IG-Maths
0580

Geometry-2
Notes

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(Suresh Goel)

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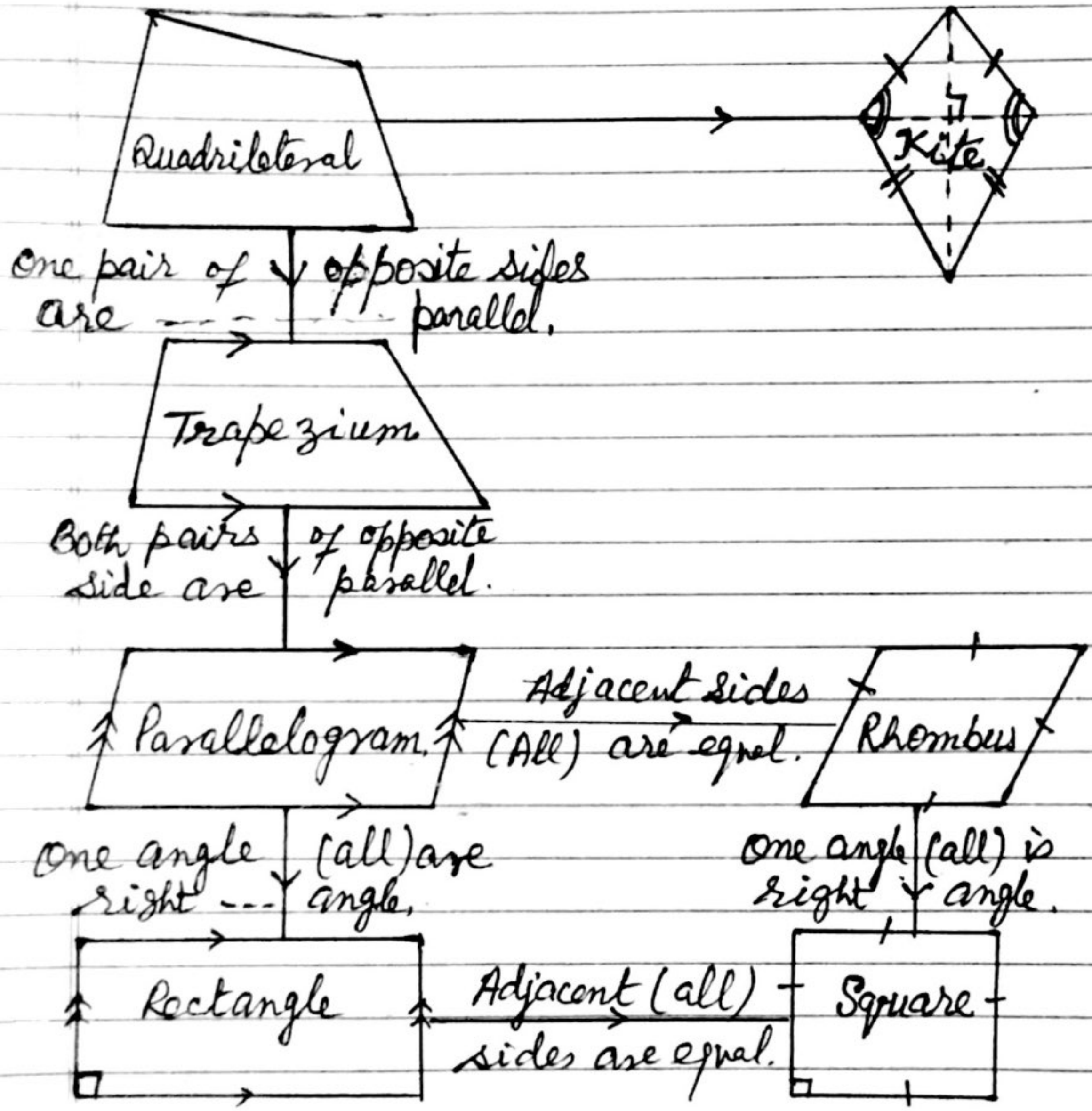
Quadrilaterals.

§17.

Types of Quadrilaterals
(Four sided closed geometric figures)



Quadrilateral: A plane figure ABCD, bounded by four line segment is called a quadrilateral.

(i) **Adjacent side:** AD and AB with a common vertex A.

(ii) Opposite sides: Two pairs: AD & BC, AB & CD

(iii) **Adjacent Angles:** $\angle A$ and $\angle B$ with a common side. (or $\angle B$ & $\angle C$) ...

(iv) **Opposite angle:** $\angle A$ and $\angle C$, $\angle B$ and $\angle D$.

(v) **Diagonals:** AC and BD

(vi) **Angle Sum Property:**
$\angle A + \angle B + \angle C + \angle D = 360^\circ$

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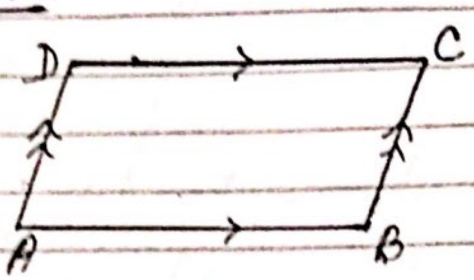
Quadrilaterals

Geometry — Date P-26 Page No.

§17. Various Types of Quadrilaterals:

(a) Parallelogram:

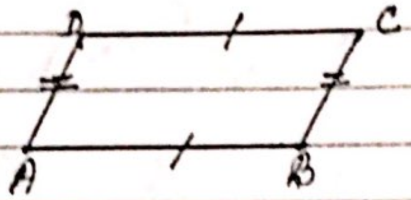
Def: A quad. in which both pairs of opposite sides are parallel.



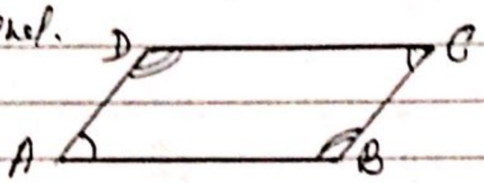
AB ∥ DC and AD ∥ BC.

Properties of a parallelogram:

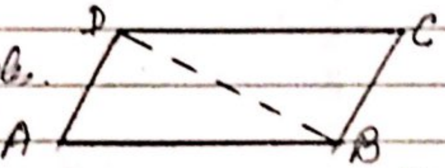
(i) Both pairs of opposite sides are equal. AB = CD and AD = BC.



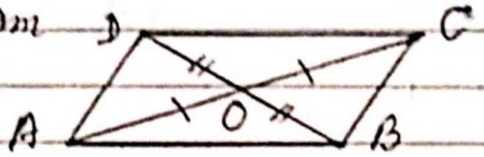
(ii) Both pairs of opp. angles are equal.
$\angle A = \angle C$ and $\angle B = \angle D$.



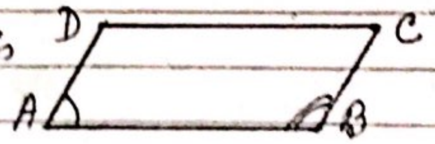
(iii) Each diagonal of a parallelogram divides it into two congruent triangles.
Triangle DAB ≅ Triangle BCA



(iv) The diagonals of a parallelogram bisect each other.
OA = OC and OB = OD.



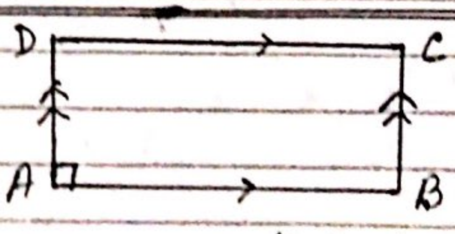
(V) Sum of two consecutive angles is 180°.



$\angle A + \angle B = 180°$

(b) Rectangle:

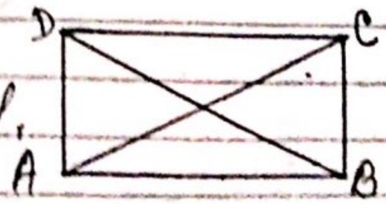
Rectangle is a parallelogram whose one angle (hence all) is right angle 90°.



∴ (i) All the properties of parallelogram are satisfied by a rectangle. Extra properties

(ii) One angle (all) is 90°

(iii) Diagonals of a rectangle are equal.
AC = BD



Quadrilaterals. Geometry

§17. **Various types of Quadrilateral:**

(C) **Rhombus**: Rhombus is a paralleogram whos adjacent (all) sides are equal.

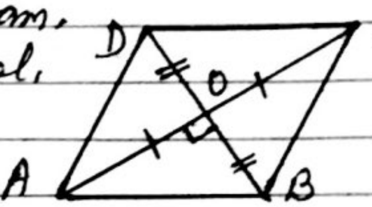
(i) All the properterties of parallel gram.

(ii) Adjacent (all) sides are equal.

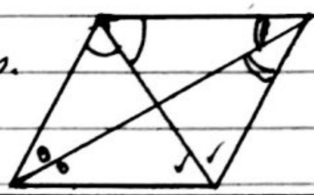
(iii) Diagonals bisect at right angle.

OA = OC and OB = OD.

angle AOB = 90°.



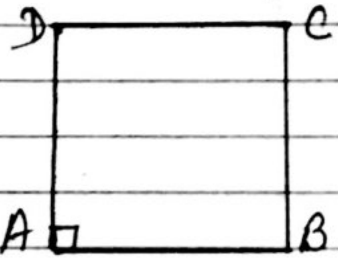
(iv) Diagonals bisect vertial angles.



(d) **Square**: It a rhombus with one (all) angle right angle.

Or

It is a rectangle with adjacent (all) sides equal.



(i) It has all the prop. of parallelogram, rectangle and rhombus.

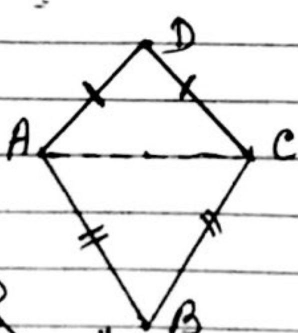
∴ • all angles are righ angles.

• all sides are equal / diagonal are equal.

• diagonals bisect eachother at right angle.

(e) **Kite**: Kite has two isosceles triangles of the opposite sides of a diagonal.

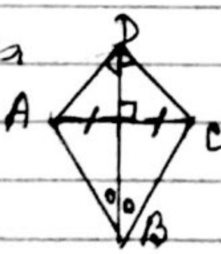
AD = DC and BA = BC.



(i) Diagonal DB is the perp. bisector of the diagonal AC.

(ii) BD bisects the vertical angles ∠B and ∠D.

(iii) △ ADB ≅ △CDB congruent.



(iv) One pair of opposite angles are equal. ∠DAC = ∠DCB.

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Geometry

Quadrilaterals and Polygons

Date P-28

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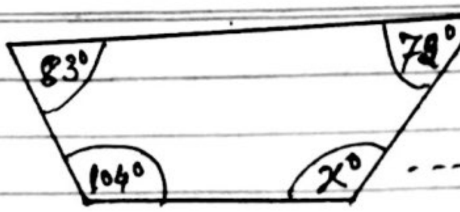
Example 26: The diagram shows a quadrilateral. Find the value of x. ---[1]



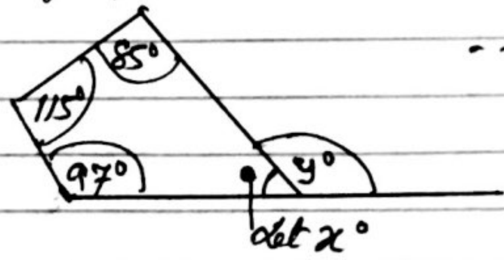
Solution: $x^\circ + 104 + 83 + 72 = 360^\circ$

$\therefore\ x = 101^\circ$ ✓

W-17/21/Q1

(Sum of Interior angles of a quad. is 360°)

Example 27 Find the value of y. ---[2]



Solution $x + 97^\circ + 115^\circ + 85^\circ = 360^\circ$

$\therefore\ x = 63^\circ$

Now $x + y^\circ = 180^\circ$

or $63 + y^\circ = 180$

$\therefore\ y = 117^\circ$ ✓

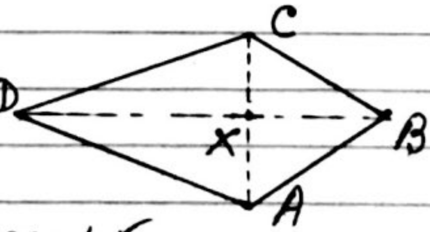
M-16/22/Q18(b)

Example 28: ABCD is a kite, the diagonals AC and BD intersect at X. AC = 12 cm, BD = 20 cm, and DX : XB = 3 : 2

(a) Calculate angle ABC ---[3]

(b) Calculate the area of the kite. ---[2]

W-13/21/Q21



Solution: $DX : XB = 3 : 2$

Let $DX = 3x$

and $XB = 2x$

$\therefore\ DB = DX + XB = 20$

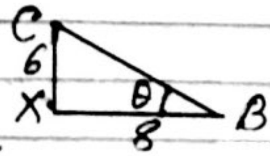
$= 3x + 2x = 20$

or $5x = 20 \rightarrow \therefore\ x = 4$ ✓

$\therefore\ XB = 2x = 2 \times 4 = 8$ ✓ cm

$\therefore\ CX = \frac{AC}{2} = \frac{12}{2} = 6\text{ cm}$ ($\because$ BD is perp. bisector of AC)

(a) $\tan\theta = 6/8 = 3/4$ (Let angle $CBX = \theta$)



$\therefore\ \theta = 36.87^\circ$

$\therefore\ \angle ABC = 2\theta = 2 \times 36.87 = \underline{73.74^\circ}$ ✓

(b) Area of kite = 2 × area of triangle DCB

$= \cancel{2} \times \frac{1}{\cancel{2}} \times DB \times CX$

$= 20 \times 6 = 120$ sq. cm. ✓

Geometry

Quadrilaterals and Polygons.

Example 29: In the hexagon ABCDEF, AB parallel to ED, and AF is parallel to CD. angle ABC = 90°, Angle CDE = 140° and angle DEF = 120°, Calculate angle EFA. ---[4]

S-15 | 43 | Q6(a)

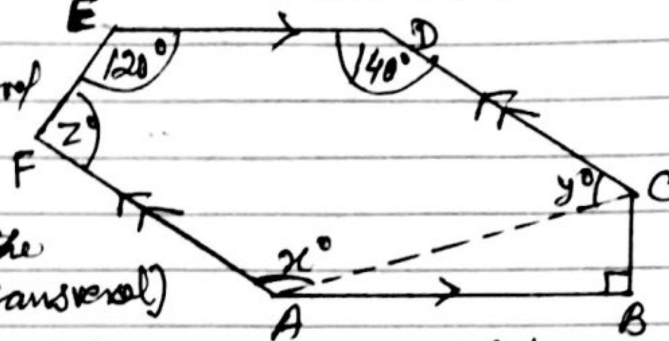
Solution: To find $z°$.

Join AC.

AF // CD cut by transversal AC.

$\angle x + \angle y = 180$ --- (i)

(Int. angles on the same side of transversal)



Now $x° + y° + 140° + 120° + z° = 540$

$180° + 140 + 120° + z° = 540$

$\therefore z° = 100°$

$\therefore$ Required angle EFA = 100° ✓

[Sum of Interior angles of n-sided poly = $(n-2) \times 180°$

In Pentagon $(n=5)$

Sum = $(5-2) \times 180° = 540°$]

Example 30 ABCDEF is a hexagon. AB is parallel to ED and BC is parallel to FE. YFE and YABX are straight lines. Angle CBX = 32° and angle EFA = 90°.

S-14 | 43 | Q8(a)

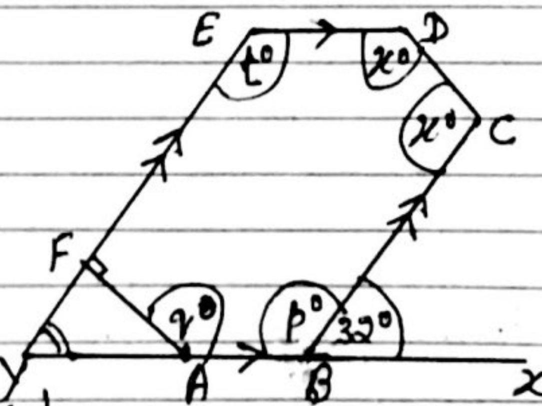
Calculate the value of:

(i) p, ---[1]

(ii) q, [2]

(iii) t, [1]

(iv) x [3]



Solution: (i) $p + 32 = 180°$ (Linear Pairs)

$\therefore p = 148°$ ✓

(ii) $\angle y = 32°$ [YE // BC, corresponding angles]

In ΔFAY

$q = y + F = 32° + 90 = 122°$ ✓ [$\because \angle F = 90°$]

(iii) $t + \angle y = 180$ [YAB // ED, Int. angle on the same side of YE]

$t + 32 = 180°$

$t = 148°$ ✓

(iv) Sum of Int. angles $= (n-2) \cdot 180°$

for hexagon $n = 6$

Sum $= (6-2) \cdot 180 = 720°$

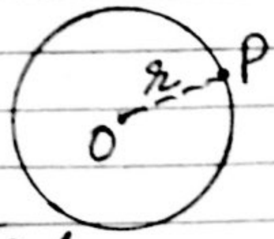
$q + p + x + x + t + 90 = 720$

$122° + 148 + 2x + 148 + 90 = 720$

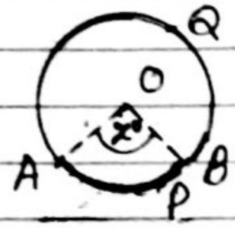
$\therefore x = 106°$ ✓

Geometry – Circles

§ 18. **Circle:** Circle is the locus of point 'P' which is at a constant distance 'r' from a fixed point 'O'.
Fixed point 'O' is the centre and the constant distance 'r' is the radius of the circle.

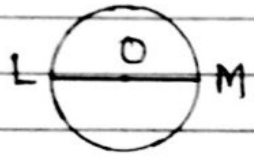


1 (i) **Arc of a circle:** If A and B are any two point of circle, then part of circle.
$\widehat{APB}$ is Minor arc.
and $\widehat{AQB}$ is the Major arc.
measure of minor arc $m\widehat{APB} = x^\circ < 180^\circ$ and $m\widehat{AQB} = (360-x)$

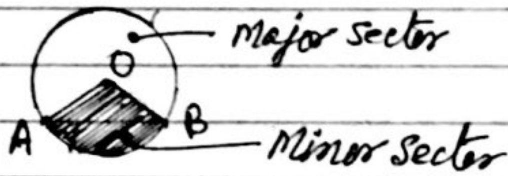


(ii) **Chord $\overline{AB}$:** Segment AB

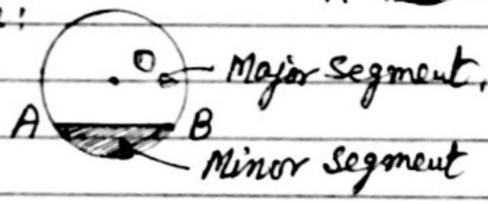
(iii) **Diameter:**
Chord LM passing through the centre.



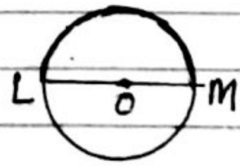
(iv) **Sector:**



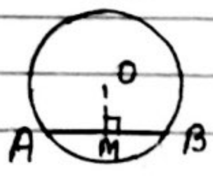
(V) **Segment of a circle:**



(VI) **Semi circle:**
Arc $\widehat{LQM}$
LM is diameter.

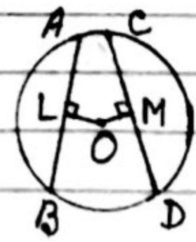


2. **Properties of a Circle:**
(i) Perpendicular from the centre to a chord bisects the chord. "OM perp AB"
Then AM = MB.



Conversely: segment joining centre to mid point 'M' of chord, is perp. to chord.

(ii) OL perp AB and $OM \perp CD$.
Equal chords are equidistant from the centre
i.e. $AB = CD$, Then $OL = OM$
Conversely: If $OL = OM$ then $AB = CD$



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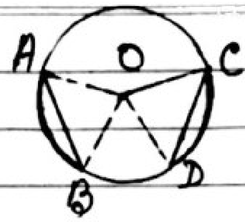
Geometry P-31

Circles

Properties of Circles:

2 (iii) If two arc are equal, then the corresponding chords are equal.



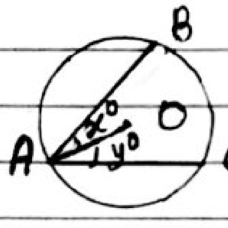
$\widehat{AB} = \widehat{CD} \Rightarrow \overline{AB} = \overline{CD}$

and conversely $\overline{AB} = \overline{CD} \Rightarrow$ arc $\widehat{AB} = \widehat{CD}$

(iv) Given AB and AC are equal. then AO is the bisector of angle BAC.

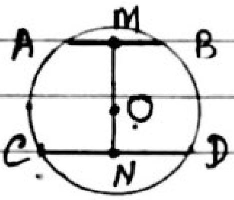
$AB = AC \Rightarrow x^\circ = y^\circ$

and conversely.



(v) $\overline{AB} \parallel \overline{CD}$ line joining their mid-points passes through centre.

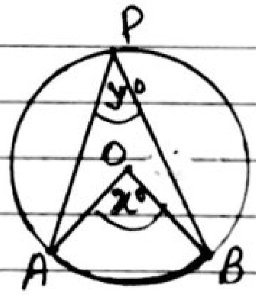
M is the mid point of AB and N is the mid point of CD. Then MN passes through the centre O.



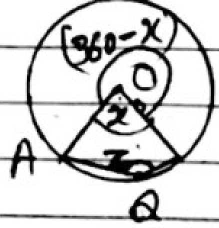
(vi) The angle subtended by an arc $\widehat{AB}$

(a) at the centre x° is double the angle subtended by it any (P) point on the remaining part of the circle y°.

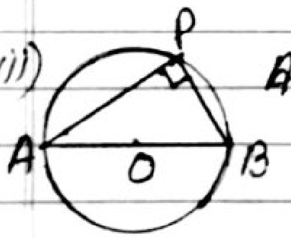
$\therefore x^\circ = 2y^\circ$ or $y = \frac{1}{2}x$



(b) $z^\circ = \frac{1}{2}(360 - x^\circ)$



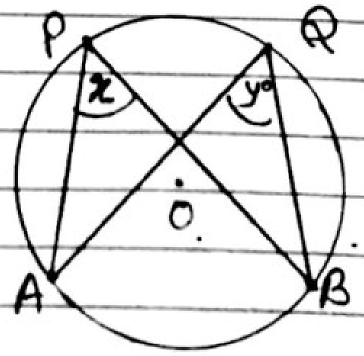
(vii)



Angle in semicircle is a right angle

$\angle APB = 90^\circ$ and conversely.

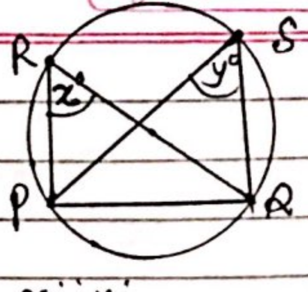
(viii) Angles in the same segment of a circle are equal $\angle x = \angle y$.



Circles

Properties of Circles:

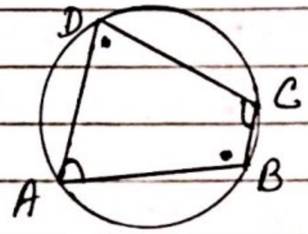
2(IX) (Converse of (viii))



If a line segment $\overline{PQ}$ subtends equal angles at two other points 'R' and 'S' lying on the same side of it, then the four points are concyclic.

(Given $x = y$ Then four points PQRS lie on the same circle.)

(X) The opposite angles of a cyclic quadrilateral are supplementary.

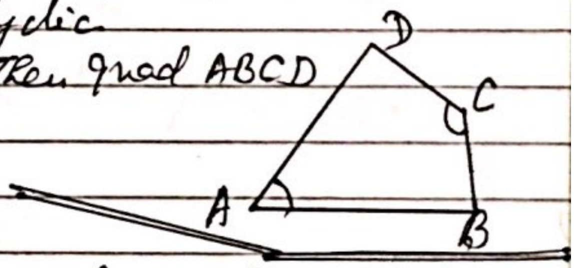


Given a cyclic quad ABCD (vertices of quad. lie on a circle) Then.

$\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$

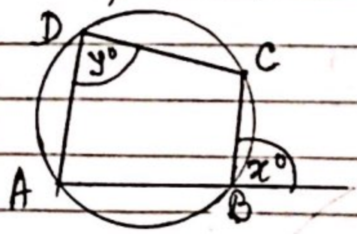
$\boxed{XI}$. Conversely: If one pair of opp. angles of a quad. ABCD are supplementary, the quad is cyclic.

∵ if $\angle A + \angle C = 180$ (or $\angle B + \angle D = 180$) Then quad ABCD is cyclic.



(xii) If one side of a cyclic quad. is produced then the exterior angle so formed is equal to the opposite interior angle.

$\angle x = \angle y$

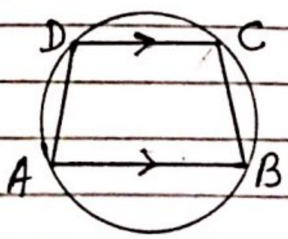


(xiii) If two sides of a cyclic quad (ABCD) are parallel (AB ∥ DC)

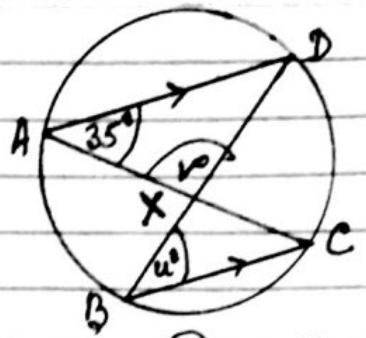
Then:

Remaing two sides are equal $AD = BC$

and diagonals are equal $AC = BD$



Example 3 (a) A, B, C and D are the point on the circle. AD is parallel to BC. The chords AC and BD intersect at X.
Find the value of u and the value of v. ---[3]



Solution: $u = 35°$ ✓ ---(i) [∵ angles subtended by arc $\widehat{DC}$ on the remaining part of circle are equal] §18·2(VII)

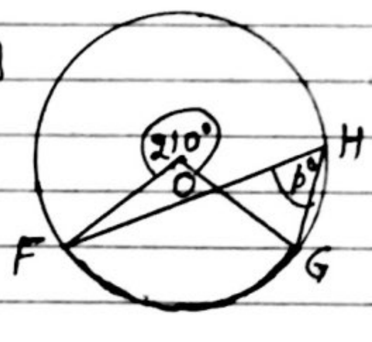
Now angle $ACB = 35°$ [AD // BC cut by transversal AC] Alt. interior angles.
or $\angle XCB = 35$ ---(ii)

Now angle $AXB = u + \angle XCB$ [Exterior angle of ΔXBC is equal to sum of opp. int. angles]
$= 35° + 35°$
angle $AXB = 70°$ ---(iii)

Now $v° + \angle AXB = 180$ (linear pairs, angles on a line)
$v + 70 = 180°$ from (iii)
or $v = 110°$ ✓

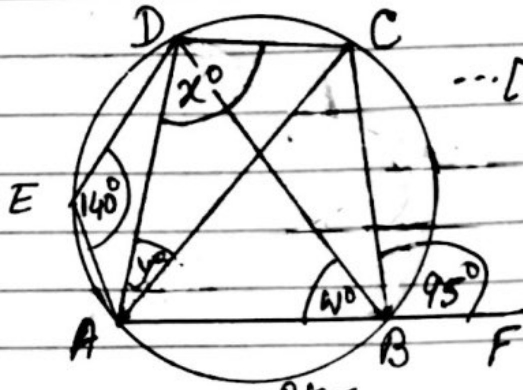
[S-17/21/Q21]

(b) F, G and F are points on a circle, Centre O. Find the value of p. ---[2]



Solution: $\angle FOG + 210° = 360°$ (Angles at a point)
∴ $\angle FOG = 150°$ ---(i)
$\angle FOG = 2\angle FHG$
or $\angle FHG = \frac{1}{2}$ angle FOG
or $p = \frac{1}{2} \times 150°$ (from i) [∵ Angle subtends by an arc $\widehat{FG}$ at the centre is double the angle FHG, subtended by it in the remain part of the circle). §18·2(VI)(a)
or $p = 75°$

Example 32: A, B, C, D and E lie on a circle. AB is extended to F. angle AED = 140° and angle CBF = 95°. Find the value of w, x and y. ---[5]



S-17/22/Q26

Solution:

$x = 95°$ ✓ [Cyclic quad. ABCD, AB is ext. to F. ∴ Exterior angle CBF = $x°$ opp. int angle, Refer. §18-2(XII)]

$\angle w + 140 = 180$ [opp. angles of cyclic quad ABDE, §18-2(X)]

$\angle w = 40°$ ✓

Now $\angle ABC + \angle CBF = 180°$ [Linear pair or angles on a line]

$w + \angle DBC + 95 = 180$

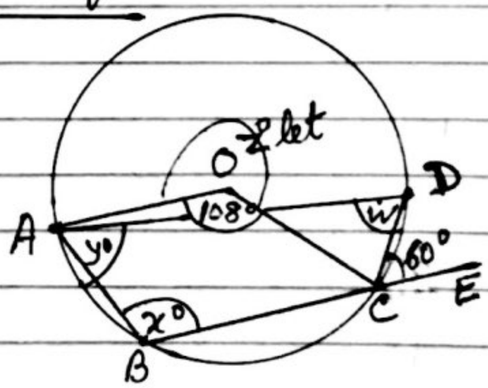
$40 + \angle DBC + 95 = 180$ $(w = 40°)$

$\angle DBC = 45°$ --- (i)

$\angle y = \angle DBC$ [Angles in the same segment of circle. or angle subtended by $\overset{\frown}{DC}$]

$= 45°$ ✓

Example 33: A, B, C, D are points on the circle, centre O. BCE is a straight line. angle ADC = 108° and angle DCE = 60°. Calculate the values of w, x and y. ---[3]



W-17/22/Q22

Solution: $\angle z + 108 = 360°$

$\therefore z = 252$; $x = \frac{1}{2}\angle z = \frac{1}{2} \times 252 = 126$ ✓

Cyclic quad DABC, BC is produced

Ext angle DCE = opp. int angle

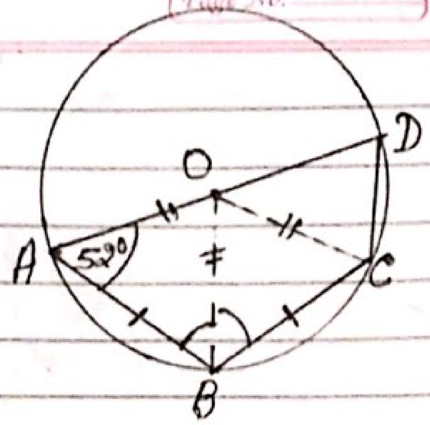
$60° = y° \Rightarrow y = 60°$ ✓ [∵ §18-2(XII) Page-32]

For cyclic quad. ABCD, $x + w = 180$ [∵ §18-2(X) Page-32]

$126 + w = 180$

$w = 54°$ ✓

Circles and cyclic quadrilaterals



Example 34 (b). The diagram shows points A, B, C and D on the circumference of a circle, centre O. AD is a straight line. AB = BC angle OAB = 52°. Find angle ADC ---[3]

Solution: Join OB and OC,

$OA = OB = OC =$ radius ---(i)

In ΔOAB, $\angle OBA = \angle OAB = 52$ (isosceles Δ)

Now $\Delta OBC \cong \Delta OAB$ (SSS)

$\therefore \angle OBC = \angle OAB = 52$ --(ii)

$\therefore \angle ABC = \angle ABO + \angle OBC$ (from (i) & ii)

$= 52 + 52 = 104°$ ... (iii)

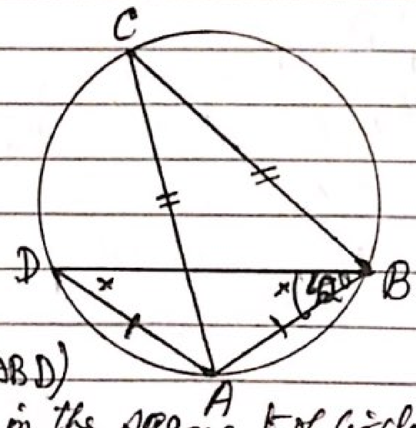
Now in cyclic quad ABCD,

$\angle ADC + \angle ABC = 180$ (opp. angle of cyclic quad) [Refer: §18-2(X)]

$\angle ADC + 104 = 180$ [from iii]

$\therefore \angle ADC = 76°$ ✓

$\boxed{M-17/42/Q6(b)}$



Example 35. The points A, B, C and D lie on the circumference of a circle. AB = AD and AC = BC and angle ABD = 42° ---[3] Find angle CAB.

$\boxed{S-17/43/Q2(b)}$

Solution: $\angle ADB = \angle ABD = 42°$ (In Iso ΔABD)

Now $\angle ACB = \angle ADB = 42°$ --(1) (Angles in the segment of circle) §18-2 (viii)

Now In isosceles ΔCAB.

$\angle CAB = \angle CBA = x$ (let)

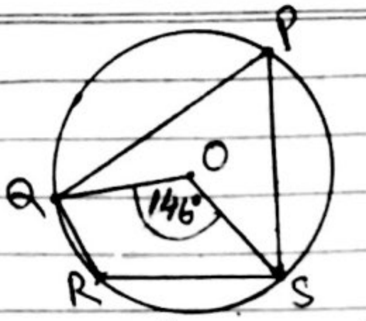
$\therefore$ Angle ACB + angle CAB + angle CBA = 180 (Angle sum prop.)

$42° + x + x = 180°$

$2x = 138°$ $\Rightarrow x = 69°$ or angle CAB = 69° ✓

Example 36: The points P, Q, R and S lie on the circumference of the circle, centre O. Angle QOS = 146°; find angle QRS. ---[2]

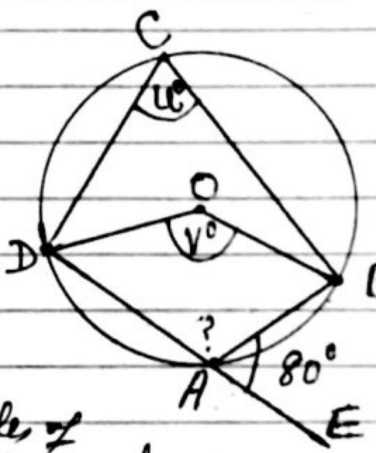
S-17/43/Q2(C)



Solution: angle QPS = $\frac{1}{2}$ angle QOS = $\frac{1}{2} \times 146° = 73°$ [§18-2(VI)]

In cyclic quad. angle QRS + angle QPS = 180 [opp. angles of a cyclic quad]
or angle QRS + 73° = 180°
or angle QRS = 107° ✓

Example 37(a) A, B, C and D lie on the circle, centre O. DAE is a straight line. Find the value of u and the value of v ---[2]

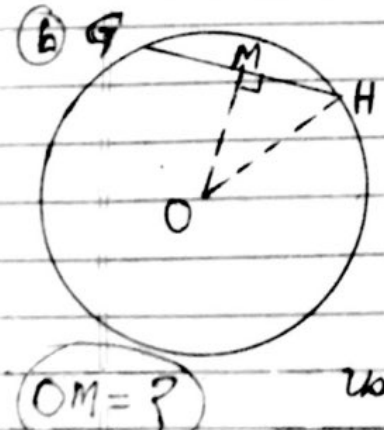


Solution: angle DAB + ∠BAE = 180° [∵ linear pair]
∴ angle DAB = 100 —— (1)
Now In cyclic quad ABCD.
∠DAB + ∠DCB = 180 [opp. angles of a cyclic quad are supplementary)
100 + u = 180 from (1)
u = 80° ✓

Now V = 2u° [angle at the centre subtended by arc $\widehat{DAB}$ is double the angle at the remaining circle]
or V = 2×8
V = 160° ✓

W-16/42/Q8

(b)



The diagram shows a circle, centre O, radius 8cm, GH is a chord of length 10cm, Calculate the length of the perpendicular from O to GH. ---[3]

Solution Draw OM perp. GH and join OH,
MH = $\frac{1}{2}$ GH = $\frac{1}{2} \times 10$ = 5cm (∵ perp. from centre to a chord bisects the chord).
and OH = radius = 8cm

OM = ?

Using Pythagoras theo. $OM^2 + MH^2 = OH^2 \Rightarrow OM^2 + 5^2 = 8^2$
$OM^2 = 39$
∴ OM = 6.24

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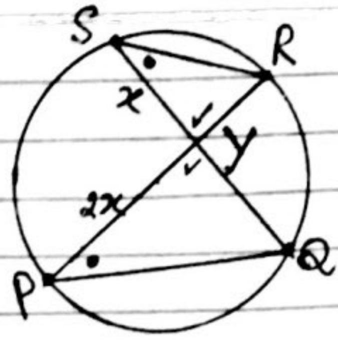
Circles and Cyclic Quad. Geometry

P-37

Example 37(d) All lengths are in centimetres.



P, Q, R and S lie on the circle. PR and QS intersect at Y. PY = 2x and YS = x

The area of triangle YRS = $\frac{5}{12}x(x-1)$

The area of triangle YQP = $x(x+1)$

Find the value of x. --[4]

Solution: Triangle YSR ~ Triangle YPQ (Similar)

(AA Similarity: angle Y - Vertically opp. angle YSR = angle YPQ angles in the same segment of circle)

$\therefore \dfrac{\text{Area of Triangle YSR}}{\text{Area of Triangle YPQ}} = \left(\dfrac{YS}{YP}\right)^2$

[Areas of similar triangles are proportional to ratio of sq. of corresponding sides]

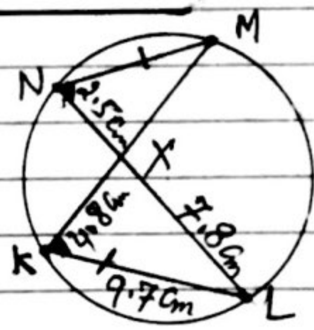
or $\dfrac{\frac{5}{12}x(x-1)}{x(x+1)} = \left(\dfrac{x}{2x}\right)^2$

$\dfrac{5(x-1)}{12(x+1)} = \dfrac{1}{4} \Rightarrow 20(x-1) = 12(x+1)$

$\Rightarrow 8x = 32$

$\Rightarrow x = 4$ ✓

(c) K, L, M and N lie on a circle.



KM and NL intersect at X,

KL = 9.7 cm, KX = 4.8 cm,

LX = 7.8 cm, NX = 2.5 cm,

Calculate MN. ---[2]

Solution: Triangle MNX ~ (Similar) Triangle LKX

Corresponding sides are in the same ratio:

[∵ AA Similarity theo. angle X, vertically opp. angles. angle MNX = angle LKX angles in the same segment of a circle]

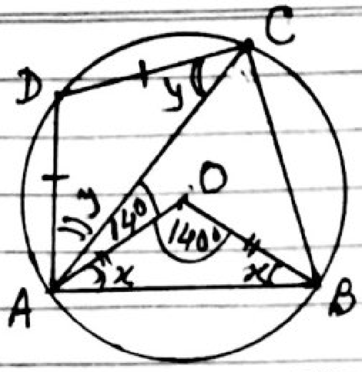
$\dfrac{MN}{KL} = \dfrac{NX}{KX}$

or $\dfrac{MN}{9.7} = \dfrac{2.5}{4.8} \Rightarrow MN = \dfrac{2.5}{4.8} \times 9.7 = 5.05\,cm$ ✓

[W-16/42/Q8]

Circles and Cyclic Quadrilaterals

Example 38: A, B, C and D are points on the circle, centre O.
Angle AOB = 140° and angle OAC = 14°.
AD = DC.
Calculate angle ACD. ----[5]



S-14/42/Q6(C)

Solution: In ΔOAB, $OA = OB$ = radius

$\therefore \angle OAB = \angle OBA = x$ (let)

$\therefore 140° + x + x = 180$ (In ΔOAB, angle sum)

$\therefore x = 20°$

$\therefore \angle CAB = 14 + x = 14 + 20 = 34°$ ---(i)

$\angle ACB = \frac{1}{2} \times 140 = 70°$ ---(ii) (Angle at the centre is double the angle at the remaing circle)

In ΔABC

$\angle ABC + \angle CAB + \angle ACB = 180$

$\angle ABC + 34° + 70° = 180$ fm ① & ②

$\therefore \angle ABC = 76$ ---(iii)

In cyclic quad. ABCD, opp. angles are supplementary,

$\therefore \angle ADC + \angle ABC = 180$

$\angle ADC + 76 = 180$ fm (iii)

$\angle ADC = 104$ ---(iv)

In ΔADC, $AD = AC$

$\angle DAC = \angle DCA = y$ (let) (isosceles triangle theo)

Now In ΔADC

$\angle ADC + \angle DAC + \angle DCA = 180$ (Angle sum prop)

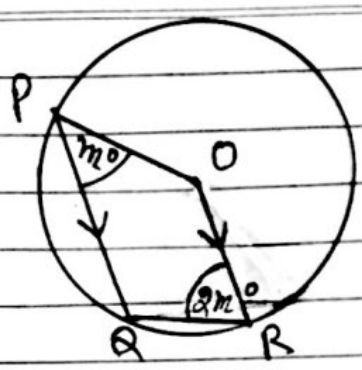
$104 + y + y = 180$ fm (iv)

$2y = 76$

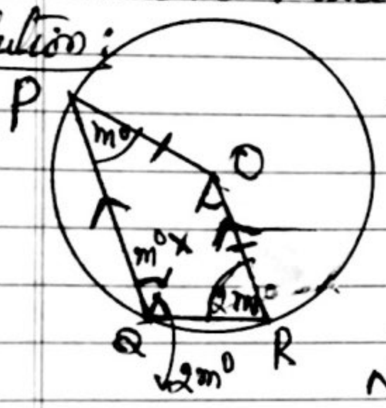
$y = 38$

$\therefore \angle ACD = 38°$ ✓

Example 39: In the diagram, P, Q and R lie on the circle, centre O. PQ is parallel to OR. Find the value of m. ---[5]



Solution:



[W-15/43/Q8(c)]

Join OQ.

OP = OQ = radius.

$\therefore$ angle OQP $= m^\circ$ --- (i)

angle QOR $= \angle OQP$ (Alt angle, PQ ∥ QR cut by OQ)

$\therefore$ angle QOR $= m^\circ$ --- (ii)

Now in isosceles Triangle OQR (QO = RO = r)

$\angle OQR = \angle ORQ = 2m$ (given)

$\angle OQR = 2m^\circ$ --- (iii)

Now in Triangle OQR

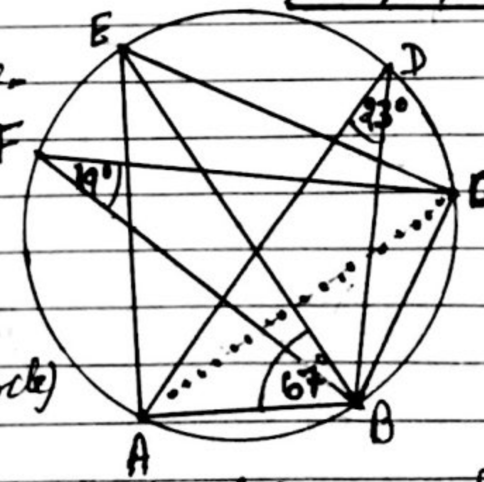
angle QOR + angle OQR + angle ORQ $= 180^\circ$ {angle sum Prop. in Triangle}

or $m + 2m + 2m = 180$ (from (ii) & (iii))

or $5m = 180^\circ \Rightarrow m = 36^\circ$ ✓

[W-17/23/Q22]

Example 40. In the diagram, points A, B, C, D, E and F lie on the circumference of the circle, angle BFC $= 19^\circ$ and angle ADB $= 23^\circ$ and angle ABE $= 67^\circ$



Work out:

(a) angle BEC ---[1]

Solution: angle BEC = angle ADB ($\because$ angles in the same segment of circle)

$= 23^\circ$ ✓

(b) work out angle ABC. ---[3]

Solution: angle ABC + angle AEC = 180 --- (i) opp. angles of a cyclic quad ABCE.

Now angle AEC = $\angle AEB + \angle BEC$

$= \angle ADB + \angle BFC$ (same seg of circle)

$= 23^\circ + 19^\circ$

$= 42^\circ$ --- (ii)

from (i) angle ABC + 42 = 180 fr (ii)

$\therefore$ angle ABC = $\underline{138^\circ}$ ✓

(c) work out angle BCE ---[2]

Join AC.

angle BCE = angle BCA + $\angle ACE$

= angle ADB + $\angle EBA$

$= 23^\circ + 67^\circ$

$= 90^\circ$ ✓

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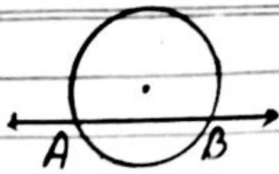
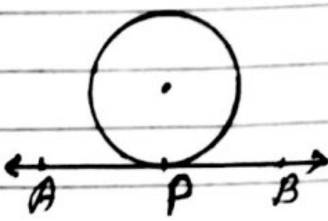
Geometry P-40

Circle and Tangent.

§19. (i) Secant of circle: A line which intersects a circle in two distinct points.



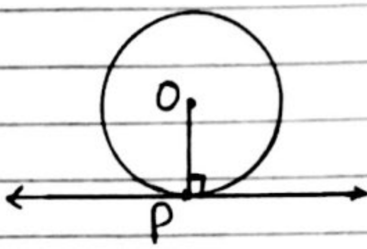
(ii) Tangent line: A line which intersect the circle at exact (one and only one) one point. 'P' and,
The point 'P' is called the point of contact.



§20. Properties of a tangent to a circle;

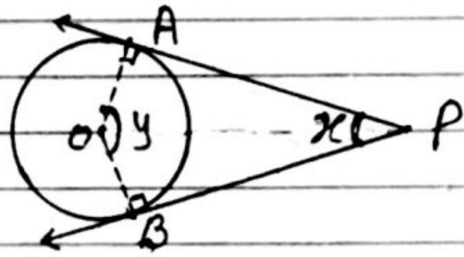
(i) Tangent to a circle is perpendicular to the radius the point of contact.

$OP \perp AB.$



(ii) Conversely; A line drawn perpendicular to end point 'P' of a radius is tangent to the circle.

(iii) The lengths of two tangents (PA and PB) drawn from an external point 'P' are equal.



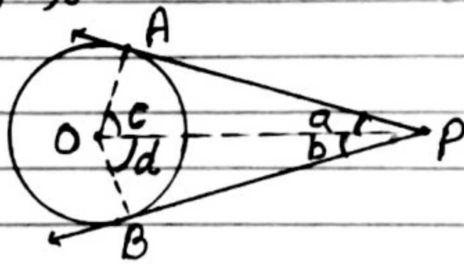
(a) $PA = PB$

(b) angle $OAP = 90°$ and angle $OBP = 90°$

(c) $OA^2 + AP^2 = OP^2$

(d) angle x + angle $y = 180$

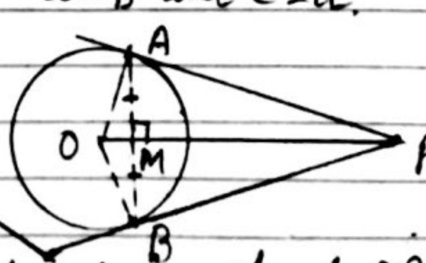
(iv) PA and PB are tangents drawn from an external point 'P' to a circle, centre O. Then line OP bisect angles APB and angle AOB; $a = b$ and $c = d$.



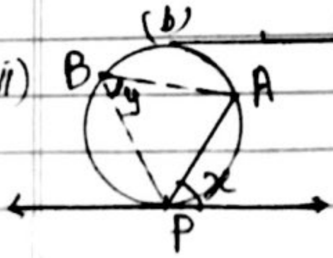
(v) OP is the perp. bisector of AB.

(a) $AM = BM$

(b) $AB \perp OP$.



(vi)



$\angle x = \angle y$

Angle between the tangent and a chord AP is equal to the angle in the alternate segment of circle.

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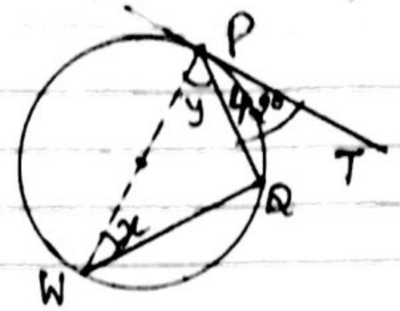
Tangents to a Circle

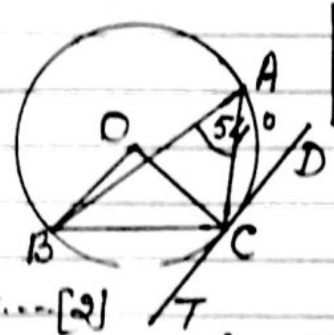
Example 41: In the diagram, PT is tangent to the circle at P. PW is a diameter and angle TPQ = 42°. ----[2]
Find angle PWQ. $\boxed{W-16/21/Q6}$



Solution $\angle PWQ = \angle QPT$ (angle between the tangent and chord is equal to the angle in alternate segment of a circle)
$= 42°$

$x + y = 90$ --(i)
$\because \angle Q = 90°$
$y + 42 = 90$ --(ii)
from (i) & (ii)

Example 42: A, B and C are the points on a circle, centre O. TCD is a tangent to a circle. Angle BAC = 54°



(a) Find angle BOC, giving a reason for your answer....[2]

Solution: angle BOC = 2.angle BAC
$= 2 \times 54 = 108°$ ✓ ($\because$ angle subtended by an arc at the centre is double the angle at the remaining part of the circle).

(b) When O is the origin, the position vector of C is $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$. --[1]
Workout the gradient of radius OC.

Solution: C (3, -4). Gradient of OC $(m_1) = \frac{y}{x} = \frac{-4}{3}$ ✓ --- (I)

(c) D is point (7, K), find the value of K.

Solution: Given C(3, -4) and D(7, K).

Gradient of CD, $m_2 = \frac{(K+4)}{(7-3)} = \frac{K+4}{4}$ --- (II)

Now tangent CD is perp to radius OC.

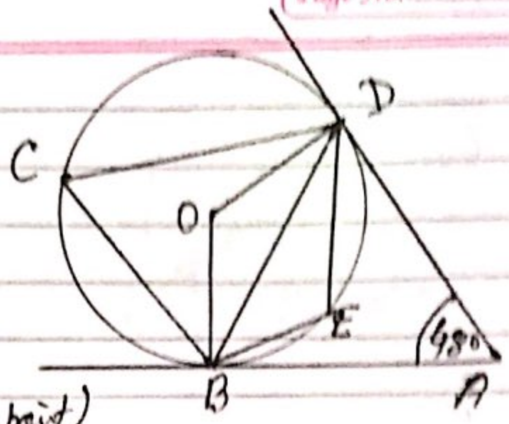
$m_1 \times m_2 = -1$

$-\frac{4}{3} \times \left(\frac{K+4}{4}\right) = -1$

or $K + 4 = +3$

$K = -1$ ✓

Example 43. In diagram, B, C, D and E lie on the circle, centre O. AB and AD are tangents to the circle. Angle BAD = 48°



(a) (i) Find angle ABD ---[1]

Solution: AB = AD (tangents from ext point)

In ΔABD, $\angle ABD = \angle ADB = x$

Now In ΔABD $\angle ABD + \angle ADB + \angle BAD = 180$ (angle sum of triangle)

$x + x + 48 = 180$

$2x = 132 \Rightarrow x = 66°$ ✓

$\therefore \angle ABD = 66°$ ✓

(iii) Find angle BCD, ---[1]

Solution: $\angle BOD + \angle BAD = 180°$ [∵ Property §20 (iii)d]

$\angle BOD + 48° = 180°$

$\angle BOD = 132°$ —— ①

Now required angle BCD = $\frac{1}{2}$ angle BOD {angle at the centre is double the angle at the remaining circle

$= \frac{1}{2} \times 132°$ fr①

$= 66°$ ✓ —— ②

(ii) Find angle OBD --[2]

In isosceles ΔOBD (OB = OD = radius)

$\therefore \angle OBD = \angle ODB = y$ let

$\therefore \Delta OBD$, $\angle BOD + \angle OBD + \angle ODB = 180°$

$132° + y + y = 180$ {$\angle BOD = 132°$ fr①

$\Rightarrow y = 24°$ ✓ $= \angle OBD$

(iii) To find angle BED ----[1]

angle BED + $\angle BCD = 180°$ [opp. angles of a cyclic quad BCDE

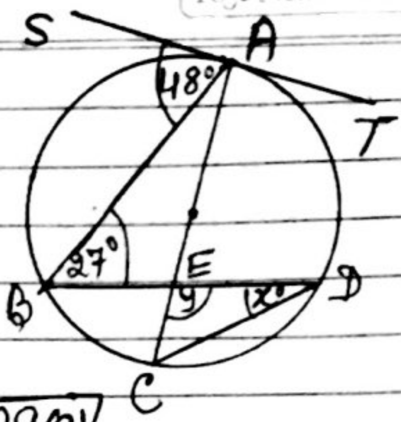
angle BED + 66° = 180° fr② angle BCD = 66°

angle BED = 114° ✓

[S-15/42/Q2(a)].

Tangent to a circle

Example 44: The points A, B, C and D lie on a circle, AC is a diameter of the circle. ST is the tangent to the circle at A. Find the value of



(a) x --[2]

(b) y --[2]

S-15/41/Q9(a)

Solution (a) $\angle SAC = 90°$ [angle between radius (AC is diameter) and tangent]

or $\angle SAB + \angle BAC = 90$

or $48° + x = 90°$ [∵ $\angle BAC = \angle BDC = x$ In the same segment of circle]

∴ $x = 42°$ ✓

(b) $\angle ACD = \angle ABD$ (Same segment of circle)

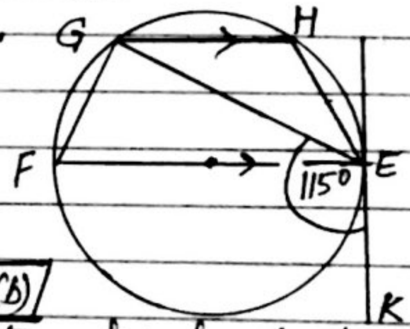
$= 27°$ —— (1)

In ΔCDE, $x + y + \angle ACD = 180°$

$42 + y + 27 = 180$ (from (1))

∴ $y = 111°$ ✓

Example 45: EFGH is a cyclic quadrilateral. EF is a diameter of the circle. KE is a tangent to the circle at E. GH is parallel to FE and angle KEG = 115°. Calculate angle GEH. --[4]



S-14/42/Q6(b)

Solution: angle GEK = angle GHE (angle between tangent and a chord is equal to the angle in the alternate circle) §20 (VI) Page-40

$115° =$ angle GHE —(1)

$\angle GEF + \angle FEK = 115$ given

$\angle GEF + 90 = 115$ [$\angle FEK = 90°$ angle between tangent and radius. §20(I)]

∴ $\angle GEF = 25°$ —(2)

$\angle HEF + \angle GHE = 180°$ [GH∥FE cut by transversal HE]

$\angle HEF + 115° = 180$ & (1)

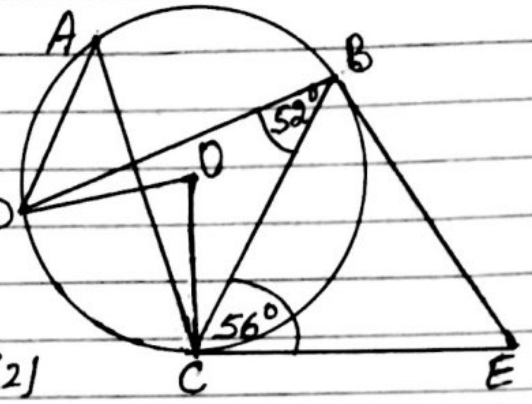
$\angle HEF = 65°$ —(3)

Now $\angle GEH + \angle GEF = \angle HEF$

$\angle GEH + 25 = 65$ from (2) & (3)

∴ $\angle GEH = 40°$ ✓

Example 46: A, B, C and D are points on a circle, centre O.
CE is tangent to the circle at C.



(a) Find the sizes of following angles and give a reason for each answer.

(i) angle DAC = --[2]

Solution: angle DAC = angle DBC (Angles in the same segment of circle)
= 52° ✓

(ii) Angle DOC = ---[2]

Solution: Angle DOC = $2 \times \angle DBC$
= $2 \times 52 = 104°$

∵ Angle subtended by arc $\widehat{DC}$ at the centre is double angle on the remaining part of the circle.

(iii) Angle BCO = --- --[2]

Solution: Radius OC is perp to the tangent at C

∴ $\angle OCE = 90°$

$\angle BCO + \angle BCE = 90°$

$\angle BCO + 56° = 90° \Rightarrow \angle BCO = 34°$ ✓

W-14/43/Q3(a)

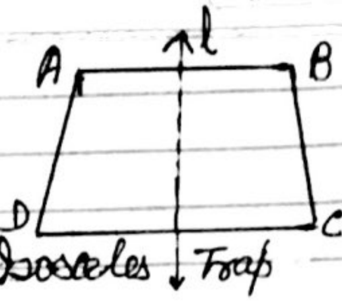
Symmetry

§20(i) **Line of Symmetry:**

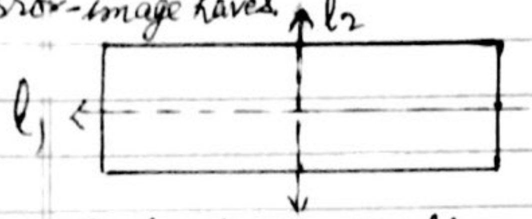
(i) A line passing through a plane shape which divides it into two congruent figures (Reflection of each other in the line) is a line of symmetry.

or

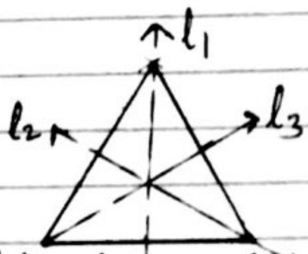
(ii) A line of Symm. divides a fig. into two mirror-image halves.



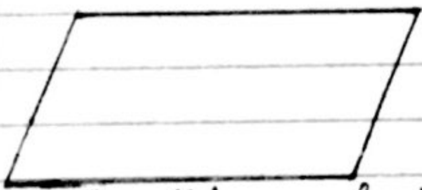
(Here 'l' is the line of symmetry of the isosceles trapezium)



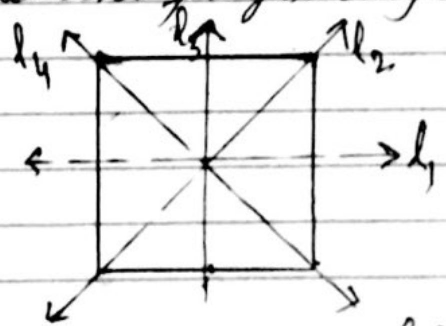
Rectangle has two lines of symmetry l_1 & l_2



Equilateral triangle has three line of symmetry l_1, l_2, l_3



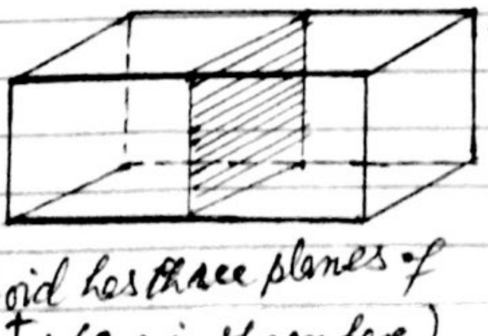
Parallelogram has 'NO' line of symmetry



A Square has for lines of symmetry.

§20 (ii) **Plane of Symmetry:**

A plane passing through a solid shape, which divides it in two congruent (refective) solid shapes.



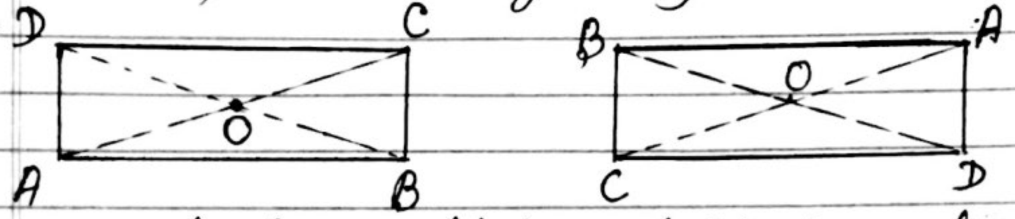
A cuboid has three planes of symmetry (One is shown here)

A sphere has infinite planes of symmetry.

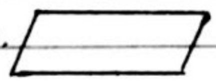
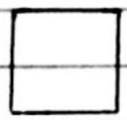
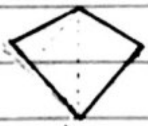
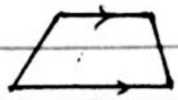
Symmetry

§20(iii) Rotational Symmetry:

If an object is rotated about a point (centre of rotation) then the image may look exactly the same, when it is rotated through 360°.
The number of times it looks the same is called **order of rotational symmetry**.

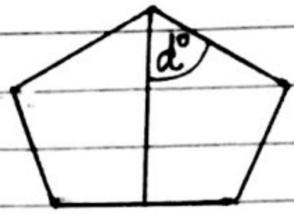


A rectangle is rotated through O (centre of rotation), the **order** of rotation is **2**.

Geometric figure		Number of lines of Symmetry	Order of Rotational Symmetry
Parallelogram.		0	2
Square		4	4
Kite		1	1
Isosceles Triangle		1	1
Trapezium		0	1
circle		Infinite	Infinite
I		2	2
N.		0	2

Symmetry

Example 47. The diagram shows a regular pentagon. AB is a line of symmetry. Work out the value of d. ---[3] [W-17/21/Q11]



Solution: Given a line of symmetry,

$$d = \frac{\text{Interior angle}}{2} \quad ---(i)$$

Interior angle of n-sided regular polygon

$$= \frac{(n-2)\cdot 180^\circ}{n}$$

for $n=5$,

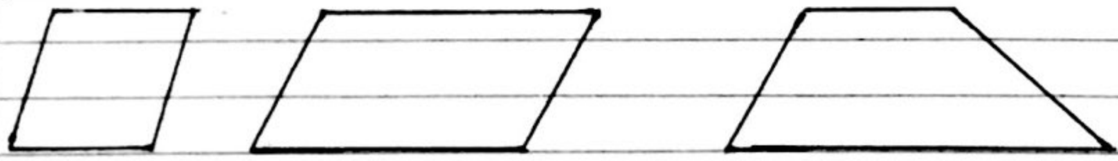
$$= \frac{(5-2)\times 180^\circ}{5} = 108^\circ$$

from (i) $d = \frac{1}{2}\times 108 = 54^\circ$ ✓

Example 48: A quadrilateral has a rotational symmetry of order 2 and no line of symmetry. Write down the mathematical name of the quadrilateral. [S-16/22/Q4] ---[1]

Solution: Parallelogram.

Example 49:

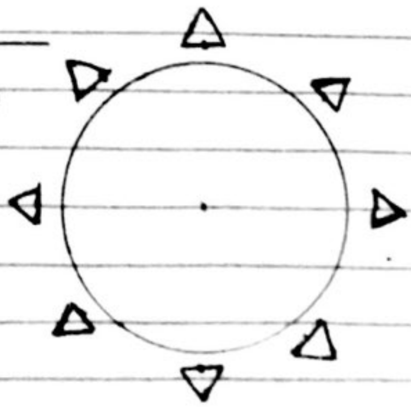


Write down: Which one of these shapes has: ---[1]

- Rotational symmetry of order 2
- No line of symmetry. [W-15/22/Q2]

Solution: Parallelogram.

Example 50: Write down the order of rotational symmetry of this shape.



Solution: 8.