IG-Maths
0580

Mensuration Notes

Contents

(Suresh Goel)

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§ Measurement of Length:

Unit: Metre (m).

Converting the units:

1 Metre = 100 centimetre (cm)

1 cm = 10 mm (millimetre)

1 m = 1000 mm.

$1\,mm = \frac{1}{10}\,cm = 0.1\,cm$

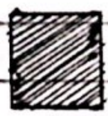
$1\,mm = \frac{1}{1000}\,m = 0.001\,m$

and $1\,cm = \frac{1}{100}\,m = 0.01\,m$

1 Kilometre (Km) = 1000 m.

$\therefore 1\,m = \frac{1}{1000}\,Km = 0.001\,Km$

§ Measurement of Area:



Unit of area = 1 sq. metre = $1\,m^2$

$1\,m^2 = 100 \times 100\,cm^2 = 10000\,cm^2$ (or sq. cm)

$1\,cm^2 = 10 \times 10\,mm^2 = 100\,mm^2$ (or sq. mm)

or $1\,cm^2 = \frac{1}{10000}\,m^2 = 0.0001\,m^2$ (or sq. m)

$1\,mm^2 = \frac{1}{100}\,cm^2 = 0.01\,cm^2$ (or sq cm.)

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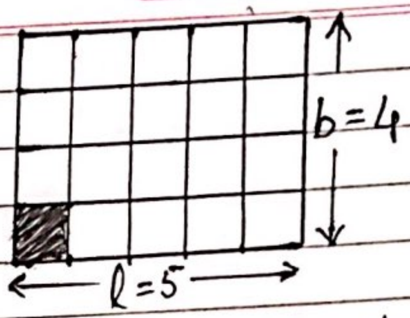
§ Area of Plane Shapes:

1. Area of a Rectangle = length × breadth
$= l \times b$ sq unit

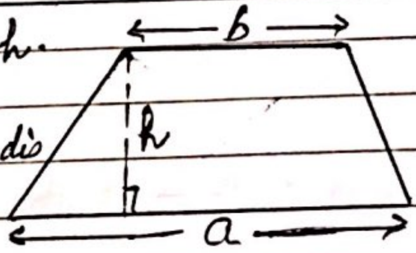


Area = $5 \times 4 = 20$ sq unit

2. Area of a Square = l^2 sq unit (l = length of one side of a square)

3. Area of a Parallelogram = $l \times h$.

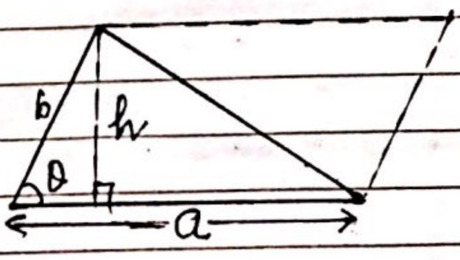
4. Area of a Trapezium = $\frac{1}{2}(a+b) \times h$.



$= \frac{1}{2}$(Sum of Parallel side) × Perp. dis
$= \frac{1}{2}(a+b) \times h$.

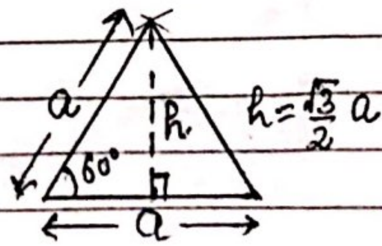
5. (i) Area of a Triangle = $\frac{1}{2} a \times h$

(diagonal of parallelogram divided it into two congruent triangles)

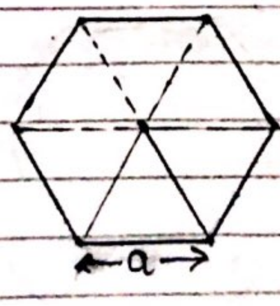


(ii) Area of Triangle = $\frac{1}{2} ab \sin\theta$

6. Area of an Equilateral Triangle:
$= \frac{\sqrt{3}}{4} a^2 \quad \left(\frac{1}{2} a \times \frac{\sqrt{3}}{2} a\right)$



7. Area of a Regular Hexagon:
$= \frac{3\sqrt{3}}{2} a^2 \left(\because 6 \times \frac{\sqrt{3}}{4} a^2\right)$



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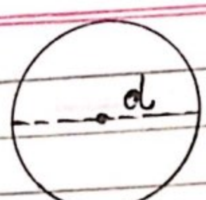
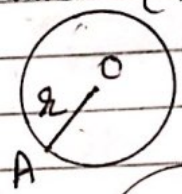
§ Circumference of a circle:



$$\frac{\text{Circumference of a Circle}}{\text{Diameter}} = \text{constant} = \pi \qquad \text{or } C = \pi d$$

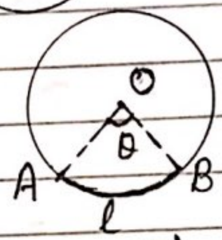
1. Circumference of a Circle $C = 2\pi r$. unit ($\because d = 2r$)

2. Area of a circle $= \pi r^2$ unit^2



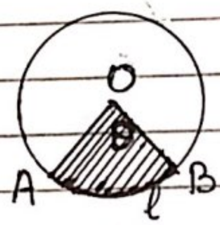
3. Lenth of Arc:

Length of arc $\widehat{AB}$ $\quad l = \frac{\theta}{360} \times 2\pi r$



(θ is the angle subtended by arc $\widehat{AB}$ at the centre) (θ is called "Sector angle")

4. Area of a Sector:



length of arc; $\widehat{AB} = l$

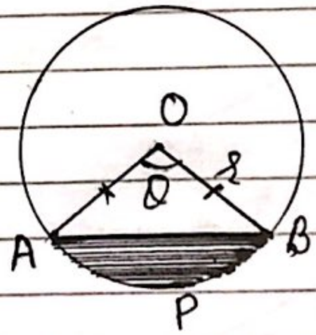
(i) Area of Sector $OAB = \frac{\theta}{360} \times \pi r^2$

(ii) Area of Sector $OAB = \frac{1}{2} l \cdot r$

5. Area of Segment of a Circle:

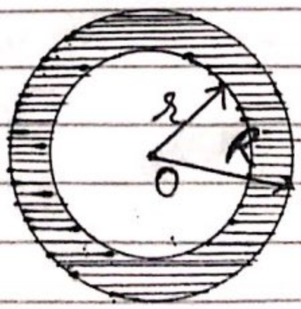
$=$ Area of Sector $OAPB$ − Area of ΔOAB

$$= \left(\frac{\theta}{360} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta\right)$$



6. Area of Circular Ring:

$$= \frac{1}{2}\pi (R^2 - r^2)$$



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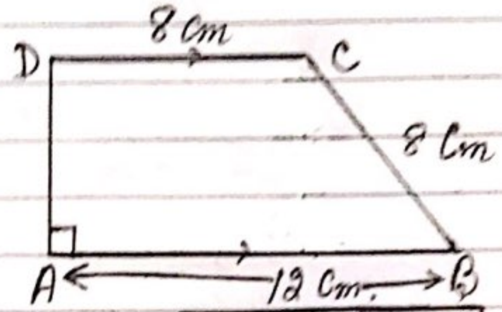
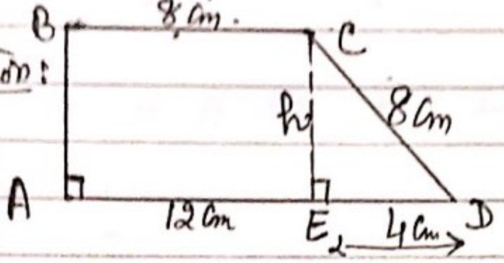
§ Area of Plane Shapes:

Example 1. Calculate the area of this trapezium. ---[4]



S-16	21	Q20
SP-2020	02	Q17

Solution:



Draw $CE \perp AD$

Let $CE = h$ Cm

Then $ED = AD - AE$

$= AD - BC$

$= 12 - 8 = 4$ Cm.

In rt ΔAED, Using Pythagoras

$CE^2 + ED^2 = CD^2$

$h^2 + 4^2 = 8^2$

$\therefore h^2 = 8^2 - 4^2 = 48 \Rightarrow h = \sqrt{48} = 4\sqrt{3} = 6.93$

$\therefore$ Area of Trap. $ABCD = \frac{1}{2}(AD + BC) \times h$

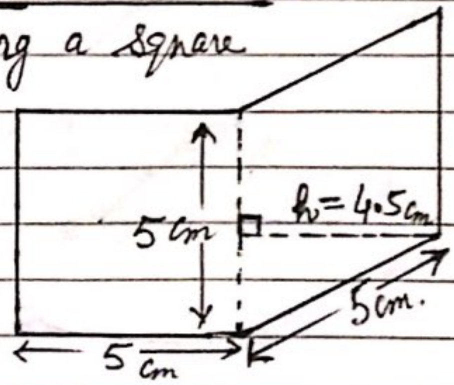
$= \frac{1}{2}(12 + 8) \times 6.93$

$= 69.3$ Cm^2 ✓

Example 2: The shape is made by joining a square and rhombus, work out,

(a) Perimeter of the shape. ---[1]

(b) The area of the shape. ---[2]



W-16	23	Q14

(a) Perimeter $= 5 \times 6 = 30$ Cm.

(b) Area = Area of Square + Area of Rhombus

$= 5^2 + 5 \times 4.5$

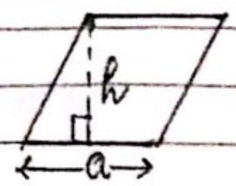
$= 25 + 22.5$

$= 47.5$ Cm^2

Note: Area of Rhombus:

= length of side × distance between the parallel sides

$= a \times h$



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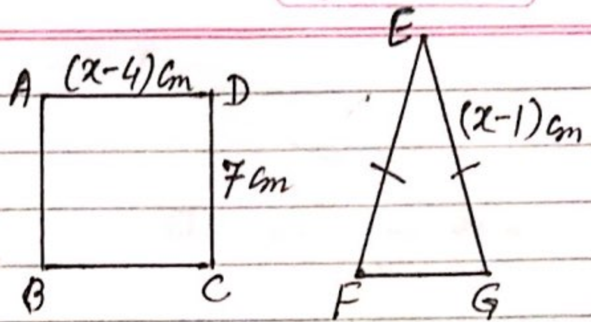
Example 3(a) ABCD is a square, Find the value of x. ---[1]

(b) Square ABCD and isosceles triangle EFG have same perimeter, Work out the length FG. ---[2]



S-15/22/Q13

Solution:

(a) $x-4=7$ (sides of square are equal)

$\therefore x = 11\text{ cm}$ ✓

(b) Perimeter of square $= 7\times 4 = 28$ cm. --(i)

and Perimeter of Triangle EFG $= EF + GF + EG$

$= 10 + FG + 10$

$= 20 + FG$ --(ii)

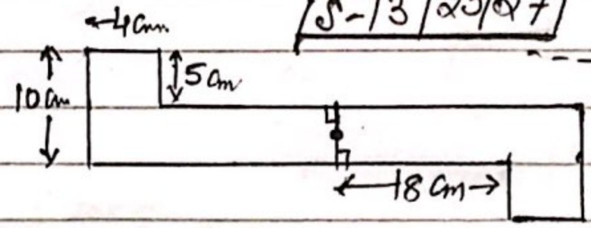
Side $EG = (x-1) = 11-1 = 10$ cm

from (i) & (ii) $20 + FG = 28$ (Given Perimeter of Sq and triangle are equal)

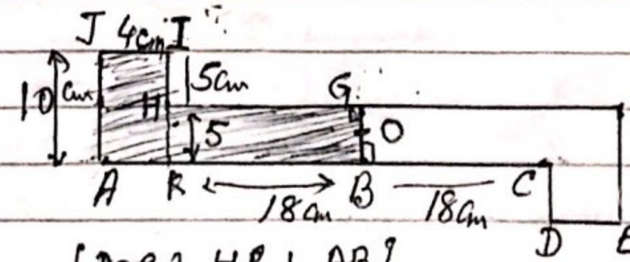
$\therefore FG = 8\text{ cm}$ ✓

Example 4: The shape has a rotational symmetry of Order 2.
Work out the area of the shape.

S-13/23/Q7 ---[3]



Solution: By Symmetry of the fig



$= 2\times$ Area ABGHIJ

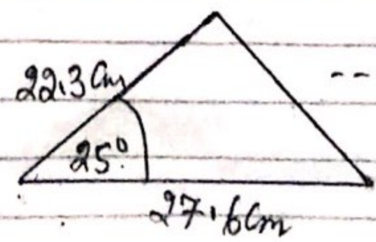
$= 2[$ area ARIJ + ar RBGH $]$

$= 2[10\times\ \ + 18\times 5]$

$= 2[40+90] = 260\text{ cm}^2$

[Draw $HR \perp AB$
$RB = BC = 18$ cm]

Example 5: Calculate the area of this triangle. ---[2]



Solution Area of triangle $= \frac{1}{2}ab\cdot\sin C$

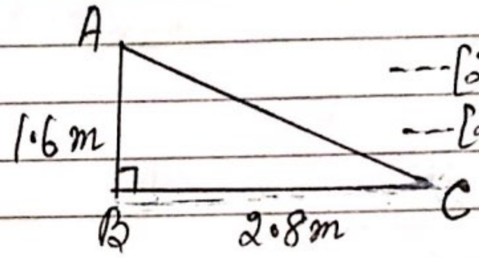
$= \frac{1}{2}\times 27.6\times 22.3\sin 25^\circ$

$= 130\text{ cm}^2$ ✓

M-16/22/Q7

Example 6 (a) Find the area of triangle ABC. ---[2]
(b) Calculate AC. ---[2]

[M-18/22/Q16]



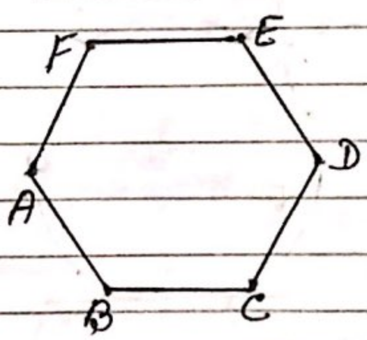
Solution: (a) Area of triangle $= \frac{1}{2} BC \times AB$

$= \frac{1}{2} \times 2.8 \times 1.6 = \underline{2.24\ m^2}$ ✓

(b) Using Pythagoras Theo.

$AC = \sqrt{(2.8)^2 + (1.6)^2} = \underline{3.22\ m}$ ✓

Example 7 (a) The diagram shows a regular hexagon ABCDEF of side 10 cm.



(i) Show that angle $BAF = 120°$. --- [2]

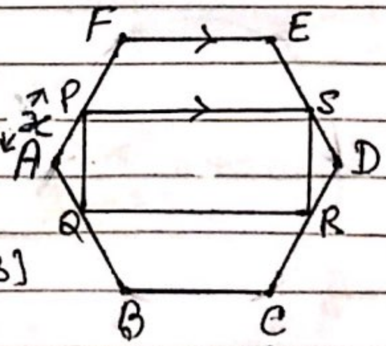
Solution: Int. angle of reg. polygon

$= \frac{(n-2) \times 180}{n}$

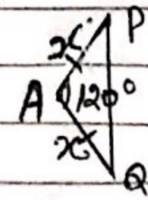
$= \frac{(6-2) \times 180}{6}$ $(n=6)$ [M-17/42/Q10]

$= \underline{120°}$ ✓

(ii) The vertices of a rectangle PQRS, touch the sides FA, AB, CD and DE. PS is parallel PS is parallel to EF and AP = x cm., Use trigonometry to find the length of PQ in terms of x. ---- [3]



Solution: In ΔPAQ,



$PQ^2 = PA^2 + QA^2 - 2 \cdot PA \cdot QA \cdot \cos PAQ$

$= x^2 + x^2 - 2 \times x \times x \times \cos 120°$

$= 2x^2 - 2x^2 \times \left(-\frac{1}{2}\right)$

$PQ^2 = 3x^2$

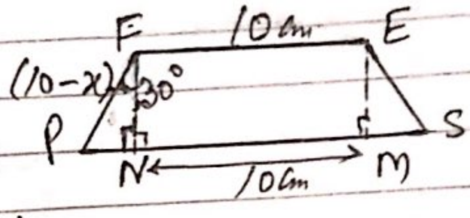
$\therefore PQ = \sqrt{3}\, x$ ✓

$\cos 120° = \cos(180 - 60) = -\cos 60° = -\frac{1}{2}$

(Continued →)

(Continued→)

Example 7(a)(iii) $PF = (10-x)$, Show that $PS = (20-x)$cm ---[3]



Solution: Draw $PN \perp PS$

In ΔPFN

$$\frac{PN}{PF} = \sin 30^\circ$$

$$\Rightarrow PN = PF \sin 30^\circ = (10-x) \times \frac{1}{2}$$

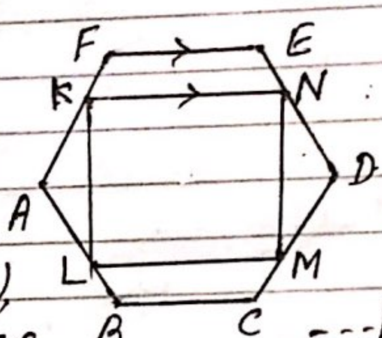
Now $PS = 2 \times PN + NM$ [$NM = FE = 10$cm]

$$= 2 \times \frac{1}{2} \cdot (10-x) + 10$$

$$= (20-x)$$ ✓

(b) The diagram shows the vertices of a square KLMN touching the sides of the same hexagon ABCDEF, with KN parallel to FE.

Use results of part (a)(ii) and part (a)(iii), to find the length of a side of the square. ---[4]



M-17/42/Q10

Solution: KLMN is square

$$KL = KN$$

$$\sqrt{3}x = 20 - x \quad \text{--- (1)}$$

[from a(ii) $KL = \sqrt{3}x$
a(iii) $KN = (20-x)$]

or $(\sqrt{3}+1)x = 20$

$$x = \frac{20}{(\sqrt{3}+1)} = 7.32$$ ✓

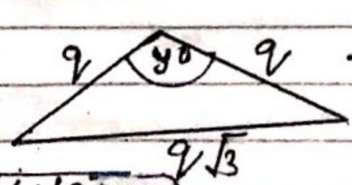
$\therefore$ from (1) $KN = 20 - x$

$$= 20 - 7.32$$

$$= 12.68$$

$\therefore$ Side of Square $= 12.7$ cm. ✓

Example 8. The perimeter of the isosceles triangle is $2q + q\sqrt{3}$, find the value of y. ---[4]



W-16/41/Q10(c)

Solution: using Cosine Rule:

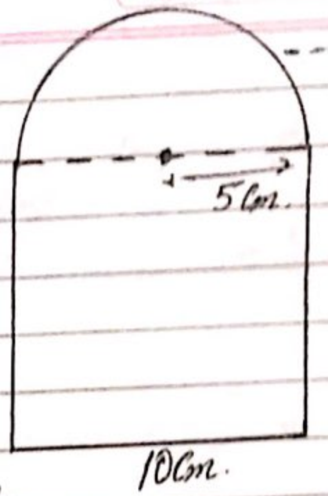
$$\cos y^\circ = \frac{q^2 + q^2 - (q\sqrt{3})^2}{2q \cdot q}$$

or $\cos y^\circ = -\frac{1}{2} = -\cos 60^\circ = \cos(180 - 60^\circ) = \cos(120^\circ)$ $\therefore y = 120^\circ$ ✓

Mensuration

§ Circle, arc, sector of a circle:

Example 9: The diagram shows a shape made from a square and a semi-circle. Calculate the area of the shape. Give the units of your answer. [W-17/23/Q20] ---[5]



Solution: Area of the shape.

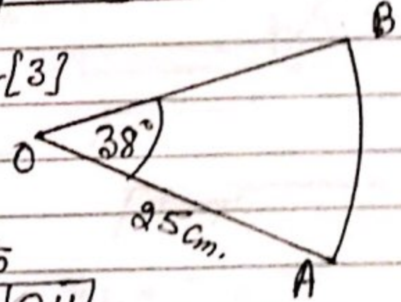
= Area of square + Area of Semicircle.

$= l^2 + \frac{1}{2} \cdot \pi r^2 \quad [r = \frac{10}{2} = 5 cm.]$

$= 10^2 + \frac{1}{2} \times \pi \times 5^2$

$= 100 + 39.27 = 139.27$ sq. cm. ✓

Example 10: The diagram shows a sector of a circle, Centre O, radius 25 cm, The sector angle is 38°. Calculate the length of arc $\widehat{AB}$. Give your answer to four significant figures. [M-16/22/Q11] ---[3]

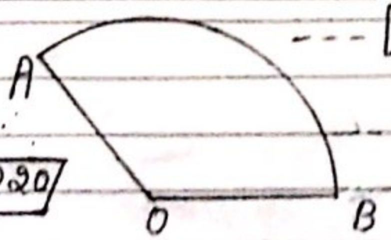


Solution: Length of arc $AB = \frac{\theta}{360} \times 2\pi r$ $\quad [\theta = 38°, \; r = 25 cm.$

$= \frac{38}{360} \times 2\pi \times 25$

$= 16.58$ cm.

Example 11: $\widehat{AB}$ is an arc of a circle, Centre O, radius 9 cm. The length of arc AB is 6π cm. The area of sector AOB is $k\pi$ cm². Find the value of k. [S-16/21/Q20] ---[3]



Solution: Area of Sector $= \frac{1}{2} l r$

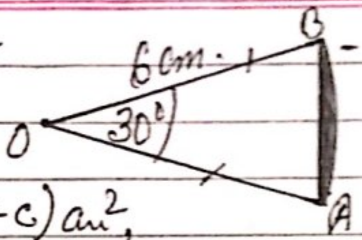
$\therefore \cancel{\pi} k = \frac{1}{2} \times 6\cancel{\pi} \times 9$

$\therefore k = 27$ ✓

[l = length of arc AB = 6π cm
Area of Sector = $k\pi$ cm²
$r = 9$ cm

§ Circle, arc, Sector of a Circle:

Example 12: The diagram shows a sector of a circle, centre O and radius 6 cm. The sector angle is 30°. The area of the shaded segment is $(k\pi - c)\ cm^2$, where k and c are integers. Find the value of k and value of c. ---[3]



[W-17/22/Q23]

Solution: Area of isosceles $\Delta OAB = \frac{1}{2}r^2 \sin\theta = \frac{1}{2}\times 6^2 \times \sin 30° = 9\ cm^2$

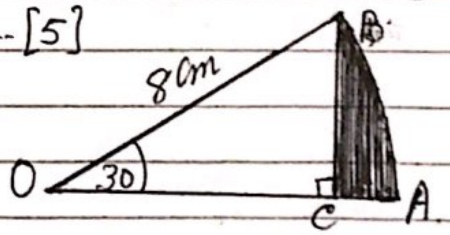
Area of Sector OAB $= \frac{\theta}{360}\times\pi r^2 = \frac{30}{360}\times\pi\times 6^2 = 3\pi\ cm^2$

$\therefore$ Area of the shaded segment = Area of Sector − area of Δ

$= (3\pi - 9) = \underbrace{(k\pi - c)}_{\text{Given}}$

$\therefore\ k = 3$ ✓ and $c = 9$ ✓

Example 13: OAB is a sector of a circle, with radius 8 cm and sector angle 30°. BC is is perp. to OA. Calculate the area of the shaded region. [W-15/23/Q25] ---[5]



Solution: Area of Shaded region = Area of sector OAB − ar of rt ΔOBC. —①

In rt ΔOBC, $\frac{BC}{OB} = \sin 30$

$\Rightarrow$ $BC = OB \cdot \sin 30 = 8\times\frac{1}{2} = 4\ cm$

and $OC = OB \cos 30 = 8\times\frac{\sqrt{3}}{2} = 4\sqrt{3}\ cm$

f① Req Area $= \frac{\theta}{360}\times\pi r^2 - \frac{1}{2}\times OC\times BC$

$= \frac{30}{360}\times\pi\times 8^2 - \frac{1}{2}\times 4\sqrt{3}\times 4$

$= 16.755 - 13.856$

$= 2.8985$

$= 2.9\ cm^2$

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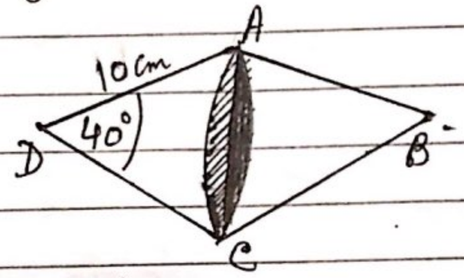
Area of Plane figures (circles, arc, sectors).

Example 14: ABCD is rhombus with side length 10 cm. and angle $ADC = 40°$, DAC is a sector of a circle with centre D and BAC is a sector of a circle, centre B. --- [4]

Calculate the shaded area. [S-17/21/19]



Solution: Area of Shaded shape = $2 \times$ area of segment DAC

$= 2[\text{area of sector } DAC - \text{area } \Delta DAC]$

$= 2\left[\frac{40}{360} \times \pi \times 10^2 - \frac{1}{2} \times 10^2 \times \sin 40°\right]$

$= 2\left[\frac{100\pi}{9} - 50 \sin 40\right]$

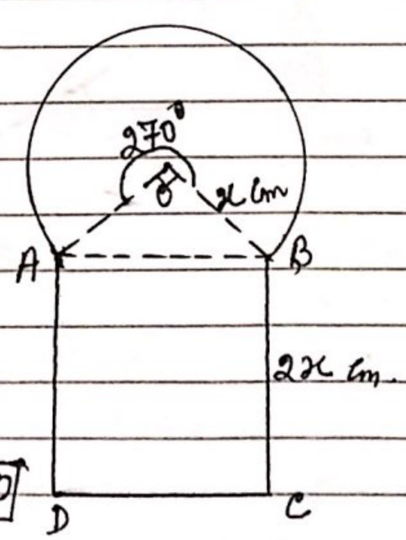
$= 2 \times [34.9 - 32.14] = 2 \times 2.767$

$= 5.53\ cm^2$

Example 15: The diagram shows a sector of a circle, a triangle and a rectangle. The sector has centre O, radius x cm. and angle 270°. The rectangle has length $2x$ cm. The total area of the shape is $kx^2\ cm^2$.

(a) Find the value of k. --- [5]

(b) Find the value of x when the total area is $110\ cm^2$. --- [2] [W-17/42/Q10]



Solution:

(a) Total area of Shape = Sector with angle 270° + area of rt ΔAOB + area of Rect ABCD

$= \frac{270}{360} \times \pi x^2 + \frac{1}{2} x \times x + 2x \times \sqrt{2}x$ $\left[\because AB = \sqrt{x^2 + x^2} = \sqrt{2}x\right]$ Using Pythagoras Theo in rt ΔAOB.

$= [2.856 + 2.8284]x^2$

$= 5.68x^2 = kx^2$ given

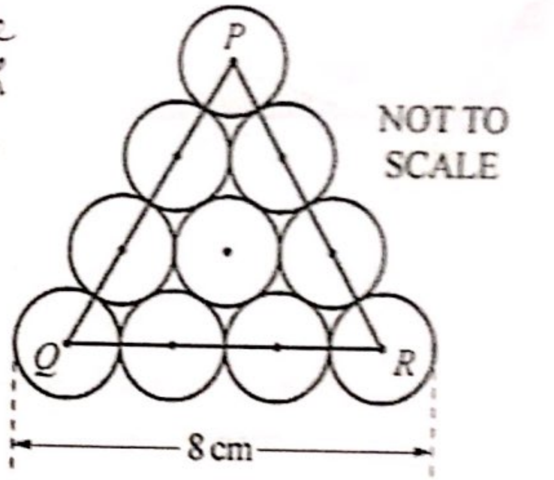
$\therefore k = 5.$

(ii) Given area $5.68x^2 = 110 \Rightarrow x^2 = \frac{110}{5.68} = 19.366$

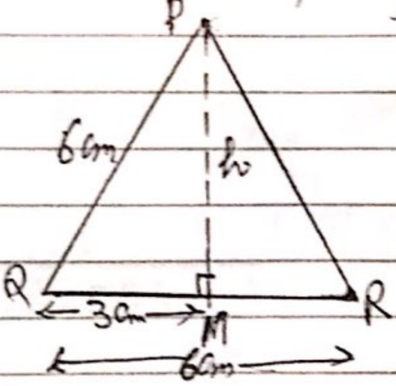
$\therefore x = \sqrt{19.366} = 4.4$

Area of Plane figures (Circle)

Example 16 (a) The ten circles in the diagram, each have radius 1cm. The centre of each circle is marked with a dot. Calculate the height of triangle PQR.



Solution: (a)
In equilateral $\triangle PQR$
Draw $PM \perp QR$



$\therefore QM = \frac{1}{2} QR = \frac{1}{2} \times 6 = 3$ cm.

$\therefore$ rt $\triangle PMQ$, using Pythagoras Theo.

$PM^2 = PQ^2 - QM^2$

$h = 6^2 - 3^2 = 27$

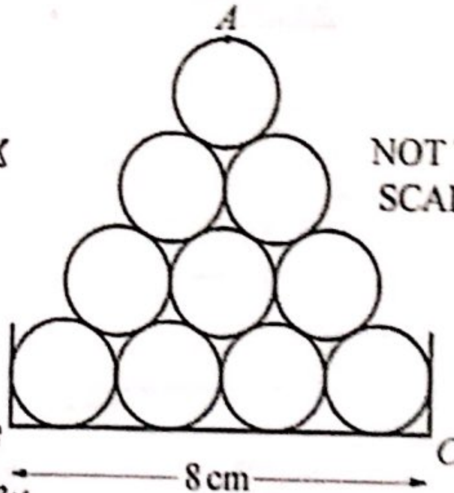
Height $h = \sqrt{27} = 5.2$ cm

.... cm [3]

M-16/42/Q10

(b) Mr Patel uses whiteboard pens that are cylinders of radius 1 cm.

(i) The diagram shows 10 pens, staked in a tray. The is 8cm wide. The point A is the highest point in the stake. Find the height of A above the base BC, of the tray. ---[1]



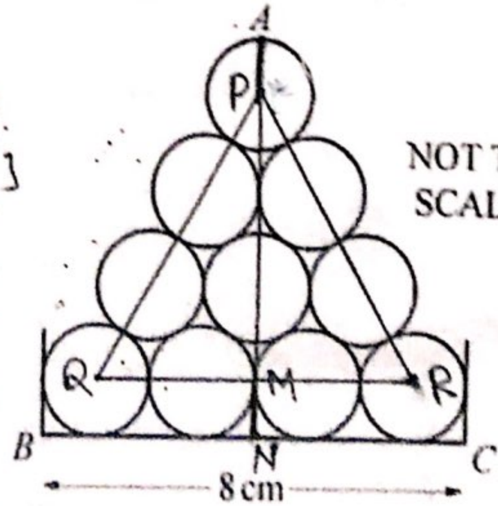
Solution Height of A above BC = AN

$= PM + AP + MN$

$= 5.2 + 1 + 1$

$= 7.2$ cm.

[PM = 5.2 from (a)]



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Mensuration

Area of Plane Shapes:

Example 17: The total area of each of the following shapes is X, The area of the shaded part of each shape is kx. For each shape, find the value of k and write your answer below each diagram. ----[10] [S-14|42|Q11]

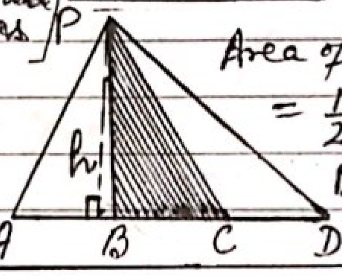
(i) [Δ's which have their bases on a line and have a common vertex, their areas are proportional to the size of bases.]

$AB = BC = CD$

$k = \text{.....}\ \frac{1}{3}$ ✓

[Area of $\Delta PAD = \frac{1}{2} \times AD \times h$]

area of $\Delta PBC = \frac{1}{2} \times BC \times h = \frac{1}{2} \times \frac{AD}{3} \times h$]

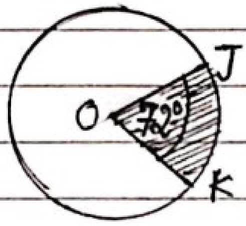


Area of Triangle $= \frac{1}{2} \times \text{base} \times \text{Alt}$ and $BC = \frac{1}{3} AD$

(ii) Angle $JOK = 72^0$

$k = \text{......}\ \frac{1}{5}$ ✓

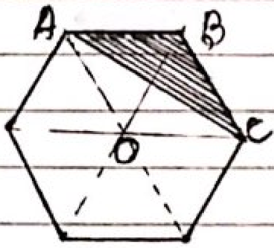
[Area of Sector $= \frac{\theta}{360} \times \pi r^2 = \frac{72}{360} \times \pi r^2 = \frac{1}{5} \pi r^2$]



(iii) The shape is a regular hexagon,

$k = \text{.....}\ \frac{1}{6}$ ✓

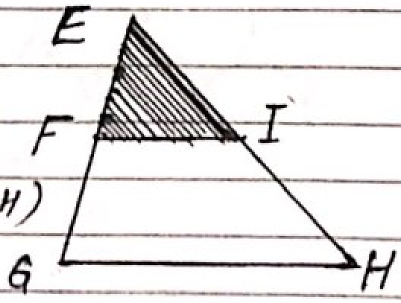
[$\because$ ar ΔABC $=$ ar ΔOAB $= \frac{1}{6}$ ar hexagon]



(iv) $EF = FG$ and $EI = IH$

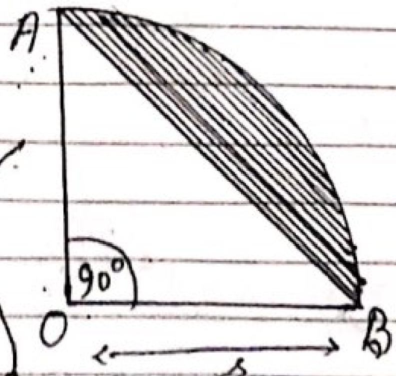
$k = \text{......}\ \frac{1}{4}$ ✓ (Similar Δ's)

[$\frac{ar\,\Delta EFI}{ar\,\Delta\ EGH} = \left(\frac{EF}{EG}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ ($\because \Delta EFI \sim \Delta EGH$)



(v) The diagram shows a sector of a circle, centre O, angle $AOB = 90^0$.

$k = \text{....}\ \left(\frac{\pi - 2}{\pi}\right)$ ✓



$\because$ [Area of Sector $= \frac{90}{360} \pi r^2 = \frac{1}{4} \pi r^2$

Area of $\Delta OAB = \frac{1}{2} OA \times OB = \frac{1}{2} r^2$

$\therefore$ Area of Segment shaded $= \frac{1}{4}\pi r^2 - \frac{1}{2} r^2$

$= \frac{1}{4}\pi r^2 \left(1 - \frac{2}{\pi}\right) = \frac{1}{4}\pi r^2 \left(\frac{\pi - 2}{\pi}\right)$

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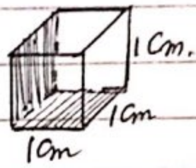
Mensuration

Three dimensial (3-D) Shapes (or Solid Shapes)

§ Units of Volume:



(i) Volume of a cube with each side measuring 1cm. $= 1$ cubic cm. $= 1\,cm^3$ or 1c.c.

(ii) 1 cubic metre or $1m^3 = 100 \times 100 \times 100 = 1000000$ c.c.

(iii) 1 Litre $= 1000$ c.c.

$1\,m^3 = 1000$ Litres

$1\,mm^3 = 0.001\,cm^3$	∵ $1\,Cm = 10\,mm$
$1\,cm^3 = 0.000001\,m^3$	$1\,m = 100\,Cm$
$1\,cm^3 = 0.001$ Litre	1 Litre $= 1000\,Cm^3$
1 Litre $= 0.001\,m^3$	$1\,m^3 = 1000$ Litres

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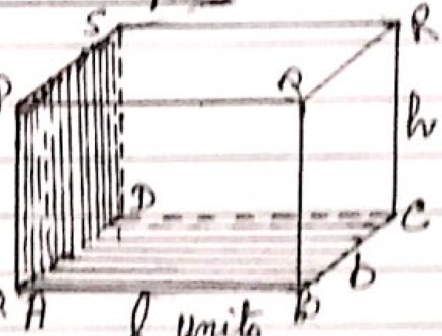
Mensuration

Date P-14
Page No

Surface Area and Volume of 3-D Shapes:

[1] Cuboide: Given length, breadth and height l, b and h respectively.



(i) Total surface area of cuboid:
$= 2(lb+bh+lh)$ unit^2

(ii) Surface area of an open box (without lid) $= [2(l+b)\times h + lb]$ unit^2

(iii) Area of four walls of a room $= 2(l+b)\times h$

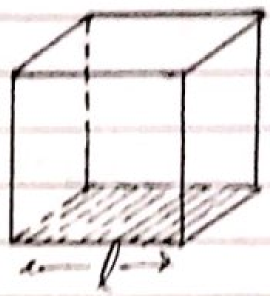
(iv) Length of the longest diagonal $AR = BS = CP = DQ = \sqrt{l^2+b^2+h^2}$

(v) Volume of a Cuboid $= l\times b\times h$ unit^3

[2] Cube:

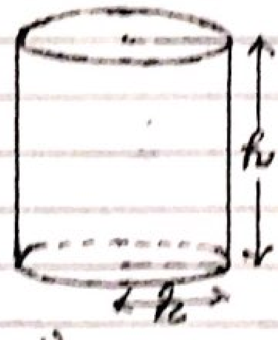
(i) Total Surface area $= 6l^2$ units^2

(ii) Volume $= l^3$ unit^3



[3] Cylinder:

(i) Volume $= \pi r^2\cdot h$ unit^3



(ii) Curved Surface area $= 2\pi r h$ unit^2

(iii) Total surface area $= 2\pi r h + 2\cdot\pi r^2$
$= 2\pi r(h+r)$ ✓

(iv) Surface area of open (at the top) box $= (2\pi r h + \pi r^2)$

(v) Volume of a hollow Cylinder $= \pi(R^2-r^2)\cdot h$

(vi) Total Surface area of a hollow Cylinder
$= 2\pi(R+r)h + \pi(R^2-r^2)$.

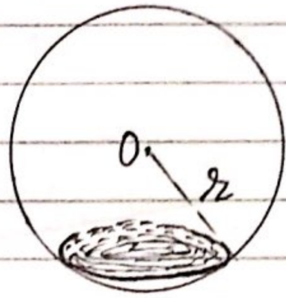
Mensuration

Surface Area and Volume of 3-D Shapes

§[4] Sphere:

(i) Surface Area $= 4\pi r^2$ unit^2

(ii) Volume $= \frac{4}{3}\pi r^3$ unit^3



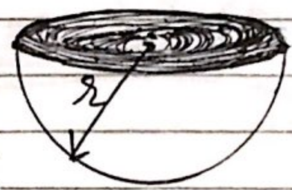
(iii) Volume of hollow sphere $= \frac{4}{3}\pi(R^3 - r^3)$

§[5] Hemisphere:

(i) Volume $= \frac{2}{3}\pi r^3$

(ii) Curved Surface area $= 2\pi r^2$

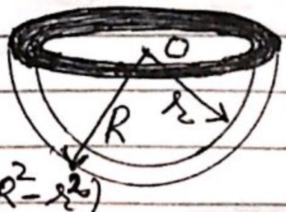
(iii) Total Surface area $= 2\pi r^2 + \pi r^2 = 3\pi r^2$.



§[6] Hemispherical Bowl:

(i) Volume $= \frac{2}{3}\pi(R^3 - r^3)$

(ii) Total Surface area $= 2\pi(R^2 + r^2) + \pi(R^2 - r^2)$

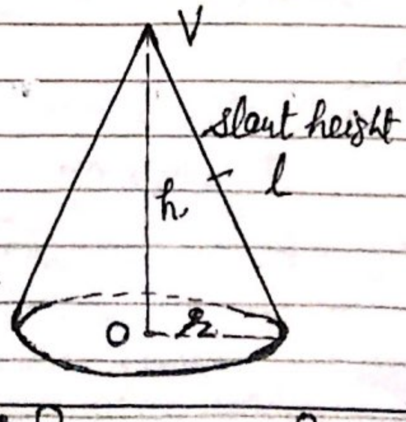


§[7] Cone:

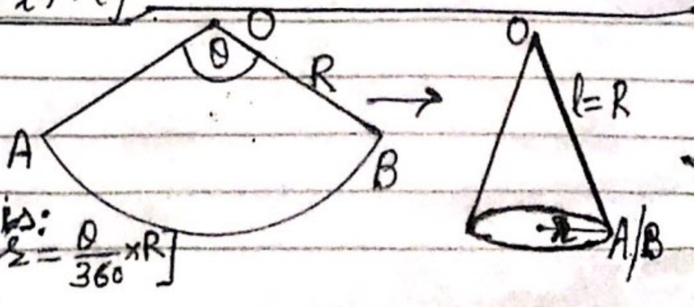
(i) Volume $= \frac{1}{3}\pi r^2 h$

(ii) Curved Surface area $= \pi r l$ $[l^2 = r^2 + h^2]$

(iii) Total Surface area $= \pi r l + \pi r^2$
$= \pi r(l + r)$



(iv) Given a sector of a circle of radius R and sector angle θ, when the arc is folded so that the edges OA & OB coincide. Then the radius 'r' of so formed cone is: $\left[r = \frac{\theta}{360} \times R\right]$



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Mensuration

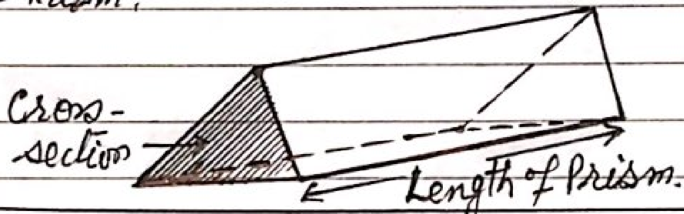
P-16

Surface area and volume of 3-D Shapes

§ [8] **Prism:**

Volume Prism = Area of Cross-section × length.

(i) Triangular Prism:

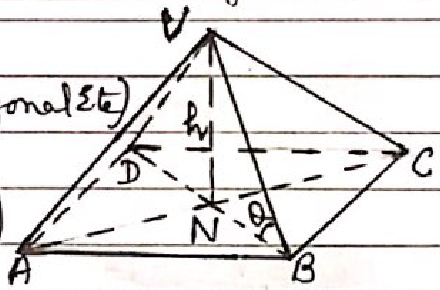


§ [9] **Pyramid:**

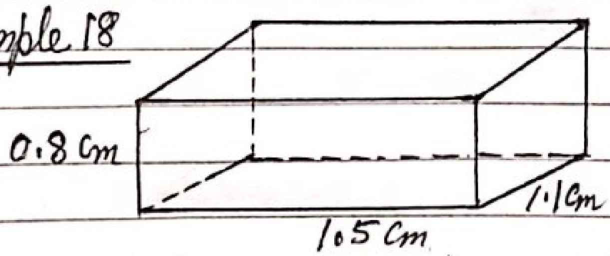
Pyramid with rectangular base.
(May be triangular base or Hexagonal Etc)

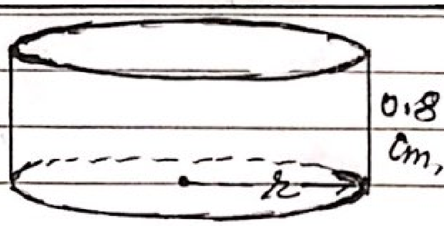
Volume $= \frac{1}{3} \times$ area of base × height.

Angle between the edge VB and base $\theta = \tan^{-1}\left(\frac{h}{NB}\right)$



Example 18





The diagram shows two sweets. The cuboid has length 1.5 cm, width 1.1 cm and height 0.8 cm. The cylinder has height 0.8 cm and same volume as the cuboid.

(i) Calculate the volume of the cuboid. -- [2] $\boxed{W\text{-}16/42/Q6(a)}$

(ii) Calculate the radius of the cylinder. --- [2]

(iii) Calculate the difference between the surface area of the two sweets. --- [5]

Solution:

(i) Volume of Cuboid $= l \times b \times h = 1.5 \times 1.1 \times 0.8$

$= 1.32 \text{ cm}^3$ ✓

(ii) Volume of Cylinder $= \pi r^2 h$

$= \pi r^2 \times 0.8 \text{ cm}^3$

Given the Volume of Cyl. = Volume of Cuboid

$\therefore \pi r^2 \times \cancel{0.8} = 1.5 \times 1.1 \times \cancel{0.8}$

$r^2 = \frac{1.5 \times 1.1}{\pi} = 0.5252$

$\therefore r = \sqrt{0.5252} = 0.725 \text{ cm}$

(iii) Surface area of Cuboid

$= 2(lb + bh + lh)$

$= 2[1.5 \times 1.1 + 1.1 \times 0.8 + 1.5 \times 0.8]$

$= 7.46 \text{ cm}^2$

Surface area of cylinder

$= 2\pi r(h + r)$

$= 2\pi \times 0.725(0.8 + 0.725)$

$= 6.9468 \text{ cm}^2$

$\therefore$ Difference between the surface area $= 7.46 - 6.9468 = 0.513 \text{ cm}^2$ ✓

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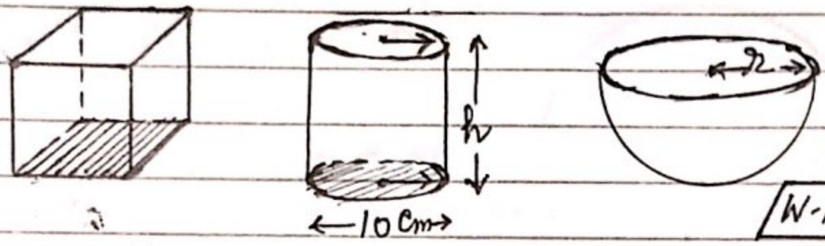
Surface area and volume of 3-D shapes.

Example 19.



W-17/43/Q6

The diagram shows a cube, a cylinder and a hemisphere, The volume of each of these solids is 2000 cm^3.

(a) Work out the height, h, of the cylinder, ---[2]

(b) Work out the radius, r, of the hemisphere. --- [3]

(c) Work out the surface area of the cube. ---[3].

Solution:

(a) Volume of cylinder $= \pi r^2 h = 2000$

$\pi \times 5^2 \times h = 2000$ [$\because$ = 10 cm] [r = 5 cm

$\therefore h = \left(\frac{2000}{25\pi}\right)$

or $\underline{h = 25.5 \text{ cm}}$ ✓

(b) Volume of hemisphere:

$\frac{2}{3}\pi r^3 = 2000$

$\therefore r^3 = \frac{2000 \times 3}{2\pi} = 954.9$

$\therefore r = (954.9)^{1/3} = 9.85 \text{ cm}$ ✓

or $\underline{r = 9.85 \text{ cm}}$

(c) Volume of cube:

$l^3 = 2000$

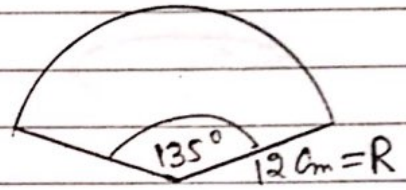
$l = (2000)^{1/3} = 12.6 \text{ cm}$

Surface area of cube $= 6l^2$

$= 6 \times (12.6)^2$

$= \underline{952 \text{ cm}^2}$ ✓

Surface area and Volume of 3-D shapes:

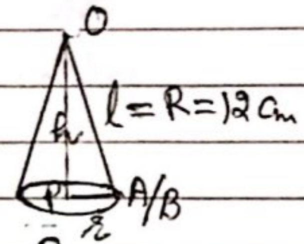


Example 20: A sector of a circle has radius 12 cm and an angle 135°.

(i) Calculate the length of the arc of this sector. Give your answer as a multiple of π --- [2]

Solution

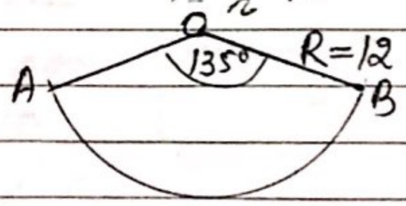
$$\text{length of arc} = \frac{\theta}{360} \times 2\pi R = \frac{135°}{360} \times 2\pi \times 12 = 9\pi \checkmark \text{ — ①}$$



(ii) The sector is used to make a cone.

(a) Calculate the base radius, r. --- [2]

(b) Calculate the height of the cone, h. --- [3]



Solution when arc OA is folded the Radii OA and OB coincide to get a cone, whose slant height:

$$l = OA = R = 12 \text{ Cm}$$

(a) and circumference of base of cone = length of arc $\widehat{AB}$

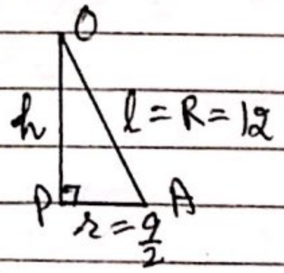
$$2\pi r = 9\pi \quad \text{from ①}$$

$$\therefore r = \frac{9}{2} = 4.5 \text{ Cm} \checkmark$$

Refer. §[7](iv) $r = \frac{\theta}{360} \times R$

$= \frac{135}{360} \times 12 = 4.5 \checkmark$

(b) In right triangle OPA,



$$h^2 + r^2 = l^2$$

$$h^2 + \left(\frac{9}{2}\right)^2 = 12^2$$

$$h^2 = 144 - \frac{81}{4} = 144 - 20.24$$

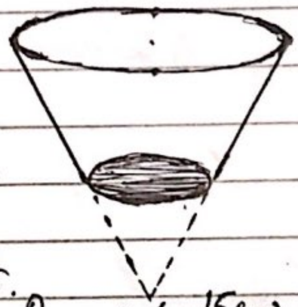
$$h^2 = 123.75$$

$$h = \sqrt{123.75} = 11.1$$

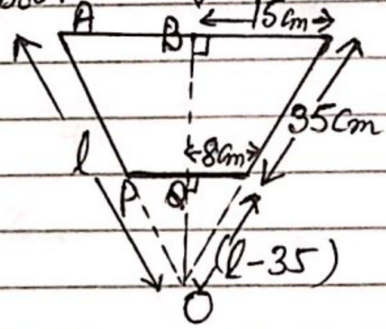
$$\therefore h = 11.1 \text{ cm} \checkmark$$

S-15/42/Q4(a)

Surface area and Volume of 3-D Shapes:



Example 21. The diagram shows a plant pot.
It is made by removing a small cone from a larger cone and adding a circular base.
This is the cross section of the plant pot.



(i) Find l. ---[3]

Solution: $\Delta OPQ \sim \Delta OAB$ (Similar)

$$\therefore \frac{OP}{OA} = \frac{PQ}{AB}$$

$$\frac{l-35}{l} = \frac{8}{15}$$

$$\therefore 15(l-35) = 8l \Rightarrow l = 75\text{cm} \checkmark$$

(ii) Calculate the total surface area of outside of the plant pot. --[3]

Solution: Total Surface area of Pot = Lateral Surface area of big cone − L.S.A of small cone + circular base

$$= \pi R l - \pi r(l-35) + \pi r^2$$
$$= \pi \times 15 \times 75 - \pi \times 8 \cdot (75-35) + \pi \times 8^2$$
$$= \pi[1125 - 320 + 64]$$
$$= 869\pi$$
$$= 2730\ \text{cm}^2 \checkmark$$

$\left\{ \begin{array}{l} R = 15\text{ cm} \\ r = 8\text{ cm} \\ l = 75\text{ cm} \end{array} \right.$

[S-15/42/Q4(b)]

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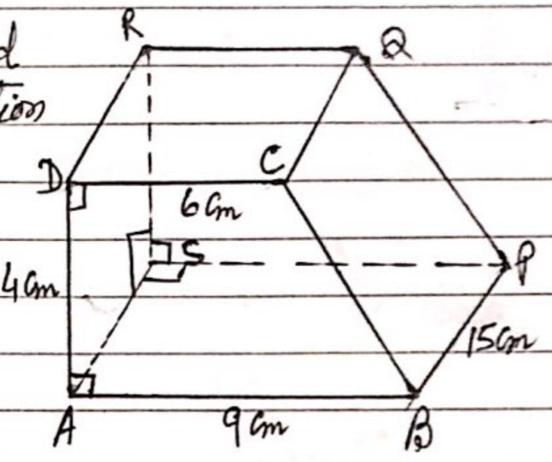
Mensuration

Surface area and Volume of 3-D Shapes:

Example 22: Diagram shows a solid prism of length 15 cm. The cross-section of the prism is the trapezium ABCD. Angle DAB = angle CDA = 90°. AB = 9 cm, DC = 6 cm and AD = 4 cm.

(i) Calculate the total surface area of the prism. ---[5]

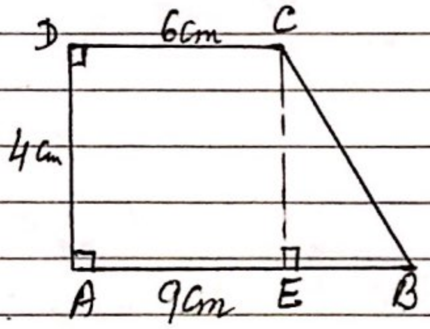
(ii) Calculate the volume. ---[3]



Solution:

(i) Draw CE perp. AB.

In rt Δ CEB $[EB = 9-6 = 3]$



$CB^2 = CE^2 + EB^2$

$= DA^2 + EB^2$

$= 4^2 + 3^2 = 5^2$

$\therefore CB = 5$ ✓

Now Total Surface area = 2 ar Trap. ABCD + ar rect BPQC + ar ADRS + ar RDCQ + ar ABPS

$= 2 \times \frac{1}{2}(9+6) \times 4 + 15 \times 5 + 15 \times 4 + 15 \times 6 + 15 \times 9$

$= 420 \text{ cm}^2$ ✓

(ii) Volume = area of cross-section ABCD × length BP

$= \left\{\frac{1}{2}(9+6) \times 4\right\} \times 15$

$= 30 \times 15$

$= 450 \text{ cm}^3$ ✓

S-13/21/Q26

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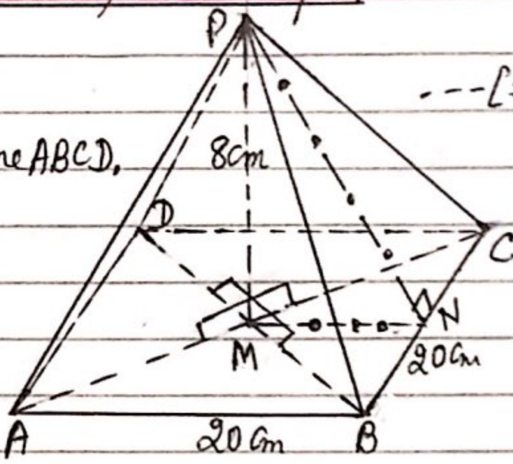
Mensuration

Date P-21

Surface area and volume of 3D-Shapes:

Example 23: The diagram shows a solid pyramid on a square horizontal plane ABCD. The diagonals AC and BD intersect at M. P is vertically above M. AB = 20 cm, and PM = 8 cm. Calculate the total surface area of the pyramid. ---[5]

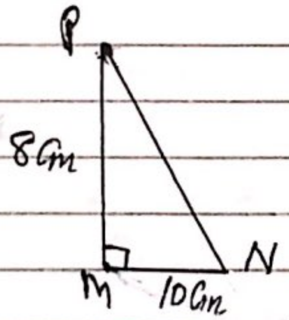
S-15/23/Q18

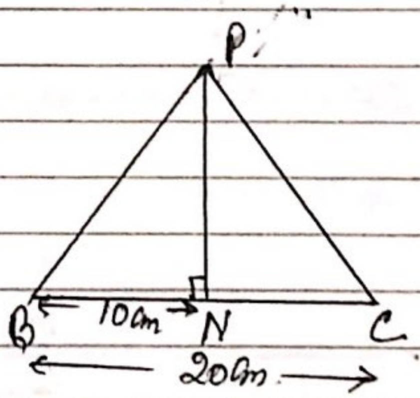


Solution: Total surface = base square + 4 × slant triangular face. —— (1)

$MN = \frac{20}{2} = 10$ cm,

In rt Δ PMN





$PN^2 = PM^2 + MN^2$
$= 8^2 + 10^2$
$= 164$

$PN = \sqrt{164} = 12.8$ cm —— (1)

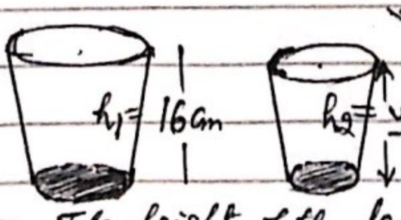
fr (1) Total Surface area = Sq. base + 4 × ar of slant face Δ

$= 20^2 + 4 \times \frac{1}{2} \times 20 \times PN$

$= 400 + 2 \times 20 \times 12.8$ [PN = 12.8 fr (1)]

$= 912\ cm^2$

Example 24.



The diagram shows two glasses that are mathematically similar. The height of the larger glass is 16 cm and its volume is 375 cm³. The height of the smaller glass is y cm, and its volume is 192 cm³. find y. ...[3]

Solution: $\frac{\text{Volume of smaller glass}}{\text{Volume of larger glass}} = \left(\frac{h_1}{h_2}\right)^3 \Rightarrow \left(\frac{y}{16}\right)^3 = \frac{192}{375} = 0.512$

$\therefore \frac{y}{16} = (0.512)^{1/3} = 0.8$

$\Rightarrow y = 16 \times .8 = 12.8$ cm.

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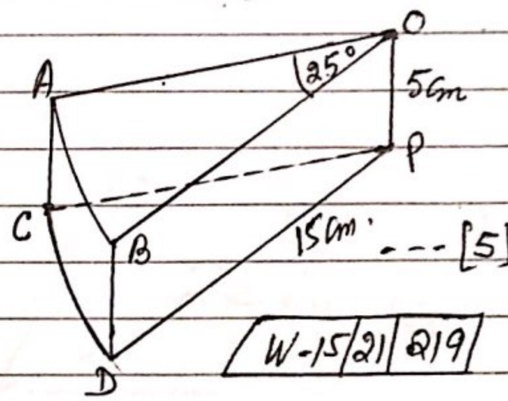
Surface area and Volume of 3-D Shapes:

Example 25: The diagram shows a wooden prism of height 5 cm. The cross-section of the prism is a sector of a circle with sector angle 25°. The radius of the sector is 15 cm. Calculate the total surface area of the prism. --- [5]



W-15/21 Q19

Solution: Area of Sector $OAB = \frac{\theta}{360} \times \pi r^2 = \frac{25}{360} \times \pi \times 15^2 = 49 \text{ cm}^2$

Length of arc $AB = \frac{\theta}{360°} \times 2\pi r = \frac{25}{360} \times 2\pi \times 15 = 6.545 \text{ cm}$

Curved Area $ACDB$ = length of arc $AB \times BD = 6.545 \times 5 = 32.72 \text{ cm}^2$

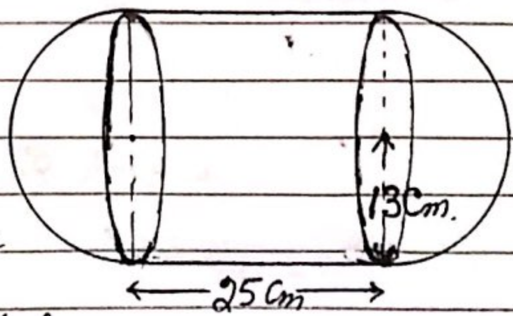
Area of Rectangle $OBDP = 15 \times 5 = 75 \text{ cm}^2$

$\therefore$ Total surface area $= 2 \times 49 + 32.72 + 2 \times 75$

$= 280.72$

$= \underline{281 \text{ cm}^2}$

Example 26: The diagram shows a solid made up of a cylinder and two hemispheres. The radius of the cylinder and the hemisphere is 13 cm. The length of the cylinder is 25 cm.



(i) One cubic centimetre of the solid has a mass of 2.3g. ---[4]
Calculate the mass of the solid. Give your answer in kilograms.

Solution: Volume of solid = 2×Vol. of hemisphere + Vol. of cylinder

$= 2 \times \frac{2}{3}\pi r^3 + \pi r^2 l$

$= \pi r^2\left[\frac{4}{3}r + l\right] = \pi \times 13^2\left[\frac{4}{3} \times 13 + 25\right]$

$= 22476 \text{ cm}^3$

$\therefore$ mass of solid $= V \times \text{density} = 22476 \times 2.3 \text{ g} = 51694.8 \text{ gr}$

$= 51.694 \text{ kg}$

$= 51.7 \text{ kg}$.

(continued→)

Mensuration

Surface area and volume of 3-D Shapes.

(Continued→)

Example 26 (ii) The surface of the solid is painted at a cost of \$ 4.70 per square metre. Calculate the cost of painting. ---[4]

Solution: Surface area = 2 × curved surface area of sphere + Curved surface area of Cylinder

$$= 2 \times 2\pi r^2 + 2\pi r l$$

$$= 2\pi r(2r + l)$$

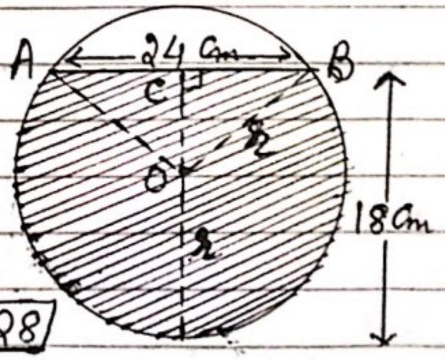
$$= 2\pi \times 13\,(2 \times 13 + 25) = 4166 \text{ cm}^2$$

$$= \frac{4166}{100 \times 100} = 0.4166 \text{ m}^2$$

Rate of painting = \$4.70 per sq.m.

$\therefore$ Cost of painting = $0.4166 \times 4.70 =$ \$1.96 ✓

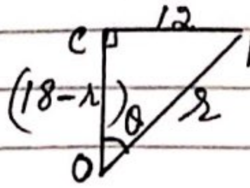
Example 27: The diagram shows the cross-section of a cylinder, Centre O, radius r, lying on its side. The cylinder contains water to a depth of 18 cm. The width, AB, is 24 cm, is the surface of the water. **W-16/43/Q8**



(a) Use algebric method to show that $r = 13$ cm. ---[4]

(b) Show that Angle AOB = 134.8° correct to one decimal place. --[2]

Solution:



(a) $BC = \frac{24}{2} = 12$

In right Δ OCB

Using Pythagoras Theo.

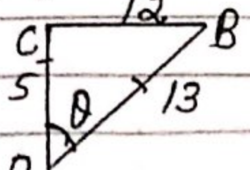
$$r^2 = (18 - r)^2 + 12^2$$

or $36r = 468$

or $r = 13$ cm ✓

(b) angle AOB = 2θ, —① if angle COB = θ

OC = 18 − 13 = 5



$$\sin\theta = \frac{BC}{OB} = \frac{12}{13}$$

$$\therefore \theta = \sin^{-1}\left(\frac{12}{13}\right) = 67.38^\circ$$

$\therefore$ angle AOB $= 2\theta = 2 \times 67.38 = 134.76$ ✓
$= 134.8^\circ$ ✓

(continued→)

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(Continued→)

Example 27(C) (i) Calculate the area of major sector OAPB --[3]

(ii) Calculate the area of the shaded segment APB ---[3]

(iii) The length of cylinder is 40 cm. Calculate the volume of water in the cylinder. --[1]

Solution: C(i) Area of Sector $OAPB = \frac{\theta}{360} \times \pi r^2$

$= \frac{225.2}{360} \times \pi \times 13^2$ $\left[\begin{array}{l} \theta = 360 - 134.8^\circ \\ = 225.2^\circ \end{array} \right.$

$= 332 \text{ cm}^2$ ✓ —— (1)

(ii)

area of triangle $AOB = \frac{1}{2} r^2 \sin(\text{angle } AOB)$

$= \frac{1}{2} \times 13^2 \times \sin 134.8^\circ = 60 \text{ cm}^2$ —— (2)

$\therefore$ Area of Shaded Segment APB = area of Sector OAPB + ar Δ OAB

$= 332 + 60$ (frm (1) & (2))

$= 392 \text{ cm}^2$.

(iii)

Volume of water = area of cross-section × length of cyl.

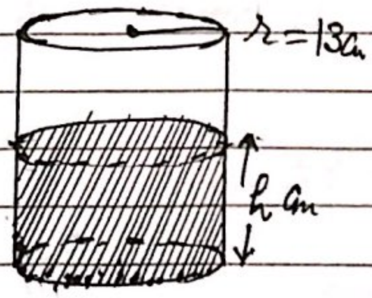
$= 392 \times 40 = 15680 \text{ cm}^3$. —— (3)

*(d) Volume of water in the cylinder $\pi r^2 h = 15680$ frm (3)

or $\pi \times 13^2 \times h = 15680$

$\therefore h = \frac{15680}{\pi \times 13^2} = 29.5 \text{ cm}$ ✓

Solution ↗



(d)* Question: The cylinder is turned so that it stands on one of its circular ends.

In this position, the depth of the water is h. Find h. --[2]

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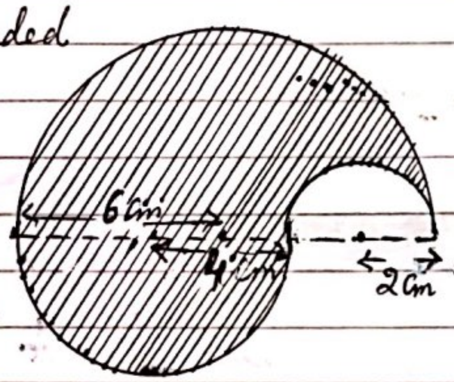
Surface area and volume of 3D-Shapes:

Example 28: The diagram shows a shaded shape formed by three semi-circular arcs. The radius of each semi-circle is shown in the diagram.



(i) Calculate the perimeter of the shaded shape. ---[2]

(ii) The shaded shape is made from metal 1.6 mm thick.
Calculate the volume of metal used to make this shape.
Give your answer in cubic millimetres. $\boxed{S\text{-}15/41/Q9(b)}$ --[5]

Solution

(i) Perimeter of the shaded shape = Sum of the circumference of the three semicircles.

$= \pi(r_1 + r_2 + r_3)$

$= \pi(6 + 4 + 2)$

$= 12\pi = 37.7\text{cm}.$ ✓

(ii) Area of the shaded shape $= \frac{1}{2}\pi[4^2 + 6^2 - 2^2]$

$= \frac{\pi}{2} \times 48 = 24\pi = 75.4 \text{ cm}^2$

$= 75.4 \times 100 \text{ mm}^2 = 7540 \text{ mm}^2$

Thickness of metal Plate = 1.6 mm

$\therefore$ Volume = area of Plate × thickness

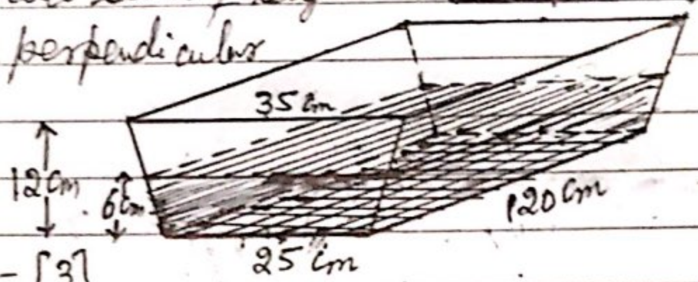
$= 7540 \times 1.6$

$= 12064 \text{ mm}^3$ ✓

Mensuration

Surface area and Volume of 3-D Shapes.

Example 29: The diagram shows a horizontal trough in the shape of a prism. The cross-section of this prism is is a trapezium. [N-15/43/Q3]
The trapezium has parallel sides of lengths 35 cm and 25 cm, and a perpendicular height of 12 cm. The length of the prism is 120 cm.



(a) Calculate the volume of the trough, ---- [3]

(b)(i) Show that the volume of water is $19800\ cm^3$. ---[2]

(ii) Calculate the percentage of the trough that contains water. --[1]

(c) The water is drained from the trough at a rate of 12 litres per hours. Calculate the time it takes to empty the trough (In hrs and mins) --[4]

Solution (a)

Volume of the trough.
= Area of cross section × length
$= \frac{1}{2}[25+35] \times 12 \times 120$
$= \frac{1}{2} \times 60 \times 12 \times 120 = \underline{43200\ cm^3}$

b(i) Volume of water
$= \frac{1}{2} \times [25+30] \times 6 \times 120 = \underline{19800\ cm^3}$

(ii) Percentage of water $= \frac{19800}{43200} \times 100 = \underline{45.83}$

(c) Volume of water $= 19800\ cm^3 = 19.8$ Litres
rate of draining = 12 lit/h
$\therefore$ Time $= \frac{19.8}{12} = 1.65$ hrs
= 1hr 39 min.

Q.(d) The water from the trough just fills a cylinder of radius r cm and height 3r cm, Calculate the value of r.

(d) Solution :

Volume of Cylinder $= \pi r^2 h$
or $\pi r^2 \times 3r = 19800\ cm^3$ (given)
or $3\pi r^3 = 19800$
$r^3 = \frac{19800}{3\pi} = 2100.8$
$r = \sqrt[3]{2100.8} = 12.8\ cm$

Q (e) The cylinder has a mass of 1.2 kg. $1\ cm^3$ of water has a mass of 1 g. Calculate the total mass of the cylinder and water (in kg)

Solution Mass of water = 19800 g.
Total mass $= 1.2\ kg + \frac{19800}{1000}\ kg$
$= (1.2 + 19.8)\ kg$
$= 21\ kg$