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IGCSE Maths.

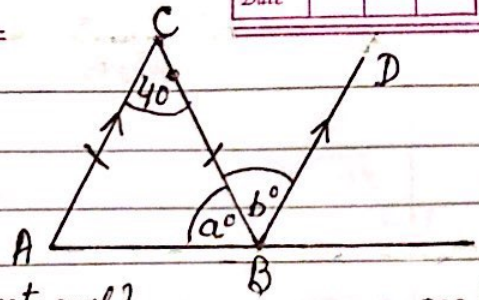
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Geometry  
Revision  
|S.P-20|M-20|S-20|W-19|

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1. Triangle ABC is isosceles.  
AC is parallel to BD.  
Find the value of a and the value of b.



Solution: angle CBD = angle ACB ( $\because$  Alternate angle)

$$\Rightarrow b = 40^\circ \checkmark$$

SP-20/02/Q8 [2]

Now Isosceles Triangle ABC, AC = BC, the opposite angles,

$$\therefore \text{angle CAB} = \text{angle CBA} = a^\circ$$

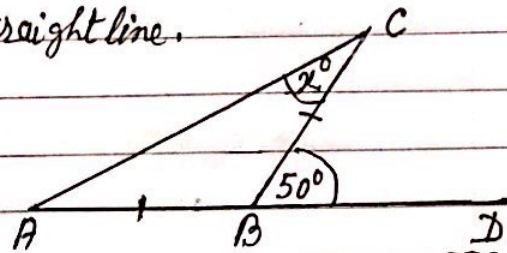
In  $\triangle ABC$ ,  $a^\circ + a^\circ + 40^\circ = 180^\circ$  (Sum of angles of a triangle is  $180^\circ$ )

$$\Rightarrow 2a + 40 = 180$$

$$\Rightarrow a = \frac{140}{2} = 70^\circ \checkmark$$

$$\therefore a = 70^\circ \text{ and } b = 40^\circ$$

2. AB = BC and ABD is a straight line.  
Find the value of x.



Solution: AB = BC in  $\triangle ABC$

the opposite angle are equal

$$\text{angle BAC} = \text{angle ACB} = x^\circ \text{ (given)} \text{ --- (1)}$$

Now angle BAC + angle ACB =  $50^\circ$  ( $\because$  Exterior angle CBD is

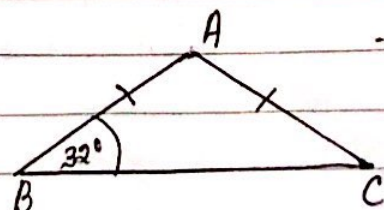
$\Rightarrow x^\circ + x^\circ = 50^\circ$  (from (1) equal to the opp interior angle)

$$2x = 50^\circ$$

$$x = \frac{50}{2} = 25^\circ$$

$$\therefore x = 25^\circ \checkmark$$

3. Triangle ABC is isosceles,  
Angle ABC =  $32^\circ$  and AB = AC  
Find angle BAC.



Solution: In  $\triangle ABC$ , AB = AC, the opposite angles  $\angle ACB = \angle ABC = 32^\circ$  --- (1)

Now in  $\triangle ABC$ ,

angle BAC + angle ACB + angle ABC =  $180^\circ$  (Angle sum property in a Triangle)

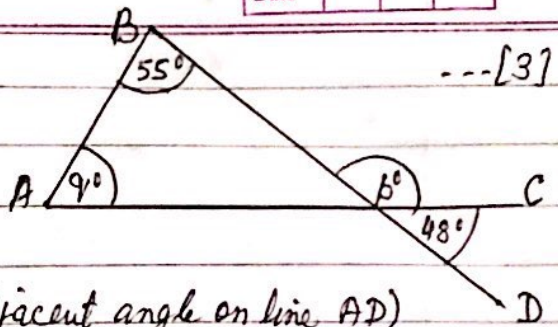
$$\therefore \text{angle BAC} + 32^\circ + 32^\circ = 180^\circ$$

$$\therefore \text{angle BAC} = 180 - 32^\circ - 32^\circ = 116^\circ \therefore \text{angle BAC} = 116^\circ$$

# Triangles/ Quadrilaterals

4.(a) In the diagram, AC and BD are straight lines.

Find the value of  $p$  and the value of  $q$ .



---[3]

Solution:  $p + 48^\circ = 180^\circ$  (Adjacent angle on line AD)

$$\therefore p = 180^\circ - 48^\circ = 132^\circ \quad \text{--- (1)}$$

Now  $q^\circ + 55^\circ = p$  (Exterior angle theorem in a triangle)

$$q + 55^\circ = 132^\circ \text{ from (1)}$$

$$\Rightarrow q = 132 - 55 = 77^\circ \quad \therefore p = 132^\circ \text{ and } q = 77^\circ$$

(b) The angles of a quadrilateral are  $x^\circ$ ,  $(x+5^\circ)$ ,  $(2x-25^\circ)$  and  $(x+10^\circ)$ . Find the value of  $x$ .

---[3]

[W-19/41/Q1(a)(b)]

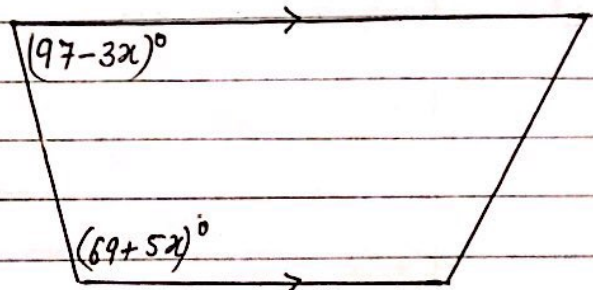
Solution:  $x + (x+5) + (2x-25) + (x+10) = 360^\circ$  (Sum of interior angles of a quad. is  $360^\circ$ )

$$\Rightarrow 5x - 10 = 360^\circ$$

$$\Rightarrow 5x = 360 + 10 = 370 \Rightarrow x = \frac{370}{5} = 74^\circ$$

$$\therefore x = 74^\circ$$

5. Work out the value of  $x$ .



Solution:

[S-20/21/Q6] ---[3]

$$97 - 3x + 69 + 5x = 180^\circ$$

$$166 + 2x = 180^\circ$$

$$2x = 180 - 166 = 14$$

$$x = \frac{14}{2} = 7$$

$$\therefore x = 7^\circ$$

(Sum of interior angle between two parallel lines made by a transversal on of the same side of it is  $180^\circ$ )

# Polygons.

6. A hexagon has five angles that each measure  $115^\circ$ .  
Calculate the size of sixth angle. ---[3]  
[SP-20/02/Q16]

Solution: let the sixth angle =  $x^\circ$   
 $\therefore x + 5 \times 115^\circ = (6-2) \times 180^\circ$  } sum of interior angles of  
 $x + 575^\circ = 720$  } an  $n$ -sided polygon  
 $x = 720 - 575$  }  $= (n-2) \times 180^\circ$   
 $\therefore x = 145^\circ$

7. Find the interior angle of a regular polygon of 24 sides. --[2]  
[M-20/22/Q4]

Solution: Interior angle of a  $n$ -sided regular polygon =  $\frac{(n-2) \times 180^\circ}{n}$   
 $\therefore$  Interior angle of a 24-sided regular polygon:  
 $= \frac{(24-2) \times 180^\circ}{24}$   
 $= \frac{22 \times 180^\circ}{24} = 165^\circ \checkmark$

8. A regular polygon has an interior angle  $176^\circ$ . ---[3]  
Find the number of sides of this polygon. [W-19/22/Q12]

Solution: let the number of sides of the polygon =  $n$   
Interior angle =  $\frac{(n-2) \times 180^\circ}{n} = 176^\circ$  (given)

$$\Rightarrow (n-2) \times 180^\circ = 176n$$

$$\Rightarrow 180n - 360 = 176n$$

$$\Rightarrow 180n - 176n = 360$$

$$4n = 360^\circ$$

$$n = \frac{360}{4} = 90$$

$$\therefore \underline{n = 90}$$

# Polygons.

9. The interior angle of a regular polygon with  $n$ -sides is  $150^\circ$ . Calculate the value of  $n$ . [S-20/43/Q8(a)] --(2)

Solution: Interior angle  $\frac{(n-2) \cdot 180^\circ}{n} = 150^\circ$  (given)

$$\Rightarrow (n-2) \cdot 180^\circ = 150 \cdot n$$

$$\Rightarrow 180n - 360^\circ = 150^\circ n$$

$$\Rightarrow 180n - 150n = 360^\circ$$

$$\Rightarrow 30n = 360^\circ \Rightarrow n = \frac{360^\circ}{30} = 12$$

$$\therefore n = 12$$

10. A regular polygon has 72 sides. Find the size of an interior angle. --[3]  
[W-19/41/1(C)]

Solution: an interior angle of  $n$ -sides polygon =  $\frac{(n-2) \cdot 180^\circ}{n}$

for  $n = 72$ ,

$$= \frac{(72-2) \times 180^\circ}{72}$$

$$= \frac{70 \times 180^\circ}{72} = 175^\circ$$

$$\therefore \text{Required angle} = 175^\circ$$

11. The diagram shows a regular octagon joined to an equilateral triangle. Work out the value of  $x$ . --[3]

Solution: Interior angle of a octagon ( $n = 8$ )

$$= \frac{(n-2) \times 180^\circ}{n}$$

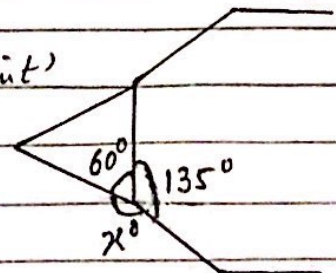
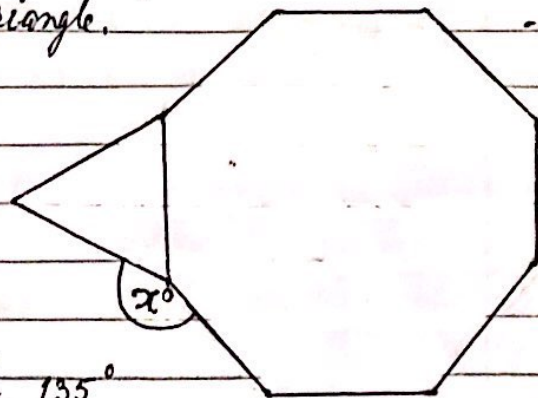
$$= \frac{(8-2) \times 180^\circ}{8} = \frac{6 \times 180^\circ}{8} = 135^\circ$$

Now  $x + 60^\circ + 135^\circ = 360^\circ$  (Angles at a point)

$$x + 195 = 360^\circ$$

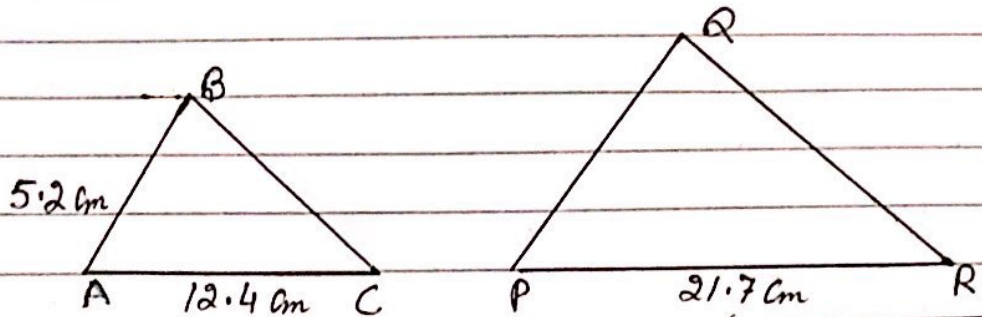
$$x = 360 - 195 = 165^\circ$$

$$\therefore x = 165^\circ$$



# Similarity

12. Triangle ABC is similar to triangle PQR.  
Find PQ. ...[2]



Solution:  $\triangle ABC \sim \triangle PQR$

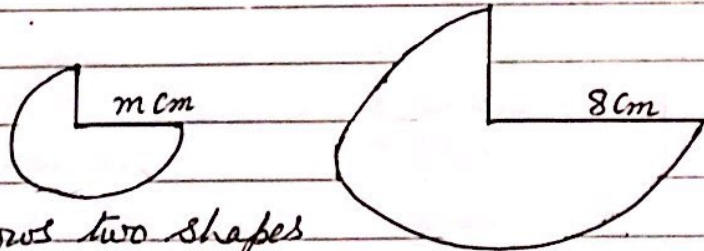
$$\frac{PQ}{AB} = \frac{PR}{AC} \quad (\text{Corresponding sides of similar triangles are proportional})$$

$$\Rightarrow \frac{PQ}{5.2} = \frac{21.7}{12.4} \Rightarrow PQ = \frac{21.7 \times 5.2}{12.4} = 9.1$$

$$\therefore PQ = 9.1 \text{ cm.}$$

[SP-20/02/Q12]

13.



The diagram shows two shapes that are mathematically similar.  
The smaller shape has area  $52.5 \text{ cm}^2$  and the larger shape has area  $134.4 \text{ cm}^2$ .

Calculate the value of  $m$ .

[S-20/23/Q17] ...[3]

Solution:  $\frac{\text{area of smaller shape}}{\text{area of larger shape}} = \left(\frac{m}{8}\right)^2$

$$\Rightarrow \frac{52.5}{134.4} = \frac{m^2}{64}$$

$$\Rightarrow m^2 = \frac{52.5 \times 64}{134.4} = 25$$

$$\therefore m = \sqrt{25} = 5$$

$$\therefore m = 5 \text{ cm.}$$

(The areas of similar shapes are proportional to the square of the corresponding sides of the shapes.)

14. The scale of a map is 1: 10 000 000.

On the map, the area of Slovakia is  $4.9 \text{ cm}^2$ .

Calculate the actual area of Slovakia.

Give your answer in square kilometres. [W-19/21/Q14] ... [3]

Solution: 
$$\frac{\text{area of the map}}{\text{actual area}} = \left( \frac{1}{10\,000\,000} \right)^2$$

$$\Rightarrow \frac{4.9}{\text{actual area}} = \frac{1}{10^{14}}$$

$$\Rightarrow \text{Actual area} = 4.9 \times 10^{14} \text{ sq cm}$$

$$= \frac{4.9 \times 10^{14}}{10^{10}}$$

$$= 4.9 \times 10^4 \text{ sq km}$$

$$\left. \begin{aligned} 1 \text{ km} &= 1000 \times 100 \\ &= 10^5 \text{ cm} \\ 1 \text{ sq km} &= (10^5)^2 \\ &= 10^{10} \text{ sq cm} \end{aligned} \right\}$$

$$\therefore \text{Actual area of Slovakia} = 49\,000 \text{ sq km.}$$

15. Two mathematical similar containers have heights of 30 cm and 75 cm. The larger container has a capacity of 5.5 litres.

Calculate the capacity of the smaller container.

Give your answer in millilitres. [W-19/22/Q13] ... [3]

Solution: 
$$\frac{V_1}{V_2} = \frac{\text{Capacity (Volume) of smaller container}}{\text{Capacity (Volume) of larger container}} = \left( \frac{h_1}{h_2} \right)^3$$

$$\Rightarrow \frac{V_1}{5.5 \times 1000} = \left( \frac{30}{75} \right)^3$$

$$1 \text{ litre} = 1000 \text{ c.c.}$$

$$V_1 = \left( \frac{2}{5} \right)^3 \times 5500 \text{ cm}^3$$

$$= \frac{8}{125} \times 5500 = 352 \text{ cm}^3$$

$$= 352 \text{ millilitre}$$

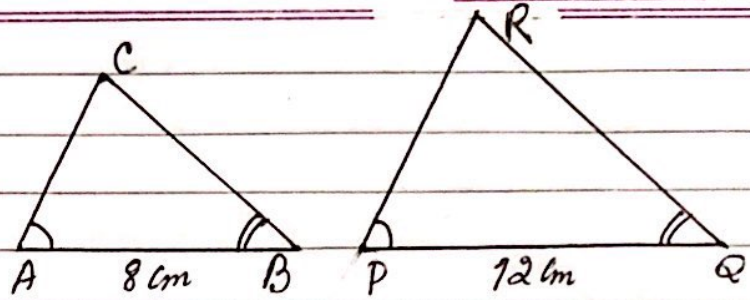
$$\left. \begin{aligned} 1 \text{ litre} &= 1000 \text{ cm}^3 \\ 1000 \text{ millilitre} &= 1000 \text{ cm}^3 \end{aligned} \right\}$$

$$\Rightarrow 1 \text{ millilitre} = 1 \text{ cm}^3$$

$$\therefore \text{Capacity of the smaller container}$$

$$= \underline{352 \text{ millilitres}}$$

16. Triangle ABC is mathematically similar to triangle PQR. The area of triangle ABC is  $16\text{cm}^2$ .



- (i) Calculate the area of triangle PQR. ---[2]
- (ii) The triangles are the cross-sections of prisms which are also mathematically similar. The volume of the smaller prism is  $320\text{cm}^3$ . Calculate the length of the larger prism. ---[3]

S-20/42/Q8(a)

Solution (i)  $\frac{\text{area of triangle PQR}}{\text{area of triangle ABC}} = \left(\frac{PQ}{AB}\right)^2$

$\Rightarrow \frac{\text{area of triangle PQR}}{16} = \left(\frac{12}{8}\right)^2$  [Area  $\Delta ABC = 16\text{cm}^2$ ]

$\Rightarrow \text{area of triangle PQR} = \frac{144}{4} \times 16 = \underline{36\text{cm}^2}$

(ii)  $\text{length of the smaller prism} = \frac{\text{Volume of small prism}}{\text{area of cross of cross-section}} = \frac{320}{16}$

$= 20\text{cm}$ . ——— (1)

$\frac{\text{length of larger prism}}{\text{length of small prism}} = \frac{PQ}{AB}$

$\Rightarrow \frac{\text{length of larger prism}}{20\text{cm}} = \frac{12}{8}$

from (1)

$\therefore \text{length of larger prism} = \frac{12}{8} \times 20 = 30\text{cm}$

$= \underline{30\text{cm}}$  ✓



## Similarity

17. Two cones are mathematically similar.  
The total surface area of the smaller cone is  $80 \text{ cm}^2$ ,  
The total surface area of the larger cone is  $180 \text{ cm}^2$ .  
The volume of the smaller cone is  $168 \text{ cm}^3$ . ---[3]  
Calculate the volume of the larger cone. [W-19/41/Q4(C)]

Solution:  $\frac{\text{area of smaller cone}}{\text{area of larger cone}} = \left(\frac{r_1}{r_2}\right)^2$   $\left\{ \begin{array}{l} r_1 \text{ and } r_2 \text{ are the} \\ \text{radii of the smaller} \\ \text{and larger cones resp.} \end{array} \right.$

$$\Rightarrow \frac{80}{180} = \left(\frac{r_1}{r_2}\right)^2 = \frac{4}{9}$$

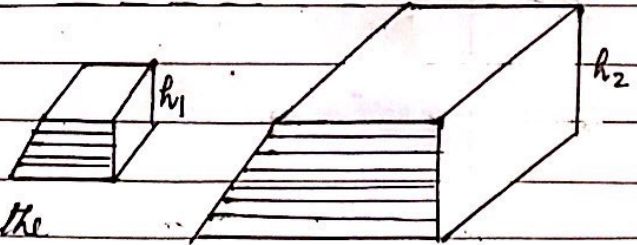
$$\Rightarrow \left(\frac{r_1}{r_2}\right)^2 = \frac{4}{9} \Rightarrow \frac{r_1}{r_2} = \sqrt{\frac{4}{9}} = \frac{2}{3} \quad \text{--- (1)}$$

Now  $\frac{\text{Volume of smaller cone}}{\text{Volume of larger cone}} = \left(\frac{r_1}{r_2}\right)^3$

$$\Rightarrow \frac{168}{\text{Volume of larger cone}} = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$$

$$\Rightarrow \text{Volume of larger cone} = \frac{168 \times 27}{8} = \underline{567 \checkmark}$$

18. The diagram shows two mathematically similar solid prisms. The volume of the smaller prism is  $648 \text{ cm}^3$  and the



Volume of the larger prism is  $2187 \text{ cm}^3$ . The area of cross-section of the smaller prism is  $36 \text{ cm}^2$ . [W-19/43/Q6(B)(i)]

Calculate the area of cross-section of the larger prism. ---[3]

Solution:  $\frac{\text{Volume of larger prism}}{\text{Volume of smaller prism}} = \left(\frac{h_1}{h_2}\right)^3 \Rightarrow \frac{h_1}{h_2} = \left(\frac{2187}{648}\right)^{\frac{1}{3}} = \frac{3}{2} \quad \text{--- (1)}$

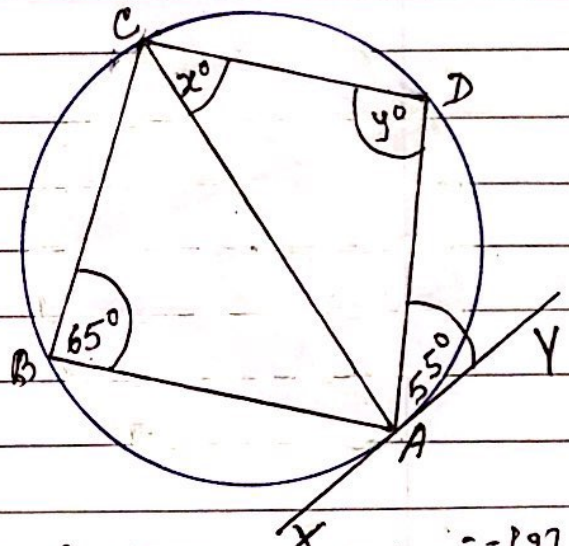
Now  $\frac{\text{area of cross-section of larger cone}}{\text{area of cross-section of smaller cone}} = \left(\frac{h_1}{h_2}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$$\frac{\text{area of cross-section of larger cone}}{36} = \frac{9}{4}$$

$$\Rightarrow \text{Area of cross-section of larger cone} = \frac{9}{4} \times 36 = \underline{81 \text{ cm}^2 \checkmark}$$

# Circles

19. A, B, C and D are points on the circumference of the circle.  
The line XY is a tangent to circle at A.



(a) Find the value of  $x$ , giving the reason for your answer. --- [2]

Solution:  $x = 55^\circ$  ✓ because angles in the alternate segment of circle are equal.

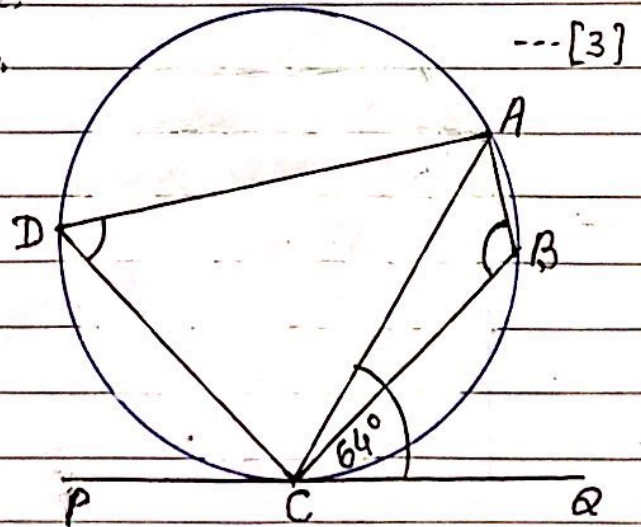
(b) Find the value of  $y$ , giving a reason for your answer. --- [2]

Solution:  $y = 115^\circ$  because the opposite angles in the cyclic quadrilateral ABCD are supplementary.  $y + 65 = 180 \Rightarrow y = 115^\circ$  ✓

[SP-20/02/Q26]

20. A, B, C and D lie on the circle.  
PCQ is a tangent to the circle at C.

Angle ACQ =  $64^\circ$   
Work out angle ABC, giving reasons for your answer.



Solution: angle ABC =  $116^\circ$  ✓

because angle ADC =  $64^\circ$   
angles in alternate circle.

and  $\angle ADC + \angle ABC = 180$ , opp. angle in the cyclic quad. ABCD.  $\therefore 64^\circ + \angle ABC = 180^\circ$

or angle ABC =  $180 - 64 = 116^\circ$

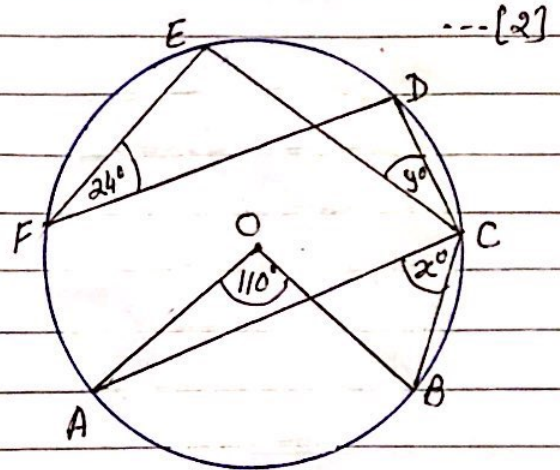
$\therefore$  Angle ABC =  $116^\circ$  ✓

[M-20/22/Q15]

21. A, B, C, D, E and F lie on the circle, centre O.

Find the value of  $x$  and the value of  $y$ .

[S-20/21 | Q10]



Solution: Angle subtended by the arc AB in the alternate segment of the circle ' $x$ ' is half the angle subtended by it at the centre.

$$\therefore x = \frac{1}{2} \times 110^\circ = 55^\circ \quad \therefore x = 55^\circ$$

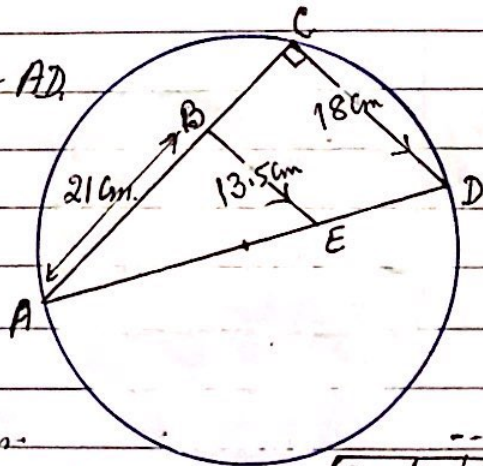
and  $y = 24^\circ$  ✓ as the angles in the same segment of the circle subtended by arc ED.

22. C lies on a circle with diameter AD.

B lies on AC and E lies on AD, such that BE is parallel to CD.

$AB = 21$  cm,  $CD = 18$  cm and  $BE = 13.5$  cm.

Work out the radius of the circle.



Solution: In  $\triangle ACD$ ,  $BE \parallel CD$

$\therefore \triangle ABE$  and  $\triangle ACD$  are similar.

$$\frac{AB}{AC} = \frac{BE}{CD} \quad (\because \text{Corresponding sides of the similar triangles are proportional})$$

$$\Rightarrow \frac{21}{AC} = \frac{13.5}{18} \Rightarrow AC = \frac{21 \times 18}{13.5} = 28$$

Now using Pythagoras theorem in  $\triangle ACD$  ( $\because AD$  is diameter)

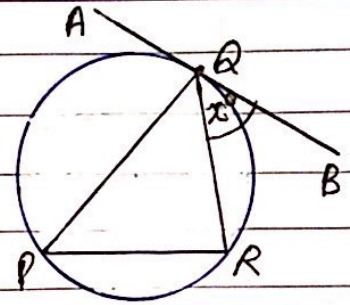
$$AD^2 = AC^2 + CD^2 = 28^2 + 18^2 = 1108$$

$$\text{Diameter } AD = \sqrt{1108} = 33.286 \Rightarrow 2r = 33.28$$

$$r = 16.64$$

$$\therefore \text{radius} = \underline{16.64 \text{ cm}}$$

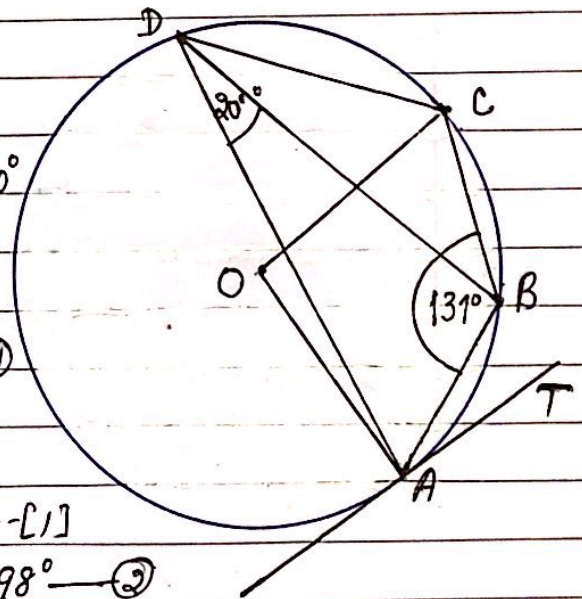
23. P, R and Q are points on the circle.  
 AB is a tangent to the circle at Q,  
 QR bisects angle PAB.  
 angle BQR =  $x^\circ$  and  $x < 60$   
 Use this information to show that  
 triangle PQR is an isosceles triangle.  
 Give a geometrical reason for each step of your work. --- [3]



[S-20/21/Q15]

Solution: angle PQR = angle BQR =  $x^\circ$  (∵ QR bisects angle PAB)  
 angle RPQ = angle BQR =  $x^\circ$  (∵ Angle in the Alternate  
 segment of a circle)  
 ∴ In Triangle PQR opposite sides PR = QR  
 ∴ Triangle PQR is an isosceles Triangle.

24. A, B, C, D lie on circle, Centre O.  
 TA is tangent to the circle at A.  
 Angle ABC =  $131^\circ$  and angle ADB =  $20^\circ$   
 Find.



(a) angle ADC. --- [1]

Answer: angle ADC =  $180^\circ - 131^\circ = 49^\circ$  (1)

(∵ ABCD is a cyclic quad.  
 angle ADC +  $131^\circ = 180^\circ$ )

(b) angle AOC. --- [1]

Answer: angle AOC =  $2 \times \angle ADC = 2 \times 49^\circ = 98^\circ$  (2)

(c) angle BAT. --- [1]

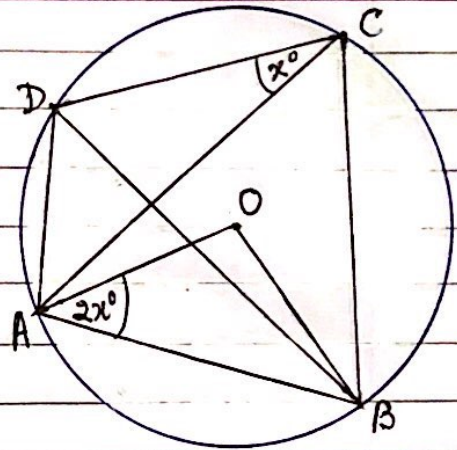
Answer: angle BAT = angle ADB =  $20^\circ$  (angles in the alternate segment) (3)

(d) angle OAB

Answer: angle OAB + angle BAT =  $90^\circ$  (∵ OA is perp AT  
 (rad ⊥ tangent)  
 at A.)  
 angle OAB +  $20^\circ = 90^\circ$   
 angle OAB =  $90 - 20 = 70^\circ$  ✓

[S-20/22/Q20]

23. In the diagram, A, B, C and D lie on the circumference of a circle, centre O. Angle  $ACD = x^\circ$  and angle  $OAB = 2x^\circ$ . Find an expression, in terms of  $x$ , in its simplest form for



(a) angle AOB --- [1]

Answer: angle AOB =  $180 - 4x$  ✓ — ①

(In  $\triangle OAB$ ,  $OA = OB = r \Rightarrow \angle OAB = \angle OBA = 2x$   
 and sum of angles of a triangle is  $180^\circ$ )  
 (angle AOB +  $2x + 2x = 180^\circ$ )

[W-19/22/Q19]

(b) angle ACB. --- [1]

Answer: angle ACB =  $\frac{1}{2}$  angle AOB =  $\frac{1}{2}(180 - 4x) = 90 - 2x^\circ$

( $\because$  Angle AOB at the centre is double the angle in alternate seg. of arc AB)

(c) angle DAB --- [2]

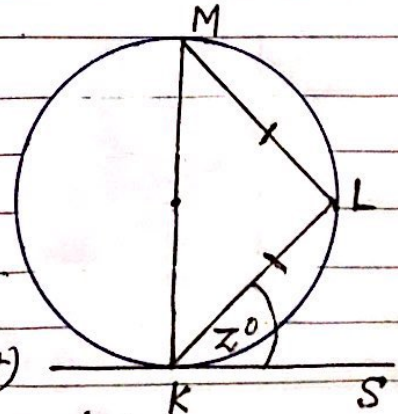
Answer: In triangle ABD,  $\angle DAB + \angle ADB + \angle ABD = 180^\circ$

$$\Rightarrow \angle DAB = 180 - (\angle ADB + \angle ABD)$$

$$= 180 - (\angle ACB + \angle ACD)$$

$$= 180 - (90 - 2x + x) = 180 - (90 - x) = (90 + x)^\circ \checkmark$$

24(a) (i) K, L and M are points on the circle, KS is a tangent to the circle at K, KM is a diameter and triangle KLM is isosceles.



Find the value of  $z$ . --- [2]

Solution:  $z = \text{angle KML}$  (Alternate segment) ①

Now angle  $KLM = 90^\circ$  (KM is diameter, angle in the semi-circle is right angle)

$\angle KML = \angle MKL$  (Isosceles) ②

$\angle KML + \angle MKL = 90$  ③ from ① (angle sum prop of  $\triangle$ )

from ② & ③  $\angle KML + \angle KML = 90 \Rightarrow 2\angle KML = 90 \Rightarrow \angle KML = 45^\circ$  ④

from ① & ②  $z = 45^\circ \checkmark$

(continued  $\rightarrow$ )

[S.20/43/Q8(b)(c)]

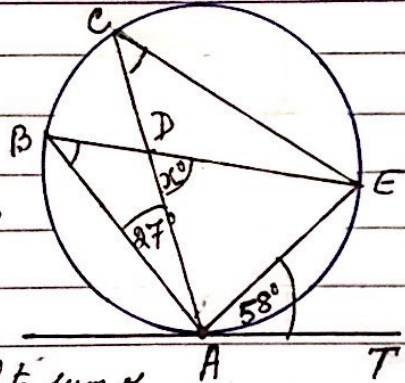
(continued →)

24(a)(ii) AT is a tangent to the circle at A.

Find the value of  $x$ . ---[2]

Solution: angle ACE = 58 (Alternate segment theorem)  
 $\angle ABE = \angle ACE$  (angles in the same segment of a circle)  
 $\therefore \angle ABD = 58^\circ$  — ①

Now  $x = 58 + 27$  (In  $\triangle DAB$ ,  $x$  is the exterior angle  $\rightarrow$  equal to sum of opp. interior angles).  
 $\therefore x = 85^\circ \checkmark$



(iii) F, G, H and J are points on the circle.

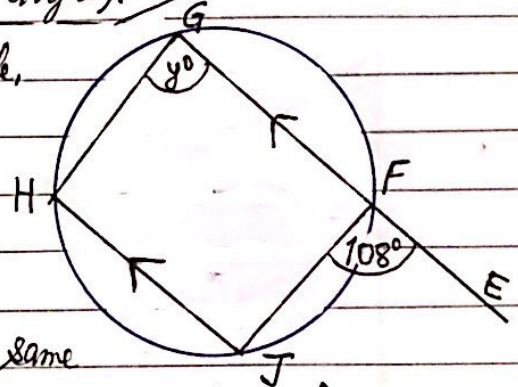
EFG is a straight line parallel to JH.

Find the value of  $y$ . ---[2]

Solution: angle H = 108°  
 $y + \text{angle H} = 180^\circ$  (Interior angles on the same side of transversal between parallel lines)

$\Rightarrow y + 108 = 180$  (Side of transversal between parallel lines)

$\Rightarrow y = 72^\circ \checkmark$

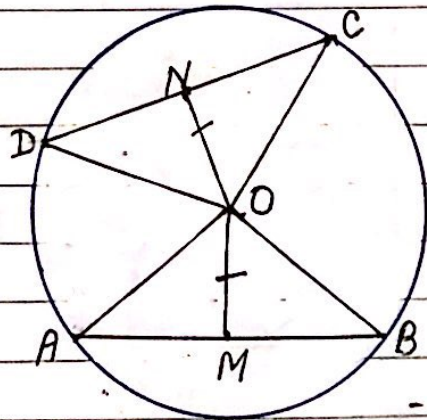


(b) A, B, C and D are points on the circle, centre O.

M is the mid point of AB and

N is the mid point of CD;  $OM = ON$ .

Explain, giving reasons, why triangle OAB is congruent to triangle OCD.



Solution:  $OM = ON \Rightarrow AB = CD$  (Chords equidistant from centre are equal)

Now  $AB = CD$  — ①

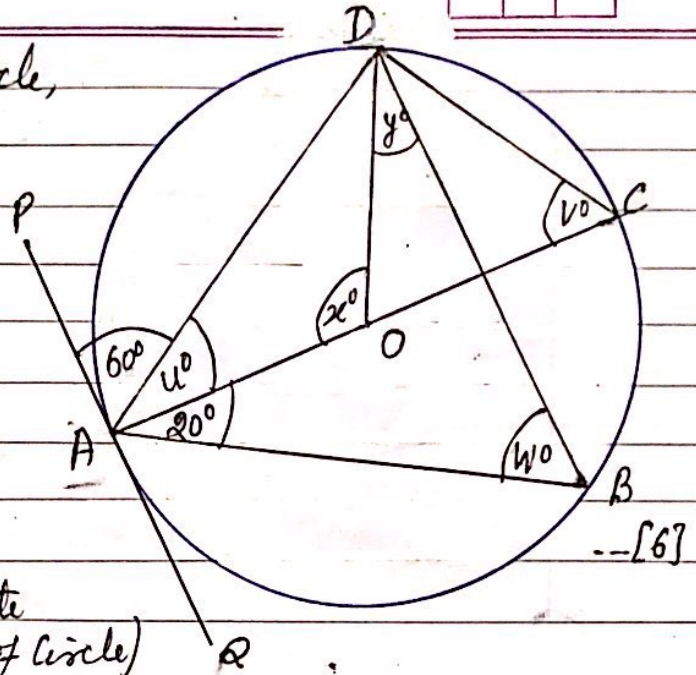
$OA = OC = r$

$OB = OD = r$

$\therefore$  Triangle OAB  $\cong$  Triangle OCD (SSS Theorem of Congruency)

S-20/43/Q8(b)(c)

25(a) A, B, C and D lie on the circle, centre O, with diameter AC. PA is a tangent to the circle at A. angle PAD = 60° and angle BAC = 20°. Find the value of u, v, w, x and y.



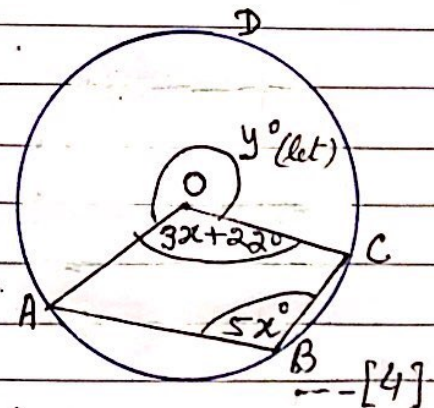
Solution:  $\underline{u = w = 60^\circ}$  (Alternate segment of circle) ---[6]

$\angle ADC = 90^\circ$  (angle in semicircle)  
 $\Rightarrow u + v = 90 \Rightarrow u + 60 = 90 \Rightarrow u = 30^\circ \checkmark$

$x = 2w = 2 \times 60 = 120^\circ$  (angle at the centre is double the angle in the remaining circle)  
 $\therefore x = 120^\circ \checkmark$

$\angle OAD = \angle ODA$  (OA = OD)  
 $30 = \angle ODA$  and  $\angle BDC = 20^\circ$   
 $\therefore y + 30 + 20 = 90 \Rightarrow y = 40^\circ \checkmark$

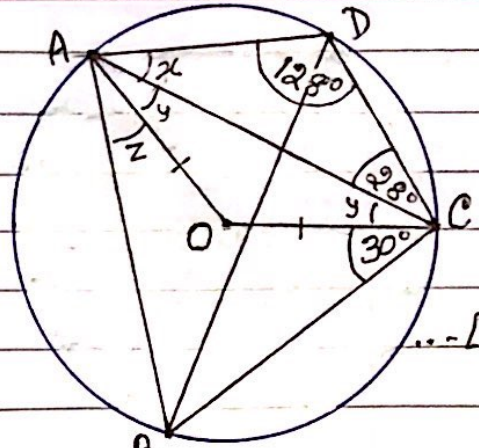
25(b) A, B and C lie on the circle, centre O. Angle ADC = (3x + 22)° and angle ABC = 5x°. Find the value of x.



Solution: angle subtended by arc ADC at the centre angle  $y^\circ = 360 - (3x + 22) = (338 - 3x)^\circ$  ---[4]

Now  $338 - 3x = 2 \times 5x$  (angle subtended by arc ADC is double the angle in the opp. segment of circle)  
 $338 - 3x = 10x$   
 $\Rightarrow 13x = 338$   
 $x = \frac{338}{13} = 26^\circ$   
 $\therefore x = 26^\circ \checkmark$

26. In the diagram A, B, C and D lie on the circle, centre O.  
 Angle  $ADC = 128^\circ$ , angle  $ACD = 28^\circ$   
 and angle  $BCO = 30^\circ$



(i) Show that obtuse angle  $AOC = 104^\circ$   
 Give a reason for each step.

Answer: angle  $ABC = 180 - 128 = 52^\circ$  — ①

$\because$  The opp. angles of cyclic

quad ABCD are supplementary

$\therefore$  obtuse angle  $AOC = 2 \times 52$  ( $\because$  angle at the centre is double  
 $= 104^\circ \checkmark$  the angle at circumference by  
 an arc  $\widehat{ADC}$ )

(ii) Find angle BAO. --- [2]

Answer:  $x = 180 - (128 + 28)$  (angle sum prop in  $\triangle ADC$ )

$$x = 24^\circ \text{ --- ②}$$

In isosceles  $\triangle OAC$ ,  $2y + 104 = 180 \Rightarrow y = 38^\circ$  — ③

Now  $\angle BAD + \angle BCD = 180$  (opp angle of cyclic quad ABCD)

$$\Rightarrow (z + 38 + 24) + (28 + 38 + 30) = 180^\circ$$

$$z + 168 = 180 \Rightarrow z = \angle BAO = 180 - 168 = 22^\circ \checkmark$$

(iii) Find angle ABD --- [1]

Answer: angle  $ABD = \text{angle } ACD = 28^\circ \checkmark$  (Angles in the same segment of circle)

(iv) The radius, OC, of the circle is 9.6 cm.

Calculate the total perimeter of sector OADC. --- [3]

Answer: Perimeter of sector OADC =  $OA + OC + \text{arc } \widehat{ADC}$

$$= 9.6 + 9.6 + \frac{104}{360} \times 2\pi \times 9.6$$

$$= 19.2 + 17.425$$

$$= 36.62 \text{ cm } \checkmark$$

[W-19/43/Q6(a)]



# Constructions

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27. Using a ruler and pair of compasses only, construct a triangle with sides 5cm, 8cm and 10cm. Leave in your construction arcs.

---[2]

SP-20/02/Q7

