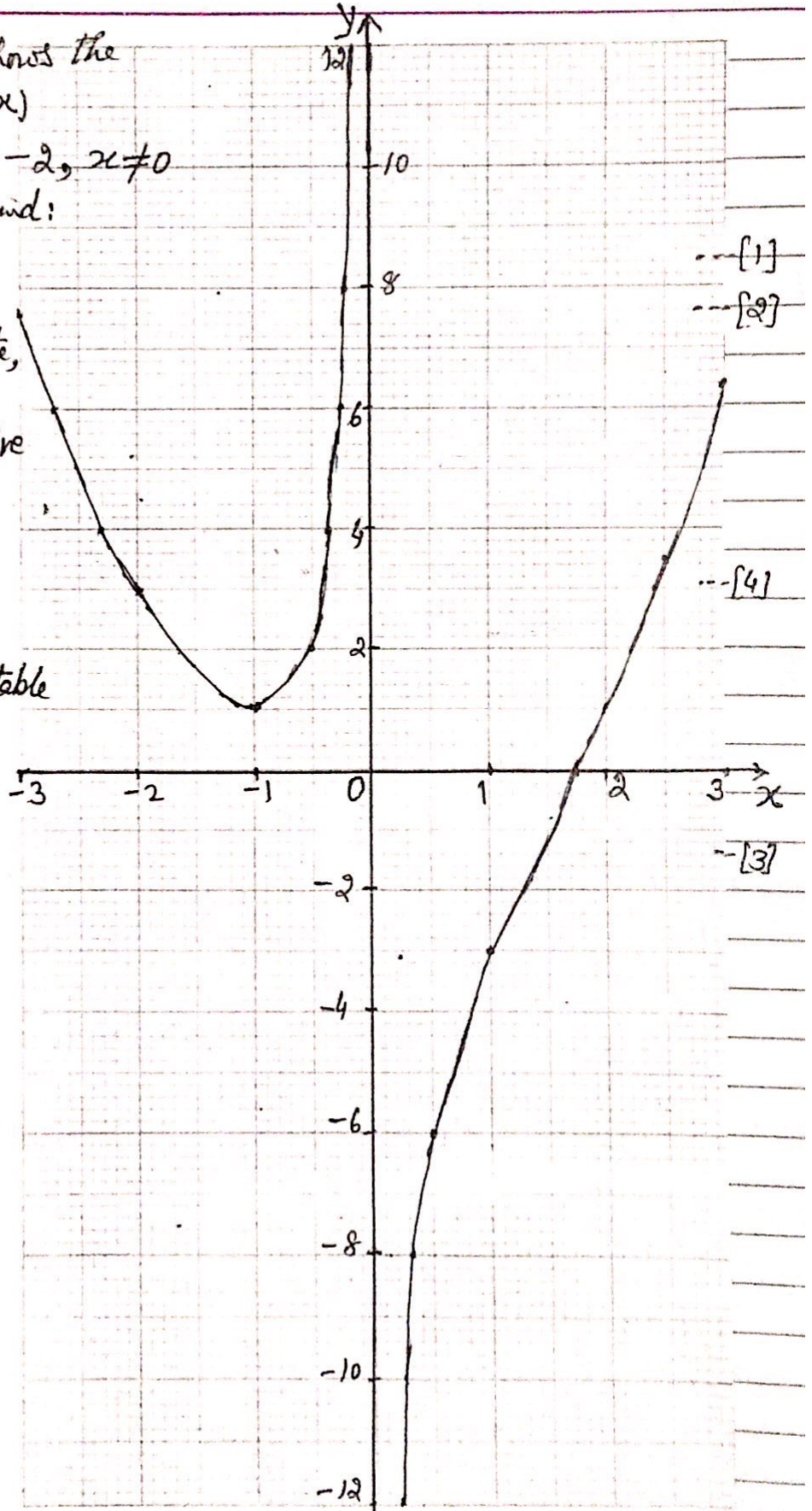
(ii) $f(f(-2))$

(b) On the grid opposite, draw a suitable straight line to solve the equation

$$x^2 - \frac{2}{x} - 7 = -3x$$

for $-3 \leq x \leq 3$

(c) By drawing a suitable tangent, find an estimate of the gradient of the curve at $x = -2$



(Continued →)



(Continued →)

(d) (i) Complete the table for $y = g(x)$ where $g(x) = 2^{-x}$ for $-3 \leq x \leq 3$

x	-3	-2	-1	0	1	2	3
y			2	1	0.5		0.125

--- [3]

(ii) On the grid opposite, draw the graph of $y = g(x)$... [3]

(iii) Use your graph to find the positive solution to the equation

$f(x) = g(x)$

[5-19/43/Q5]

--- [1]

10. The graph of $y = 10 - 8x^2$ for $-1.5 \leq x \leq 1.5$ is drawn

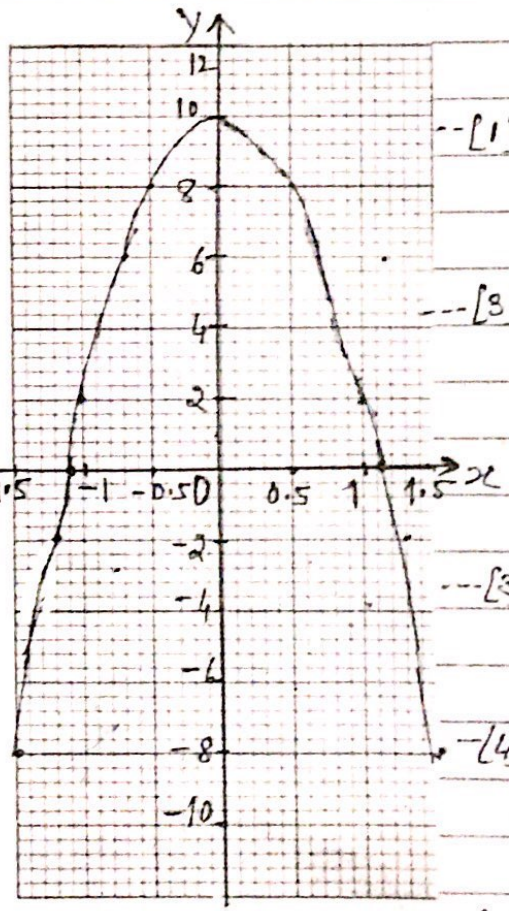
on the grid. equation of the

(a) write down the line of symmetry of the graph.

(b) on the grid opposite, draw the tangent to the curve at the point where $x = 0.5$. Find the gradient of this tangent.

(c) The table shows some values for $y = x^3 + 3x + 4$

x	-1.5	-1	-0.5	0	0.5	1	1.5
y	-3.9				5.6	8	11.9



--- [1]

--- [3]

--- [3]

--- [4]

(i) Complete the table

(ii) on the grid opposite, draw the graph of $y = x^3 + 3x + 4$ for $-1.5 \leq x \leq 1.5$

(d) Show that the values of x where the two curves intersect are the solutions to the equation $x^3 + 8x^2 + 3x - 6 = 0$

--- [1]

(e) By drawing a suitable line, solve the equation

$x^3 + 5x + 2 = 0$ for $-1.5 \leq x \leq 1.5$

--- [3]

[5-18/47/Q7]

11(a) Complete the table of values for $y = \frac{x^3}{3} - \frac{1}{2x^2}$, $x \neq 0$

x	-3	-2	-1	-0.5	-0.3	0.3	0.5	1	2	3
y	-9.1	-2.8	-0.8		-5.6	-5.5	-2.0			8.9

--- [3]

(Continued →)



11(b) On the grid draw the graph of $y = \frac{x^3}{3} - \frac{1}{2x^2}$, $x \neq 0$ $-3 \leq x \leq 3$... [5]

(c)(i) By drawing a suitable tangent, find an estimate of the gradient of the curve at $x = -2$ [3]

(ii) Write down the equation of the tangent to the curve at $x = -2$. Give your answer in the form $y = mx + c$ [2]

(d) Use your graph to solve the equations.

(i) $\frac{x^3}{3} - \frac{1}{2x^2} = 0$... [1]

(ii) $\frac{x^3}{3} - \frac{1}{2x^2} + 4 = 0$... [3]

(e) The equation $\frac{x^3}{3} - \frac{1}{2x^2} + 4 = 0$ can be written in the form, $ax^n + bx^{n-3} - 3 = 0$. Find the values of a , b , and n [3]

[S-18/42/Q6]

12. $f(x) = 8 - 3x$ $g(x) = \frac{10}{x+1}$, $x \neq -1$, $h(x) = 2^x$
(a) Find (i) $hf(\frac{2}{3})$... [2]

(ii) $gh(-2)$... [2]

(iii) $g^{-1}(x)$... [3]

(iv) $g^{-1}f(5)$... [1]

(b) Write $f(x) + g(x)$ as a single fraction in its simplest form. ... [3]



13(a) (i) $y = 2^x$

Complete the table:

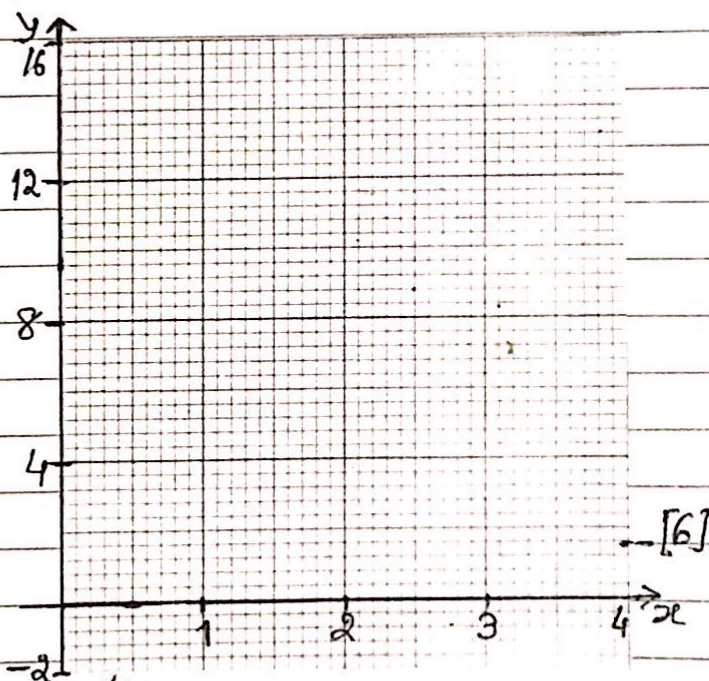
x	0	1	2	3	4	
y		2	4	8		--- [2]

(ii) $y = 14 - x^2$

Complete the table:

x	0	1	2	3	4	
y		13	10	5		--- [2]

(b) On the grid, draw the graphs of $y = 2^x$ and $y = 14 - x^2$ for $0 \leq x \leq 4$



(c) Use your graphs to solve the equations:

(i) $2^x = 12$ --- [1]

(ii) $2^x = 14 - x^2$ --- [1]

(d) (i) On the grid, draw the line from the point (4,2) that has a gradient of -4. --- [1]

(ii) Complete the statement.

This straight line is a ----- to the graph of $y = 14 - x^2$ at the point (---, ---) --- [2]

[5-18/43/22]



14.(a) $f(x) = 2x - 3$

$g(x) = x^2 + 1$

(i) find $gg(2)$ ---[2]

(ii) Find $g(x+2)$, giving your answer in its simplest form. ---[2]

(iii) Find x when $f(x) = 7$ ---[2]

(iv) Find $f^{-1}(x)$ ---[2]

(b) $h(x) = x^x, x > 0$

(i) Calculate $h(0.3)$

Give your answer correct to 2 decimal places. ---[2]

(ii) Find x when $h(x) = 256$ ---[1]

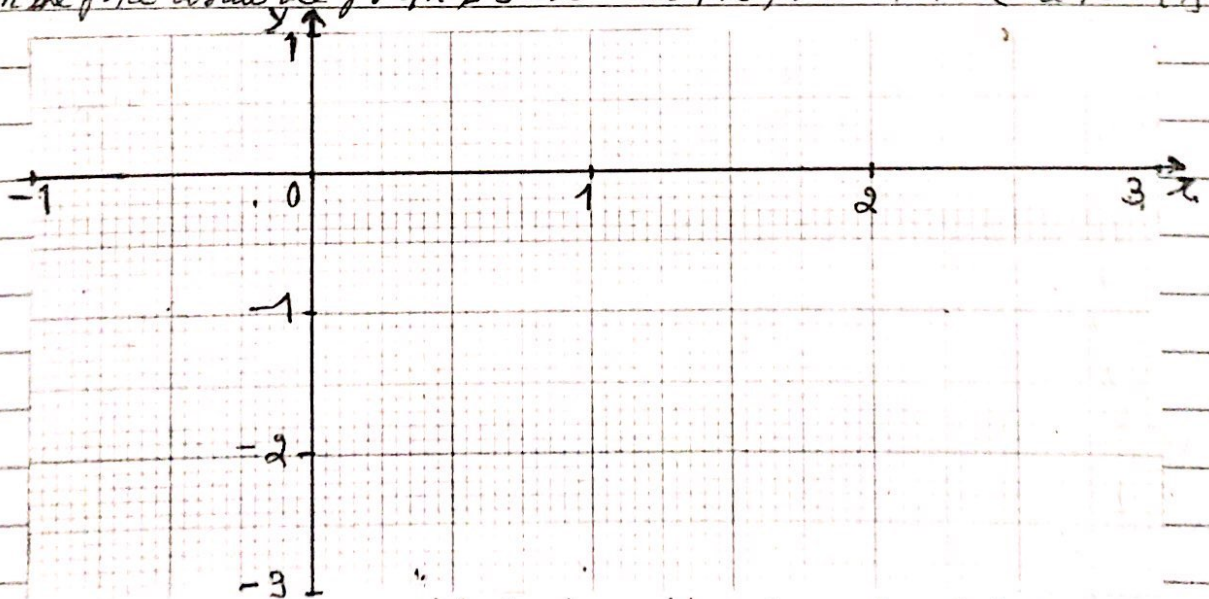
5-18/43/210

15. The table shows some values of $y = x^3 - 3x^2 + x$

x	-0.75	-0.5	-0.25	0	0.5	1	1.5	2	2.5	2.75
y	-2.9	-1.4	-0.5		-0.1	-1	-1.9		-0.6	

(a) Complete the table. ---[3]

(b) On the grid draw the graph of $y = x^3 - 3x^2 + x$ for $-0.75 \leq x \leq 2.75$ ---[4]



(c) Use your graph to complete the inequalities in x for which $y > -1$

--- $< x <$ --- and $x >$ --- ---[3]

(d) The equation $x^3 - 3x^2 + 2x - 1 = 0$ can be solved by drawing a straight line on the grid,

(i) Write down the equation of this line. ---[2]

(ii) On the grid, draw this line and use it to solve the equation,

$x^3 - 3x^2 + 2x - 1 = 0$

(Continued →) ---[3]

(Continued →)

15 (e) By drawing a suitable tangent, find an estimate for the gradient of the graph of $y = x^3 - 3x^2 + x$ at $x = -0.25$... [3]
 [W-18/41/Q3]

16. The table shows some values of $y = x^3 - 3x - 1$

x	-3	-2.5	-2	-1.5	-1	0	1	1.5	2	2.5	3
y	-19	-9.1		0.1	1	-1	-3	-2.1	1	7.1	

(a) Complete the table of values. ... [2]

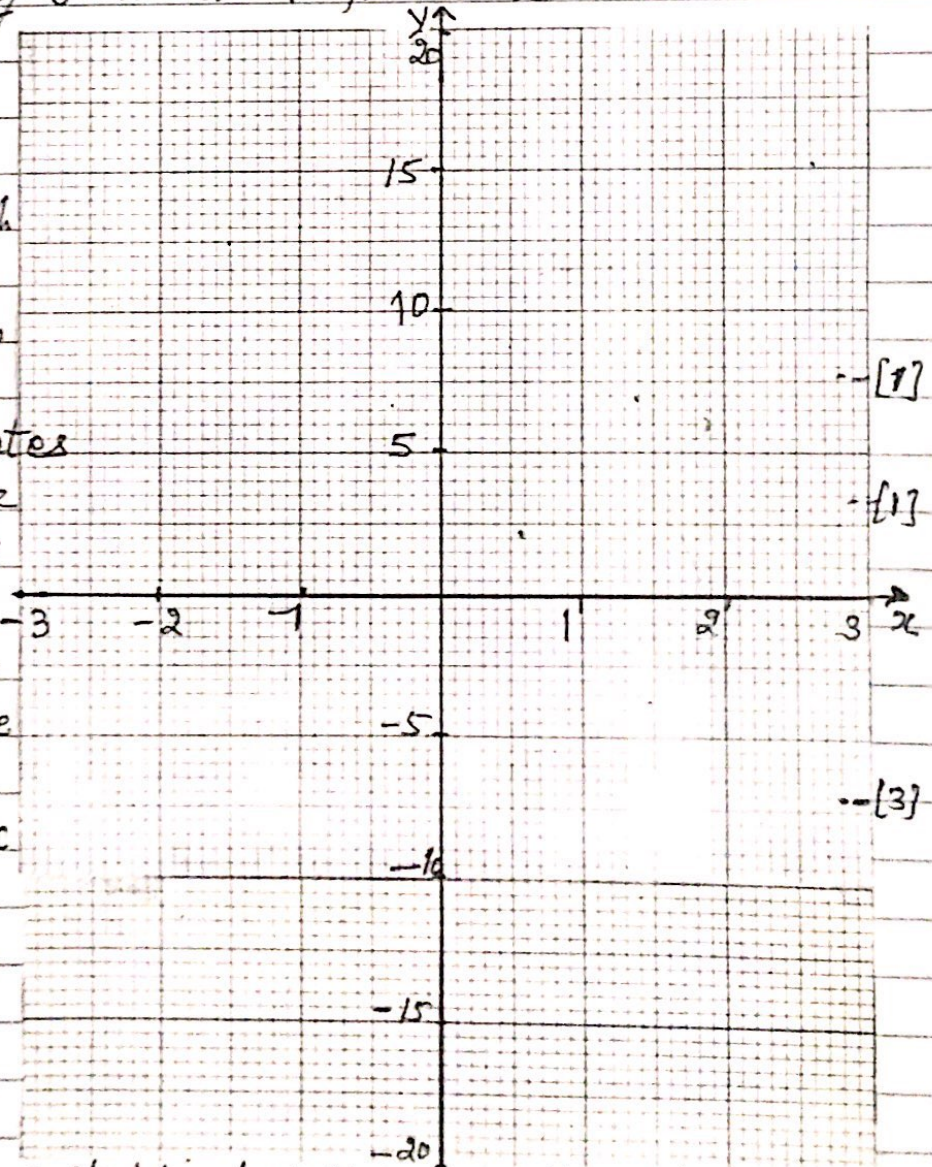
(b) Draw the graph of $y = x^3 - 3x - 1$ for $-3 \leq x \leq 3$... [4]

(c) A straight line through $(0, -17)$ is tangent to the graph of $y = x^3 - 3x - 1$.

(i) On the grid, draw this tangent. ... [1]

(ii) Find the coordinates of the point where the tangent meets your graph. ... [1]

(iii) Find the equation of the tangent. Give your answer in the form $y = mx + c$ [3]



(d) By drawing a suitable straight line on the grid, solve the equation $x^3 - 6x - 3 = 0$... [4]

[W-18/42/Q5]

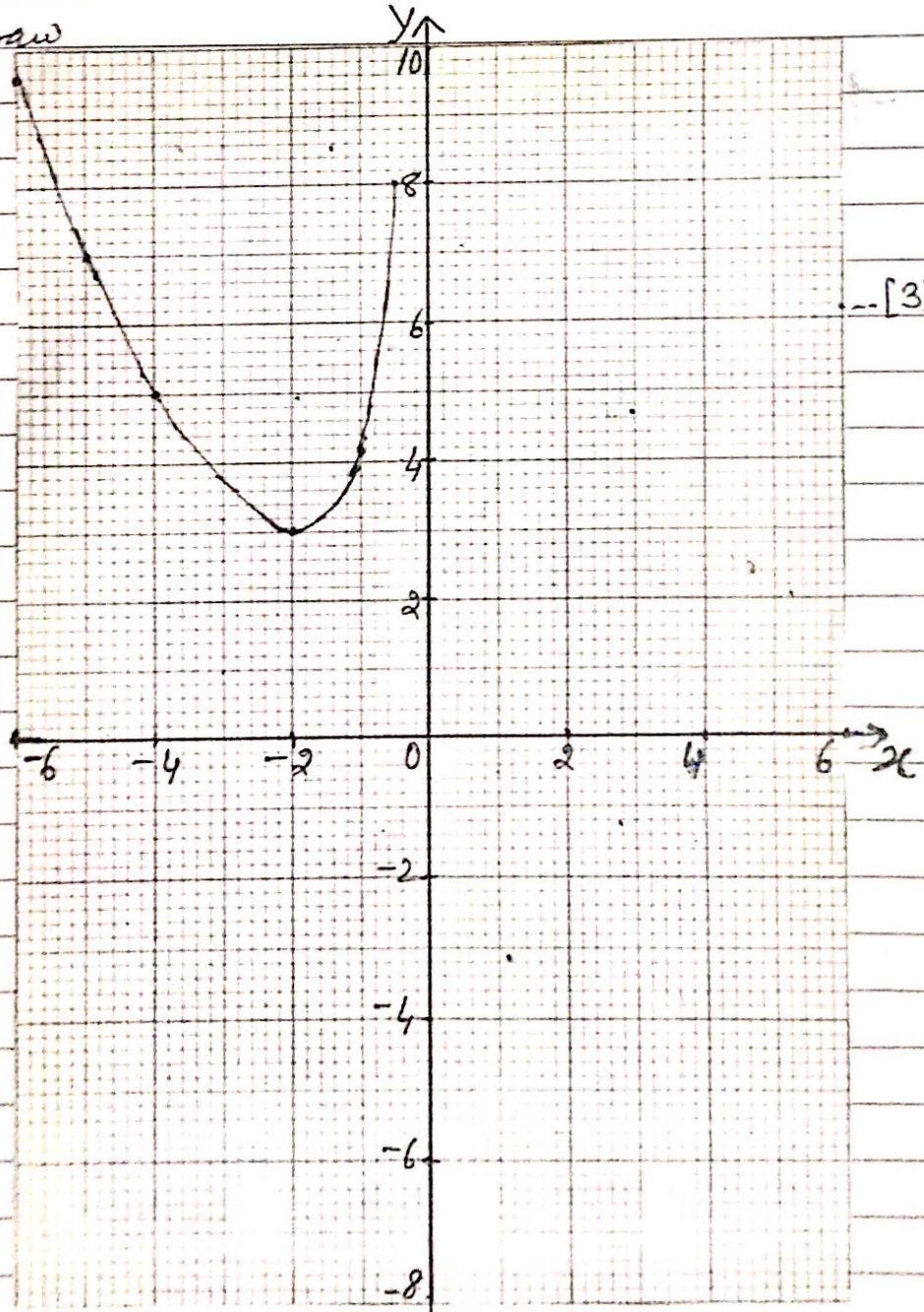
17. $f(x) = \frac{x^2}{4} - \frac{4}{x}, x \neq 0$

(a) Complete the table for $f(x)$.

x	0.5	1	2	3	4	5	6	
$f(x)$	-7.9	-3.8		0.9		5.5	8.3	... [2]

(b) The graph of $y = f(x)$ for $-6 \leq x \leq -0.5$ is drawn on the grid.

on the grid draw
the graph of
 $y = f(x)$
for $0.5 \leq x \leq 6$



(c) By drawing a suitable tangent, estimate the gradient of the graph of $y = f(x)$ at the point $(-4, 5)$ --- [3]

(continued →)

(Continued →)

17(d) $g(x) = \frac{9}{x}, x \neq 0$

Complete the table for $g(x)$

x	-4	-3	-2	-1	1	2	3	4	
$g(x)$	-2.3		-4.5	-9	9	4.5		2.3	...[1]

(e) On the same grid, draw the graph of $y = g(x)$ $-4 \leq x \leq -1$ and $1 \leq x \leq 4$...[1]

(f) (i) Use your graph to find the value of x when $f(x) = g(x)$...[1]

(ii) Write down an inequality to show the positive values of x for which $f(x) > g(x)$[1]

(g) The exact answer to part (f) (i) is $\sqrt[3]{k}$
Use algebra to find the value of k[2]

[W-18/43/R4]

18. $f(x) = 3x + 4$ $g(x) = 2x - 1$ $h(x) = 3^x$

(a) Find $g(\frac{1}{2})$...[1]

(b) Find $fh(-1)$...[2]

(c) Find $f^{-1}(x)$...[2]

(d) Find $ff(x)$ in its simplest form. ...[2]

(e) Find $(f(x))^2$ in the form $ax^2 + bx + c$...[2]

(f) Find x when $h^{-1}(x) = g(2)$...[2]

[W-18/43/R9]



1. A curve has equation $y = x^3 - 6x^2 + 16$
- (a) Find the coordinates of the two turning points. --- [6]
- (b) Determine whether each of the turning points is a maximum or a minimum. --- [3]
- Give reasons for your answers. SP-20/04/Q11
2. Find the equation of the tangent to the curve at the indicated point.
- $$y = x^4 - 6x^3 + 13x^2 - 10x + 5 \quad \text{at } (0, 5)$$
3. Find the equation of tangent line to the curve $y = x^2 - 2x + 7$ which is,
- (a) Parallel to the line $2x - y + 9 = 0$
- (b) perpendicular to the line $5y - 15x = 13$
4. Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$.
5. A curve has equation. $y = x^3 - 3x + 3$
- (a) Find the coordinates of the two turning points
- (b) Determine whether each of the turning points is a maximum or a minimum.
6. (a) Find the coordinates of the turning points on the curve $y = 3x^4 + 4x^3 - 12x^2 + 12$
- (b) Determine whether each of the turning points is a maximum or minimum.
- Give reasons for your answers.
7. Find the gradient of the tangent to the curve,
- $$y = 3x^4 - 4x \quad \text{at } x = 4$$
8. Find the points at which the tangent to the curve,
- $$y = x^3 - 3x^2 - 9x + 7$$
- is parallel to the x -axis.



	Exercise-1	Answers	Exercise-2
1.	(a) $x=7, y=-3$ (b) 2 (c) $x > -5$		1.(a) $y = \frac{1}{2}(80-2x)$ (i) $A = \frac{1}{2}x(80-2x) \cdot x$ $\rightarrow x^2 - 40x + A = 0 \checkmark$ (ii) $(x-30)(x-10)$ $\rightarrow 30; 10 \checkmark$ (iii) 5.86, 34.14
2.	(a) $\pm \sqrt{k-5}$ (b) $(x-5)(x+5)$ (c) $\frac{4x^2-7x-8}{x(x+1)}$ (d) 3, 4, 5, 6		(b) (i) $\frac{200}{x} - \frac{200}{x+10}$ $= \frac{2000}{x(x+10)} \checkmark$ (ii) 16 min 40s \checkmark
3.	(a) $y > x; x \geq 15; y < 50; x+y \leq 70$ (b) Four correct ruled lines and correct region indicated. (c) 189		2(a) (i) $1.991 \times 10^3 \checkmark$ (ii) $2(5-ut) \checkmark$ t^2 (b) (i) $(2x+3)(x-1) - (x+1)(x-2)$ $= 62$ $\rightarrow x^2 + 2x - 63 = 0 \checkmark$ (ii) $(x+9)(x-7) \checkmark$ (iii) 20 \checkmark
4.	(a) $x^2 + 4x - 21$ (b) (i) $59^2(3p^2 - 5q)$ (ii) $(2g+5k)(2f+3h)$ (iii) $(9k+m)(9k-m)$ (c) 5.5		3(a) 1-2 (b) (i) $(1-\lambda)(1.3-\lambda) = 0.4 \checkmark$ (ii) $10\lambda^2 - 23\lambda + 9 = 0 \checkmark$ (iii) $(5\lambda-9)(2\lambda-1) = 0$ $\lambda = \frac{9}{5} \text{ or } \lambda = \frac{1}{2} \checkmark (\text{or } 0.5)$ (iv) $1.3 - 0.5 = 0.8 \checkmark$
5.	(a) -10 (b) -1, 0, 1, 2		4 (i) a^9 (ii) $125x^3y^6$ (iii) $\frac{48}{32x^4}$
6.	(a) (i) $(2x+m)(m-3)$ (ii) $(2y-9)(2y+9)$ (iii) $(t-4)(t-2)$ (b) $x = \frac{2m}{k+1}$ (c) $x=6$ $y=-2$		5(a) (i) $\frac{x-1}{x+2} \checkmark$ (ii) (a) $\frac{x}{x+3} \times \frac{(x-1)}{x+2} = \frac{7}{15}$ $\rightarrow 4x^2 - 25x - 21 = 0 \checkmark$ (b) $(4x+3)(x-7) = 0$ $\rightarrow x = 7 \text{ and } -\frac{3}{4} \checkmark$ (c) 7 \checkmark
7.	(a) $5(m-2p^2)(m+2p^2)$ (b) $P = \frac{100A}{100+TR}$		
8.	(a) -1.5 (b) $6ab^2(2b+3a^2)$ (c) (i) $10a^5c^9$ (ii) $\frac{8a^6}{c^9}$ (d) 0.5 (e) $\frac{7x-x^2}{2(x-2)}$		



Exercise-2 (continued) / Answers

6 (i) $3m - 4(m+4) = 6m(m+4)$

$\rightarrow 6m^2 + 25m + 16 = 0$

(ii)
$$\frac{-25 \pm \sqrt{\left(\frac{25}{12}\right)^2 - \frac{16}{6}}}{12}$$

 $= -0.79 \text{ and } -3.38 \checkmark$

7 (a) (i) $243p^{10}$ (ii) $9xy^4$ (iii) $\frac{m^2}{25}$

(b) $10 \checkmark$

8 (a) $x^2 + (2x-3)^2 = 6^2$

$\rightarrow 5x^2 - 12x - 27 = 0$

(b) $\frac{12 \pm \sqrt{\left(\frac{12}{5}\right)^2 + \frac{27}{5}}}{5}$
 $x = -1.42 \text{ and } 3.82 \checkmark$

(c) $14.4 \checkmark$

(d) $39.5 \checkmark$

9 (a) 15.6 (b) (i) $(y+12)(y-7)$ (ii) 38

(c) (i) $168(m-0.75) + 207m = 100m(m-7.5)$

or $50m^2 - 225m + 63 = 0 \checkmark$

(ii) $(10m-3)(5m-21)$

$\rightarrow m = 4.2 \checkmark$

10 (a) (i) 116.5

(ii) 13.4

(iii) 10.4

(iv) 130

(b) (i) 3 (ii) 51.3

11 (a) (i) $\frac{72}{m}$ (ii) $\frac{72}{m+0.9} \checkmark$

(b) $\frac{72}{m} - \frac{72}{m+0.9} = 4$

$\Rightarrow 72(m+0.9) - 72m = 4m(m+0.9)$

$\rightarrow 10m^2 + 9m - 162 = 0 \checkmark$

(c) (i) 3.6 and $-4.5 \checkmark$

(ii) 20

12 (a) $\frac{1}{2} \times 4(x-1)(2x+5) \sin 90^\circ = 30$

$\rightarrow 2x^2 + 3x - 20 = 0 \checkmark$

(b) $(2x-5)(x+4)$

2.5 and $-4 \checkmark$

(c) $11.7 \checkmark$

Exercise-3

1 (a) (i) $77, 243$

(ii) (a) $2n^2+5$ (b) 3^{n-1}

(b) (i) 21 (ii) 11

2 (a) $25, 9, 16$

(b) (i) $(n-1)^2$

(ii) $n+3$

(c) 25

(d) (i) n^2-3n-2

(ii) 808

3 (a) $40, 54$

$26, 34$

(b) n^2+3n

(c) 100

(d) $a = \frac{1}{2}, b = \frac{5}{2}$

4 (a) (i) $3, -1$

(ii) $23-4n$

(iii) 22

(b) 23

5 (a) $\frac{18}{28}$

(b) $2n+3$ (c) 45 (d) $a = \frac{3}{2}, b = \frac{13}{3}$

6 (a) $\frac{8}{15}; \frac{n+2}{2n+3}$

(b) (i) $1-2n$

(ii) n^2+1



Exercise-4

Answers

- 1(a) 9, 10.5
 (b) Fully correct curve.
 (c) 2.1 to 2.6
 8.5 to 9
 (d) 2, 3, 5, 7
 (e) (-2, -12)
 (f) (i) $20 + x^2 = x^3$
 or $x^3 - x^2 - 20 = 0$

- (ii) Fully correct curve $y = x^2$
 (iii) 3.1 to 3.6
 (iv) 3.0 to 3.1

- 2(a) 2
 (b) $\frac{x-1}{2}$
 (c) $4x^2 + 4x + 5$
 (d) -1

- 3(a) -2.1, 1.6, -1.7, 2.1
 (b) Fully correct curve
 (c) line $y = \frac{1}{2}(1-x)$ ruled
 -2.15 to -2.01
 -0.45 to -0.2
 2.25 to 2.45

(d) number of intersections of their curve and line $y=1$

- 4(a) -3 (b) $\frac{12}{11}$ (c) $64x - 45$
 (d) $\frac{x+5}{8}$ (e) $\frac{8x^2 + 11x - 13}{x+2}$
 (f) (i) $(8x-5)^2 + 6 = 19$
 $\rightarrow 16x^2 - 20x + 3 = 0$
 (ii) 0.17 and 1.08

- 5(a) 0, -0.17, 2.4
 (b) Fully correct smooth curve.
 (c) $x \leq 0.17$ to 0.25
 and $x \geq 2.25$ to 2.3
 (d) (i) $y = 4 - x$
 (ii) Correct ruled line
 0.125 to 0.2 and 2.15 to 2.2

- 6(a) 2, 2, 6
 (b) Correct graph
 (c) -3.3 to -3.1
 (d) $y = -2x$ ruled
 -2.6 to -2.45
 (e) 3 or 4 or 5

- 7(a) 82 (b) $\frac{x+2}{7}$ (c) $a=1, b=2, c=2$
 (d) $\frac{6}{7}$

- 8(a) 2.45 ; 0.25 ; -0.25
 (b) Fully correct smooth curve.
 (c) 0.7 to 0.8
 (d) (i) Correct ruled line
 (ii) both intersecting of their (b) and their (d) (i)
 (e) Substitute $x = \sqrt{2}$ into $\frac{1}{2x} - \frac{x}{4}$
 or identifies $y=0$

- 9(a) (i) -3 (ii) 6.2 to 6.4
 (b) $y = 5 - 3x$ ruled
 -0.3 to -0.2
 1.65 to 1.8
 (c) tangent ruled at $x = -2$
 -4.5 to -2.5
 (d) (i) 8, 4, 0.25
 (ii) Correct graph
 (iii) 1.8 to 1.9



Exercise-4 (continued) Answers

10. (a) $x=0$
 (b) Tangent ruled at $x=0.5$
 -9 to -6.5
 (c) (i) $0, 2.4, 4$
 (ii) Correct smooth curve
 (d) $x^3+3x+4=10-8x^2$ and correctly completed.
 (e) line $y=-2x+2$ drawn and -0.45 to -0.35 .

11. (a) $-2, -0.2, 2.5$
 (b) Fully correct curve
 (c) (i) correct tangent and $3 \leq \text{grad} \leq 5$
 (ii) $y = \text{their (c) (i) } x + \text{their } y\text{-intercept.}$

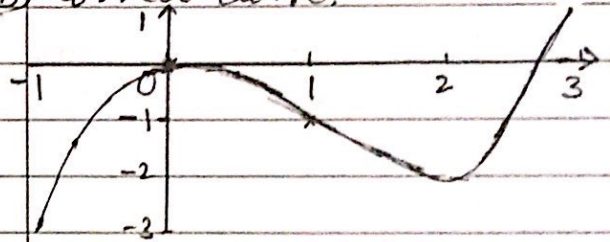
- (d) (i) 1.05 to 1.25
 (ii) -2.3 to -2.2
 -0.4 to -0.3
 0.3 to 0.4
 (e) $a=2, b=24$ and $n=5$

12. (a) (i) 1 (ii) 8 (iii) $\frac{10-x}{x}$
 (iv) 5
 (b)
$$\frac{-3x^2+5x+18}{x+1}$$

13. (a) (i) $1, \dots, 16.$
 (ii) $14, \dots, -2$
 (b) Fully correct smooth curves.
 (c) (i) 3.5 to 3.7 (ii) 2.65 to 2.8
 (d) (i) correct line
 (ii) tangent -- at $(2, 10)$

14. (a) (i) 26 (ii) x^2+4x+5
 (iii) 5 (iv) $\frac{x+3}{2}$
 (b) (i) 0.70 (ii) 4

15. (a) $0, -2, 0.9$
 (b) Correct curve.



- (c) -0.45 to $-0.35 < x < 1$
 and $x > 2.35$ to 2.45

- (d) (i) $y=1-x$
 (ii) Correct ruled line and 2.25 to 2.4

- (e) correct tangent and 1.7 to 3.7

16. (a) $-3, 17$

- (b) Fully correct curve.

- (c) (i) Correct ruled tangent to the curve through $(0, -17)$

- (ii) $(1.7$ to $2.2, -1$ to $2.5)$

- (iii) $y=9x-17$

- (d) $y=3x+2$ ruled correctly and -2.2 to $-2.1,$
 -0.6 to -0.4
 2.6 to 2.8

17. (a) $-1, 3$

- (b) correct graph

- (c) correct ruled tangent and $-2 \leq \text{gradient} \leq -1.5$

- (d) $-3, 3$

- (e) correct graph.

(continued →)



Exercise 4 (continued) / Answers

17(f) (i) 3.6 to 3.85

(ii) $x >$ their $f(i)$

(g) $\frac{x^2}{4} = \frac{9}{x} + \frac{4}{x}$ or $\frac{x^3}{4} - 4 = 9$

18(a) 0 (b) 5 (c) $\frac{x+1}{2}$

(d) $9x+16$

(e) $9x^2+24x+16$

(f) 27

Exercise - 5

1(a) (0, 16), (4, -16)

(b) (0, 16) maximum

(4, -16) minimum.

with correct reason.

2. $10x + y = 5$

3. (a) $y - 2x = 3$

(b) $36y + 12x = 227$

4. (2, -9)

5. (a) (1, 1), (-1, 5)

(b) minimum at $x=1$

and Max at $x=-1$.

6(a) $x=0$, $x=1$, $x=2$

(b) Max at $x=0$

Min at $x=2$ andneither max nor minimum
at $x=1$ (point of inflexion).

7. 764.

8. (3, -20), (-1, 12)

