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IG-Maths 0580

Geometry-1 Notes

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(SURESH GOEL.)

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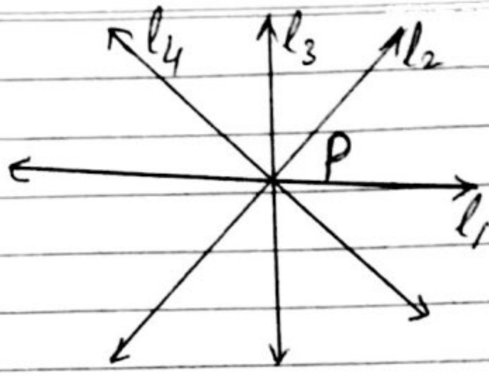
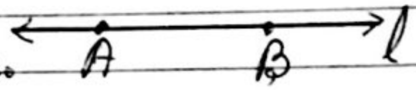
Geometry Notes

Lines and Angles:

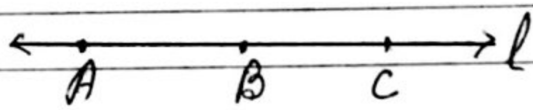


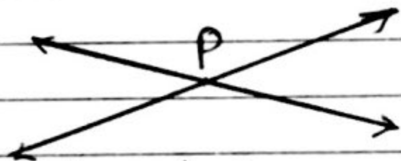
§1. (i) Through a given point, there pass infinitely many line. Three or more such lines are called **concurrent** lines.

(ii) Given two distinct points, there is **one** and only one line containing them.



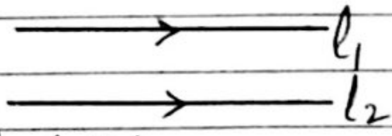
(iii) Three or more points lying on the same line are called **collinear** points.





(iv) Two lines are intersecting if they have **one** point in common. The common point is called the "**point of intersection**".

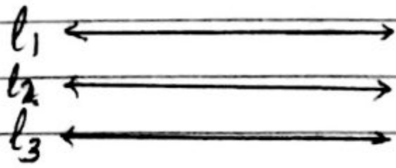
§2. (i) Parallel Lines: Two line in a plane which do not intersect when extendend.



we write: $l_1 \parallel l_2$.

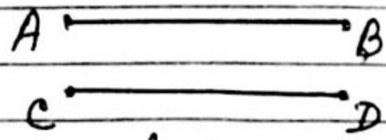
(ii) Two lines which are parallel to the same line are parallel to each other

$l_2 \parallel l_1$ and $l_3 \parallel l_1 \Rightarrow l_2 \parallel l_3$



§3. Line Segments:

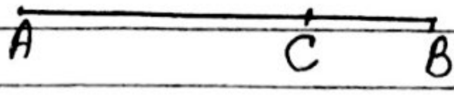
(i) Two congruent line segments are of equal length.



$\overline{AB} \cong \overline{CD} \Leftrightarrow$ length AB = length CD.

(ii) If a point 'C' lies in the interior of segment $\overline{AB}$.

Then $l(AC) + l(CB) = l(AB)$.

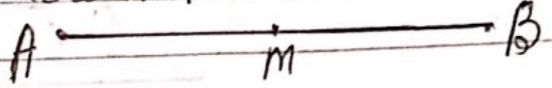


Geometry

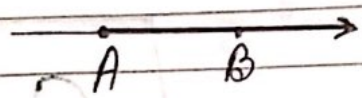
Lines and Angles:

Line Segments:

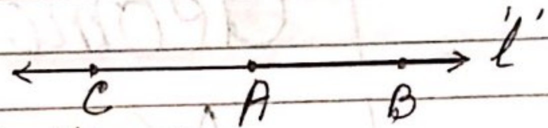
§3 (iii) Mid point 'M' of line segment $\overline{AB}$.
$AM = MB$.



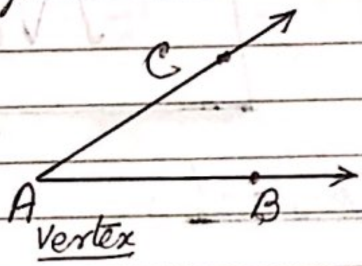
(iv) Ray $\overrightarrow{AB}$.
A is the initial point of ray $\overrightarrow{AB}$



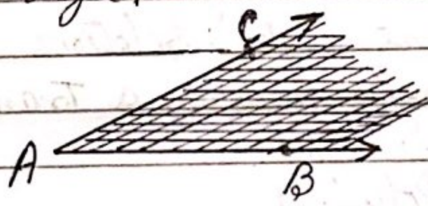
(v) Opposite Rays:
Point 'A' lies on line 'l'.
Then $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are opposite rays.



§4. (i) Angle: Angle is formed by two non-collinear rays with a common initial point, called Vertex of the angle, and rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are called the arms of the angle.

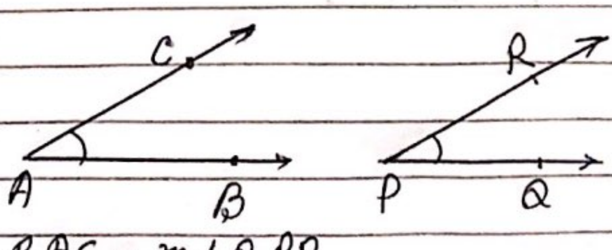


(ii) Interior of angle:



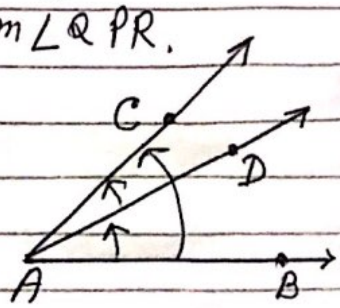
(iii) Measure of an angle:
Every angle has a measure.
Unit of angle measure is "Degree". 1 right angle $= 90^\circ$

(iv) Congruent angles:
congruent angle have equal measure.



$\angle BAC \cong \angle QPR \Leftrightarrow$
$m\angle BAC = m\angle QPR$.

(v) Angle addition axiom:
If D is point in the interior of $\angle BAC$. Then
$m\angle BAC = m\angle BAD + m\angle DAC$.



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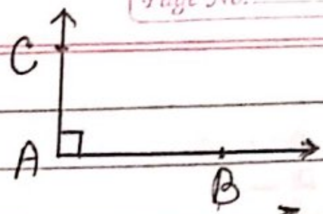
Geometry

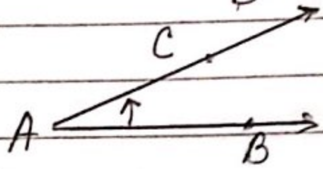
Lines and Angles:

§ 5 Types of Angles:

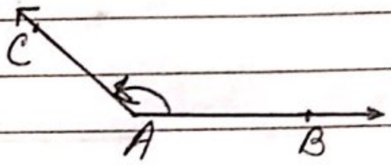
(i) Right angle: whose measure is 90°.



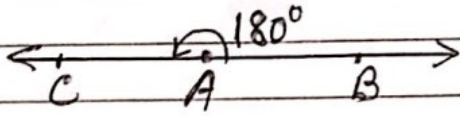
(ii) Acute angle: Whose measure is less than 90°.



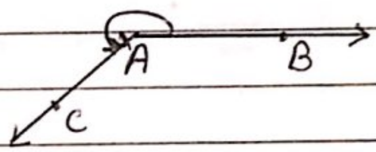
(iii) Obtuse angle: whose measure is more than 90° but less than 180°.



(iv) Straight angle: whose measure is 180°



(v) Reflex Angle: Whose measure is more than 180° but less than 360°.

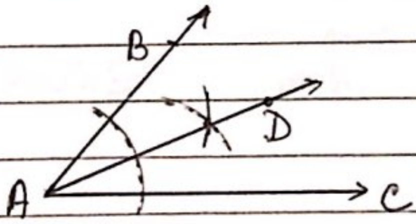


§ 6 Some Angles Relations:

(i) Angle Bisector: If D is point in the interior of $\angle BAC$, so that:

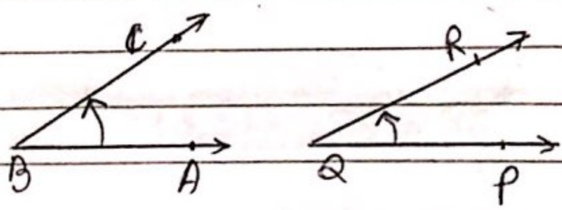
$m\angle CAD = m\angle BAD$

Ray $\overrightarrow{AD}$ is the bisector of angle BAC.



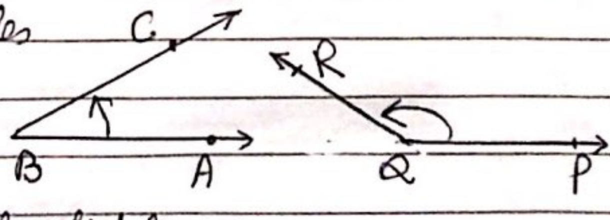
(ii) Complementary Angles:
Two angles ABC and PQR whose sum of measures is 90°

$m\angle ABC + m\angle PQR = 90°$

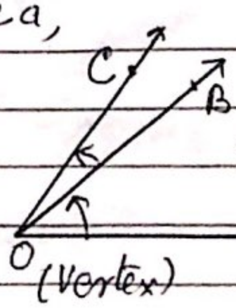


(iii) Supplementary Angles: Two angles sum of whose measures is 180°

$m\angle ABC + m\angle PQR = 180°$



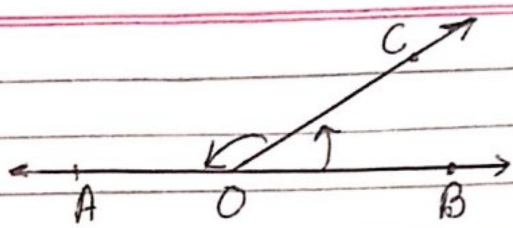
(iv) Adjacent angles: Two angles which have a,
(i) common Vertex
(ii) common arm
(iii) the uncommon arms are on the either side of the common arm,
angle AOB and $\angle BOC$ are adjacent angles.



$\overrightarrow{OB}$ is the common arm

§ 6. Some Angles Relations

(V) Linear Pair of Angles:



Two adjacent angles whose sum of measures is 180°.

$m\angle BOC + m\angle COA = 180°$

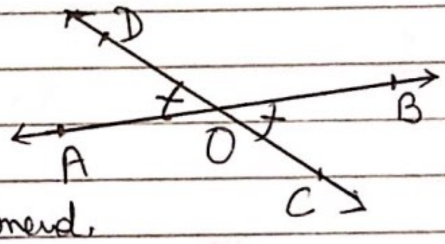
Then $\angle BOC$ and $\angle COA$ form a linear pair,

or The points A, O and B are collinear. ($\overrightarrow{OA}$ and $\overrightarrow{OB}$ are opp. rays)

Conversely: If on a line $\overleftrightarrow{AB}$ a ray $\overrightarrow{OC}$ stands

$m\angle BOC + m\angle AOC = 180°$

(Vi) Vertically opposite angles:

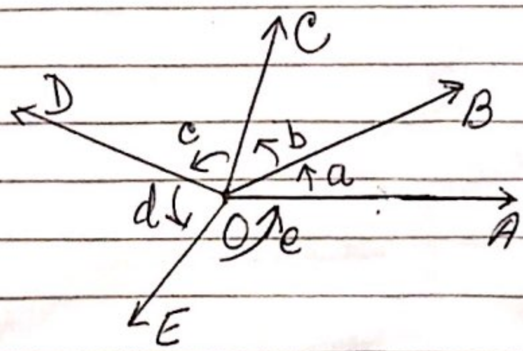


If two lines intersect at a point. Then two pairs of vertically opposite angles are formed.

(i) $\angle AOD$ and $\angle BOC$ (ii) $\angle BOD$ and $\angle COA$.

Vertically opposite angles are equal. $\begin{cases} \text{(i) } \angle AOD = \angle BOC \\ \text{(ii) } \angle BOD = \angle COA. \end{cases}$

(Vii) The sum of all angles around a point is 360°.



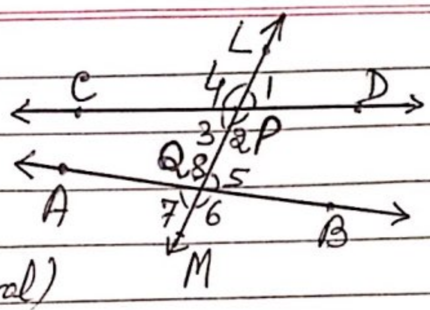
$m\angle AOB = a$, $m\angle BOC = b$

--- ---.

$a + b + c + d + e = 360°$

Geometry

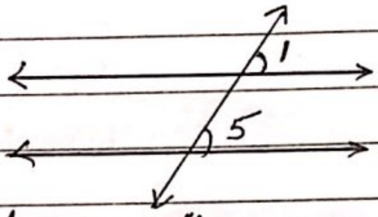
§7. **Angles made by a Transversal with two lines:**



Transversal:

Two lines AB and CD are intersected by a line LM (Transversal) at two distinct points P and Q.

(i) **Corresponding angles:**

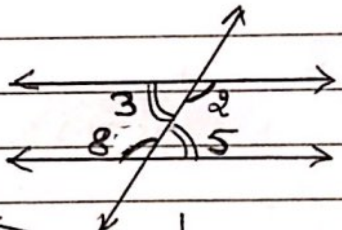


Two angles ($\angle 1$ and $\angle 5$) on the same side of the transversal and both of them are either above the two lines (or below the two lines)

There are four pairs of corresponding angles.

(i) $\angle 1$ and $\angle 5$ (ii) $\angle 2$ and $\angle 6$ (iii) $\angle 4$ and $\angle 8$ (IV) $\angle 3$ and $\angle 7$.

(ii) **Alternate Interior angles:**

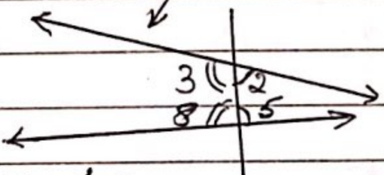


Two pairs (i) $\angle 2$ and $\angle 8$

(ii) $\angle 3$ and $\angle 5$

(iii) **Consecutive interior angles:**

(or Co-Interior angles)

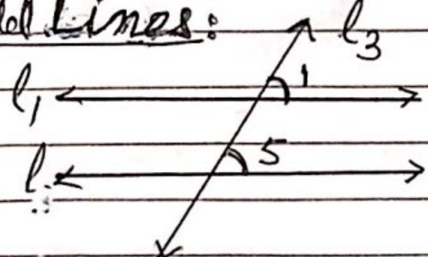


Two pair (i) $\angle 2$ and $\angle 5$

(ii) $\angle 3$ and $\angle 8$.

§8. **Transversal intersecting Parallel Lines:**

(i) **Corresponding angles axiom:**



If a transversal l_3 intersects two parallel lines ($l_1 \parallel l_2$), then: the corresponding angles are equal. $\angle 1 = \angle 5$

so all the four pairs.

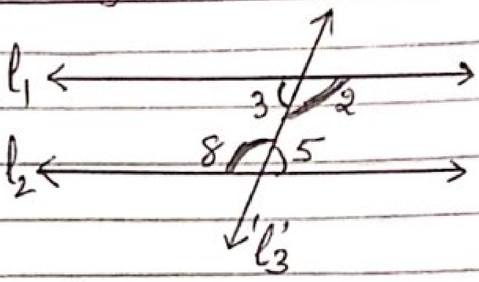
Conversely:

If two lines l_1 and l_2 are intersected by a transversal such that one pair of corresponding angles are equal then: lines are parallel, $l_1 \parallel l_2$

Geometry

§8 Relation between the angles formed by two lines intersected by a transversal l:

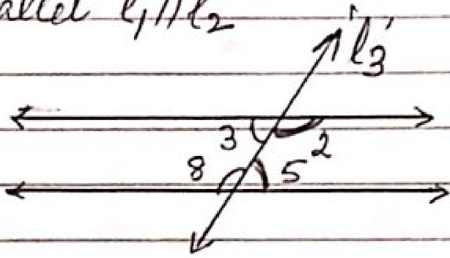
(ii) **Alternate angles:**



If two parallel lines ($l_1 \parallel l_2$) are intersected by a transversal 'l_3' then both the pairs of alternate angles are equal. (i) $\angle 3 = \angle 5$ and (ii) $\angle 2 = \angle 8$.

Conversely: If any one pair of alternate angles are equal then: lines are parallel $l_1 \parallel l_2$

(iii) **Co-Interior angles:**

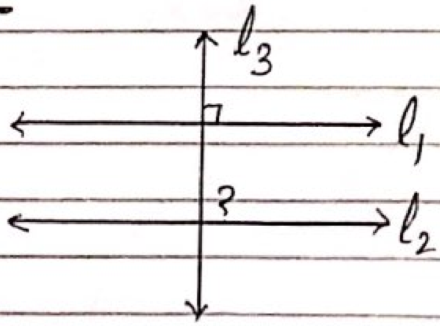


If two parallel lines ($l_1 \parallel l_2$) are cut by a transversal 'l_3' Then so formed interior angles on the same side are supplementary. { (i) $\angle 2 + \angle 5 = 180°$ (ii) $\angle 3 + \angle 8 = 180$

Conversely: If any one-pair of interior angles on the same side are supplementary,

(i) $\angle 2 + \angle 5 = 180$ (or $\angle 3 + \angle 8 = 180$)

Then: $l_1 \parallel l_2$



(IV) If $l_1 \parallel l_2$ and $l_3 \perp l_1$

Then $l_3 \perp l_2$.

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Geometry

P-7

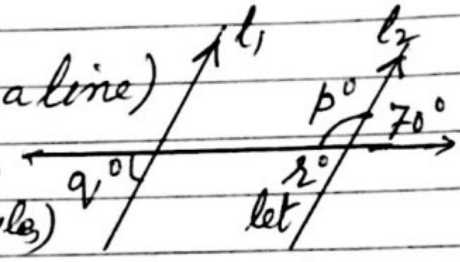
Lines and Angles.

Example 1. The diagram shows a straight line intersecting two parallel lines. Find the value of p and the value of q. --- [2]

S-17/22/Q8

Solution: (i) $p + 70° = 180$ (linear pair or angles on a line)

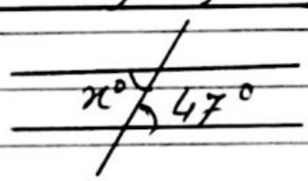
$\therefore p = 110°$ ✓



(ii) angle $r = 70°$ (Vertical angles)

$\therefore \angle q = \angle r$ ($l_1 \parallel l_2$ cut by trans. l_3
$= 70°$ ✓ Corresponding angles)

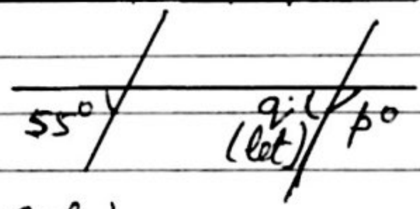
Example 2. Find the value of x. ---[1]



Ans: $x = 47°$ (Alternate Int angles)

M-16/22/Q4(a)

Example 3. Find the value of p. ---[2]



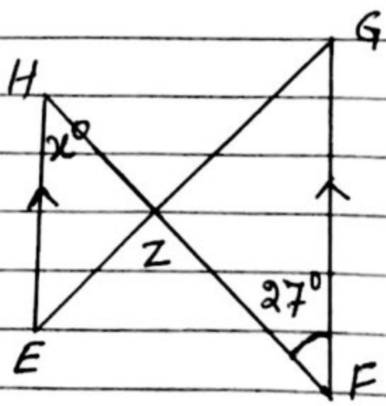
Solution: $p° + q° = 180°$ --(i) (linear pair)

$\therefore q = 55°$ (corresponding angles)

W-13/21/Q3

from (i) $p° + 55 = 180° \Rightarrow p = 125°$ ✓

Example 4. Given EH is parallel to FG.
$\angle HFG = 27°$, find x ---[1]



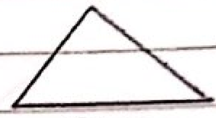
Ans: $EH \parallel FG$ cut by transversal HF.

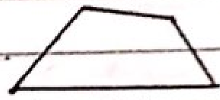
$\therefore x = 27°$ (Alternate angles)

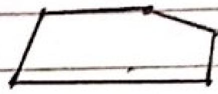
M-17/42/Q6(a)(i)

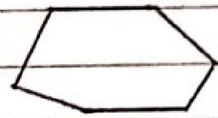
Polygons

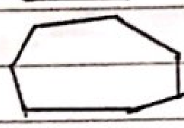
§9 (i) **Polygons:** A closed figure bounded by three or more line segments is called a polygon.

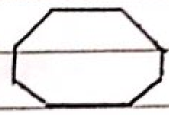
ⓐ Triangle - No. of sides - 3. -----


ⓑ Quadrilateral. No of sides - 4. -----


ⓒ Pentagon - No of sides 5. -----


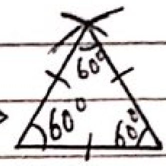
ⓓ Hexagon - No of sides - 6. -----


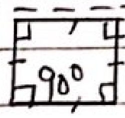
ⓔ Heptagon - No of sides - 7 -----


ⓕ Octagon - No of sides - 8. -----


(ii) **Regular Polygon.**

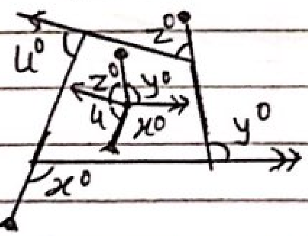
A polygon whose all sides and angles are equal.

ⓐ Equilateral triangle ----->


ⓑ Square ----->


ⓒ Regular Pentagon -----


(iii) Sum of Exterior angles of any polygon $= 360°$



(iv) Sum of Interior angles of n-sided polygon $= (n-2)\cdot 180°$

(v) One (each) exterior angle of n-sided regular polygon $= \dfrac{360°}{n}$

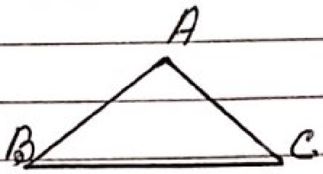
(vi) One (Each) interior angle of n-sided regular polygon $= (n-2)\cdot 180°/n$

Triangles

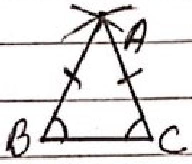
Geometry

§10. Types of Triangles:

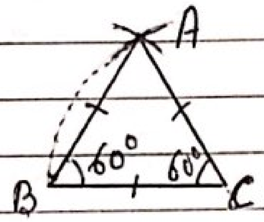
(i) On the basis of sides:



ⓐ Scalene triangle,
No two sides are equal,

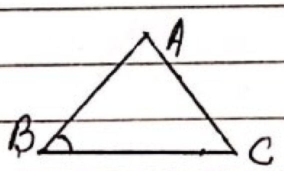


ⓑ Isosceles triangle. Two of whose sides are equal: $AB = AC$ and the opposite angles are equal $\angle B = \angle C$.

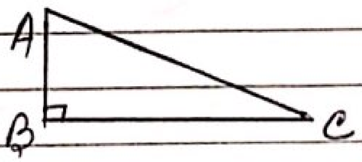


ⓒ Equilateral Triangle:
All three sides are equal, $AB = BC = CA$
and each angle $= 60°$

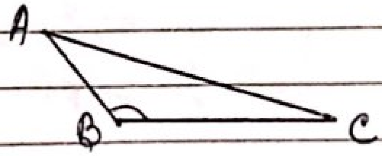
(ii) On the basis of angles:



ⓐ Acute triangle: All the three angles are acute.

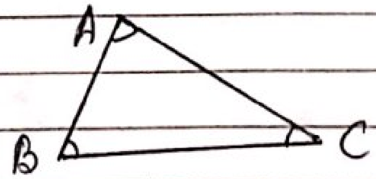


ⓑ Right angled Triangle: One angle is $90°$, $\angle B = 90°$

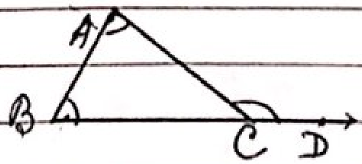


ⓒ Obtuse Triangle: One angle is obtuse angle. $90° < \angle B < 180°$

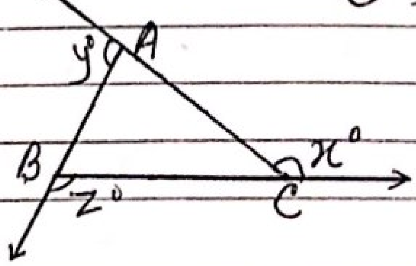
(iii) Properties of Triangle:



ⓐ Angle sum property:
Sum of the three angles is $180°$.
$\angle A + \angle B + \angle C = 180°$



ⓑ Exterior angle Theorem:
If one side of a triangle is produced, the ext. angle $\angle ACD = \angle A + \angle B$

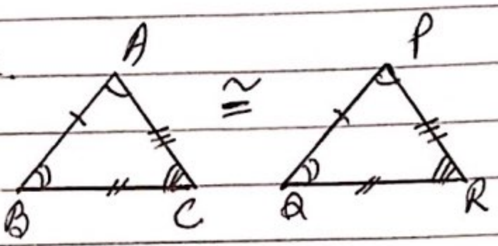


ⓒ Sum of three exterior angles of a triangle is $360°$.
$x + y + z = 360°$

Triangles.

§ 11 Congruent triangles:

(i) Given two congruent triangles,
$\Delta ABC \cong \Delta PQR$ Then.



(a) The corresponding sides are congruent:

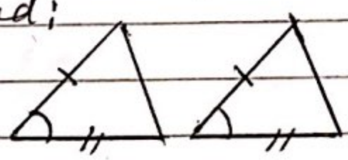
$AB = PQ$; $BC = QR$ and $AC = PR$

and, (b) The corresponding angles are equal:

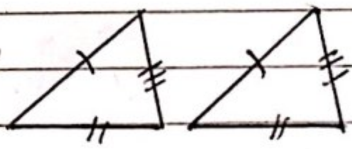
$\angle A = \angle P$; $\angle B = \angle Q$ and $\angle C = \angle R$.

(ii) Conditions required for two triangles to be congruent:
One of the following four is required;

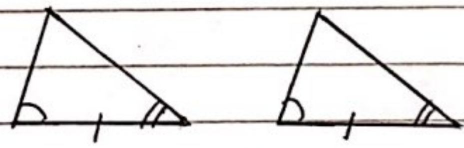
(a) SAS Two sides and included angle.



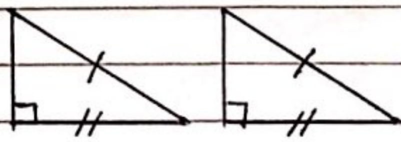
(b) SSS All three corresponding sides be equal.



(c) ASA (or SAA) One side and the two corresponding angles,

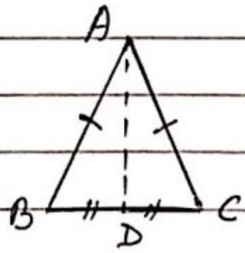


(d) RHS, In two right angled triangles, Hypentenuse and one of the corresponding sides,



§ 12 Theorems:

(i) In an isosceles triangle the line joing the vertex to the mid point of the base is perp to the base.



Given: Isosceles ΔABC, $AB = AC$ and D is the mid point of BC ($BD = DC$).

T.P Then: $AD \perp BC$.
(or $\angle ADB = 90°$)

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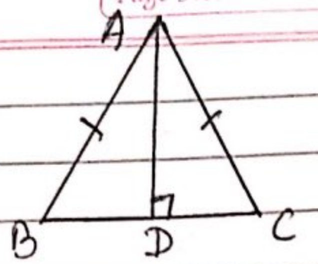
Triangles

Geometry. P-11



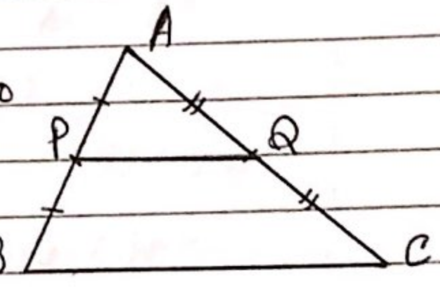
§ 12(ii) In an isosceles triangle, perp from vertex, bisects the base.

Given: $AD \perp BC$; $AB = AC$

To Prove: $BD = DC$.

(Converse of 12(i))

(iii) Mid point theorem:



The line joining the mid points of two sides of a triangle is parallel to the third and half of it.

Given: P and Q are the mid points of sides AB and AC resp.

To Prove: $PQ \parallel BC$ and $PQ = \frac{1}{2} BC$.

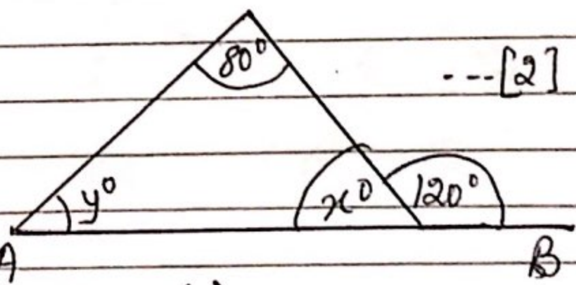
(iv) Converse of (iii)

Given P is the mid point of AB and $PQ \parallel BC$,

Then, To Prove: Q is the mid point of AC.

← x → x →

Example 5: Find the value of x and the value of y. ---[2]



Solution

(i) $x + 120° = 180°$ (Straight angle)

$\therefore x = 60°$ ✓

(ii) $y + 80 = 120$ (Ext angle Theo. in a triangle)

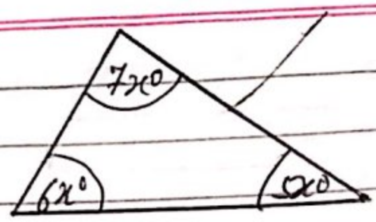
or $y = 40°$ ✓

W-17/22/Q3

Geometry

Triangles and Polygon

Example 6: The three angles in a triangle are $5x^\circ$, $6x^\circ$ and $7x^\circ$



(a) Find the value of x --- [2]

(b) Work out the size of the largest angle in the triangle. --- [1]

S-17 / 23 / 10

Solution (a) $5x + 6x + 7x = 180^\circ$ (Sum of Interior angle of a triangle is 180°)

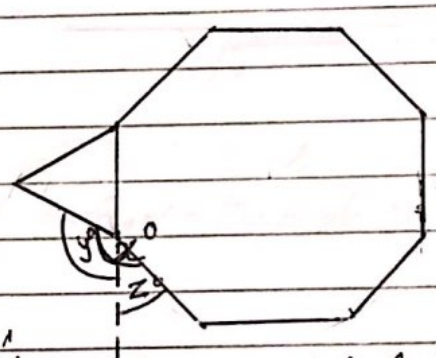
or $18x = 180$

$x = 10$ ✓

(b) largest angle $= 7x^\circ = 7 \times 10 = 70^\circ$ ✓

Example 7. The diagram shows a regular octagon joined to an equilateral triangle.

Workout the value of x.



Solution:

$x = y + z$

$= 120^\circ + 45^\circ$

$= 165^\circ$ ✓

Ext angle of n-sided regular polygon $= \frac{360^\circ}{n}$

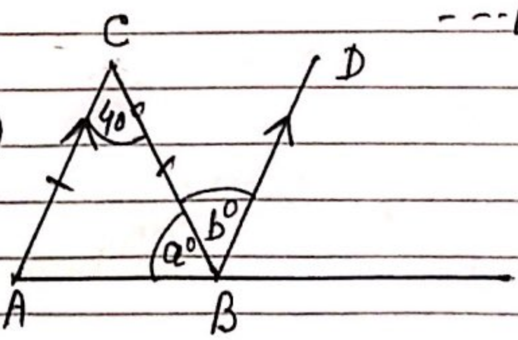
$y =$ Ext angle of Equilateral triangle $= \frac{360}{3} = 120^\circ$

$z =$ Ext. angle of Octagon $= \frac{360}{8} = 45^\circ$

S-17 / 21 / Q14

Example 8: Triangle ABC is isosceles and AC is parallel to BD. --- [2]

Find the value of a and value of b.



Solution: $b = 40^\circ$ ✓ (Alt interior angle) AC // BD.

In iso $\triangle ABC$ $\angle A = \angle B = a$

$\therefore \angle A + \angle B + C = 180^\circ$

$a + a + 40 = 180^\circ$

$\therefore 2a = 140$ or $a = 70$ ✓

S-16 / 21 / Q9

Example 9. Five angles of hexagon are each 115°. Calculate the size of the sixth angle. [S-16/21/Q17] --[3]

Solution: Let sixth angle $= x^\circ$

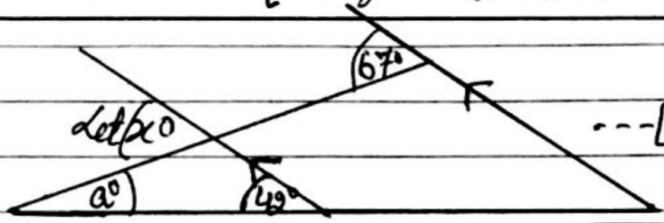
$\therefore x + 115^\circ \times 5 = 720^\circ$

$\therefore x = 720^\circ - 575^\circ$

$x = 145^\circ$ ✓

[$\because$ For n-sided polygon sum of int. angle $= (n-2)180^\circ$
$\because$ Sum of six angle of hexagon $= (6-2) \times 180^\circ = 720^\circ$]

Example 10.
Find the value of a. --[2]



[W-16/22/Q2]

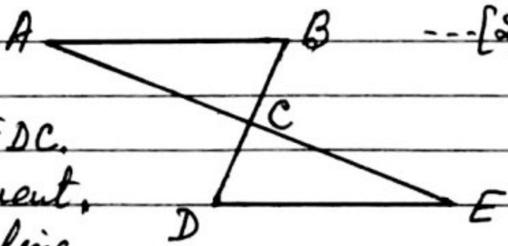
Solution: $x^\circ = 67^\circ$ (corresponding angles)

also $x^\circ = a + 42^\circ$ (Ext angle is equal to the sum of opp. int. angles of a triangle)

$\therefore a + 42 = 67^\circ$

$\therefore a = 25^\circ$ ✓

Example 11: The diagram shows two straight lines, AE and BD, intersecting at C. Angle ABC = angle EDC. Triangles ABC and EDC are congruent. Write down two properties of the line segments AB and DE. --[2]



[S-15/22/Q7]

Solution: Given $\Delta ABC \cong \Delta EDC$

$\therefore AB = ED$ ✓ (corresponding sides)

and angle ABC = angle EDC

$\therefore AB \parallel DE$

($\because$ angle formed by lines AB and DE cut by transversal BD. are Alternate angles equal.)

$\therefore$ Segments AB and DE are equal and parallel.

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Similar Figures — Geometry

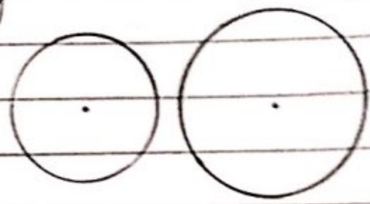
Date P-14

Page No.

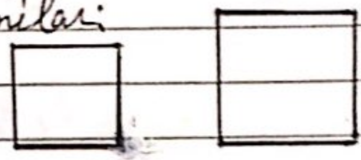
§13. **Similar figures:**

Geometric figures having the same, size may vary.

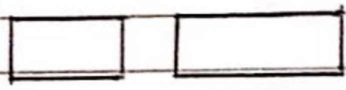
(i) (a) any two circles are similar.



(b) any two squares are similar:

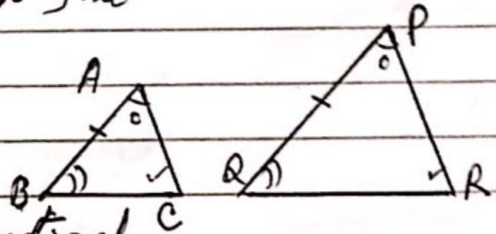


(c) Are any two rectangles similar? why/why not



(ii) **Similar triangles:**

(a) $\Delta ABC \sim \Delta PQR$ (Given)



Then:

Corresponding sides are proportional.

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = K$$

and the corresponding angles are equal.

$$\angle A = \angle P, \quad \angle B = \angle Q \text{ and } \angle C = \angle R$$

Conversely:

(b) Conditions required for similar triangles:

One of the following is required for two triangles be similar;

"AAA" (i) Corresponding angles are equal.

$$\angle A = \angle P, \angle B = \angle Q \text{ and } \angle C = \angle R$$

'SSS'

Or (ii) Corresponding sides are proportional.

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = K.$$

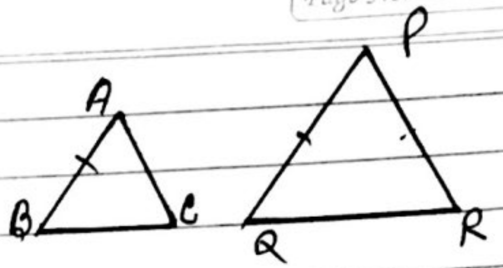
SAS

or (iii) Two pairs of sides are proportional and the respective included angles are equal.

$$\frac{AB}{PQ} = \frac{BC}{QR} \text{ and } \angle B = \angle Q$$

Properties of Similar figs

§14 Given $\triangle ABC \sim \triangle PQR$



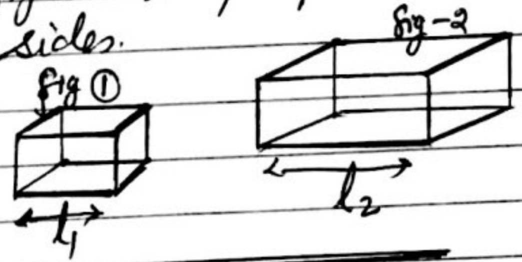
(a) Corresponding sides are in the same ratio

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = k$$

(b) Areas of similar triangles are proportial to the square of the corresponding sides.

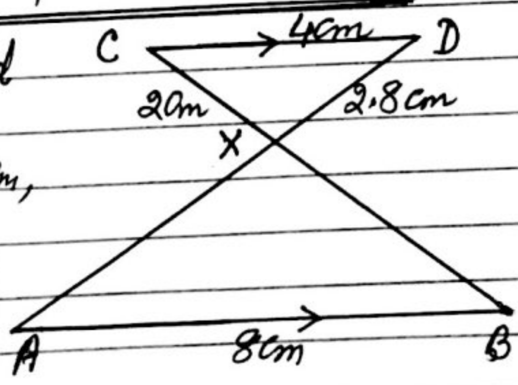
$$\frac{\text{area of } \triangle ABC}{\text{area of } \triangle PQR} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2 = k^2$$

(c) Volumes of two solid figures are proportial to the cubes of the corresponding sides.



$$\frac{\text{Volume of fig 1}}{\text{Volume of fig 2}} = \left(\frac{l_1}{l_2}\right)^3$$

Example 12: In the diagram, AB and CD are parallel. AD and BC intersect at X. AB = 8 cm, CD = 4 cm, CX = 2 cm and DX = 2.8 cm.



(a) Complete the statement: ---[1]
Triangle ABX is --- to Triangle DCX.

(b) Calculate AX ---[2]

(c) The area of triangle ABX is $y\,cm^2$. Find the area of triangle DCX in terms of y. --[1]

S.17/22/Q24

Solution (a) similar ✓ (AAA similarity theo. → { ∴ $\angle A = \angle D$ Alt angle; $\angle B = \angle C$ Alt angle; $\angle X = \angle X$ Vertical }

(b) $\frac{AX}{DX} = \frac{AB}{CD}$ (corresponding sides of similar $\triangle$s)

$\frac{AX}{2.8} = \frac{8}{4}$

$\therefore AX = 5.6\,cm$ ✓

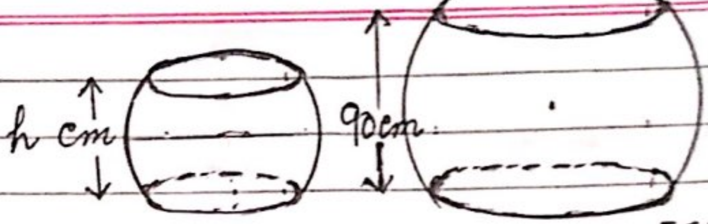
(c) $\frac{\text{Area of } \triangle ABX}{\text{Area of } \triangle DCX} = \left(\frac{AB}{DC}\right)^2$

or $\frac{y}{\text{area } DCX} = \left(\frac{8}{4}\right)^2 = 4$

$\therefore \text{Area } DCX = \frac{y}{4}\,cm^2$

Example 13:

The two barrels in the diagram are mathematically similar.



--- [3]

The small barrel has a height 'h' cm. and a capacity of 100 litres.
The larger barrel has a height of 90 Cm, and a capacity of 160 litres,
workout the value of h. [S-17/21/Q11]

Solution:

$$\frac{\text{Volume of small barrel}}{\text{Volume of larger barrel}} = \left(\frac{h_1}{h_2}\right)^3$$

$$\text{or} \quad \frac{100000}{160,000} = \left(\frac{h}{90}\right)^3 \qquad \left[\, 1\text{ Litre} = 1000\text{ c.c.}\right.$$

$$\Rightarrow \frac{5}{8} = \frac{h^3}{729000}$$

$$\Rightarrow h^3 = \frac{5}{8} \times 729000 = 455,625$$

$$\therefore h = \sqrt[3]{455625} = 76.95\text{ cm}$$

Example 14. A model of a house is made using a scale of 1:30.
The model has a volume of 2400 cm³. Calculate the volume of the actual house. Give your answer in cubic metres. ----- [3]
[W-17/23/Q10]

Solution:

$$\frac{\text{Volume of model house}}{\text{Volume of actual house}} = \left(\frac{l_1}{l_2}\right)^3 = \left(\frac{1}{30}\right)^3$$

$$\text{or} \quad \frac{2400}{V} = \frac{1}{27000}$$

$$\Rightarrow V = 2400 \times 27000 \text{ c.c}$$

$$= \frac{2400 \times 27000}{1000000} \text{ cu. m} \qquad [1\text{ cu. m} = 1000000\text{ cc}]$$

$$V = 64.8 \text{ cubic metres}$$

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Similarity

Example 15: A map is drawn on a scale of 1 : 1000000. A forest on the map has an area of 4.6 cm². Calculate the actual area of the forest in square kilometres. [S-16/22/Q7] ---[2]

Solution:

$$\frac{\text{Area of forest on the map}}{\text{Actual Area of the forest}} = \left(\frac{l_1}{l_2}\right)^2$$

$$\therefore \frac{4.6}{\text{Actual area}} = \left(\frac{1}{1000000}\right)^2$$

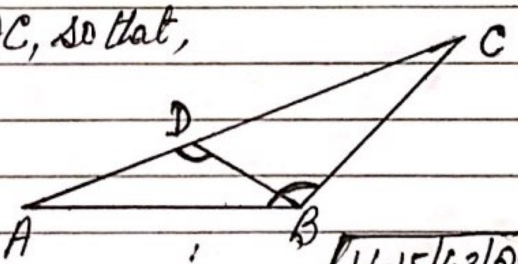
[1 Km = 100000 cm
[1 Km² = (100000)²

$$\therefore \text{Actual area} = 4.6 \times 1000000 \times 1000000 \text{ cm}^2$$

$$= \frac{4.6 \times 1000000 \times 1000000}{100000 \times 100000}$$

$$= 4.6 \times 10 \times 10 = 460 \text{ sq. kilometres}$$

Example 16 In the diagram, D is on AC, so that, angle ADB = angle ABC.



(i) Show that angle ABD = angle ACB ---[2]

(ii) Complete the statement,
triangle ABD and ACB are ---- [1] [W-15/43/Q8]

(iii) AB = 12 cm, BC = 11 cm and AC = 16 cm, Calculate the length of BD -- [2]

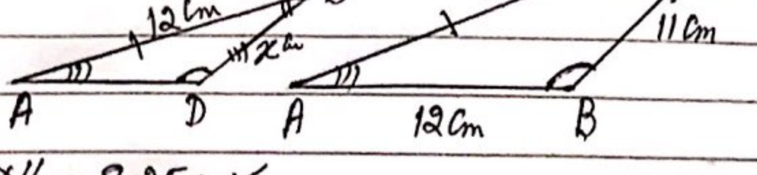
Solution (i) (a) $\angle ADB = \angle ABC$ (Given)

(b) $\angle CAB$ is common.

$\therefore$ The third angle ABD = angle ACB ($\because$ Sum of three angles of a triangle is 180°)
[and if two angles are equal, then third angles will also be equal.

(ii) Similar ($\because$ AAA similarity Theorem)

(iii) Given triangle ADB ~ triangle ACB (from (ii))
Corresponding sides are proportional.



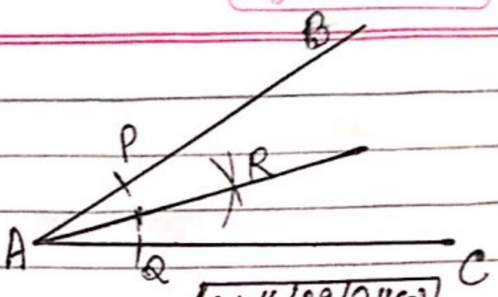
Let BD = x cm

$$\frac{BD}{BC} = \frac{AB}{AC}$$

$$\Rightarrow \frac{x}{11} = \frac{12}{16} \Rightarrow x = \frac{12 \times 11}{16} = 8.25 \text{ cm}$$

IG-Maths 0580 — **Constructions** — **Geometry** — Date P-18 — Page No.

§15(i) To draw bisector of an angle:



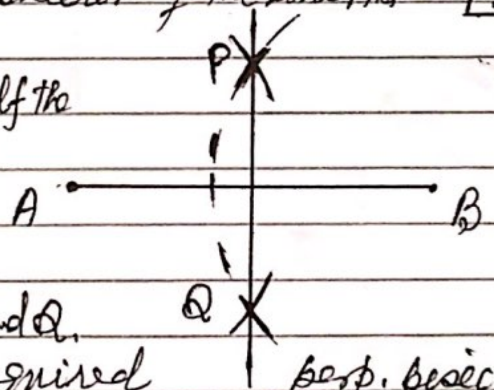
Example 17. Using compasses and a straight edge only, construct the bisector of angle BAC. ---- [2]

[W-16/22/Q11(a)]

With the vertex A as centre draw an arc, intersecting the sides AB and AC at points 'P' and 'Q' respectively.
Now taking radius more than half the distance PQ, draw the arcs with centres 'P' and 'Q', intersecting at the point 'R'.
Join AR and produce, AR is the required angle bisector.

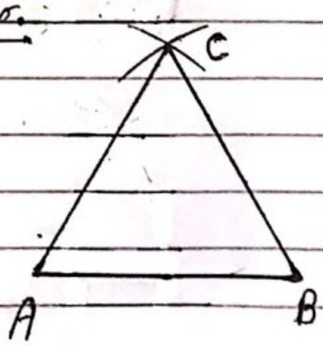
§15(ii) To draw the Perpendicular bisector of a line segment:

Example 18. Using a straight edge and compasses only, construct the perpendicular bisector of the line AB [S-16/23/Q6] --[2]



Taking more than half the length AB, as radius draw arcs with centres 'A' and B, to intersect at P and Q.
Join PQ, is the required perp. bisector.

§15(iii) To construct an Equilateral Triangle:



Example 19. The line AB is one side of an equilateral triangle ABC. Using a straight edge and compasses only, construct triangle ABC. --- [2]

Taking length AB as radius draw the arcs with centre A and as centre B; The two arcs intersect at C.
Join AC and BC. Triangle ABC is the required equilateral triangle.

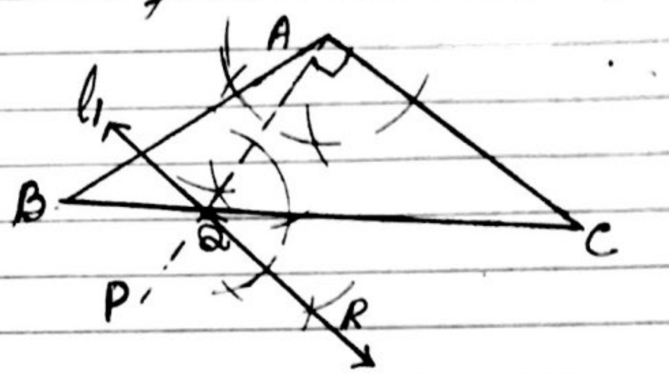
[M-17/22/Q2]

IG-Maths 0580

Constructions

§ 15(iv) Given a line 'l_1'. To draw another line 'l_2' at a distance of 'p' units from l_1. ($l_2 \parallel l_1$ and p is the perp. distance between l_1 and l_2)

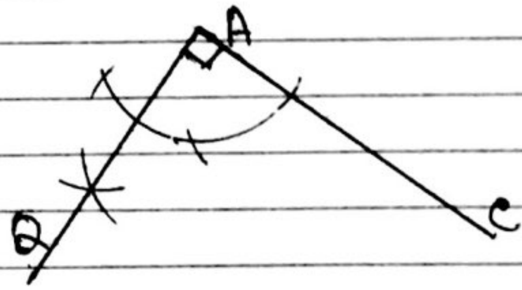
Example 20. Draw a line on B-side of line AC that is 3 cm from AC. --[1]



Draw a line AP through A and perp. to AC. Take a point 'Q' on AP so that AQ = 3 cm, again draw QR perp. to line AQ at Q.

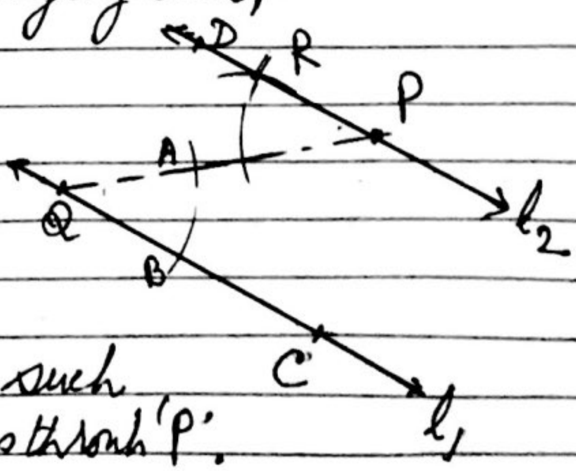
Then 'QR' or l_1 is the required line at a distance 3 cm from line AC on the B-side.

To draw a line AQ perp. to AC at A



AQ is perp. to AC.

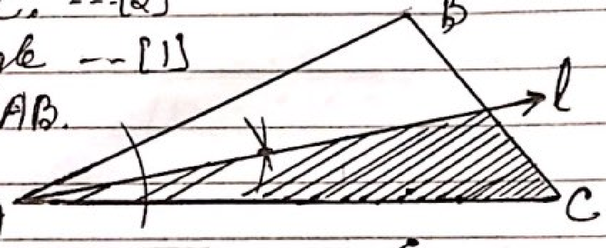
§15(v) To draw a line 'l_2' parallel to a given line 'l_1', passing through a point 'P' not lying on l_1.



Take point Q on line l_1.
Join PQ.
Draw angle DPQ equal to angle PQC.
Then line PD or 'l_2' is such that $l_2 \parallel l_1$ and passes through 'P'.

§16. Locus: It is a set of points satisfying a given rule (a geometric figure).
Conversely: Every point satisfying the rule lies on the locus.

Example 21 (a) Using a straight edge and compasses only, construct the bisector of angle BAC. --[2]
(b) Shade the region in the triangle --[1] that is nearer to AC than to AB.
[S-17/22/Q16]

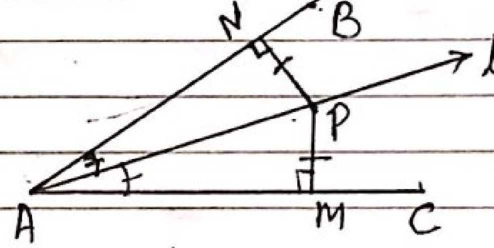


Ans: Line 'l' is the bisector of ∠BAC

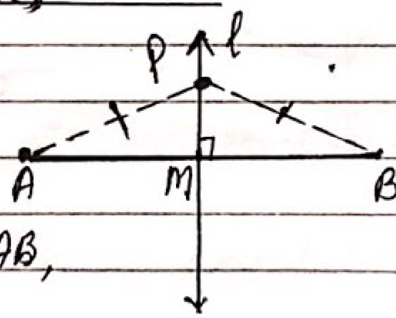
§ Locus Theorem 1.

Given an angle BAC and 'l' is its bisector, every point on the angle bisector is equidistant from its sides, $PM = PN$. [$PM \perp AC$ and $PN \perp AB$]

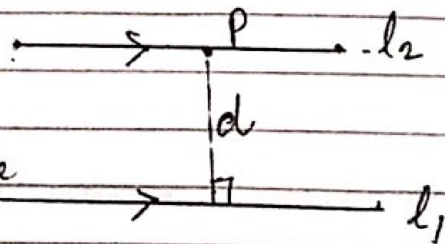
or The locus of a point equidistant from two lines is the angular bisector 'l'; $PM = PN$



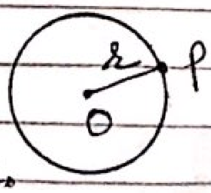
§ Locus Theorem 2: Given a segment AB.
The locus of a point 'P' equidistant from the end points of segment AB, ($PA = PB$), is the perp. bisector of AB, line l.



§ Locus Theorem 3: Given a line l_1, P is a point at distance 'd' (perp. distance) from line l_1.
Then the locus of point P is line ~~line~~ 'l_2' passing through P and ($l_2 \parallel l_1$) parallel to l_1.



§ Locus Theo 4: Locus of point 'P' which is always at at a distance 'r' from a fixed point 'O' is a circle.



Example 22 (a) Construct the locus of points inside the triangle, that are 5 cm from B. --- [1]

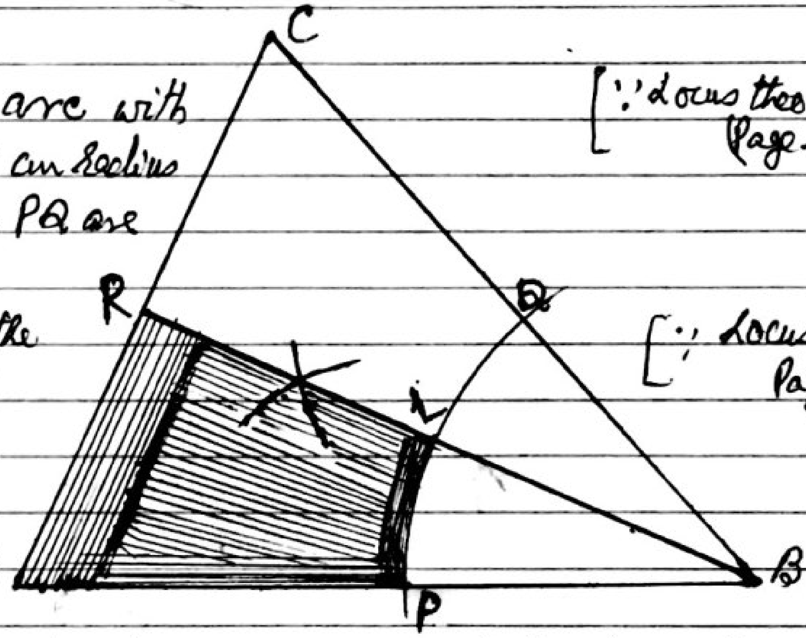
(b) Construct the locus of point, inside the triangle, that are equidistant from AB and BC. --- [2]

(c) Shade the region, inside the triangle, containing points that are ;

- more than 5 cm from B

and • nearer to AB than to BC. [W-16/21/Q17] --- [1]

Solution:



(a) Draw circular arc with B as centre and 5 cm radius then points on the arc PQ are 5 cm from B [∵ Locus theo 4] (Page-20)

(b) Draw BR as the angular bisector of angle ABC, the point of BR are equidistant from AB and BC. [∵ Locus theo 1 Page-20]

(c) • the region outside the arc PQ and inside the triangle is more than 5 cm from B

- A- side of angular bisector BR is nearer to AB.

Shaded region APLR is required.

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Locus

Geometry

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Example 23: The diagram shows the plan, ABCD, of a park, the scale is : 1 cm = 20 m.

(a) Find the actual distance BC. --- [2]

(b) A Fountain, F, is to be placed,

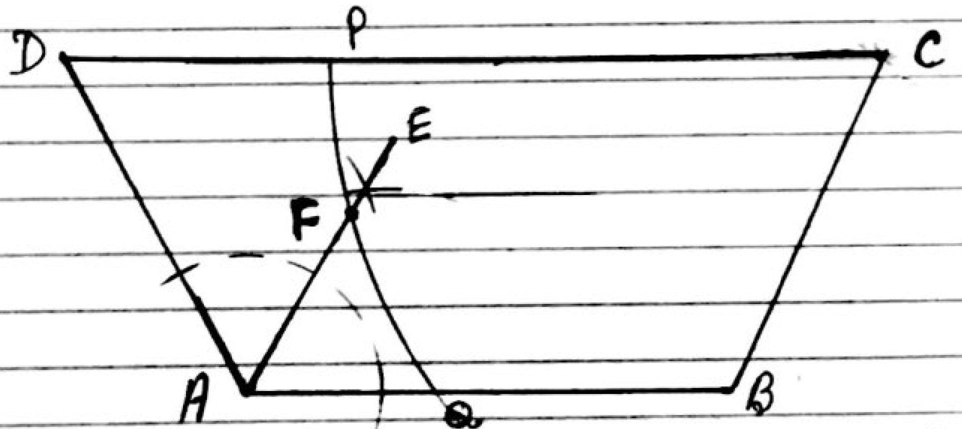
- 160 m from C

and

- equidistant from AB and AD.

On the diagram, using a ruler and compasses only, construct and mark the position of F.
Leave in all your construction lines. --- [5]

[W-14/22/Q20]



Solution (a) On the plan BC = 5.1 cm

Scale is [1 cm = 20 m

∴ Actual distance = 5.1 × 20
= 102 metres ✓

(b) • 160 m from C

draw an arc "PQ" with C as centre (∵ 20 m = 1 cm, ∴ 160 m = 8 cm)
and radius 8 cm, (all the points on the arc are at a distance of 160 m from C) (Locus theo 4)

• Draw angular bisector of angle DAB. (AE) [Locus theo-1]

Then the intersection of arc PQ and angular bisector of angle BAD at point 'F' is the location of fountain.

Locus (Loci)
(Plural of Locus)

Example 24: In this question use a ruler and compasses only. Show all your construction arcs. The diagram shows a triangular field ABC.
The scale is 1 cm. represent 50 metres. [M-16/42/Q2]

(a) Construct the locus of the points that are equidistant from A and B. -- [2]

(b) Construct the locus of points that are equidistant from the lines AB and AC. -- [2]

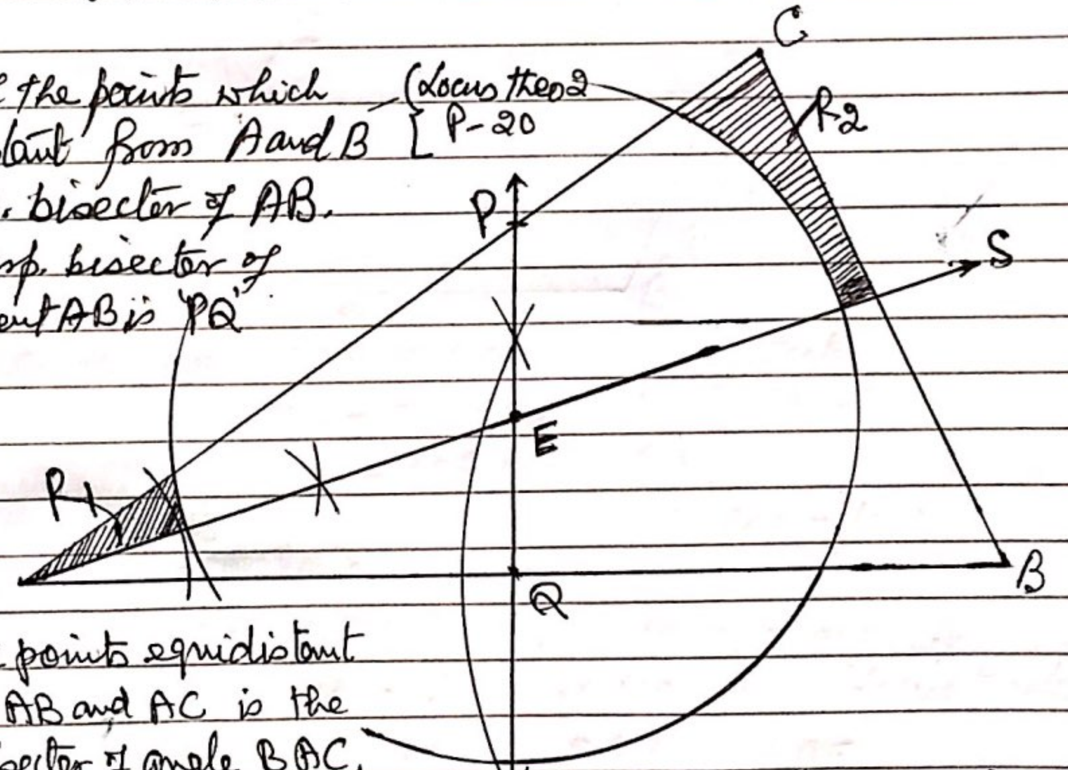
(c) The loci intersect at the point E. Construct the locus of points that are 250 m from E. -- [2]

(d) Shade any region inside the field ABC that is: -- [2]
- more than 250 m from E

and
- closer to AC than to AB.

Solution:

(a) Locus of the points which are equidistant from A and B is the perp. bisector of AB. (Locus theo 2 [P-20]
Draw the perp. bisector of line segment AB is 'PQ'



(b) Locus of points equidistant from lines AB and AC is the angular bisector of angle BAC. Draw 'AS' angular bisector of angle BAC. [∵ Locus Theo 1 Page 20]

(c) 250 m = 5 cm on the map.
The locus of points at 250 m from E is the circle with Centre E and radius 5 cm. [Locus Theo 4, Page 20]

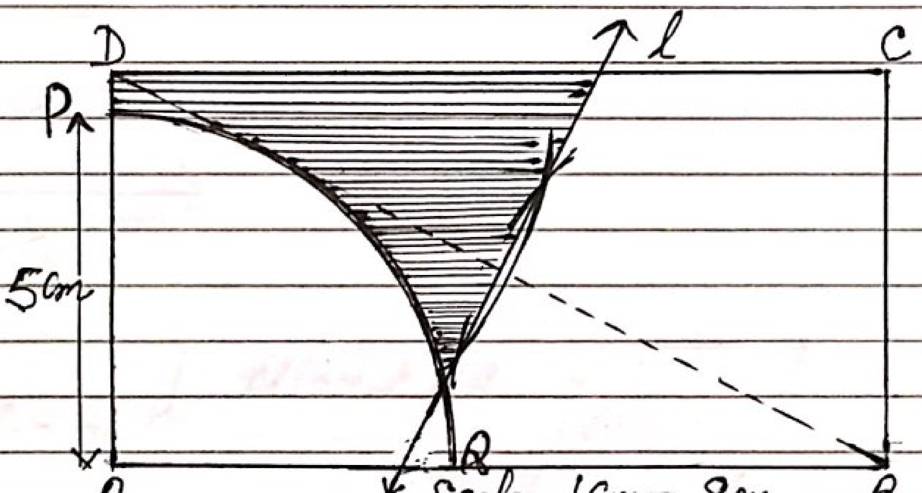
(d) more than 250 m from E is out side the circle and closer to AC is C-side of 'AS'.
Two such regions are R_1 and R_2

Example 25: The rectangle ABCD is a scale drawing of a rectangular football pitch.

(a) Construct the locus of points 40 m from A and inside the rectangle. ---[2]

(b) Using a edge and compasses only, construct the perpendicular bisector of DB. --[2]

(c) Shade the region on the football pitch which more than 40 m. from A and nearer to D than B. [S-13/22/Q19] ---[1]



Scale 1cm = 8m.

(a) Draw a circular arc $\widehat{PQ}$ with Centre at A and radium 5cm. { ∴ 40m = 5cm on drawing

[Points on arc $\widehat{PQ}$ are 40m from A. Locus theorem 4/Page 20]

(b) Draw line 'l' perp. bisector of DB, points of 'l' are equidistant from the points D and B [∵ Locus Theo-2/Page 20]

(c) Shade out side the circular arc $\widehat{PQ}$ and on D-side of perp. bisector 'l' of DB.