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0580

IGCSE - Maths

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Mensuration Revision

SP 20	M-20	S-20	W-19

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Mensuration

Chage of units.

1. Chai says thet $8 cm^2$ is same as $80 mm^2$.
Explain why Chai is wrong. [SP-20/02/Q4] ---[1]

Answer: $1 cm = 10 mm$

$\Rightarrow 1 cm^2 = 10 \times 10 mm^2 = 100 mm^2$

$\Rightarrow 8 cm^2 = 8 \times 100 = 800 mm^2$ and not $80 mm^2$

$\therefore$ Chai is wrong.

Note:

1. (i) $1 m^2 = 100 \times 100 = 10\,000 cm^2$
(ii) $1 Km^2 = 1000 \times 1000 = 1\,000\,000 m^2$

2. (i) $1 cm^3 = 10 \times 10 \times 10 = 1000 mm^3$
(ii) $1 m^3 = 100 \times 100 \times 100 = 1\,000\,000 cm^3$

3. (i) 1 Litre $= 10 \times 10 \times 10 = 1000 cm^3$ (c.c) $= (1 dm^3)$
(ii) $1 m^3 = 100 \times 100 \times 100 cm^3 = 1000 \times 1000 cm^3$
$= 1000$ litres.

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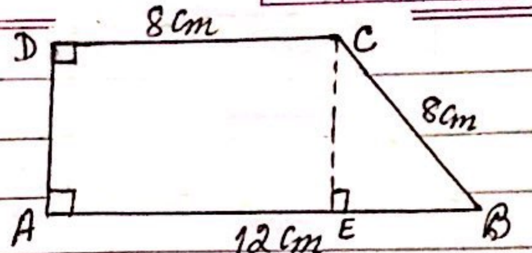
2. Calculate the area of this trapezium.



[SP-20/02/Q17] ...[4]

Solution: Draw CE perpendicular to AB.

$BE = AB - AE = AB - CD = 12 - 8 = 4$

Now In rt triangle using Pythagoras theorem,

$CE^2 + BE^2 = BC^2 \Rightarrow CE = \sqrt{BC^2 - BE^2} = \sqrt{8^2 - 4^2} = \sqrt{48}$

$\Rightarrow CE = 6.928$ cm

Area of Trapezium $= \frac{1}{2}$ (Sum of Parallel sides) x distance between parallel sides

$= \frac{1}{2}(12+8) \times 6.928$

$= \frac{1}{2} \times 20 \times 6.928 = 69.28$

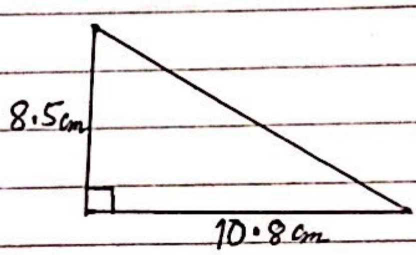
$= 69.3\ cm^2$ ✓

---

3. The diagram shows a right-angled triangle.

(a) Calculate the area. ...[2]

(b) Calculate the perimeter. ...[3]



Solution (a) Area of right-angled triangle $= \frac{1}{2} \times$ the product of perp. sides.

$= \frac{1}{2} \times 10.8 \times 8.5$

$= 45.9\ cm^2$

(b) Hypotenuse $= \sqrt{10.8^2 + 8.5^2} = \sqrt{188.89} = 13.74$ cm.

$\therefore$ The perimeter of the triangle $= 13.74 + 10.8 + 8.5$

$= 33.04$

$= 33$ cm. ✓

[M-20/22/Q7]

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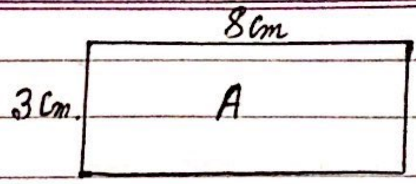
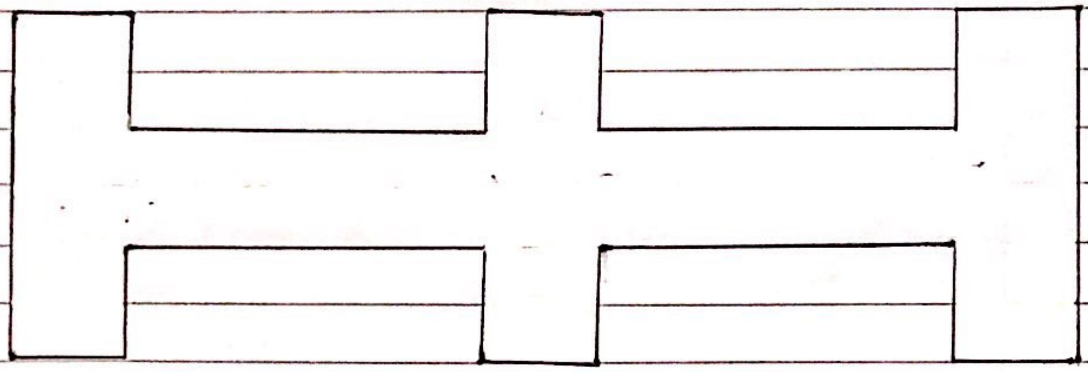
4. Rectangle A measures 3 cm by 8 cm.



Five rectangles congruent to A are joined to make a shape.



Work out the perimeter of this shape. [S-20/21/Q 1] ---[2]

<u>Solution:</u> Perimeter of a rectangle $= 2(8+3) = 22$ cm.

Perimeter of Shape $= 5\times 22 - 8\times 3 = 86$ cm ✓

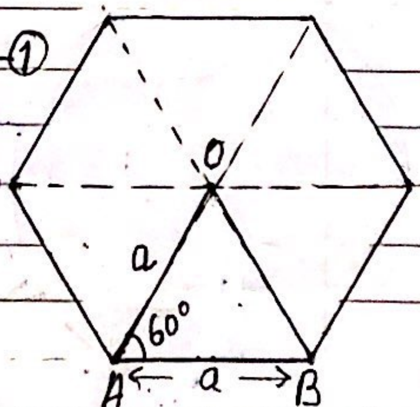
(or $8\times 6 + 5\times 4 + 3\times 6 = 86$)

5. Find the area of a regular hexagon with side length 7.4 cm.

[S-20/21/Q22] ---[3]

<u>Solution:</u> Area of a regular hexagon

$= 6\times$ area of equilateral ΔOAB —①



Let the side $= a$.

Now Area of Equilateral Triangle OAB

$= \frac{1}{2} ab \sin C$

$= \frac{1}{2}\times a\times a\times \sin 60^\circ$

$= \frac{1}{2}a^2\times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}a^2$

from ①

Area of hexagon $= 6\times \frac{\sqrt{3}}{4}a^2 = \frac{3\sqrt{3}}{2}a^2$

for $a = 7.4$ cm $= \frac{3\sqrt{3}}{2}\times(7.4)^2$

$= 142.27$

$\therefore$ Required Area $= 142$ cm^2 ✓

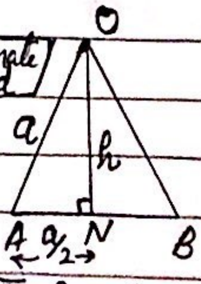
Alternate method

$h = \sqrt{a^2 - \left(\frac{a}{2}\right)^2}$

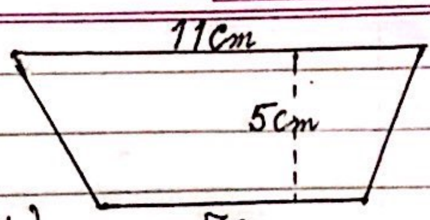
$= \frac{\sqrt{3}}{2}a$

Area of $\Delta = \frac{1}{2}\times a\times h$

$= \frac{1}{2}\times a\times \frac{\sqrt{3}}{2}a = \frac{\sqrt{3}}{4}a^2$ ✓



6. Calculate the area of the trapezium.



[S-20/22/Q6] ---[2]

Solution:

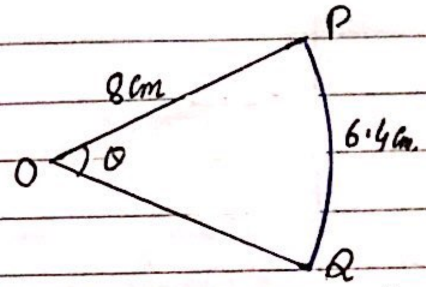
Area of Trapezium $= \frac{1}{2}$(Sum of Parallel sides) x (distance between the parallel sides)

$= \frac{1}{2}(11+7) \times 5$

$= \frac{1}{2} \times 18 \times 5 = \underline{45\ cm^2}$ ✓

---

7. The diagram shows a sector of a circle of radius 8 cm. The length of arc PQ is 6.4 cm. Find the area of the sector.



[S-20/22/Q24] ---[4]

Solution: Let the sector angle POQ = θ

Length of arc PQ $= \frac{\theta}{360} \times 2\pi r = 6.4$

$\Rightarrow \frac{\theta}{360} \times \pi r = \frac{6.4}{2} = 3.2$ —— (1)

Now area of Sector $= \frac{\theta}{360} \times \pi r^2$

$= \frac{\theta}{360} \times \pi r \times r = 3.2 \times 8$ for (1)

$= \underline{25.6\ cm^2}$ ✓

[Alternatively: Area of Sector $= \frac{1}{2} l r$ [l is the length of arc)

$= \frac{1}{2} \times 6.4 \times 8$

$= \underline{25.6\ cm^2}$

---

8. Calculate the area of the sector of a circle with radius 65 mm, and sector angle 42°. Give your answer in cm^2. ---[3]

[S-20/23/Q9]

Solution: Area of Sector $= \frac{\theta}{360°} \times \pi r^2$

[$r = 65\ mm = 6.5\ cm$]

$= \frac{42}{360} \times \pi \times (6.5)^2$

$= 15.485$

∴ Area of Sector $= \underline{15.5\ cm^2}$. ✓

9. The total perimeter of a semicircle is 19.02 cm. ---[3]
Calculate the radius of the semicircle. [S-20/23/Q12]

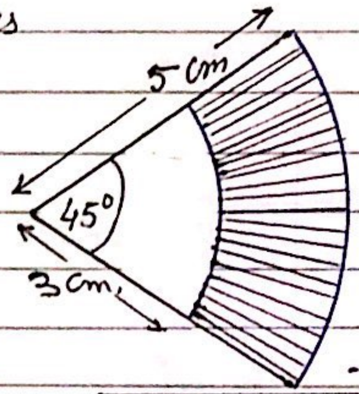
Solution: Perimeter of semicircle $= 2r + \pi r = 19.02$

$$\Rightarrow r(2+\pi) = 19.02$$

$$\Rightarrow r = \frac{19.2}{(\pi+2)} = \underline{3.7\ cm}$$

---

10. The diagram shows two sectors of circles with the same centre.
Calculate the shaded area.



---[3]
[W-19/21/Q17]

Solution: Area of shaded region

$$= \frac{\theta}{360} \times \pi (r_1^2 - r_2^2)$$

$$= \frac{45}{360} \times \pi (5^2 - 3^2)$$

$$= \frac{45}{360} \times \pi \times 16 = 2\pi = \underline{6.28\ cm^2}$$

10'. A pipe is completely full of water. Water flows through the pipe at a speed of 1.2 m/s into a tank.
The cross-section of the pipe has an area of $6\ cm^2$.
Calculate the number of litres of water flowing into the tank in 1 hour. ---[4]
[W-19/21/Q22]

Solution: Speed of water $= 1.2\ m/s = 120\ cm/s$

Area of cross-section $= 6\ cm^2$

∴ The volume flown in one second $= 6 \times 120\ cm^3$

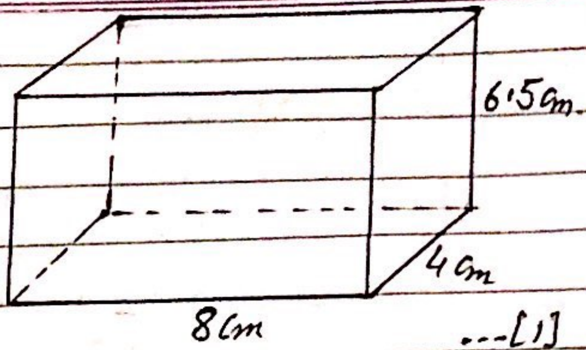
∴ Volume flown in one hour $= (6 \times 120) \times 60 \times 60\ cm^3$

$$= 2592000\ cm^3$$

$$= \underline{2592\ litres}\ [1\ litre = 1000\ cm^3]$$

Cuboids

11. The diagram shows a cuboid.
Calculate the volume of cuboid



---[1]

[S-20/23/Q2]

Solution: $V = lbh$
$= 8 \times 4 \times 6.5$
$= \underline{208\ cm^3}$. ✓

12. A cuboid measures 5cm by 7cm by 9.5cm.
Work out the surface area of the cuboid. ---[3]
[W-19/23/Q9]

Solution: Surface area of cuboid $= 2[lb + bh + lh]$
$= 2[5\times7 + 7\times9.5 + 5\times9.5]$
$= 298\ cm^2$

13 Water from Manjeet's shower flows at a rate of 12 litres per minute. The water from the shower flows into a tank that a cuboid of length 90cm and width 75cm.
Calculate the increase in the level of water in the tank when the shower is used of 7 minutes. [M-20/42/Q3(C)]·[3]

Solution: Water flown through the shower in 7 minutes $= 12\times7$ litre
$= 84$ litres — (1)

Let the increase in water level in the tank $= h$ cm.
$\therefore$ Increase in Volume in cuboid tank $= l\times b\times h$
$= 90\times75\times h\ cm^3$
$= 6750h\ cm^3$ — (2)

from (1) & (2) $6750h = 84\times1000$ [$\because$ 1 litre $= 1000\ cm^3$]
$h = \frac{84000}{6750} = 12.44$ cm

$\therefore h = \underline{12.4\ cm}$. ✓

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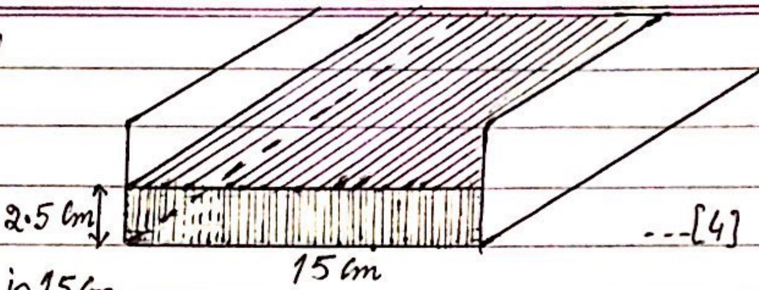
Cuboid / Cylinder.

14. Water flows at a speed of 20 cm/s along a rectangular channel into a lake.



---[4]

The width of the channel is 15 cm.
The depth of the water is 2·5 cm. Calculate the amount of water from the channel into the lake in 1 hour.
Give your answer in litres. [S-20/43/Q6(b)] ---[4]

Solution: Water flown in 1 hour through the channel (length of water column)
$= 20 \times 60 \times 60$ cm
$l = 72000$ cm.

$\therefore$ Volume of water through the channel in 1 hr.
$= l \cdot b \cdot h$
$= 72000 \times 15 \times 2.5$
$= 2700\,000 \text{ cm}^3$
$= \underline{2700 \text{ litres}}$ ✓ ($\because$ 1 litre $= 1000 \text{ cm}^3$)

15 A solid cylinder has radius 3 cm and height 4·5 cm.
Calculate the total surface area of the cylinder. --[4]
[S-20/23/Q 10]

Solution: Total surface area of cylinder $= 2\pi r h + 2\pi r^2$
$= 2\pi r(h + r)$
$= 2\pi \times 3\,(4.5 + 3)$ $\left[\because r = 3 \text{ cm},\ h = 4.5 \text{ cm}\right.$
$= 2\pi \times 3 \times 7.5$
$= 141.37 \text{ cm}^3$
$= \underline{141 \text{ cm}^3}$ ✓

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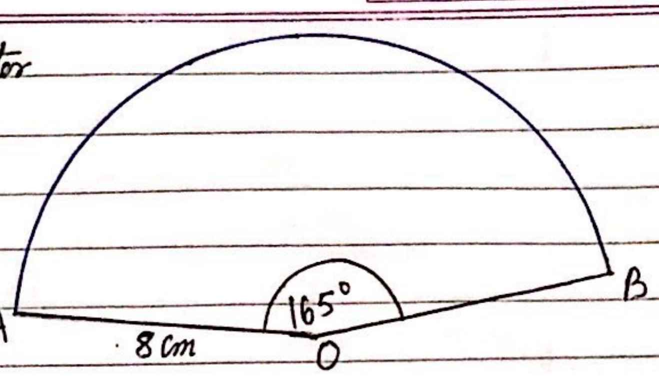
Spheres

16. The diagram shows a sector of a circle with centre O, radius 8 cm and sector angle 165°. The surface area of a sphere is same as the area of the sector. Calculate the radius of the sphere.



Solution: Area of sector $= \frac{\theta}{360} \times \pi r^2 = \frac{165}{360} \times \pi \times 8^2$ ——— (1)

Surface area of sphere $= 4\pi R^2$ (Radius of sphere = R (let))

$\therefore \; 4\pi R^2 = \frac{165}{360} \times \pi \times 64$ frm (1)

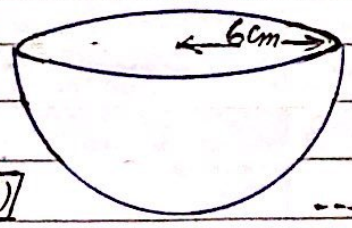
$R = \frac{165}{360} \times 64 \times \frac{1}{4} = \frac{10560}{1440} = 7.333$

$R = \sqrt{7.333} = 2.7079$

$\therefore R = 2.71 \text{ cm}$ ✓

17. The diagram shows a hemisphere with radius 6 cm. Calculate the volume. Give units of your answer.

[W-19/42/Q4(a)] ---[3]



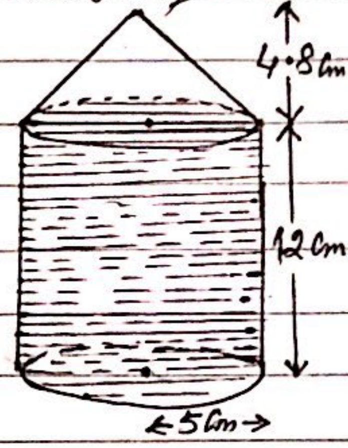
Solution: Volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3}\pi \times 6^3$

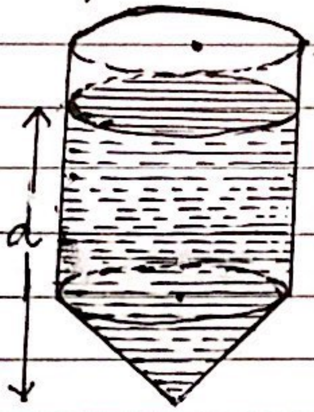
$= 452.389.. \text{ cm}^3$

$= 452.4 \text{ cm}^3$ ✓

18. A container is made from a cylinder and a cone, each of radius 5cm. The height of the cylinder is 12 cm and the height of the cone is 4.8 cm.
The cylinder is filled completely with water.
The container is turned upside down.



Calculate the depth, d, of the water.



---[5]

W-19/23/Q22

Solution: Case I: Cylinder is full of water Volume $V = \pi r^2 h$
$= \pi \times 5^2 \times 12$
$= 300\pi \text{ cm}^3$ —— ①

Case II Cone is full of water $V_1 = \frac{1}{3}\pi r^2 h_1$
$V_1 = \frac{1}{3}\pi \times 5^2 \times 4.8 = 40\pi$ —— ②

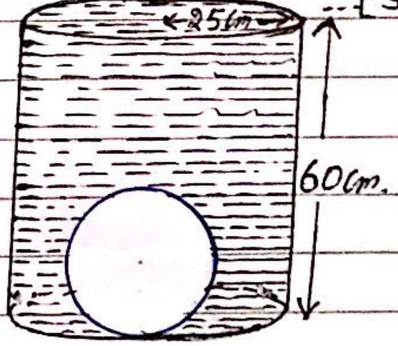
Now let the height of water level in the cylinder $= h_1$
Now Volume of water in cylinder $= \pi r^2 h_1$
$V_2 = \pi \times 5^2 \times h_1 = 25\pi h_1$ —— ③

$V_1 + V_2 = V$
$40\pi + 25\pi \cdot h_1 = 300\pi$
$\Rightarrow 25\pi h_1 = 300\pi - 40\pi = 260\pi$
$h_1 = \frac{260\pi}{25\pi} = 10.4$
$\therefore d = h_1 + 4.8$
$= 10.4 + 48 = \underline{15.2 \text{ cm}}$ ✓

19. (a) Show that the volume of a metal sphere of radius 15cm is $14140\,cm^3$, correct to 4 significant figures. ---[2]

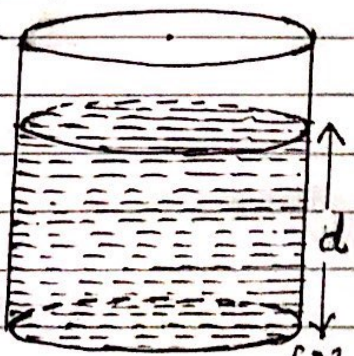
Solution: Volume of Sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 15^3 = 14137.16\ cm^3$.
$= 14140\ cm^3$ (4s.f.) — (1)

(b) (i) The sphere is placed inside an empty cylindrical tank of radius 25cm and height 60cm. ---[3]
The tank is filled with water. Calculate the volume of water needed to fill the tank.



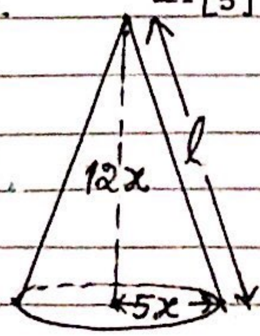
Solution: Volume of Water = Volume of Cylinder − Volume of sphere
$= \pi R^2 H - 14137$ (from (1))
$= \pi \times 25^2 \times 60 - 14137$
$= 117810 - 14137 = 103673\ cm^3$ ✓ — (2)

(b) (ii) The sphere is removed from the tank. Calculate the depth, d, of water in the tank.

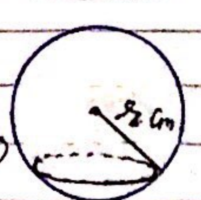


solution: Volume of water $= \pi r^2 d = 103673$ from (2)
$\pi \times 25^2 \cdot d = 103673$
$d = \frac{103673}{625\pi} = 52.8\ cm$ ✓ ---[2]

(C) A cone has radius $5x$ cm and height $12x$ cm. ---[5]
The sphere has radius r cm.
The cone has the same total surface area as the sphere. Show that $r^2 = \frac{45}{2}x^2$



Solution: Total surface area of sphere $= 4\pi r^2$ — (1)
Let the slant height of cone is $l = \sqrt{(5x)^2 + (12x)^2} = 13x$
Total surface area of cone $= \pi R^2 + \pi R l$
$= \pi \cdot (5x)^2 + \pi \times 5x \times 13x = 90x^2\pi$ — (2)



from (1) & (2) $\therefore$ $4\pi r^2 = 90x^2 \cdot \pi$
$\Rightarrow r^2 = \frac{90}{4}x^2 = \frac{45}{2}x^2$ ✓ SP-20/04/Q10

$\therefore r^2 = \frac{45}{2}x^2$ ✓

20 A solid metal cone has radius 1.65 cm and slant height 4.70 cm.

(a) Calculate the total surface area of the cone. ---[2]

(b) Find the angle the slant height makes with the base of the cone. ---[2]

Solution: (a) Total surface area of cone $= \pi r^2 + \pi r l$ $\boxed{M-20/42/Q4}$

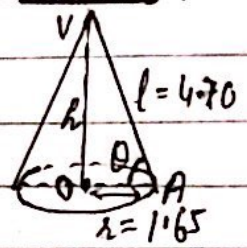
$= \pi r (r + l)$

$= \pi \times 1.65 \times (1.65 + 4.70)$

$= \pi \times 1.65 \times 6.35 = \underline{32.91\, cm^2}$ ✓

(b) $\cos\theta = \frac{r}{l} = \frac{1.65}{4.70} = 0.351$

$\therefore \theta = \cos^{-1}(0.351) = \underline{69.44^\circ}$ ✓



(C) (i) Calculate the volume of the cone. ---[4]

Solution: The perpendicular height h of the cone: $h = \sqrt{l^2 - r^2}$

$= \sqrt{(4.7)^2 - (1.65)^2} = 4.4\, cm$

$\therefore$ Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times (1.65)^2 \times 4.4 = \underline{12.54\, cm^3}$ ✓ —— (1)

(ii) A metal sphere with radius 5cm is melted down to make cones identical to this one. Calculate the number of complete identical cones that are made. -----[4]

Solution: Volume of sphere $= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi \times 5^3 = 523.6\, cm^3$ —— (2)

Number of cones $= \dfrac{\text{Volume of sphere}}{\text{Volume of one cone}}$

$= \dfrac{523.6}{12.54}$ (from (1) & (2))

$= 41.75$

$\therefore$ Complete number of cones $= \underline{41.}$

20. (a) A cylinder with radius 6cm. and height h cm has the same volume as a sphere with radius 4.5 cm. Find the value of h. ---[3]

(b) A solid metel cube of side 20 cm is melted down and made into 40 solid spheres r cm. Find the value of r. ---[3]

Solution (a) Volume of cylinder $= \pi r^2 h = \pi \times 6^2 \times h = 36\pi h$ — (1)

Volume of sphere $= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi \times (4.5)^3 = 121.5\pi$ — (2)

from (1) and (2) $121.5\pi = 36\pi h$

$\therefore h = \frac{121.5}{36} = \underline{3.375 \text{ cm}}$ ✓

(b) Volume of cube $= 20^3 = 8000 \text{ cm}^3$ — (3)

Volume of one sphere $= \frac{4}{3}\pi r^3$ — (4)

Given $40 \times \frac{4}{3}\pi r^3 = 8000$ (from (3) & (4))

$\Rightarrow r^3 = \frac{8000 \times 3}{160 \times \pi} = \frac{24000}{502.65} = 47.74$

$r = \sqrt[3]{47.74} = 3.627$ ✓

(c) A solid cylinder has radius x cm. and height $\frac{7x}{2}$.
The surface area of a sphere with radius R cm is equal to the total surface area of the cylinder.
Find an expression for R in terms of x. --[3]

[S-20/42/Q8(b,c,d)]

Solution: Surface area of sphere $= 4\pi R^2$ — (5)

Total surface area of Cylinder $= 2\pi r h + 2\pi r^2$

$= 2\pi r(h + r)$

$= 2\pi \times x\left(\frac{7}{2}x + x\right)$ $\left(\begin{array}{l} r = x \text{ cm} \\ h = \frac{7}{2}x \text{ cm}\end{array}\right)$

$= 9\pi x^2$ — (6)

from (5) & (6) $\Rightarrow 4\pi R^2 = 9\pi x^2$

$\Rightarrow R^2 = 9\pi x^2 \times \frac{1}{4\pi}$

$R^2 = \frac{9}{4}x^2$

$\Rightarrow \underline{R = \frac{3}{2}x}$ ✓

21. (a) (i) Calculate the external curved surface area of a cylinder with radius 8m and height 19m. --[2]
(ii) The surface is painted at a cost of \$0.85 per square metre. Calculate the cost of painting this surface. --[2]

Solution: curved surface area of a Cylinder $= 2\pi r h$

(a)(i) $= 2\pi \times 8 \times 19 = \underline{955\ m^2}$ ✓

(ii) Cost of painting $= 955 \times 0.85 = \$\underline{811.75}$ (or \$812) ✓

(b) A solid metal sphere with radius 6cm is melted down and all the metal is used to make a solid cone, with radius 8 cm and height h cm.
(i) Show that $h = 13.5$ ---[2]
(ii) Calculate the slant height of the cone. ---[2]
(iii) Calculate the curved surface area of the cone. --[1]

[W-19/41/Q4]

Solution (i) Volume of Sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 6^3 = 288\pi\ cm^3$ — ①

Volume of Cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 8^2 \times h = \frac{[illegible]}{3}\pi h$ — ②

from ① & ② $\frac{64}{3}\pi h = 288\pi$

$\Rightarrow h = \frac{288\pi \times 3}{64\pi} = \underline{13.5}$ ✓ cm.

(ii) Slant height 'l' of cone $l = \sqrt{h^2 + r^2}$

$= \sqrt{(13.5)^2 + 8^2} = \underline{15.69}$ cm

(iii) Curved surface area of Cone $= \pi r l$

$= \pi \times 8 \times 15.69$

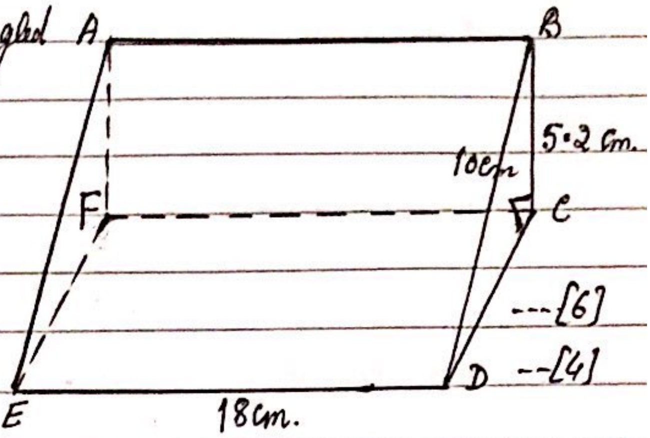
$= 394.39$

$= \underline{394.4}\ cm^2$ ✓

Scanned with CamScanner

Prism

21. The diagram shows a prism ABC DEF.
The cross-section is a right-angled triangle BCD.
BD = 10 cm, BC = 5.2 cm and
ED = 18 cm.



(i) (a) Work out the volume of the prism. ---[6]
(b) Calculate angle BEC. --[4]
(ii) The point G lies on the line ED and GD = 7 cm. Work out angle BGE. ---[3]

[W-19/42/4(b)]

Solution (i)(a) In rt triangle BCD, $CD = \sqrt{BD^2 - BC^2}$

$= \sqrt{10^2 - (5.2)^2}$

$= 8.54$

Area of cross-section of the prism = area of triangle BCD

$= \frac{1}{2} \times CD \times BC$

$= \frac{1}{2} \times 8.54 \times 5.2 = 22.2\ cm^2$ — (1)

$\therefore$ Volume of the prism = area of cross-section × length

$= 22.2 \times 18 = 399.749$

$= \underline{400\ cm^3}$ ✓

(i)(b) $EC = \sqrt{ED^2 + DC^2} = \sqrt{18^2 + 8.54} = \sqrt{396.9} = 19.92\ cm$

$\tan BEC = \_\_\_ = \frac{5.2}{19.92} = 0.261$

$\therefore$ angle $BEC = \tan^{-1}(0.261) = \underline{14.6^\circ}$ ✓

(ii) angle $BGE = 180 - \theta$ — (2)

$\tan\theta = \frac{BD}{GD} = \frac{10}{7}$

$\theta = \tan^{-1}\frac{10}{7} = 55^\circ$

$\therefore$ from (2) angle $BGE = 180 - 55^\circ$

$= \underline{125^\circ}$ ✓