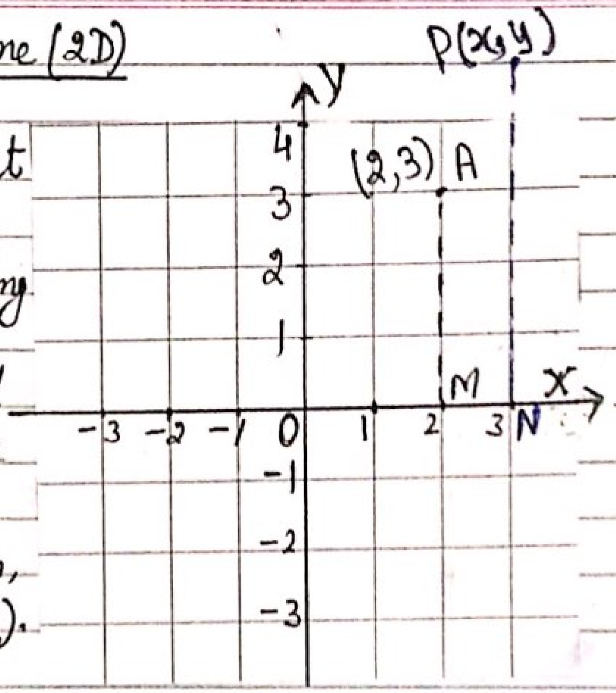
IG-Maths 0580

Coordinate Geometry Notes

(Suresh Goel)

[1] § Coordinates of a point in plane (2D)

To fix the position of a point in a plane, consider two perpendicular lines intersecting at a point 'O' called origin.
The horizontal line OX is called x-axis, and the vertical line OY is called y-axis.
If OM = 2 and AM = 3 Then,
The coordinates of point A are (2,3).



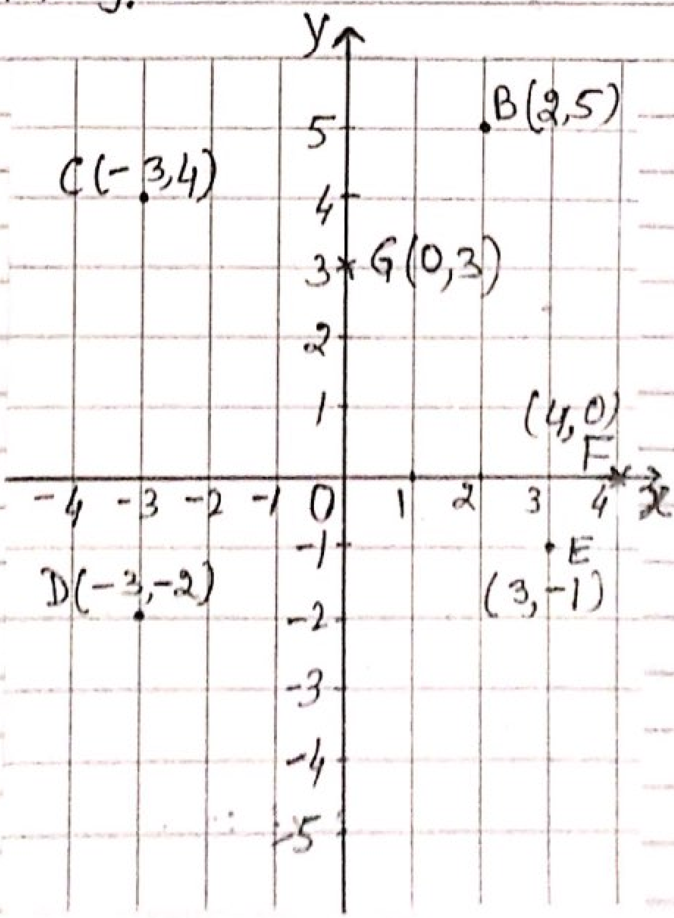
In general the coordinates of any point P are (x, y) where ON = x and PN = y.

Read the coordinates of the Point.
B(2,5)
C(−3,4)
D(−3,−2)
E (3,−1)

A point on x-axis F(4,0)

A point on y-axis G(0,3).

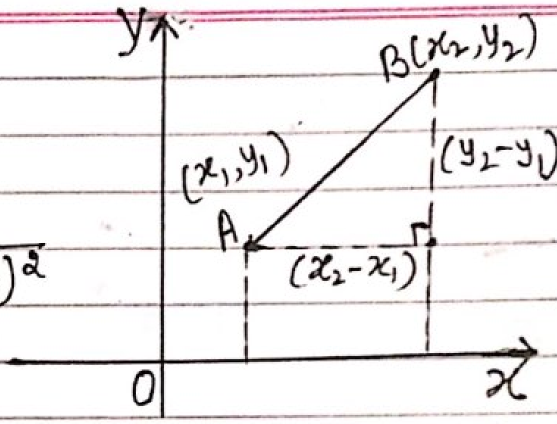
Coordinates of Origin O(0,0).



[2] § Distance Formula:

distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$.

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$



Example 1. The point A has co-ordinates $(-4,6)$ and the point B has co-ordinates $(7,-2)$. Calculate the length of line AB.

[S-15|21|Q8] --- [3]

Solution: distance $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Given $A(-4,6)$, $B(7,-2)$

$$\therefore AB = \sqrt{(7-(-4))^2+((-2)-6)^2}$$

$$= \sqrt{11^2+(-8)^2}$$

$$= \sqrt{121+64} = \sqrt{185} = 13.6$$

Example 2: A line join the points $A(-2,-5)$ and $B(4,13)$. Calculate the length AB.

--- [3] [S-16|42|Q9(a)]

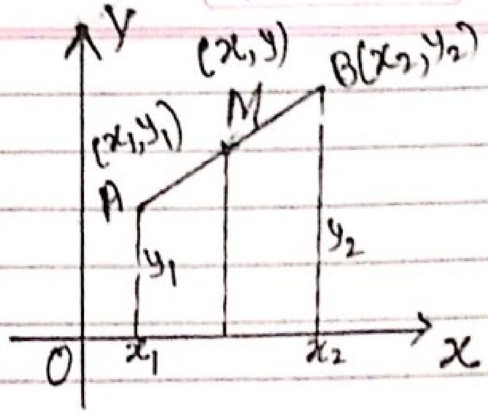
Solution $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$= \sqrt{(4-(-2))^2+(13-(-5))^2}$$

$$= \sqrt{6^2+18^2} = \sqrt{360} = 19$$

Coordinate Geometry

[3]§ **Mid point formula:**



Given the coordinates of the end points of a line AB.

$A(x_1,y_1)$ $B(x_2,y_2)$

Then the the coordinates of the mid point $M(x,y)$ of AB. are given by:

$$M\left(\frac{x_1+x_2}{2},\ \frac{y_1+y_2}{2}\right)$$

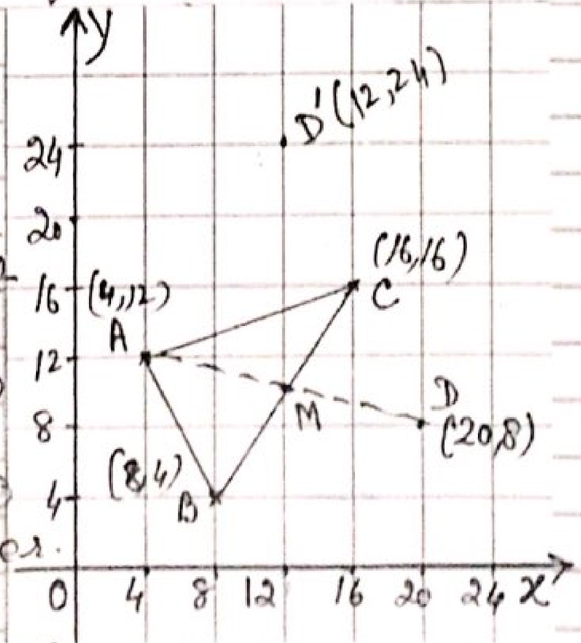
Example3: A(5, 23) and B(−2, 2) are two points, --- [2]
Find the coordinates of the mid point of line AB.

Solution: Mid point of AB $= \left(\frac{x_1+x_2}{2},\ \frac{y_1+y_2}{2}\right)$

$$= \left(\frac{5+(-2)}{2},\ \frac{23+2}{2}\right)$$

$$= \left(\frac{3}{2},\ \frac{25}{2}\right) = (1.5,\ 12.5)$$

Example 4 The vertices of a parallelogram are at --- [2]
A(4,12), B(8,4) and C(16,16).
Write down the co-ordinates of two possible positions of the fourth Vertex 'D'.



Solution: Let BC is diagonal of the parallelogram Mid point of BC $= \left(\frac{8+16}{2},\ \frac{4+16}{2}\right)$

$= (12, 10)$ — ①

Let the coord. of D are (x, y)

∴ Mid point of AD $= \left(\frac{4+x}{2},\ \frac{12+y}{2}\right)$ — ②

diagonals of a parallelogram bisects eachother.

fr ① & ② $\frac{4+x}{2} = 12$ & $\frac{12+y}{2} = 10$

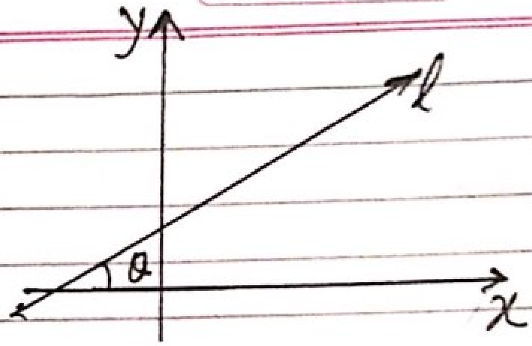
$\Rightarrow x = 20, y = 8$ D(20,8)

Similarly By taking AC as diagonal D'(12, 24)

[4] § Inclination of a line:

In a line 'l' is inclined at an angle 'θ' with the positive direction of X-axis, Then

Inclination of line $l = \theta$.



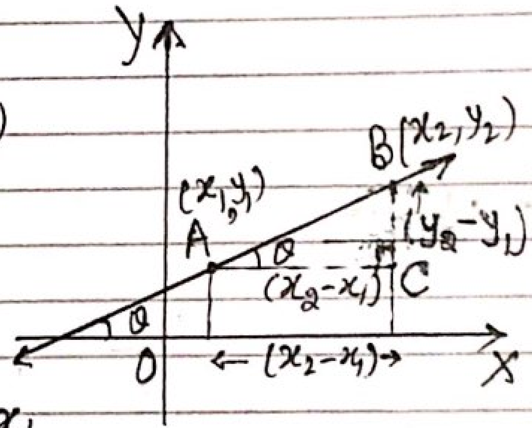
[4] § Gradient (or slope) of line 'l' (denoted by 'm'):

Give a line 'l' passing through two points $A(x_1, y_1)$ and $B(x_2, y_2)$

Gradient = $\tan\theta$ ✓

or $m = \dfrac{\text{rise}}{\text{run}} = \dfrac{BC}{AC}$

or $m = \dfrac{(y_2 - y_1)}{(x_2 - x_1)}$ $\quad x_2 \neq x_1$



Example 5: The co-ordinates of P are (−4, −4) and the co-ordinates of Q are (8, 14). Find the gradient of line PQ. ---[2]

$\boxed{W-13/43/Q7(a)}$

Solution: P(−4, −4), Q(8, 14)

gradient of line $PQ = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{14 - (-4)}{8 - (-4)}$

$m = \dfrac{18}{12} = \dfrac{3}{2}$ ✓

Example 6: The point P has coordinates (10, 12) and the point Q has coordinates (2, −4). Find of the gradient of line PQ.

$\boxed{M-18/42/Q10}$ ---[2]

Solution P(10, 12), Q(2, −4)

Gradient of line $PQ = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-4 - 12}{2 - 10} = \dfrac{-16}{-8} = 2$ ✓

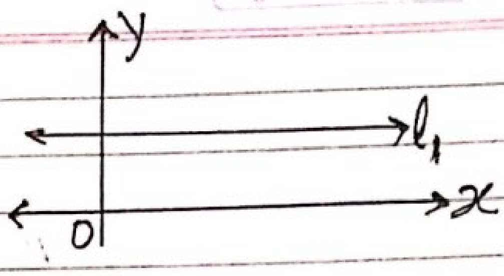
$m = 2$ ✓

IG-Maths 0580

Coordinate Geometry

Gradient of a line

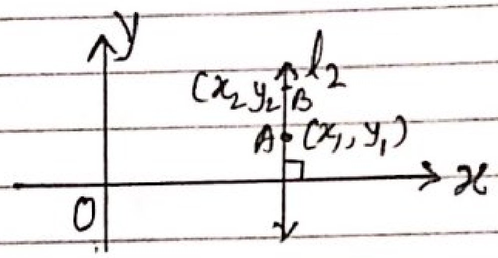
§[4]. Some Special Cases:



(i) Gradient of line ∥ x-axis

$$m = 0$$

(i′) Gradient of x-axis $= 0$



(ii) Gradient of line $l_2 \parallel$ y-axis.
(or $l_2 \perp$ x-axis)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

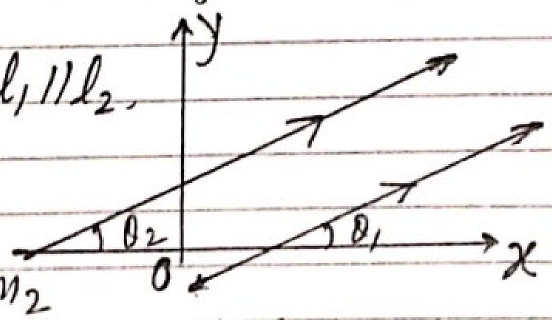
$$m = \frac{a}{0} \text{ not defined } [\because x_1 = x_2]$$

(ii′) Gradient of y-axis is not defined.

§[5] (i) Relation between the Gradients of Parallel lines $l_1 \parallel l_2$.

Let Grad. of $l_1 = m_1$
and Grad. of $l_2 = m_2$

Then $l_1 \parallel l_2 \Leftrightarrow m_1 = m_2$

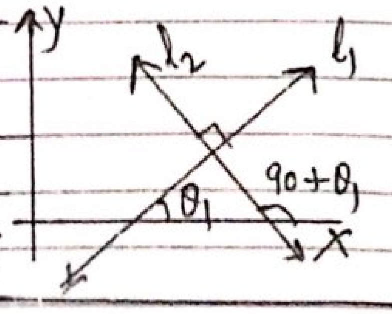


here $l_1 \parallel l_2 \Leftrightarrow \theta_1 = \theta_2$

(ii) Relation between the Gradients of perpendicular lines.

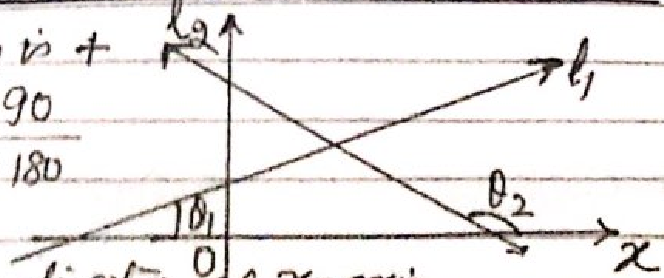
l_1 perp $l_2 \Leftrightarrow m_1 \times m_2 = -1$

or $m_2 = -\frac{1}{m_1}$



Note: (i) Gradient of line 'l_1' m_1 is + or $m_1 > 0$ then $0 < \theta_1 < 90$

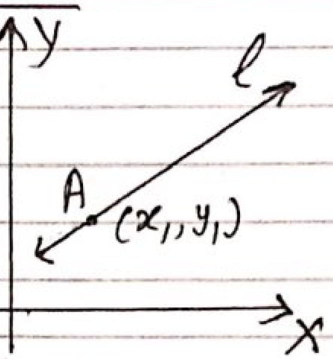
(ii) If m_2 is −ve, $90 < \theta_2 < 180$
line is inclined at an obtuse angle with the positive direction of x-axis



Coordinate Geometry

Different forms of equation of a line:

§[6]



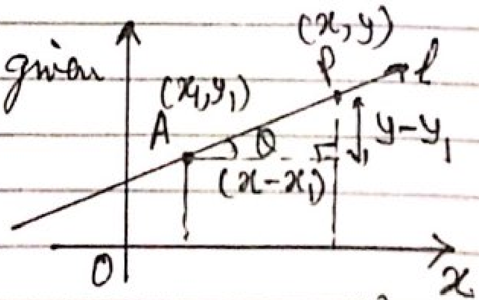
What do we understand by the Equation of a line. (or any curve):

(i) General Equation of a line is,
$ax + by + c = 0$: 'a', and 'b' both are not equal to zero.

Example: (i) $2x - 3y + 10 = 0$
(ii) $3x - 7 = 0$
(iii) $y = 4$

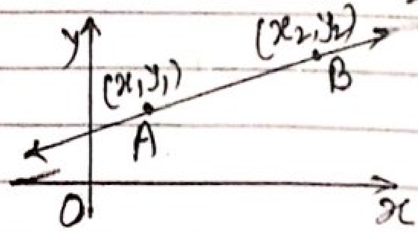
Note: By equation of a line we mean that the coordinates of every point on the line should satisfy the equation of line.
and conversely if the coordinates of any point satisfy the equation then the point must lie on the line.

(ii) Equation of a line passing through a given point $A(x_1, y_1)$ and gradient 'm'.



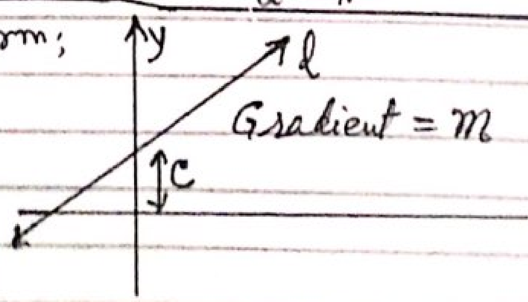
$$y - y_1 = m(x - x_1)$$

(iii) Two Point form; Equation of line passing through two points $A(x_1, y_1)$ and $B(x_2, y_2)$ $m = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$



$\therefore$ Eqn of line: $y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$.

(iv) Gradient 'm' and y-Int c form;
Eqn of line $y = mx + c$.



(v) To Reduce the Gen. Eqn $ax + by + c = 0$ To Gradient form $y = -\frac{a}{b}x + \left(-\frac{c}{b}\right)$

Example 7: A(5,23) and B(−2,2) are two points,

(a) Find the coordinates of the mid point of line AB --- [2]

(b) Find the equation of line AB. ---[3]

(c) Show that pt(3,17) lies on the line AB. W-13/22/18 --[1]

Solution: (a) Mid point of AB. $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$$= \left(\frac{5+(-2)}{2}, \frac{(23+2)}{2}\right) = \left(\frac{3}{2}, \frac{25}{2}\right)$$

(b) Given A(5,23), B(−2,2)

Let the eqn of line AB is

$y = mx + c$ --- (1)

point A(5,23) lies on line (1) must satisfy. ∴ $23 = 5x + c$ --- (2)

Similarly B(−2,2) lies on (1)

$2 = -2x + c$ --- (3)

Solving (2) & (3), $m = 3$ & $c = 8$

∴ from (1) eqn of AB, $y = 3x + 8$ --- (4)

(B) Alternate method:

Eqn of line $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

or $y - 23 = \frac{(2-23)}{-2-5}(x-5)$

or $y - 23 = 3(x-5)$

or $y = 3x + 8$

(c) To find if the point (3,17) lies on line AB,

Put $x = 3$ and $y = 17$ in the eqn of line AB,

from (4) $17 = 3 \times 3 + 8$

$\Rightarrow 17 = 17$ True

∴ Point (3,17) lies on the line AB.

To find the Eqn of perp

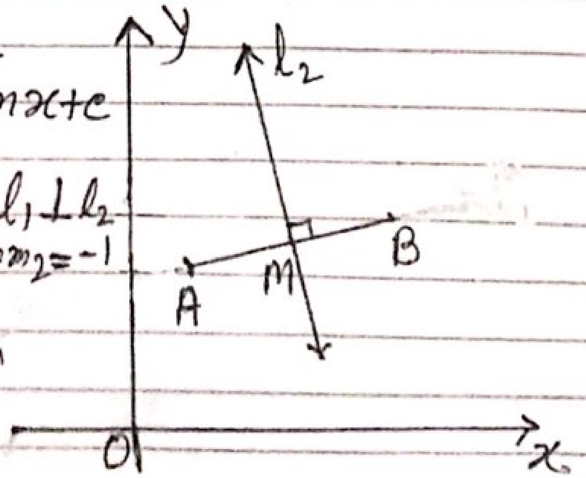
§[7] bisector of line AB: $y = mx + c$

Gradient of line AB = m

Gradient of perp line = $-\frac{1}{m}$ $\left[\because l_1 \perp l_2 \right.$, $m_1 \times m_2 = -1$

Find Mid point M of AB.

and using point-Gradient eqn of line. Find the required eqn of perp-bisector of AB.



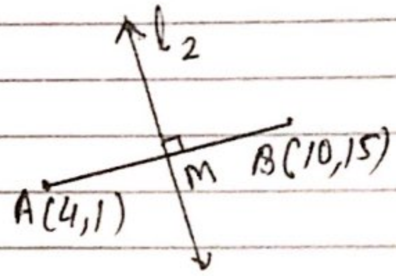
Coordinate Geometry

§ To find the Equation of Perpendicular Bisector of line AB.

Example 8: A is the point (4,1) and B is the point (10,15). --[6]
Find the equation of the perpendicular bisector of line AB.

[S-16/21/Q25]

Solution: Given $A(4,1)$, $B(10,15)$



Mid point M. of AB, $M\left(\frac{4+10}{2}, \frac{1+15}{2}\right)$

$= M(7,8)$ --- ①

Gradient of line AB,

$$m_1 = \frac{y_2-y_1}{x_2-x_1} = \frac{15-1}{10-4} = \frac{14}{6} = \frac{7}{3} \checkmark \text{ — ②}$$

∴ Gradient of Req. perp. bisector,

$$m_2 = -\frac{1}{m_1}$$

[AB ⊥ Perp bisector ∴ $m_1 \times m_2 = -1$ or $m_2 = -\frac{1}{m_1}$

$$= \frac{-1}{\frac{7}{3}} \quad \text{fr ②}$$

$$\therefore m_2 = -\frac{3}{7}$$

∴ Eqⁿ of perp. bisector passing through $M(7,8)$, and gradient $m_2 = -\frac{3}{7}$

∴ Eqⁿ $y - y_1 = m_2(x - x_1)$

or $y - 8 = -\frac{3}{7}(x-7)$

or $y - 8 = -\frac{3}{7}x + 3$

or $y = -\frac{3}{7}x + 11 \checkmark$

(or $3x + 7y = 77 \checkmark$)

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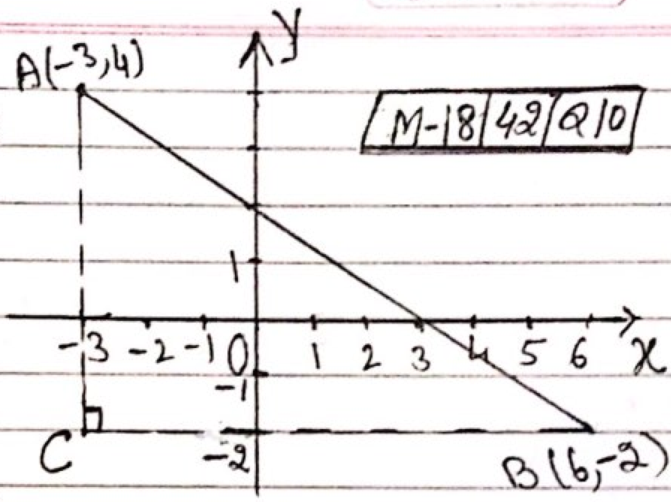
Example 9:

(a) Calculate length AB. ---[3]

M-18/42/Q10



Solution: In the figure.

$BC = 6-(-3) = 9$

$AC = 4-(-2) = 6$

Using Pythagoras Theorem,
In rt Δ ACB

$$AB^2 = BC^2 + AC^2$$
$$= 9^2 + 6^2 = 81 + 36 = 117$$
$$\therefore AB = \sqrt{117} = 10.8$$

(b) The point P has coordinates (10,12) and Q has coordinates (2,−4). Find. ---[2]

(i) the coordinates of the mid-point of the line PQ.

Solution: Mid point of PQ $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$$= \left(\frac{10+2}{2}, \frac{12+(-4)}{2}\right) = (6,4)$$

(ii) Find the gradient of line PQ. ---[2]

Solution: Gradient of PQ $= \frac{y_2-y_1}{x_2-x_1} = \frac{(-4)-12}{2-10} = \frac{-16}{-8} = 2$ —(ii)

(iii) Find the equation of a line perpendicular to PQ and passing through the point (2, 3). ---[3]

Solution: Gradient of PQ, $m_1 = 2$ from (ii)

$\therefore$ Gradient of Req. perp. line $m_2 = -\frac{1}{m_1}$ $[m_1 \times m_2 = -1]$

$$= -\frac{1}{2}$$

$\therefore$ Equation of line passing through (2,3) and perp. to PQ, is

$$y - y_1 = m_2(x - x_1)$$
$$y - 3 = -\frac{1}{2}(x-2)$$
$$y - 3 = -\frac{1}{2}x + 1$$
or $$y = -\frac{1}{2}x + 4$$

Example 10: Line 'A' has equation $y = 5x - 4$ and Line B has equation $3x + 2y = 18$, [W-17/43/Q8]

(a) Find the gradient of (i) line A (ii) line B. --- [2]

Solution: (i) Line A: $y = 5x - 4$

Gradient $m_1 = 5$ ✓ [line $y = mx + c$, Gradient $= m$]

(ii) Line B: $3x + 2y = 18$

or $2y = -3x + 18$

or $y = -\frac{3}{2}x + 9$ (divide by 2)

Gradient $m_2 = -3/2$ ✓

(b) Write down the coordinates of the point where line 'A' crosses the x-axis. --- [2]

Solution: Line A: $y = 5x - 4$ —①

Line crosses x-axis then $y = 0$

Put $y = 0$ in ① $0 = 5x - 4$

or $x = 4/5$

$\therefore$ Req. Point $(\frac{4}{5}, 0)$ ✓

(c) Find the equation of the line perpendicular to line 'A' which passes through the point (10, 9). Give your answer in the form: $y = mx + c$ --- [4]

Solution: from Part (a)(i)

Gradient of line 'A' $m_1 = 5$

$\therefore$ Gradient of perp. line $m_2 = -\frac{1}{m_1}$

or $m_2 = -\frac{1}{5}$

$\therefore$ Req. Eqn of line $y = mx + c$

or $y = -\frac{1}{5}x + c$ —②

Passes through (10, 9),

$\therefore$ $9 = -\frac{1}{5} \times 10 + c \Rightarrow c = 11$

$\therefore$ from ② Req line eqn; $y = -\frac{1}{5}x + 11$ ✓✓

(d) Work out the coordinates of the point of intersection of line A and line B. --- [3]

Line A: $y = 5x - 4$ ----③

Line B: $3x + 2y = 18$ —④

Solving put value of y from ③ in ④

$3x + 2(5x - 4) = 18$

or $13x = 26$

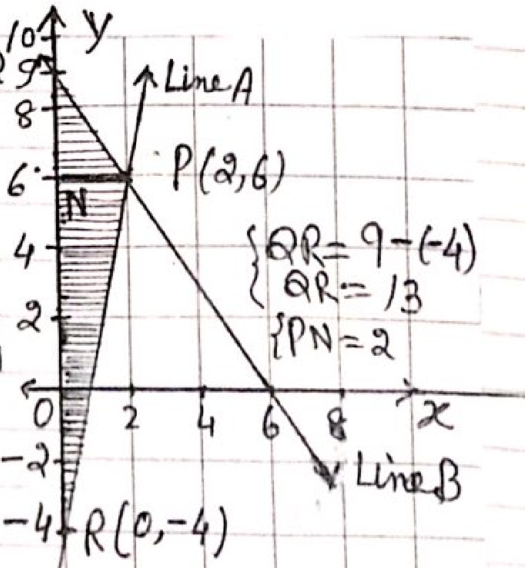
or $x = 2$

from ③ $y = 6$

$\therefore$ Point of Int P(2, 6) ✓

(e) Work out the area enclosed by line A, line B and the y-axis. --- [3]

Solution:



Req. Area of ΔPQR

$= \frac{1}{2} QR \times PN$

$= \frac{1}{2} \times 13 \times 2$

$= 13$ ✓

{$QR = 9 - (-4)$, $QR = 13$, $PN = 2$}

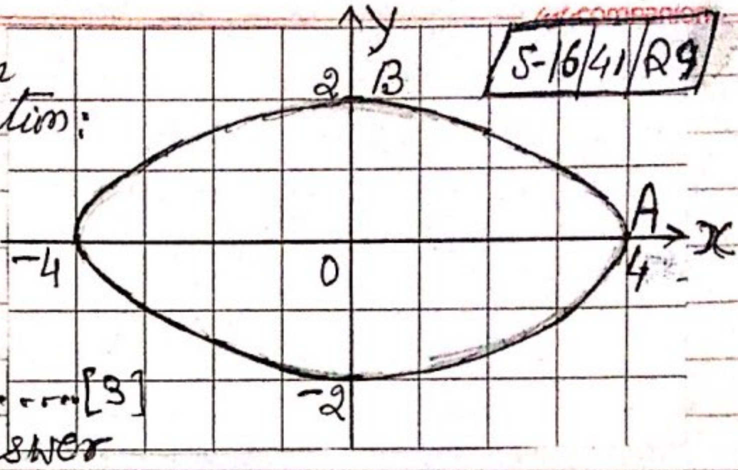
IG-Maths 0580

Coordinate Geometry

5-16/41/Q9

Example 12: The diagram shows a curve with equation:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$



(a) A is the point (4,0) and B is the point (0,2).

(i) Find the equation of line AB. Give your answer in the form $y=mx+c$ ....[3]

(ii) Show that $a^2=16$ and $b^2=4$. ....[2]

Solution (a)(i) $A(4,0)$, $B(0,2)$

Let Equ^n of AB is $y=mx+c$ — ①

Gradient of AB, $m=\frac{y_2-y_1}{x_2-x_1}$

$$=\frac{2-0}{0-4}=-\frac{1}{2}$$

from ① Equ^n of AB,

$$y=-\frac{1}{2}x+c \quad \text{— ②}$$

Passes through $A(4,0)$ in ②

$$0=-\frac{1}{2}\times 4+c \Rightarrow c=2$$

from ② Equation of line AB is

$$y=-\frac{1}{2}x+2 \quad \text{— ③}$$

(ii) Equation of curve:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \text{— ④}$$

A(4,0) lies on curve ④

$$\frac{4^2}{a^2}+\frac{0}{b^2}=1$$

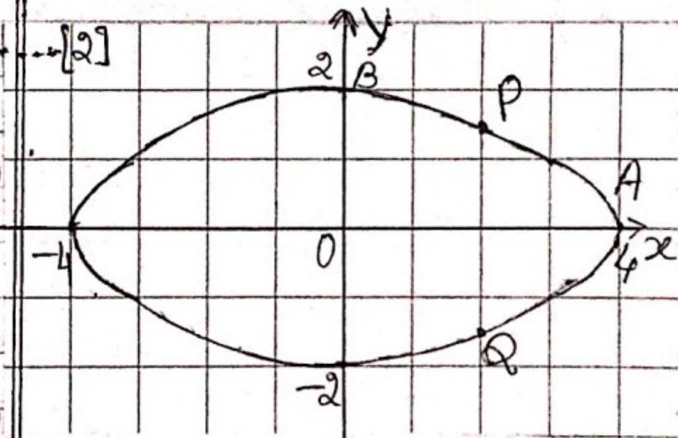
$$\Rightarrow a^2=16$$

Also B(0,2) lies on ④

$$\frac{0}{a^2}+\frac{2^2}{b^2}=1$$

$$\Rightarrow b^2=4$$

Proved



(b) $P(2,k)$ and $Q(2,-k)$ are points on the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ — ④

(i) Find the value of k ---[3]

(ii) Calculate angle POQ. ---[3]

Solution (b)(i) $(2,k)$ lies on curve ④

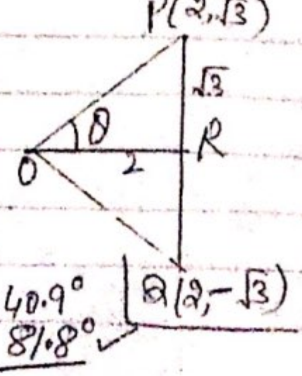
$$\frac{4}{a^2}+\frac{k^2}{b^2}=1$$

or $\frac{4}{16}+\frac{k^2}{4}=1$ { ∵ from (a)(ii) $a^2=16$, $b^2=4$ }

$$\therefore k^2=3$$

$$k=\sqrt{3}$$

(ii) angle $POQ=2\theta$



$$\tan\theta=\frac{\sqrt{3}}{2}$$

$$\theta=\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$=40.9^\circ$$

$\therefore$ Angle $POQ=2\theta=2\times 40.9^\circ$

$$=81.8^\circ$$

(Continued →)

(Continued →)

Example 12 (C) The area enclosed by the curve with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab.

(i) Find the area enclosed by the curve $\frac{x^2}{16} + \frac{y^2}{4} = 1$ --- [1]
give your answer as a multiple of π.

Solution: $\left.\begin{array}{l} a^2 = 16 \\ b^2 = 4 \end{array}\right\} \Rightarrow \begin{array}{l} a = 4 \\ b = 2 \end{array}$

$\therefore$ Area enclosed $= \pi ab = \pi \times 4 \times 2 = 8\pi$ ✓

(ii) A curve mathematically similar to the one in the diagrams, intersects X-axis at (12,0) and (-12,0). Workout the area enclosed by this curve, giving your answer as a multiple of π. --- [2]

Solution: $\frac{\text{Area enclosed by new curve}}{\text{Area enclosed by given curve}} = \left(\frac{a_1}{a_2}\right)^2$

or $\frac{\text{Area enclosed by new curve}}{8\pi} = \left(\frac{12}{4}\right)^2$

$= 3^2 = 9$

$\therefore$ Area of enclosed by new curve $= 9 \times 8\pi$

$= 72\pi$ ✓