IG- Maths
0580

Matrices

Notes

Matrices Notes

Introduction:

In a class there are three friends Simran, Akriti and Joyce they have 15, 10 and 13 note books and 6, 2 and 5 pens respectively, we can represent this information in a rectangular array as follows

(i)
$$\begin{array}{c} \\ \text{Simran} \\ \text{Akriti} \\ \text{Joyce} \end{array} \begin{array}{c} \begin{array}{cc} \text{Note books} & \text{pens} \end{array} \\ \begin{pmatrix} 15 & 6 \\ 10 & 2 \\ 13 & 5 \end{pmatrix}_{3\times 2} \end{array} = A \text{ (let)}$$

Then A is called a matrix of Order 3×2, 3 rows and 2 colums.

The same information can also be expressed as

(ii)
$$\begin{array}{c} \\ \text{Notebooks} \\ \text{Pens} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Simran} & \text{Akriti} & \text{Joyce} \end{array} \\ \begin{pmatrix} 15 & 10 & 13 \\ 6 & 2 & 5 \end{pmatrix}_{2\times 3} \end{array} = B \text{ (let)}$$

2×3
row by column

The matrix B is of the Order 2×3. 2 rows and 3 columns.

(iii) **Squared Matrices:** Number of rows and columns are equal.

$C = \begin{pmatrix} 2 & 3 \\ 5 & -7 \end{pmatrix}_{2\times 2}$ is a square Matrix of Order 2.

$D = \begin{pmatrix} 5 & -3 & 7 \\ 0 & 1 & 4 \\ 2 & -6 & 1 \end{pmatrix}$ is a sq. Matrix of Order 3.

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(iv) **Diagonal matrix:** A squared matrix in which all the non-diagonal elements are zero.

$$E = \begin{pmatrix} 4 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 6 \end{pmatrix} \quad \text{or} \quad F = \begin{pmatrix} 3 & 0 \\ 0 & 5 \end{pmatrix}$$

(v) **Row Matrix:** only one row.

$$G = \begin{pmatrix} 2 & -5 & 7 \end{pmatrix}_{1\times 3}$$

$$\text{or} \quad H = \begin{pmatrix} 1 & 6 \end{pmatrix}_{1\times 2}$$

(vi) **Column Matrix:** Only one column.

$$I = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}_{3\times 1} \quad \text{and} \quad J = \begin{pmatrix} 2 \\ -3 \end{pmatrix}_{2\times 1}$$

(vii) **Zero Matrix:** All the elements are zero.

$$K = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}_{2\times 2} \quad L = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{2\times 3}$$

(viii) **Identity Matrix:** A square matrix in which all the diagonal elements are 1 each and the rest are zero.

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}_{2\times 2}$$ is an identity matrix of order 2.

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§ **Equal Matrices:** Two matrices are said be equal if (i) they have the same order
(ii) corresponding element are equal.

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}_{2\times 3} ; B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2\times 3}$$

(i) both the matrices have same order 2×3

and (ii) $a_{11} = b_{11}$, $a_{12} = b_{12}$, $a_{13} = b_{13}$

$a_{21} = b_{21}$, $a_{22} = b_{22}$ and $a_{23} = b_{23}$

Example: Given equal matrices

$$\begin{pmatrix} 2 & x \\ -3 & 5 \end{pmatrix} = \begin{pmatrix} 2 & -5 \\ y & 5 \end{pmatrix}$$

Find the value of x and y.

$x = -5$ ✓

$y = -3$ ✓

§ **Addition of Matrices:** (i) both should be of the same order.

(ii) add the corresponding elements.

Example: (i) $\begin{pmatrix} 2 & -4 \\ 5 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 6 \\ -2 & 7 \end{pmatrix} = \begin{pmatrix} 2+1 & -4+6 \\ 5+(-2) & 0+7 \end{pmatrix}$

$= \begin{pmatrix} 3 & 2 \\ 3 & 7 \end{pmatrix}$ ✓

(ii) $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + \begin{pmatrix} -1 \\ 5 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \\ 3 \end{pmatrix}$ ✓

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§ Scalar Multiplication:

To multiply a matrix by a number, multiply each element of the matrix by this number.

$$A=\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}, \text{ Then } kA=\begin{pmatrix} ka & kb & kc \\ kd & ke & kf \end{pmatrix}$$

Example:

$$A=\begin{pmatrix} 2 & -3 \\ 5 & 7 \end{pmatrix}, \text{ Then } 5A=\begin{pmatrix} 10 & -15 \\ 25 & 35 \end{pmatrix}$$

Example: Find the value of x and y for the following:

$$2\begin{pmatrix} x & 1 \\ 2 & y-3 \end{pmatrix}+\begin{pmatrix} 5 & -4 \\ 1 & 2 \end{pmatrix}=\begin{pmatrix} 7 & -2 \\ 5 & 14 \end{pmatrix}$$

Solution: $\Rightarrow \begin{pmatrix} 2x & +2 \\ 4 & 2y-6 \end{pmatrix}+\begin{pmatrix} 5 & -4 \\ 1 & 2 \end{pmatrix}=\begin{pmatrix} 7 & -2 \\ 5 & 14 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} 2x+5 & -2 \\ 5 & 2y-4 \end{pmatrix}=\begin{pmatrix} 7 & -2 \\ 5 & 14 \end{pmatrix}$$

$\therefore$ $2x+5=7$ and $2y-4=14$

or $x=1$ ✓ and $y=9$ ✓

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§ Determinant of a Matrix:

To each squared matrix there corresponds a value. Determinant of a matrix A is denoted by |A|

Given a sq. matrix of order 2.

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Then det A or $|A| = (ad - bc)$

Example (i) $A = \begin{pmatrix} 2 & -3 \\ 5 & 7 \end{pmatrix}$

$$\therefore |A| = 2 \times 7 - (5 \times -3)$$
$$= 14 + 15 = 29 \checkmark$$

(ii) $A = \begin{pmatrix} -1 & 3 \\ 4 & 6 \end{pmatrix}$

$$\therefore |A| = (-1) \times 6 - 4 \times 3$$
$$= -6 - 12 = -18 \checkmark$$

(iii) $A = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ Identity Matrix.

$$\therefore |I| = 1 \times 1 - 0 \times 0 = 1 \checkmark$$

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§ Multiplication of two Matrices:

Condition for the multiplication of two matrices.

$$A_{m\times n} \times B_{n\times p} = C_{m\times p}$$

the number of columns of the first matrix A must be same as the the number of rows of the second matrix.

$$A_{2\times 3} \times B_{3\times 2} = C_{2\times 2}$$

$$\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}_{2\times 3} \times \begin{pmatrix} p & q \\ r & s \\ u & v \end{pmatrix}_{3\times 2}$$

multiply row to column

$$= \begin{pmatrix} ap+br+cu & aq+bs+cv \\ dp+er+fu & dq+es+fv \end{pmatrix}$$

Example1. $A = \begin{pmatrix} 3 & -2 \\ 1 & 4 \end{pmatrix}$ $B = \begin{pmatrix} 2 & 0 \\ -5 & 7 \end{pmatrix}$ [W-14/22/Q11]

(a) Calculate BA. --[2]

$$BA = \begin{pmatrix} 2 & 0 \\ -5 & 7 \end{pmatrix}\begin{pmatrix} 3 & -2 \\ 1 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 2\times 3 + 0\times 1 & 2\times(-2)+0\times 4 \\ -5\times 3 + 7\times 1 & -5\times(-2) + 7\times 4 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & -4 \\ -8 & 38 \end{pmatrix} \checkmark$$

(b) Find determinant of A. ---[1]

$$|A| = \begin{vmatrix} 3 & -2 \\ 1 & 4 \end{vmatrix} = 12-(-2)$$

$$= 14 \checkmark$$

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Example 2 $M = \begin{pmatrix} 7 & u \\ 2 & 3 \end{pmatrix}$ and $|M| = 1$ W-15/23/Q13

Find the value of u. ---[2]

Solution: $M = \begin{pmatrix} 7 & u \\ 2 & 3 \end{pmatrix}$ $\therefore$ $|M| = 7\times3 - 2\times u$

$= 21 - 2u = 1$ given

$\Rightarrow 2u = 20$

or $u = 10$ ✓

Example 3. $M = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ $N = \begin{pmatrix} 4 & 3 \\ 1 & k \end{pmatrix}$ $P = \begin{pmatrix} 1 & 3 \\ 0 & 6 \end{pmatrix}$

(i) Work out $M+P$ S-16/41/Q2(b) --[1]

(ii) Work out PM --[1]

(iii) $|M| = |N|$ find the value of k. [3].

Solution (i) $M+P = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ 0 & 6 \end{pmatrix} = \begin{pmatrix} 1+1 & 2+3 \\ 3+0 & 4+6 \end{pmatrix}$

$= \begin{pmatrix} 2 & 5 \\ 3 & 10 \end{pmatrix}$ ✓

(ii) $PM = \begin{pmatrix} 1 & 3 \\ 0 & 6 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1\times1+3\times3 & 1\times2+3\times4 \\ 0\times1+6\times3 & 0\times2+6\times4 \end{pmatrix}$

$= \begin{pmatrix} 10 & 14 \\ 18 & 24 \end{pmatrix}$ ✓

(iii) $|M| = |N|$

$\therefore \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} 4 & 3 \\ 1 & k \end{vmatrix}$

or $4 - 6 = 4k - 3$

or $4k = 1$ $\Rightarrow$ $k = \frac{1}{4}$ ✓

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§ Inverse of a Matrix:

Given two Sq. Matrices A and B of the same order, such that:

$$A \cdot B = I$$ (where I is identity matrix.)

Then inverse of matrix A, denoted by $A^{-1} = B$.

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$\boxed{\text{Then } A^{-1} = \frac{1}{|A|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}}$$

where $|A| = (ad - bc)$

Note (i) interchang the position of a and d.
(ii) Change the signs of b and c.

Imp. Note: $|A| \neq 0$ for A^{-1} to exist.

Example 4. $M = \begin{pmatrix} 3 & 1 \\ -11 & -2 \end{pmatrix}$ Find M^{-1} --- [2] [S-15/22/Q11]

Solution $|M| = 3 \times (-2) - (-11) \times 1$
$= -6 + 11$
$= 5.$

$$\therefore M^{-1} = \frac{1}{5}\begin{pmatrix} -2 & -1 \\ 11 & 3 \end{pmatrix}$$ ✓

To Verify $MM^{-1} = I$

$$MM^{-1} = \frac{1}{5}\begin{pmatrix} 3 & 1 \\ -11 & -2 \end{pmatrix}\begin{pmatrix} -2 & -1 \\ 11 & 3 \end{pmatrix}$$

$$= \frac{1}{5}\begin{pmatrix} -6+11 & -3+3 \\ 22-22 & 11-6 \end{pmatrix}$$

$$= \frac{1}{5}\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

Note: It is not a part of the question.

§ Alternate method: Let $M^{-1} = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$

$$\therefore \begin{pmatrix} 3 & 1 \\ -11 & -2 \end{pmatrix}\begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

or $$\begin{pmatrix} 3p+r & 3q+s \\ -11p-2r & -11q-2s \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

or $\begin{cases} 3p + r = 1 \\ -11p - 2r = 0 \end{cases}$ and $\begin{cases} 3q + s = 0 \\ -11q - 2s = 1 \end{cases}$

Solving $p = -2/5$, $r = \frac{11}{5}$; $q = -\frac{1}{5}$, $s = 3/5$ ✓

$$\therefore M^{-1} = \begin{pmatrix} -\frac{2}{5} & -\frac{1}{5} \\ \frac{11}{5} & \frac{3}{5} \end{pmatrix}$$ ✓

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Example 5. $A=\begin{pmatrix}4 & 2\\ 2 & 1\end{pmatrix}$ $B=\begin{pmatrix}7 & -3\\ 4 & 5\end{pmatrix}$ $C=\begin{pmatrix}-2 & 3 & 1\\ 4 & 5 & -1\end{pmatrix}$ $D=\begin{pmatrix}-9\\ 0\end{pmatrix}$

(a) Which of these four matrix calculation is not possible.
$A+B$ $\quad 3C \quad CB \quad AD$ --- [1]

(b) Calculate AB $\boxed{W\text{-}16/23/Q25}$ --- [2]

(c) Workout B^{-1} --- [2]

(d) Explain why A does not have an inverse, --- [1]

Solution: (a) $C_{2\times 3}$ & $B_{2\times 2}$ $\therefore$ CB is not possible.
$2 \neq 2\times2$... (∵ No. of Columns of C ≠ No. of rows of B)

(b) $AB=\begin{pmatrix}4 & 2\\ 2 & 1\end{pmatrix}\begin{pmatrix}7 & -3\\ 4 & 5\end{pmatrix}$

$$=\begin{pmatrix}4\times7+2\times4 & 4\times(-3)+2\times5\\ 2\times7+1\times4 & 2(-3)+1\times5\end{pmatrix}$$

$$=\begin{pmatrix}36 & -2\\ 18 & -1\end{pmatrix} \checkmark$$

(c) $B=\begin{pmatrix}7 & -3\\ 4 & 5\end{pmatrix}$; $|B| = 7\times5-4(-3)$
$= 35+12 = 47$

$\therefore B^{-1}=\frac{1}{47}\begin{pmatrix}5 & 3\\ -4 & 7\end{pmatrix}$ [(i) Interchange the position of Main diagonal elements
(ii) Change the sign of elements of other diagonal]

(d) $A=\begin{pmatrix}4 & 2\\ 2 & 1\end{pmatrix}$

$\therefore |A| = \begin{vmatrix}4 & 2\\ 2 & 1\end{vmatrix} = 4\times1-2\times2$
$= 4-4$
$= 0$
{ for A^{-1}; $|A|\neq 0$

$\because |A|=0$ A^{-1} does not exist,

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Example 6. $A = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$ $B = \begin{pmatrix} 6 & -4 \end{pmatrix}$ $C = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$ $D = \begin{pmatrix} 2 & 9 \\ -1 & -3 \end{pmatrix}$

(a) Calculate the result of each of the following, if possible, If a calculation is not possible, write "not possible" in the answer space.

(i) $3A$ -- [1]

(ii) AC -- [1]

(iii) BA -- [2]

(iv) $C+D$ $\boxed{S\text{-}13/42/Q7}$ -- [1]

(v) D^2 -- [2]

(b) Calculate C^{-1} -- [2]

Solution (a) (i) $3A = 3\begin{pmatrix} 5 \\ 7 \end{pmatrix} = \begin{pmatrix} 3\times5 \\ 3\times7 \end{pmatrix} = \begin{pmatrix} 15 \\ 21 \end{pmatrix}$ ✓

(ii) $A_{2\times1}$ and $C_{2\times2}$ no. of Columns of $A \neq$ No of rows of C.

$\therefore$ AC is not possible.

(iii) $BA = \begin{pmatrix} 6 & -4 \end{pmatrix}\begin{pmatrix} 5 \\ 7 \end{pmatrix}$

$= \begin{pmatrix} 6\times5 + (-4)\cdot7 \end{pmatrix} = (2)$ ✓

(iv) $C + D = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix} + \begin{pmatrix} 2 & 9 \\ -1 & -3 \end{pmatrix} = \begin{pmatrix} 2+2 & 4+9 \\ 1+(-1) & 3+(-3) \end{pmatrix} = \begin{pmatrix} 4 & 13 \\ 0 & 0 \end{pmatrix}$ ✓

(v) $D^2 = \begin{pmatrix} 2 & 9 \\ -1 & -3 \end{pmatrix}\begin{pmatrix} 2 & 9 \\ -1 & -3 \end{pmatrix} = \begin{pmatrix} 2\times2 + 9\cdot(-1) & 2\times9 + 9\cdot(-3) \\ -1\times2 + (-3)(-1) & (-1)\cdot9 + (-3)(-3) \end{pmatrix}$

$= \begin{pmatrix} -5 & -9 \\ 1 & 0 \end{pmatrix}$ ✓

(b) To Calculate C^{-1}; $C = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$

$|C| = 2\times3 - 1\times4 = 2$ ✓

$C^{-1} = \frac{1}{2}\begin{pmatrix} 3 & -4 \\ -1 & 2 \end{pmatrix}$ ✓

((i) Interchange the elements of main diagonal. (ii) Change the Signs of the nos. on the other diagonal)

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Example 7. $P = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ $Q = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$ $R = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$

(a) Work out. (i) $4P$ ---[1]

(ii) $P-Q$ ---[1]

(iii) P^2 $\boxed{W-14/43/Q5}$ ---[2]

(iv) QR

(b) Find the matrix S, ~~so~~ that $QS = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ---[3]

Solution (a) (i) $4P = 4\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -4 \\ 4 & 0 \end{pmatrix}$ ✓

(ii) $P-Q = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0-1 & -1-(-2) \\ 1-0 & 0-1 \end{pmatrix}$

$= \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$ ✓

(iii) P^2

$P \times P$

$= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

$= \begin{pmatrix} 0\times 0 + (-1)\cdot 1 & 0\times(-1) + (-1)\cdot 0 \\ 1\times 0 + 0\times 1 & 1\times(-1) + 0\times 0 \end{pmatrix}$

$= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ ✓

(iv) QR

$= \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}_{2\times 2} \cdot \begin{pmatrix} -3 \\ 5 \end{pmatrix}_{2\times 1}$

$= \begin{pmatrix} 1\times(-3) + (-2)\times 5 \\ 0\times(-3) + 1\times 5 \end{pmatrix}$

$= \begin{pmatrix} -13 \\ 5 \end{pmatrix}$ ✓

(b) Find. $QS = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

$\therefore S = Q^{-1}$, $Q = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$

Now $|Q| = \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$ ✓

$\therefore S = Q^{-1} = \frac{1}{1}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ ✓

(i) Interchange the numbers in the main diagonal.
(ii) Change the signs of the numbers on the other diagonal.

$\boxed{\text{Note: } AB = I \text{ Then. } B = A^{-1}}$