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Number System

§ Set of Natural Numbers $N = \{0, 1, 2, 3, 4, 5 ----\}$

§ Set of Integers $Z = \{---3, -2, -1, 0, 1, 2, 3, --\}$

§ Rational Numbers are the numbers which can be expressed in form p/q, $(q \neq 0)$ where p and q are integers. The set of rational numbers is denoted by Q.

Example:

$4, -6, \frac{3}{5}, \frac{12}{7}, -\frac{4}{9}, 0, 0.125, 0.333-- (\text{or } 0.\dot{3})$

Decimal representation of rational numbers:

(i) Rational numbers are either terminating decimals:

Example: 0.125 (or $\frac{1}{8}$) ; 0.37 (or $37/100$)

or (ii) Rational numbers are repeated decimals: (or Recurring decimals)

Example: $0.333---$ (or $0.\bar{3}$) (or $1/3$) ;

$0.353535--$ (or $35/99$)

§ Irrational Numbers: The numbers which can not be expressed as p/q $(q \neq 0)$ where p and q are integers.

Irrational numbers are non-terminating and non-recurring decimals. Example $\sqrt{2} = 1.4142135 ----$; $\sqrt{3} = 1.732050807---$

$\pi = 3.14159265 ----$

$e = 2.718281828 ----$

§ Real Numbers: Rational or irrational number combined are real numbers.

§ Prime Numbers: It is a positive integer which is divisible by itself or by 1. Example: $2, 3, 5, 7, 11, 13, ----$

Note: (i) 1 is not a prime number.

(ii) 2 is the only even prime number.

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§ To express a recurring decimal as fraction.

Example 1. Write the recurring decimal $0.\dot{4}$ as a fraction. $\boxed{M\text{-}16/22/Q6}$ ---[2]

Solution: Let $x = 0.\dot{4}$ ----(i)

or $= 0.4444\ldots$

$\therefore 10x = 4.444\ldots$

or $10x = 4.\dot{4}$ ----(ii)

Now subtracting (i) from (ii). $\Rightarrow 9x = 4$

or $x = 4/9$

$\therefore 0.\dot{4} = 4/9$ ✓

Example 2. Write the recurring decimal $0.1\dot{7}$ as a fraction. ---[2] $\boxed{M\text{-}17/22/Q6}$

Solution: Let $x = 0.1\dot{7}$

$\therefore 10x = 1.\dot{7}$ ----(i)

or $10x = 1.777\ldots$

$\therefore 100x = 17.777\ldots$

or $100x = 17.\dot{7}$ ----(ii)

Now subtracting (i) from (ii) $90x = 16$

$\Rightarrow x = \frac{16}{90} = \frac{8}{45}$

$\therefore 0.1\dot{7} = 8/45$ ✓

Example 3. Write your recurring decimal $0.\dot{6}\dot{3}$ as a fraction in its lowest term. You must show all your working. $\boxed{S\text{-}17/23/Q14}$ ---[3]

Solution: Let $x = 0.\dot{6}\dot{3}$ ----(i)

$= 0.636363\ldots$

$\therefore 100x = 63.636363\ldots$

or $100x = 63.\dot{6}\dot{3}$ ------(ii)

Now subtracting (i) from (ii) $99x = 63$

or $x = \frac{63}{99} = \frac{7}{11}$

$\therefore 0.\dot{6}\dot{3} = \frac{7}{11}$ ✓

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§ To express a given natural number as product of prime factors:

Example (i) Write 30 as a product of its prime factors. [S-15/21/Q2(a)] ---[2]

$\therefore$ $30 = 2 \times 3 \times 5$ $\quad \because$ $2 | 30$, $3 | 15$, 5

(ii) Write 2016 as a product of prime factors. ---[3] [M-16/22/Q13]

$2016 = 2\times2\times2\times2\times2\times3\times3\times7$ $\quad \because$ $2|2016$, $2|1008$, $2|504$, $2|252$, $2|126$, $3|63$, $3|21$, 7.

$= 2^5 \times 3^2 \times 7$

§ Highest common factor (H.C.F):

To find the H.C.F of two (or more) number, express them as product of prime factors, Then H.C.F is product of common factors with least exponents.

Example 1. Find the highest common factor (H.C.F) of 56 and 70. ---[2] [S-16/23/Q10]

Solution: $56 = 2^3 \times 7$

$70 = 2 \times 5 \times 7$

$H.C.F = 2 \times 7 = 14$ ✓

$\because$ $2|70$, $5|35$, 7 $\qquad$ $2|56$, $2|28$, $2|14$, 7

Example 2:

Find the H.C.F of 60 and 90. [0607/W-15/23/Q1] ---- [1]

Solution: $60 = 2^2 \times 3 \times 5$

$90 = 2 \times 3^2 \times 5$

$2|60$, $2|30$, $3|15$, 5 $\qquad$ $2|90$, $3|45$, $3|15$, 5

$\therefore$ $H.C.F = 2 \times 3 \times 5 = 30$ ✓

§ Lowest Common Multiple (L.C.M): To find the L.C.M of two (or more) numbers express each one of them as product of prime factors. Then their L.C.M is the product all the factors with highest highest exponents.

Example 1. Find the lowest common multiple (L.C.M) of 36 and 48. ---[2]
S-16/21/Q6

Solution: $36 = 2^2 \times 3^2$

$48 = 2^4 \times 3$

$\therefore L.C.M = 2^4 \times 3^2 = 16 \times 9 = 144$ ✓

$$\begin{array}{r|l} 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array} \qquad \begin{array}{r|l} 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

(a) Write 90 as product of prime factors -[2]

Example 2(b) Find the L.C.M of 90 and 105 W-14/22/Q15 ---[2]

Solution (a). $90 = 2 \times 3^2 \times 5$

(b) $105 = 3 \times 5 \times 7$

$\therefore$ L.C.M of 90 and 105 $= 2 \times 3^2 \times 5 \times 7$

$= 630$ ✓

$$\begin{array}{r|l} 3 & 105 \\ \hline 5 & 35 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 2 & 90 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

§ Basic Arithmetic operations with fractions: (i) Addition (ii) Subtraction (iii) Multiplication (iv) Division.

Example 1. Without using your calculator, work out $1\frac{7}{12} + \frac{13}{20}$ ---[3]
M-16/22/Q9

Solution: $1\frac{7}{12} + \frac{13}{20}$

$= \frac{19}{12} + \frac{13}{20}$

$= \frac{19}{12} \times \frac{5}{5} + \frac{13}{20} \times \frac{3}{3}$

$= \frac{95}{60} + \frac{39}{60}$

$= \frac{95 + 39}{60} = \frac{134}{60}$

$= \frac{67}{30}$

$= 2\frac{7}{30}$ ✓

Denominators:
$12 = 2^2 \times 3$
$20 = 2^2 \times 5$
$L.C.M = 2^2 \times 3 \times 5 = 60$

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Example 2. Without using your calculator, workout $2\frac{1}{4} - \frac{11}{12}$.
You must show all your working and give your answer a a fraction in its lowest terms. W-15/22/Q12 --- [3]

Solution: $2\frac{1}{4} - \frac{11}{12}$

$= \frac{9}{4} - \frac{11}{12}$

$= \frac{9}{4}\times\frac{3}{3} - \frac{11}{12}$

$= \frac{27}{12} - \frac{11}{12}$

$= \frac{27-11}{12}$

$= \frac{16}{12} = \frac{4}{3}$ ✓

$4 = 2^2$
$12 = 2^2 \times 3$
L.C.M of 4 & 12 $= 2^2 \times 3 = 12$

Alternatively

$$\begin{array}{r|l} 2 & 4, 12 \\ \hline 2 & 2, 6 \\ \hline & 1, 3 \end{array}$$

$\therefore$ L.C.M $= 2 \times 2 \times 3 = 12$

Example 3. Workout $\frac{2}{3} + \frac{1}{6} - \frac{1}{4}$, giving your answer as a fraction in its lowest form. Do not use a calculator and show all your steps of working. W-15/23/Q15 --- [3]

Solution:

$\frac{2}{3} + \frac{1}{6} - \frac{1}{4}$

$= \frac{2}{3}\times\frac{4}{4} + \frac{1}{6}\times\frac{2}{2} - \frac{1}{4}\times\frac{3}{3}$

$= \frac{8}{12} + \frac{2}{12} - \frac{3}{12}$

$= \frac{8+2-3}{12}$

$= \frac{10-3}{12}$

$= \frac{7}{12}$

find L.C.M of 3, 6, 4

$$\begin{array}{r|l} 2 & 3, 6, 4 \\ \hline 3 & 3, 3, 2 \\ \hline & 1, 1, 2 \end{array}$$

L.C.M $= 2 \times 3 \times 2 = 12$ ✓

or
$3 = 3^1$
$6 = 2 \times 3$
$4 = 2^2$
$\therefore$ L.C.M $= 2^2 \times 3 = 12$ ✓

Example 4. Without using a calculator workout $\frac{4}{5} \div 2\frac{2}{3}$ S-15/22/Q12 --- [3]

Solution $\frac{4}{5} \div 2\frac{2}{3} = \frac{4}{5} \div \frac{8}{3}$

$= \frac{4}{5} \times \frac{3}{8} = \frac{12}{40}$

$= \frac{3}{10}$ ✓

§ H.C.F and L.C.M.

Example (a) (i) Write 180 as a product of its prime factors. ---[2]

(ii) Find the L.C.M of 180 and 54 ---[2]

(b) An integer, X, written as a product of its prime factors is $a^2 \times 7^{b+2}$

Another inter Y, written as a product of its prime factors is $a^3 \times 7^2$

The highest common factor (H.C.F) of X and Y is 1225.

The lowest common multiple (L.C.M) of X and Y is 42875

Find the value of X and the value of Y. $\boxed{W\text{-}17/43/Q10}$ --[4]

Solution (a) (i)

$180 = 2^2 \times 3^2 \times 5$ ✓

(ii) $54 = 2 \times 3^3$

$\therefore$ L.C.M of 180 and 54 $= 2^2 \times 3^3 \times 5$

$= 540$ ✓

$$\begin{array}{r|l} 2 & 54 \\ 3 & 27 \\ 3 & 9 \\ & 3 \end{array} \qquad \begin{array}{r|l} 2 & 180 \\ 2 & 90 \\ 3 & 45 \\ 3 & 15 \\ & 5 \end{array}$$

(b) $X = a^2 \times 7^{b+2}$

$Y = a^3 \times 7^2$

$\therefore$ H.C.F of X and Y $= a^2 \times 7^2 = 1225$ (Given)

$\Rightarrow a^2 = \frac{1225}{49} = 25 \Rightarrow a = 5$ ✓ --- (I)

and L.C.M of X and Y $= a^3 \times 7^{b+2}$

$= 5^3 \times 7^b \times 7^2 = 42875$ (Given) $\left\{ \because \text{ from (1) } a = 5 \right.$

$\Rightarrow 7^b = \frac{42875}{5^3 \times 7^2} = 7$

$\Rightarrow b = 1$ ✓ ——(ii)

$\therefore X = a^2 \times 7^{b+2} = 5^2 \times 7^{1+2} = 5^2 \times 7^3 = \underline{8575}$ ✓

and $Y = a^3 \times 7^2 = 5^3 \times 7^2 = \underline{6125}$ ✓

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§ Ordering of operations:
(i) Bracket
(ii) Division
(iii) Multiplication
(iv) addition/subtraction.

Example 1: Workout. $16-8\div2+2\times4$ [0607/W-15/23/Q2] ...[1]

Solution: $16-8\div2+2\times4$ (division)

$=16-8\times\frac{1}{2}+2\times4$ (multiplication)

$=16-4+8$ (addition/subsraction)

$=24-4$

$=20$ ✓

Example 2 Insert one pair of brackets to make the statement correct. -- [1)

$5-2+3\times2=-5$ [0607/W-15/23/Q2]

Solution: $5-2+3\times2=5$

should be $5-(2+3)\times2=-5$

Check
$5-(2+3)\times2$ bracket
$=5-5\times2$ Multiply
$=5-10$ Subtract
$=-5$ ✓

Example 3. Work out. $12\div(3+1)-5\times3$

Solution: $12\div(3+1)-5\times3$ [Bracket first]

$12\div4-5\times3$ [divide]

$12\times\frac{1}{4}-5\times3$ [Multiply]

$=3-15$ [add/sub]

$=-12$ ✓

Example 4 Without using your calculator workout. $\frac{5}{6}-\left(\frac{1}{2}\times1\frac{1}{2}\right)$ ---[3] [S-14/22/Q8]

Solution. $\frac{5}{6}-\left(\frac{1}{2}\times1\frac{1}{2}\right)=\frac{5}{6}-\left(\frac{1}{2}\times\frac{3}{2}\right)$

$=\frac{5}{6}-\frac{3}{4}$

$=\frac{5}{6}\times\frac{2}{2}-\frac{3}{4}\times\frac{3}{3}$

$=\frac{10}{12}-\frac{9}{12}=\frac{10-9}{12}=\frac{1}{12}$ ✓

$6=2\times3$
$4=2^2$
$L.C.M=2^2\times3=12$ ✓

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§ Four rules for calculations with decimals:

Example 1 Evaluate, without using a calculator; $1.25 + 0.043 + 52.3$

Solution:

$$1.25 + 0.043 + 52.3 = \begin{array}{r} 1.250 \\ 0.043 \\ 52.300 \\ \hline 53.593 \end{array} ✓$$

Example 2. Without using a calculator, evaluate. $23.37 - 1.45$

Solution

$$\begin{array}{r} 23.37 \\ -\ 1.45 \\ \hline 21.92 \end{array} ✓$$

Example 3. Evaluate. 2.34×0.5

Solution:

$$\begin{array}{r} 2.34 \\ \times 0.5 \\ \hline 1.170 \end{array}$$

$\therefore 1.17$ ✓

Multiply. and place a decimal after 3 places.

Example 4 Evaluate: $\frac{4.52}{0.4}$

Solution: $\frac{4.52}{0.4} \times \frac{10}{10} = \frac{45.2}{4} = 11.3$ ✓

$$\begin{array}{r} 11.3 \\ 4\overline{)45.2} \\ 4 \\ \hline 5 \\ 4 \\ \hline 12 \\ 12 \\ \hline \times \end{array}$$

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§ Rules of Indices:

Consider the product $3\times3\times3\times3\times3 = 3^5$

In 3^5; 3 is the base and 5 is index (or exponent)

In general: a is a real number and n is positive integer.

Then $a\times a\times a$ --- n times $= a^n$ here n is called index

§ Laws of Indices (or Exponents): Given a is real number and m and n are positive integers.

(i) $a^m \times a^n = a^{m+n}$

(ii) $\dfrac{a^m}{a^n} = a^{m-n}$

(iii) $(a^m)^n = a^{m\cdot n}$

(iv) $(ab)^m = a^m \cdot b^m$

(v) $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$

Example: Evaluate:

(i) $3^2\times 3^4 = 3^{2+4} = 3^6 = 729$ ✓

(ii) $\dfrac{2^5}{2^3} = 2^{5-3} = 2^2 = 4$ ✓

(iii) $(5^3)^2 = 5^{3\times2} = 5^6 = 15625$ ✓

§ Note (i) $a^0 = 1$

[$\because \dfrac{a^m}{a^n} = a^{m-n}$

for $n=m \Rightarrow \dfrac{a^m}{a^m} = a^{m-m}$

$\Rightarrow 1 = a^0$]

(ii) Negative Index:

$$a^{-m} = \frac{1}{a^m}$$

[$\because a^{-m} = a^{0-m} = \dfrac{a^0}{a^m} = \dfrac{1}{a^m}$ ✓

§ Rational Index: Given 'a' is a positive real number and p/q is rational $q>0$. Then. $a^{p/q} = (a^p)^{1/q}$

Example: (i) $(25)^{3/2} = (25^3)^{1/2} = (25\times25\times25)^{1/2} = (125\times125)^{1/2}$
$= 125$ ✓

or $(25)^{3/2} = (5^2)^{3/2} = (5)^{2\times 3/2} = 5^3 = 125$ ✓

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§ $a^{p/q} = (a^p)^{1/q} = (a^{1/q})^p$ **Rules of Indices**

or $= \sqrt[q]{a^p} = (\sqrt[q]{a})^p$

Example 1. Work out (a) $125^{2/3}$ --- [1]

(b) $\left(\frac{1}{3}\right)^{-2}$ $\boxed{S-17/22/Q14}$ --- [1]

Solution: (a) $(125)^{2/3} = (5^3)^{2/3} = 5^{3\times 2/3} = 5^2 = 25$ ✓

(b) $\left(\frac{1}{3}\right)^{-2} = (3^{-1})^{-2} = 3^{-1\times -2} = 3^2 = 9$ ✓

Example 2 (a) Simplify. $(16x^{16})^{3/4}$ --- [2]

$\boxed{S-17/22/Q25}$

(b) $2p^{3/2} = 54$; find the value of p. --- [2]

Solution: (a) $(16x^{16})^{3/4} = [2^4 \cdot (x^4)^4]^{3/4} = [(2x^4)^4]^{3/4}$

$= (2x^4)^{4\times 3/4}$

$= (2x^4)^3$

$= 2^3 \cdot (x^4)^3$

$= 8 \cdot x^{4\times 3}$

$= 8x^{12}$ ✓

(b) $2p^{3/2} = 54$

$\Rightarrow p^{3/2} = \frac{54}{2} = 27$

$\Rightarrow (p^{1/2})^3 = 3^3$

$\Rightarrow p^{1/2} = 3$

$\Rightarrow p = 3^2 = 9$ ✓

Example 3 (a) $2^r = \frac{1}{16}$; find the value of r. --- [1]

$\boxed{S-17/21/Q6}$

(b) $3^t = \sqrt[5]{3}$, find the value of t. -- [1]

Solution (a) $2^r = \frac{1}{16} = \frac{1}{2^4} = 2^{-4}$

$\Rightarrow r = -4$ ✓

(b) $3^t = \sqrt[5]{3} = 3^{1/5}$

$\Rightarrow t = \frac{1}{5}$ ✓

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§ **Standard Form:** $A \times 10^n$ where n is a positive or negative integer, and $1 \leq A < 10$

Example 1. Write in Standard form. (a) 2470000 --[1]
(b) 0.0079 W-16/23/Q4 --[1]

Solution: (a) $2470000 = \frac{247\cancel{0000}}{1000000} \times 10^6 = 2.47 \times 10^6$ ✓

(b) $0.0079 = \frac{79}{10000} = \frac{79}{10 \times 10^3} = 7.9 \times 10^{-3}$ ✓

Example 2. Work out $(8 \times 10^{-4}) \times (2 \times 10^{-3})$ 0607/W-15/22/Q1(b) --[2]
giving your answer in standard form.

Solution:

$$(8 \times 10^{-4}) \times (2 \times 10^{-3}) = 8 \times 2 \times 10^{-4} \times 10^{-3} = 16 \times 10^{(-4)+(-3)}$$
$$= 16 \times 10^{-7}$$
$$= 1.6 \times 10^1 \times 10^{-7}$$
$$= 1.6 \times 10^{-7+1}$$
$$= 1.6 \times 10^{-6}$$ ✓

Example 3. Write 1.7×10^{-4} as an ordinary number ——— [1]
W-15/23/Q3

Solution:

$$1.7 \times 10^{-4} = \frac{1.7}{10000} = 0.00017$$ ✓

Example 4. Calculate giving your answer in standard form.
(a) $2 \times (5.5 \times 10^4)$ --[2]
(b) $(5.5 \times 10^4) - (5 \times 10^4)$ S-13/23/Q9 --[2]

Solution: (a) $2 \times (5.5 \times 10^4)$
$= 2 \times 5.5 \times 10^4$
$= 11 \times 10^4$
$= 1.1 \times 10 \times 10^4$
$= 1.1 \times 10^5$ ✓

(b) $(5.5 \times 10^4) - (5 \times 10^4)$
$= (5.5 - 5) \times 10^4$
$= 0.5 \times 10^4$
$= .5 \times 10 \times 10^3$
$= 5 \times 10^3$ ✓

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§ Simple Vulger, decimal fractions and percentage:

1.(i) Fraction to percentage: (i) $\frac{3}{5} = \frac{3}{5} \times 100\% = 60\%$

(ii) $\frac{1}{8} = \frac{1}{8} \times 100\% = 12.5\%$

(iii) $\frac{1}{10} = \frac{1}{10} \times 100 = 10\%$

2. Percentage to fraction:

(i) $20\% = 20 \times \frac{1}{100} = \frac{1}{5}$

(ii) $35\% = 35 \times \frac{1}{100} = \frac{7}{20}$

(iii) $16\% = 16 \times \frac{1}{100} = \frac{4}{25}$

3. Decimal to percentage:

(i) $0.5 = 0.5 \times 100\% = 50\%$

(ii) $0.35 = 0.35 \times 100\% = 35\%$

(iii) $0.1 = 0.1 \times 100\% = 10\%$

4. Percentage to decimal:

(i) $25\% = \frac{25}{100} = 0.25$

(ii) $40\% = 40 \times \frac{1}{100} = 0.4$

(iii) $5\% = 5 \times \frac{1}{100} = 0.05$

Example: Write the following in order, smallest first ---[2]

$\sqrt{0.1}$, $\frac{43}{201}$, $2\frac{1}{2}\%$, 0.2 S-14/22/Q7

Solution: $\sqrt{0.1}$, $\frac{43}{201}$, $\frac{5}{2}\%$, 0.2 Converting to decimal fractions.

$\Rightarrow$ $0.316\ldots$, 0.213 , 0.025 , 0.2

$\therefore$ in order, smallest first: $2\frac{1}{2}\%$; 0.2 ; $\frac{43}{201}$, $\sqrt{0.1}$

§ Accuracy:

§ Rounding of the numbers:

(a) Round the following numbers to the nearest 10.

(i) 782 = 780
(ii) 725 = 730
(iii) 788 = 790
(iv) 1093 = 1090

Rule: If the unit digit is less than 5, then replace it by zero (0). But if unit digit ≥ 5 then increase the ten's digit by 1 and unit digit be 0.

(b) Rounding the following numbers to the nearest 100.

(i) 782 = 800
(ii) 725 = 700
(iii) 788 = 800
(iv) 10.93 = 1100

Rule: If Ten's digit is less than 5 than write the unit and ten's digit at 00. [Rounding down] but if the ten's digit ≥ 0 then increase the Hundred's digit by 1. and unit/ten's digit by 00.

(c) Rounding the following numbers to nearest 1000

(i) 58963 = 59000
(ii) 59781 = 60000
(iii) 63251 = 63000
(iv) 4021 = 4000

Observe the hundre's digit.

(d) Rounding the following numbers to 1 decimal place (1dp)

(i) 231·735 = 231·7
(ii) 53·157 = 53·2
(iii) 157·78 = 157·8
(iv) 48·23 = 48·2

If the digit at the 2nd decimal pla is less than 5 then drop it. But If the digit at 2nd deci place is 5 or more then increase the digit at 1dp by 1.

(e) Round to 2d.p.

(i) 231·735 = 231·74
(ii) 53·123 = 53·12

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Accuracy:

§ Significant figures (S.f):

(a) Write the following numbers to 1 s.f.:

(i) 25.73 = 30 ✓
(ii) 335 = 300 ✓
(iii) 4930 = 5000 ✓
(iv) 2.78 = 3 ✓
(v) 0.312 = 0.3 ✓
(vi) 0.351 = 0.4 ✓
(vii) 0.0271 = 0.03 ✓
(viii) 0.0512 = 0.05 ✓
(ix) 0.00671 = 0.007 ✓

(b) Write the following number to 2 s.f:

(i) 25.73 = 26 ✓
(ii) 335 = 340 ✓
(iii) 4930 = 4900 ✓
(iv) 2.78 = 2.8 ✓
(v) 6753 = 6800 ✓
(vi) 0.312 = 0.31 ✓
(vii) 0.357 = 0.36 ✓
(viii) 0.0271 = 0.027 ✓
(ix) 0.0517 = 0.52 ✓
(x) 0.00686 = 0.0069 ✓

(c) Write the following numbers to 3 s.f:

(i) 25.73 = 25.7 ✓
(ii) 5237 = 524 ✓
(iii) 76891 = 769 ✓
(iv) 2.795 = 2.80 ✓
(v) 2.513 = 2.51 ✓
(vi) 0.02357 = 0.236 ✓
(vii) 0.3501 = 0.350 ✓
(viii) 0.1297 = 0.130 ✓
(ix) 0.001278 = 0.00128 ✓
(x) 75981 = 76000 ✓

Example 1: Write 23.4571 correct to. [S-17/21/Q3]
(a) 4 significant figures ---[1]
(b) to the nearest 10. ---[1]

Solution: (a) 23.4571 = 23.46 ✓ 4 s.f
(b) 20 ✓ To nearest 10.

Example 2: Write 71496 correct to 2 s.f. [S-16/23/Q2] --[1]
Solution 71496 = 71000 ✓ 2 s.f.

[W-16/21/Q2]

Example 3: Write 0.0401907 correct to (a) 3 s.f (b) 3 decimal places. [1+1]
Solution: (a) 0.0401907 = 0.0402 ✓ (b) 0.0401907 = 0.040 ✓ 3 d.p

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Accuracy:

Rounding and significant figure.

Example 4: Without using calculator and by rounding each number correct to 1 s.f., estimate the value of: $\frac{10.3 \times 19.5}{88.9 - 43.2}$ ---[2] [M-17/22/Q3]

Solution

$$\frac{10.3 \times 19.5}{88.9 - 43.2}$$

$$= \frac{10 \times 20}{90 - 40}$$ reducing each number to 1 s.f.

$$= \frac{200}{50} = 4 ✓$$

Example 5: $p = \frac{4.8 \times 1.98276}{16.83}$

(a) In the spaces provided write each number in this calculation to 1 significant figure (1 s.f). $\frac{\text{---} \times \text{---}}{\text{---}}$ --[1]

(b) Use your answer to part (a) to estimate the value of p. ---[1]

Solution: [S-14/21/Q4]

(a) $\frac{5 \times 2}{20}$ ✓

(b) $\frac{10}{20} = 0.5$ ✓

Example 6: Write 15.0782 correct to. [W-14/21/Q6]

(a) one decimal place, ---[1]

(b) to nearest 10 ---[1]

Solution:

(a) $15.0782 = 15.1$ ✓ (Correct to 1 d.p)

(b) $15.0782 = 20$ ✓ (Correct to nearest 10)

Scanned by CamScanner

Number

Accuracy

§ Upper and Lower bound:

(a) Upper and Lower bounds to nearest whole number.

Example 1. The length l metres, of football pitch is 96 m, correct the nearest metre. Complete the statement about the length of this football pitch. $\ldots \leq l < \ldots$ [W-14/22/Q6] --[2]

Solution. $95.5 \leq l < 96.5$

Here lower bound $= 95.5$

and upper bound $= 96.5$.

Example 2: The length of a car is 4.2 m, correct to 1 decimal place. Write down the upper bound and the lower bound of the length of car. [W-16/23/Q8] ---[2]

Solution: Upper bound $= 4.25$

and lower bound $= 4.15$ (or $4.15 \leq$ length < 4.25)

Example 3: An equilateral triangle has sides 6.2 cm, correct to the nearest millmetre. Complete the statement about the perimeter, --[2] P cm, of the triangle. $\ldots \leq P < \ldots$ [S-15/23/Q4]

Solution: millimeter is 0.1 cm or length 6.2 cm is correct to one decimal place.

∴ length l $6.15 \leq l < 6.25$

Now perimeter of triangle $P = 3l \Rightarrow$ $18.45 \leq 3l < 18.75$

or $18.45 \leq P < 18.75$ ✓

Example 4: Rice is sold in 75 gram packs and 120 gram pack. ---[2]

Difference

The masses of both packs are given correct to the nearest gram. Calculate the lower bound for the difference in mass between the two packs. [S-15/21/Q6]

Solution. First pack, P_1, $74.5 \leq P_1 < 75.5$

Second pack P_2, $119.5 \leq P_2 < 120.5$

∴ Lower bound of the difference in masses = lower bound of P_2 − upper bound of P_1

∴ $119.5 - 75.5 = 44$ ✓

Number

Accuracy

§ Upper and Lower bound:

Example 5: The length of a rectangle is 9.3 cm, correct to 1 decimal place, Its width is 7.7 cm, correct to one decimal place. Write down the lower bound and the upper bound for the area of the rectangle. ---[3]

M-17/22/Q7

Solution: length 9.3 cm; $9.25 \leq l < 9.35$

width 7.7 cm, $7.65 \leq W < 7.75$

Product

Area of rectangle = length × width.

Now lower bound of area = product of the lower bounds of l and w.

$= 9.25 \times 7.65 = 70.7625$ ✓

and upper bound of area = product of both upper bounds

$= 9.35 \times 7.75 = 72.4625$ ✓

Example 6: A metal pole is 500 cm long, correct to the nearest centimetre, The pole is cut into rods each of length 5.8 cm, correct to nearest millimetre, Calculate the largest number of rods the pole can be cut into.

M-16/22/12 ---[3]

Division

Solution $499.5 \leq \text{Pole length} < 500.1$

$5.75 \leq \text{rod} < 5.85$

Largest Number of rods = upper bound of Pole length ÷ lower bound of rod length.

$= 500.1 \div 5.75$

$= 87.0434\text{--}$

Largest Number = 87 ✓

Example 7: The base of a triangle is 9 cm correct to the nearest cm. The area of this triangle is 40 cm² correct to the nearest 5 cm², Calculate the upper bound for the perpendicular height of this triangle. ---[3]

S-16/22/Q13

Solution: base 9 cm correct to nearest cm. $8.5 \leq \text{base} < 9.5$

Area 40 cm² correct to nearest 5 cm². $37.5 \leq \text{Area} < 42.5$

Division

Area of Triangle = $\frac{1}{2} \times \text{base} \times \text{height}$

$\therefore \text{height} = \frac{2\,\text{Area}}{\text{base}}$. $75 \leq 2\,\text{Area} < 85$

$\therefore$ upper bound of height = $\frac{\text{upper bound of } 2\cdot\text{Area}}{\text{Lower bound of base}} = \frac{85}{8.5} = 10$ ✓

Number

Accuracy:

§ Upper and Lower bound:

Example 8: The volume of a cuboid is $878 cm^3$, correct to the nearest cubic centimetre. The length of the base of the cuboid is 7 cm, correct to the nearest centimetre. The width of the base of the cuboid is 6 cm, correct to the nearest centimetre. Calculate the lower bound for the height of the cuboid. ---[3] W-15/23/Q20

Solution: Volume $878 cm^3$. $877.5 \leq V < 878.5$

base is 7 cm. $6.5 \leq b < 7.5$

Width is 6 cm $5.5 \leq W < 6.5$

$\text{height} = \frac{V}{b \times W}$

$$\text{Lower bound of height} = \frac{\text{lower bound of Volume}}{\text{upper bound of base} \times \text{upper bound of width}}$$

$$= \frac{877.5}{7.5 \times 6.5} = 18$$ ✓

Example 9: Joe measures the side of a square correct to 1 decimal place. He calculate the upper bound for the area of square is $37.8225 cm^2$. Work out Joe's measurement for the side of the square. S-13/22/Q8 ---[2]

Solution: upper bound of area of square = (upper bound of side)^2

$= 37.8225$ given

$\therefore$ upper bound of side $= \sqrt{37.8225}$

$= 6.15$

$\therefore$ Side of square $= 6.1$ cm ✓

Example 10: The length, p cm, of a car is 440 cm, correct to the nearest 10 cm, complete the statement about p. $\cdots \leq p < \cdots$ W-13/21/Q7 --[2]

Solution $435 \leq p < 445$.

Example 11: In 206, a company sold 9600 cars, correct to the nearest 100. write down the low. bound for the number of cars sold. --- [1]

Solution: 9550 $(9550 \leq No < 9650)$ S-17/43/Q1

Numbers

§ Directed Numbers:

Example 1: Write down the temperature which is 5°C below −2°C. ---[1]

W-16/21/Q1

Solutio Required temperature = $-2-5=-7$°C

Example 2: The table shows the temperatures in five places at 10 am, one day in January.

(a) Which place was coldest. ---[1]

(b) At 2 pm, the temperature in Helsinki had increased by 4°C. Write down the temperature in Helsinki at 2pm. --[1]

Place	Temperature °C
Helsinki	−7
Chicago	−10
London	3
Moscow	−4
Bangkok	26

S-17/21/Q4

Solution: (a) Chicago ✓ (−10°C)

(b) Temp. in Helsinki at 2PM = $-7+4=-3$°C ✓

Example 3. At noon the temperature was 4°C, at midnight the temperature was −5.5°C. Workout the difference in temperature between noon and mid-night. ---[1]

S-15/21/Q1

Solution: diff in temp. = $4-(-5.5)=4+5.5=9.5$°C ✓

Example 4. At mid night the temperature in Newton was −8°C. At noon the next day the temperature in Newton was 9°C. Workout the rise in temperature from mid-night to noon. ---[1]

N-15/21/Q1

Solution: Rise in temp = $9-(-8)=17$°C.

Example 5: The roof top of a building is 35 m above the ground level, and the car parking is 4 m below the ground level. Find the height of the roof top from the surface of the car parking.

Solution: Height of roof top above the ground = 35 m

depth of the surface of car parka = (4 m) or −4 m

∴ height of roof top from car parka = $35-(-4)$

= $\underline{39\ m}$

Number

§ Squares, Square roots, cubes and cube roots of a number:

1. (a) Squares: $a \times a = a^2$ (a square) ; $5 \times 5 = 5^2 = 25$
$(-6) \times (-6) = (-6)^2 = 36$

(b) Square roots (i) If $4^2 = 16$
we say square root of $16 = \sqrt{16} = 4$

(ii) $(-4)^2 = 16$

$\therefore$ square root $16 = \pm\sqrt{16} = \pm 4$

Note: We can not find the square root of −ve numbers

2. (a) Cube: $a \times a \times a = a^3$; $5 \times 5 \times 5 = 5^3 = 125$

(b) Cube root: (i) if $5^3 = 125$ then cube root of $125 = \sqrt[3]{125} = 5$
(ii) $\sqrt[3]{-8} = -2$. $(\because (-2)^3 = -8)$

Example 1:

Complete this table of squares and cubes: --- [3]

June 2003/2/Q8

Number	Square	Cube
3	9	27
---	121	---
---	---	2744
---	---	−343

Solution: Second Row Number $= \sqrt{121} = 11$ and $11^3 = 1331$
Third row: Number $= \sqrt[3]{2744} = 14$; $14^2 = 196$
Fourth row: Number $= \sqrt[3]{-343} = -7$; $(-7)^2 = 49$

Number	Sqruence	Cube
3	9	27
11	121	1331
14	196	2744
−7	49	−343

Example 2: Write the following in Order of size, Smellest first. ---[2]
0.5^2, 0.5, 0.5^3, $\sqrt[3]{0.5}$

Solution: 0.5^3; 0.5^2; 0.5, $\sqrt[3]{0.5}$

S-14/21/Q1 $(\because 0.5^3 = 0.125;\ 0.5^2 = 0.25$ and $\sqrt[3]{0.5} = 0.793$--

§ Money and converting from one currency to another:

Example 1. Omar changes 2000 Saudi Arabian riyals (SAR) into euros (€), when the exchange rate is €1 = 5.087 SAR. Work out how much Omar receives, giving your answer correct to the nearest euro. --[2]

S-16/21/Q5

Solution: $5.087\text{ SAR} = €1 \Rightarrow 1\text{ SAR} = \frac{1}{5.087}\,€$

$\therefore 2000\text{ SA} = €\ \frac{1}{5.087} \times 2000$

$= €\ 393.159$

$= €\ 393$ ✓ (to the nearest eu.)

Example 2: The price of a toy is 12 euro's (€) in Germany and 14 swiss francs in Switzerland. 1 Swiss franc = €0.905. Calculate the difference between two prices. Give your answer in Euro's. W-16/21/Q3 --[2]

Solution: Price of toy in Germany = €12 --- (i)

Price of toy in Switzerland = 14 Swiss franc $\left[\because 1\text{ swiss fract} = €0.905\right.$

$= €0.905 \times 14$

$= €12.67$ --- (ii)

from (i) & (ii) difference = 12.67 − 12 = €0.67 ✓

Example 3: The table shows how the dollar to euro conversion rate changed during one day:

Time	10 00	11 00	12 00	13 00	14 00	15 00	16 00
\$1	€1.3311	€1.3362	€1.3207	€1.3199	€1.3200	€1.3352	€1.3401

--[3]

Khalil changed \$500 into euros (€) W-13/22/Q10

How many more euros did Khalil receive if he changed his money at the highest rate compared to the lowest rate?

Solution: Khalil has \$500.

Highest rate is $\$1 = 1.3401 \Rightarrow \$500 = 500 \times 1.3401 = 670.05$ --- (i)

Lowest rate is $\$1 = 1.3199 \Rightarrow \$500 = 500 \times 1.3199 = 659.95$ --- (ii)

$\therefore$ difference at the two rates $= 670.05 - 659.95 = \$10.10$ ✓

Number

§ Time :

Unit Time. 1 hour = 60 minute
and 1 min = 60 seconds.

12 Hour Clock: and 24 Hour clock.

Time:	12 Hour Clock		24 Hour Clock
	5 a.m	→	05 00
	7:15 am	→	07 15
	8:00 pm	→	20 00
	9:15 pm	→	21 15

Example 1: A train leaves Zurich at 2240 and arrives in Vienna at 0732 the next day, Work out the time taken. [S-16/21/Q1] ---[1]

Solution: Time from 2240 to 0732 next day
is 2240 to midnight 2400 and midnight 0000 to 0732
= 1 hr 20 min and 7 hr 32 min
= 8 hr 52 min ✓

Example 2: A bus company has the operating times:

Day	Starting	Finishing time
Saturday	06 00	24 00
Sunday	06 00	24 00
Monday	06 00	24 00
Tuesday	06 00	24 00
Wednesday	06 00	24 00
Thursday	06 00	24 00
Friday	13 00	24 00

(a) Calculate the total number of hours that the bus company operates in a week. --[3]
(b) Write the starting time on Friday in the 12-hour clock. ---[1]

Solution: (a) Saturday to Thursday Time = 18 × 6
= 108 Hr
Time of Friday = 11 hrs
∴ Total number of working hours
= 108 + 11 = 119 Hours ✓

[S-14/21/Q9]

(b) On Friday starting time = 1300 = 1:00 pm. ✓

Example 3: A plane flies from London to Los-Angeles, the flight takes 11 hrs and 10 min. The plane leaves at 0935 local time from London, The local time in Los-Angeles is 8 hrs behind the local time in London. Calculate the local time when the plane arrives in Los-Angeles. ---[2]

Solution: 0935 + 11 hr 10 min − 8 hr = 1245 ✓ [S-16/42/Q1(a)]

Number

§ Time:

Example 4: Noma flies from Johannesburg to Hong Kong. Her plane leaves Johannesburg at 1845 and arrives in Hong Kong 13hrs and 25 minutes later. The local time in Hong Kong is 6 hours ahead of the time in Johannesburg. W-13/42/Q7

At what time does Noma arrives in Hong Kong? ---[2]

Solution: $1845 + 13hr\ 25min + 6hr = 3810 = 1410$ Next day or 2 10 PM.

Ratio

§ If a quantity of magnitude 'M' unit is divides in the ratio $a:b$

Then the first part $= \frac{a}{(a+b)} \times M$

second part $= \frac{b}{(a+b)} \times M$.

Example 1: Ahmed, Batuk and Chand share \$1000 in the ratio 8:7:5

Calculate amount each receives. M-15/22/Q9 ---[3]

Solution: $8:7:5 \Rightarrow$ Total parts $8+7+5 = 20$

$\therefore$ Ahmed's Share $= \frac{8}{20} \times 1000 = \$400$ ✓

Batuk's Share $= \frac{7}{20} \times 1000 = \$350$ ✓

and Chand's Share $= \frac{5}{20} \times 1000 = \$250$ ✓

Example 2: Ralf and Susie share \$57 in the ratio 2:1

(a) Calculate the amount Ralf receives. ---[2]

(b) Ralf gives \$2 to Susie. Calculate the new ratio,

Ralf's money : Susie's money. Give your answer in simplest form. --[2]

W-16/22/Q12

Solution: $2:1 \Rightarrow$ Total parts $2+1 = 3$

$\therefore$ Ralf's money $= \frac{2}{3} \times 57 = \$38$ ✓

and Susie's money $= \frac{1}{3} \times 57 = \$19$.

(b) Now Ralf gives \$2 to Susie $\Rightarrow$ Ralf's money $= 38 - 2 = \$36$

and Susie's money $= 19 + 2 = \$21$

$\therefore$ New Ratio $= 36:21$ or $12:7$ ✓ $\quad [\because \frac{36}{21} = \frac{12}{7}$

Number

Ratio

Example 3: Pedro and Eva do their home work. Pedro takes 84 minutes to do his home work. The ratio Pedro's time : Eva's time = 7 : 6 ---[2]
Work out the number of minutes Eva takes to do her home work. [5-13/21/Q3]

Solution: Pedro : Eva = 7 : 6 Total part 7+6 = 13, Let Total Time = T

$\therefore$ Pedro's time $= \frac{7}{13} \times T = 84$ (Given)

$\Rightarrow T = 84 \times \frac{13}{7} = 12 \times 13 = 156$ min.

$\therefore$ Eva's time $= \frac{6}{13} \times 156 = 6 \times 12 = \underline{72 \text{ min}}$ ✓

Example 4: There are three different areas A, B and C for seating in a theatre. The ratio of no. of seats in area A : seats in area B = 5 : 6 and the ratio of number of seats in area B : no. of seats in area C = 4 : 3
Given that the number of seats in area A are 200. Find the number of seats in area C. ---[3]

Solution: $A : B = 5 : 6 \overset{\times 2}{=} 10 : 12$

and $B : C = 4 : 3 \overset{\times 3}{=} 12 : 9$

Find L.C.M of 6, 4 = 12 for B

$\therefore A : B : C = 10 : 12 : 9$ Total parts 10+12+9 = 31

Let the total seats in the theatre = x

$\therefore$ Seats in area $A = \frac{10}{31} \times x = 200$ (given)

$\Rightarrow x = \frac{200 \times 31}{10} = 620$

$\therefore$ No. of seats in area $C = \frac{9}{31} \times 620 = \underline{180}$ ✓

§ **Increase by a given ratio:**

Example: Increase 80 by ratio 7 : 5 $= 80 \times \frac{7}{5} = 112$ ✓

§ **Decrease by a given ratio:**

Example: Decrease 50 by ratio 3 : 5 $= 50 \times \frac{3}{5} = 30$ ✓

Number

Proportion

§ **Direct Proportion:**

(i) Cost of shirts is directly proportional to number.

(ii) distance travelled is directly prootional to the speed.

Numerically: Four number a, b, c and d such that:

ratio a : b is same as c : d [:: denotes is same as]

or $a : b :: c : d$ { we say, a, b, c and d are directly proportional

or $\frac{a}{b} = \frac{c}{d}$

$\Rightarrow ad = bc$

Example 1: The cost of five shirts is \$70, what is the cost of 7 shirts.

Solution: let the cost of 7 shirt is \$ x.

Ratio of shirts 5 : 7

Ratio of cost $70 : x$

$\Rightarrow 5 : 7 :: 70 : x$

$\Rightarrow \frac{5}{7} = \frac{70}{x}$

$\Rightarrow x = \frac{7 \times 70}{5} = \$98$ ✓

Example 2: A bicycle takes 3 hours to cover 36 km, how much time it will take cover 60 km, keeping the same speed.

Solution: Let the time required is x hours.

$\therefore 36 : 60 :: 3 : x$

or $\frac{36}{60} = \frac{3}{x} \Rightarrow 36x = 3 \times 60$

$\Rightarrow x = \frac{3 \times 60}{36} = \underline{5 \text{ hrs}}$ ✓

Number

§ Inverse Proportion: If two quantities are interconnected so that the increase in one makes the decrease in the other and conversly.

Note: If the value of one quantity is 'a' and the corresponding value of other is 'b' then $a \cdot b$ remains constant.

or $a_1 b_1 = a_2 b_2 = a_3 b_3 - - - = k$ (Constant)

Example: If 6 men can do a job in 4 days then 12 men will finish the job in 2 days only

i.e. $6 \times 4 = 12 \times 2$

Example 1: If a and b vary inversly, fill in the blanks:

a	6	12	---	2
b	4	---	3	---

Solution: Since a & b vary inversely ∴ product ab should be constant

Now $ab = 6 \times 4 = 24$

∴ first blank $= \frac{24}{12} = 2$

Second blank $= \frac{24}{3} = 8$ and the third Blank $= \frac{24}{2} = 12$

Example 2: 120 men can finish a work in 5 days. If 30 more men join them, then in how many days the work can be done.

Solutions: If number of men is increase they can finish the work in lesser number of days, hence it is a case of inverse proportion.

Let number of required days $= x$

number of men now $= 120 + 30 = 150$

$\therefore 150 \times x = 120 \times 5 \Rightarrow x = \frac{120 \times 5}{150} = 4$ days

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Number

Sets and Venn diagrams

§ **Set:** A set is well defined collection of objects.

Example: 1. Set of all natural numbers,
2. Set of positive integers.
3. Set of all vowels in English alphabet.

Note: $n(A)$ = Number of elements in A

§ **Representation of a set:**

§ 1. Roster or tabular form:

(i) Set of natural numbers $= \{0, 1, 2, 3, 4, ---\}$
(ii) Set of positive integers $= \{1, 2, 3, 4, ---\}$
(iii) Set of vowels in English alphabet $= \{a, e, i, o, u\}$
(iv) Set of prime numbers less then 12 $= \{2, 3, 5, 7, 11\}$

§ 2. Set Builder form:

(i) Set of vowels in English alphabet, $V = \{x : x$ is a vowel in English alphabet$\}$
(ii) Set of integers between -1 & 3, $A = \{x ; x$ is an integer and $-1 < x < 3\}$

§ "belongs to" or "is an element of" is denoted by "$\in$" symbol

Example 1. $V = \{a, e, i, o, u\}$
we say. $e \in V$
'$\notin$' "does not belong to" $m \notin V$

Example 2: Consider a set $N = \{0, 1, 2, 3, 4, --\}$
$3 \in N$ but $-2 \notin N$.

§ Empty set (or Void set or Null set) is denoted by ϕ or $\{\}$ is a set which does not contain any element.

Example 1. $A = \{x ; 2x + 3 = 0$ and x is an integer$\}$

2. $B = \{x ; x$ is an even prime greater than $2\}$

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Sets

§ **Subset:** Given two sets A and such that every element of set A is also an element of B, we say set A is a subset of B. $A \subseteq B$.

Example 1 $A = \{1, 3, 4, 7\}$
$B = \{1, 2, 3, 4, 5, 6, 7, 8\}$
$C = \{2, 3, 4, 5, 6, 7\}$

Then $A \subseteq B$; but $A \not\subseteq B$.

$\not\subseteq$ denotes not a subset
Here $1 \in A$ but $1 \notin C$
$\therefore A \not\subseteq C$,

§ **Equal Sets:** Two sets A and B are said to be equal iff $A \subseteq B$ and $B \subseteq A$

Example 2. $A = \{1, 3, 4, 7\}$ $\Rightarrow A = B$
$B = \{3, 1, 7, 4\}$

§ **Proper Subset:** If $A \subseteq B$ but $A \neq B$.

Then we say that A is proper sub of B. $A \subset B$.

Example 3: $A = \{2, 3, 5, 7\}$ $\Rightarrow A \subset B$
$B = \{1, 2, 3, \dots 7\}$

§ **Universal Set** ($\mathcal{E}$) is set such that all other sets in that problem are subsets of it.

Example: $\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A = \{1, 3, 5, 9\}$
$B = \{2, 5, 7, 10\}$
$C = \{6, 8\}$

Number

Sets

§ **Complement of set A (denoted by A'):**

Given is set A, and universal is $\mathcal{E}$.
Then the set of all the elements of $\mathcal{E}$, but not in A, is the Complement of set A and is denoted by A'.

Note: $(A')' = A$

Example: $\mathcal{E} = \{1,2,3,4,5,6,7,8,9,10\}$

$A = \{2,3,7,10\}$

Then complement of A, $A' = \{1,4,5,6,8,9\}$ ✓

§ **Intersection of two set A and B (denoted by $A \cap B$)**

Given two sets A and B, then the set of common elements which are in A and also in B.

Example, $A = \{1, 2, 4, 5, 9\}$

$B = \{2, 3, 5, 7, 8\}$

Then intersection of A and B, $A \cap B = \{2,5\}$ ✓

§ **Union of two Sets (denoted by $A \cup B$):**

Given two sets A and B then the set of elements which are in A or in B or in both.

Example: $A = \{1, 2, 4, 5, 9\}$

$B = \{2, 3, 5, 7, 8\}$

$A \cup B = \{1, 2, 3, 4, 5, 7, 8, 9\}$

De Morgan's Laws:
(i) $(A \cap B)' = A' \cup B'$
(ii) $(A \cup B)' = A' \cap B'$

§ **Difference of Sets (denoted by $A - B$):**

Given two sets A and B. Then $(A-B)$ is set of elements of A but not in B. and
$(B-A)$ is set of elements of B but not in A.

Note (i) $A - B = A \cap B'$ (ii) $B - A = B \cap A'$

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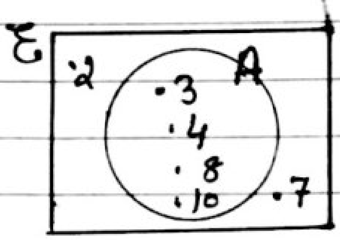
Sets and Venn diagram:

§

Venn diagrams: Venn diagrams consist of rectangles and closed curves usually circles. The universal set is represented by rectangle and its subsets by circles inside.

Example 1.

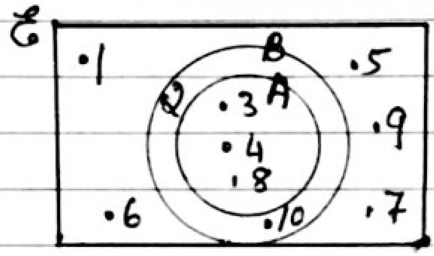


$\mathcal{E} = \{2, 3, 4, 7, 8, 10\}$

$A = \{3, 4, 8, 10\}$

Complement of $A = A' = \{2, 7\}$

Example 2.

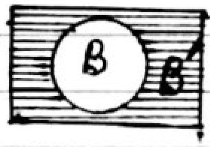


$\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

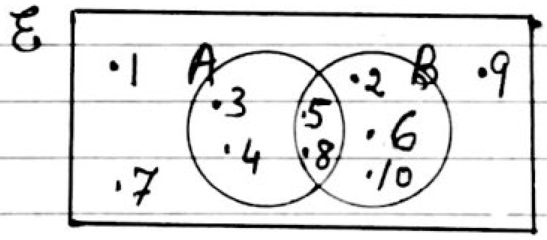
$A = \{3, 4, 8\}$

$B = \{2, 3, 4, 8, 10\}$

$A \subset B$ and $B' = \{1, 5, 6, 7, 9\}$



Example 3.



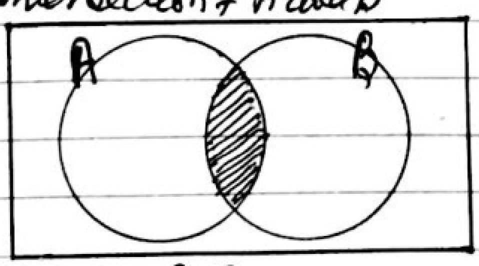
$\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{3, 4, 5, 8\}$

$B = \{2, 5, 6, 8, 10\}$

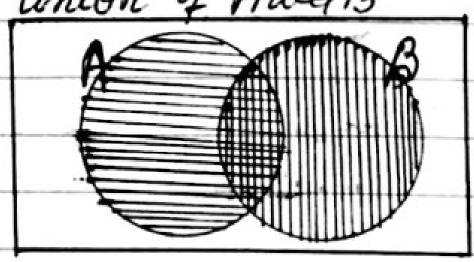
$A \cap B = \{5, 8\}$; $A \cup B = \{2, 3, 4, 5, 6, 8, 10\}$

Intersection of A and B



$A \cap B$

Union of A and B



$A \cup B$

Number

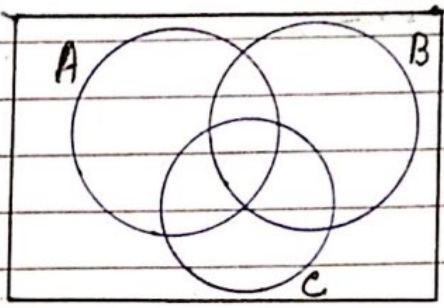
Sets and Venn diagrams

Example 4. $\mathcal{E} = \{21, 22, 23, 24, 25, 26, 27, 28, 29, 30\}$

$A = \{x : x \text{ is a multiple of } 3\}$

$B = \{x : x \text{ is prime}\}$

$C = \{x : x \leq 25\}$



(a) Complete the venn diagram. --- [4]

(b) Use set notation to complete the statement.

(i) 26 --- B --- [1]

(ii) $A \cap B =$ .... --- [1]

(c) List the element of $B \cup (C \cap A)$ -- [2]

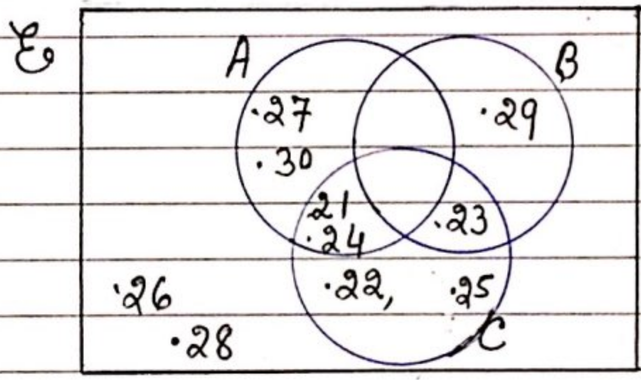
(d) Find (i) $n(C)$ --- [1]

(ii) $n(B' \cup (B \cap C))$ --- [1] S-17/43/Q10

(e) $(A \cap C)$ is a subset of $(A \cup C)$.

Complete this statement using set notation $(A \cap C) \cdots (A \cup C)$ -- [1]

Solution: (a)



(b) (i) $\notin$

(ii) ϕ

(C) 21, 23, 24, 29

(d) (i) 5 (ii) 9

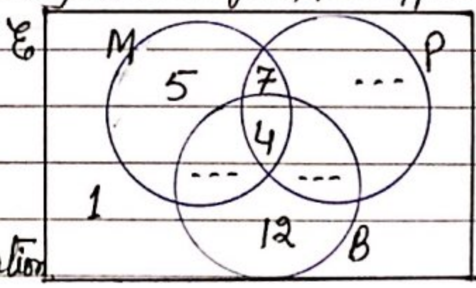
(e) $\subset$ or $\subseteq$

Example 5: Davinder asked som people if they ate mangoes, pineapples or bananas last week.

M = { people who ate mangoes}

P = { people who ate pineapples}

B = { people who ate bananas}



The venn diagram shows some information.

19 people said they ate mangoes.

6 people saide they ate only pineapples. M-16/42/Q3(a)

18 people said they ate exactly two of the three types of fruits.

(i) Write the three missing values in the venn diagram. --- [3]

(ii) Total number of people Davinder asked. --- [1]

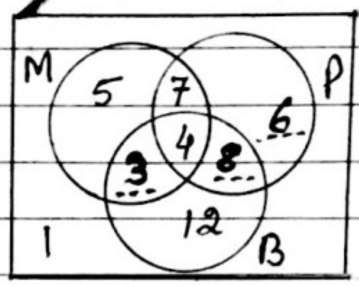
(continued →)

Sets and Venn Diagrams

(Continued)

Example 5. (iii) $n(M \cap P)$ --- [1]

(iv) One person is chosen at random from the people who ate mangoes. Write down the probability that this person also ate bananas. --- [2]



Solution (i) Ist Blank $= 19 - (5+7+4)$
$= 19 - 16 = 3$ ✓

IInd Blank = only pineapple = 6

IIIrd Blank = n(Exactly 2 fruits) $- (7+3)$
$= 18 - 10 = 8$ ✓

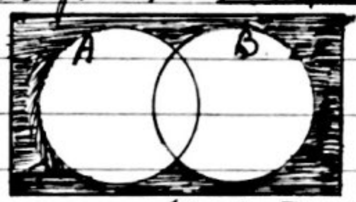
(ii) $n(Total) = (5+7+6+3+4+8+12)+1$
$= 46$ ✓

(iii) $n(M \cap P) = 7+4 = 11$ ✓

(iv) $n(M) = 19$, $n(M \cap B) = 7$
$\therefore$ Prob $= 7/19$

Example 6 Shade the required region on each Venn diagram. S-13/22/Q1

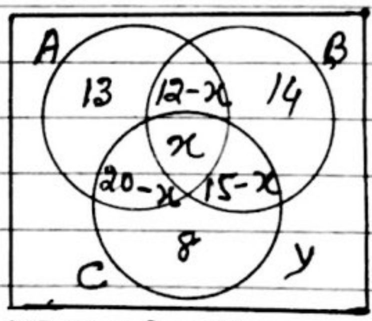
$A' \cup B$



--- [2]

$A' \cap B'$ [$= (A \cup B)'$]

Example 7: The Venn diagram shows the number of elements in sets A, B and C.



(a) $n(A \cup B \cup C) = 74$, find x, --- [2]

(b) $n(\mathcal{E}) = 100$ find y. --- [1]

(c) Find the value of $n((A \cup B)' \cap C)$ --- [1]

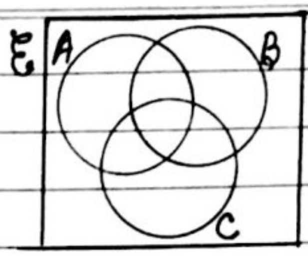
Solution (a) $13 + (12-x) + 14 + (20-x) + x + 15 - x + 8 = 74$
$\Rightarrow 82 - 2x = 74 \Rightarrow x = 4$ ✓

(b) $n(\mathcal{E}) = 100 \Rightarrow 74 + y = 100 \Rightarrow y = 26$ ✓ S-13/23/Q15

(c) $n((A \cup B)' \cap C) = 8$ ✓

Number

Sets and Venn diagrams.



Example 8 (a). x is an integer.

$\mathcal{E} = \{x : 1 \leq x \leq 10\}$

$A = \{x ; x \text{ is a factor of } 12\}$

$B = \{x : x \text{ is an odd number}\}$

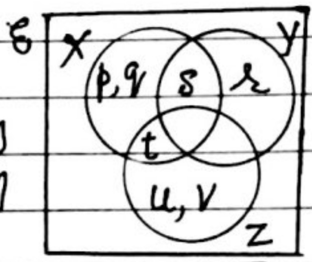
$C = \{x : x \text{ is a prime number}\}$

(i) Complete the Venn diagram to show this information. ---[3]

(ii) Use set notation to complete each statement:

$6 \;---\; A$; $A \cap B \cap C = ---$; $A \cap A' = ---$. ---[3]

(iii) Find $n(B)$



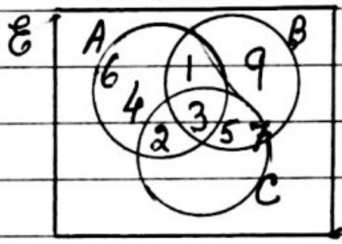
(b)(i) Use set notation to complete the the statement,

$\{u, v\} \;---\; Z$ ---[1]

(ii) Shade $X \cap (Z \cup Y)'$ ---[1]

M-15/42/Q2

Solution: (a)(i)

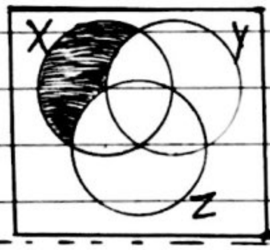


(ii) $\mathcal{E}$; $\{3\}$; ϕ or $\{\}$

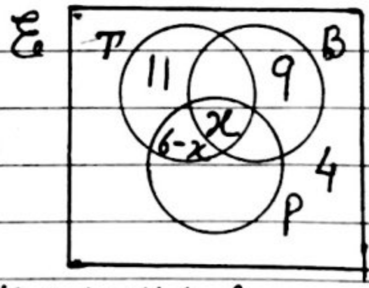
(iii) 5

(b) (i) $\subset$

(ii) $X \cap (Z \cup Y)'$



In the Venn diagram:



Example 9: $\mathcal{E} = \{\text{Children in a nursery}\}$

$B = \{\text{Children who received a book for their birthday}\}$

$T = \{\text{children who received a toy for their birthday}\}$

$P = \{\text{children who received a puzzle for their birthday}\}$

x children received a book, a toy and a puzzle.

6 children received a toy and a puzzle.

(a) 4 children received a book and a toy. 5 children received a book and a puzzle. 7 children received a puzzle but not a book and not a toy. Complete the Venn diagram above. (continued →) ---[3]

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Number

Sets and Venn diagrams

(Continued →)

Example 9 (b) There are 40 children in the nursery. Using Venn diagram, write down and solve an equation in x. ---[3]

(c) Work out, (i) The probability that a child, chosen at random, received a book but not a toy and not a puzzle. --[1]

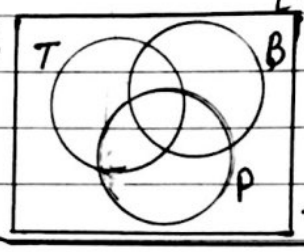
(ii) the number of children who received a book and a puzzle but not a toy. --[1]

(iii) $n(B)$ --[1]

(iv) $n(B \cup P)$ --[1]

(v) $n(B \cup T \cup P)$ --[1]

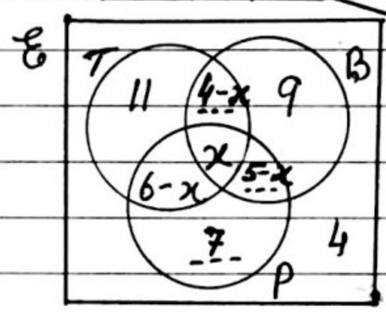
(d) Shade the region, $B \cap (T \cup P)'$.



--[1]

S-14/41/Q4

Solution (a)



$n(B \cap T) = 4$ given

$\therefore \text{---} = (4-x)$ ✓

$n(B \cap P) = 5$

$\therefore \text{---} = 5-x$ ✓

$\text{---} = 7$ (Given)

(b) $\left(11 + (4-x) + 9 + (6-x) + x + (5-x) + 7\right) + 4 = 40$

$\Rightarrow 46 - 2x = 40 \Rightarrow x = 3$ ✓

(c) (i) $9/40$ ✓

(ii) $5 - x = 5 - 3 = 2$ ✓

(iii) $n(B) = (4-x) + x + (5-x) + 9$

$= 9 + 9 - x$

$= 18 - 3 = 15$ ✓

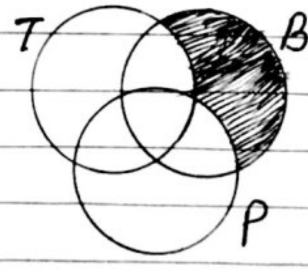
(iv) $n(B \cup P) = (4-x) + 9 + x + (6-x) + (5-x) + 7$

$= 31 - 2x$

$= 31 - 6 = 25$ ✓

(v) $n(B \cup T \cup P)' = 4$

(d) $B \cap (T \cup P)'$



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Number

Percentage

Per Cent means per hundred, for example if we say that a student scored 80% marks in Maths, that means score is 80 out of hundred.

§ Finding a percentage of a given number:

Example: 40% of 70 $= \frac{40}{100} \times 70 = 28$ ✓

§ Finding a Number when percentage is given;

Example: Find a number whose 15% is 72.
Let number is x. $\Rightarrow \frac{15}{100} \times x = 72 \Rightarrow x = \frac{72 \times 100}{15} = 480$ ✓

§ Finding how much percent one quantity is of the second quantity.

Example: (i) what percent of a is b let $x\%$.
Then $\frac{x}{100} \times a = b \Rightarrow x = \frac{b}{a} \times 100\%$

Example: what percent of 56 is 14
$\% = \frac{14}{56} \times 100 = 25\%$

Example 1. In a sale, the cost of a coat is reduced from \$85 to \$67.50.
Calculate the percentage reduction in the cost of the coat. S-15/23/Q10 [3]

Solution
original cost = \$85
reduced cost = \$67.50
reduction = 85 − 67.50 = \$17.50
∴ Percentage reduction $= \frac{17.50}{85} \times 100 = 20.6\%$ ✓

Number

Percentage

Example 2: Indira buys a television in a sale for \$924. This was a reduction of 12% on the original price. Calculate the original price of the television. M-17/22/Q10 ---[3]

Solution: Let the original price = \$ x

reduction = 12%

$\therefore$ reduced price = (100 − 12) = 88% of original

$$\Rightarrow \frac{88}{100} \times x = 924 \text{ (given)}$$

$$\Rightarrow x = \frac{924}{88} \times 100 = \$1050 \checkmark$$

Example 3: In 2012 the cost of a ticket to an art festival was \$30. This was 20% more than the ticket cost in 2011. ---[3]
Calculate the cost of the ticket in 2011. W-13/23/Q6

Solution: Let the cost of ticket in 2011 = \$ x

In 2012 cost is 20% more

$\Rightarrow$ (100 + 20)% of the cost in 2011

$$\Rightarrow \frac{120}{100} \times x = 30 \text{ (given)}$$

$$\Rightarrow x = 30 \times \frac{100}{120} = \$25 \checkmark$$

Note:

1. Let Original cost = \$ x
   Increased by = i%
   New cost = $(100+i)\% = \frac{(100+i)}{100} \times x$ --- (1)

   $$\boxed{\text{and Original cost } x = \frac{\text{New cost} \times 100}{(100+i)}}$$

2. Now if there is decrease (loss) = d% (or discount)

   New cost = $(100-d)\% = \frac{(100-d)}{100} \times x$

   $$\boxed{\therefore \text{Original cost} = \frac{100}{(100-d)} \times \text{New cost}}$$

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Number

Percentage

§ Profit and Loss :

Shop keeper buys an article from wholesaler = Cost Price (C.P)

Shop keeper sells to the custmer = Selling Price (S.P)

Case I. If S.P > C.P ⇒ Profit = S.P − C.P

$$\text{Profit \%} = \frac{\text{Profit}}{C.P} \times 100$$

Case II If C.P > S.P (or S.P < C.P) ⇒ Loss = C.P − S.P

$$\Rightarrow \text{Loss \%} = \frac{\text{Loss}}{C.P} \times 100$$

If Profit (or Loss) is given as P% (or L%)

$$\text{Then } S.P = \frac{(100+P)}{100} \times C.P$$

$$\text{or } \frac{(100-L)}{100} \times C.P$$

$$\text{or } C.P = \frac{100}{(100+P)} \times S.P \quad \left(\text{or } C.P = \frac{100}{(100-L)} \times S.P\right)$$

---

§ Original Cost and discounted (or reduced by r %).

Let original cost = \$ x

rate of reduction (or discount) = r %

Case I ∴ $\text{Reduced Cost} = \frac{(100-r)}{100} \times x$

Case II If given reduced cost then

$$\boxed{\text{Original Cost } x = \frac{100}{(100-r)} \times \text{Reduced Cost}}$$

Percentage

Example 4: Meena sells her car for \$6000. This a loss of 4% on the price she paid. Calculate the price Meena paid for the car. ---[3]

[M-16/42/Q(a)]

Solution: Loss L = 4% , S.P = \$6000

$$\therefore C.P = \frac{100}{(100-L)} \times S.P = \frac{100}{(100-4)} \times 6000$$

$$= \frac{100}{96} \times 6000 = \$6250 \checkmark$$

Example 5: Kristian and stephanie share some money in the ratio 3:2, Kristian receive \$72.

(i) Work out how much Stephanie receives. ---[2]

(ii) Kristian spends 45% of his \$72 on a computer game. Calculate the price of the computer game. ---[1]

(iii) Kristian also buys a meal for \$8.40. Calculate the fraction of the \$72 Kristian has left after buying the computer game and the meal. Give your answer in its lowest terms. ---[2]

(iv) Stephanie buys a book in a sale for \$19.20. This sale price is after a reduction of 20%. Calculate the original price of the book--[3]

[S-17/43/Q10(a)]

Solution: Kristian Share : Stephanie Share = 3 : 2 [Total part 3+2 = 5]

Let total money = \$ x

$\therefore$ Kristian Share = $\frac{3}{5} \times x = 72$ (given)

$\Rightarrow x = 72 \times \frac{5}{3} = \$120$

(i) $\therefore$ Stephaie share $= \frac{2}{5} \times 120 = \$48 \checkmark$

(ii) Price of computer game ---= 45% of 72

$= \frac{45}{100} \times 72 = \$32.4 \checkmark$

(iii) Meal cost = \$8.40

$\therefore$ (Meals + computer game) cost = 8.40 + 32.40 = \$40.8

$\therefore$ Money Left with Kristian = 72 − 40.8 = 31.2

$$\therefore \frac{\text{Money Left}}{72} = \frac{31.2}{72} = \frac{312}{720} = \frac{13}{40} \checkmark$$

(Continued →)

(Continued→) Percentage

Example 5:

Solution (iv) Let the original (C.P) price of book = $\$x$

Reduction = 20% (r%)

S.P = $19.20

$\therefore C.P = \frac{100}{(100-r)} \times S.P = \frac{100}{(100-20)} \times 19.20$

$= \frac{100}{80} \times 19.20 = \$24$ ✓

Example 6 (b) There are three types of tickets economy, business and first class. The price of these tickets is in the ratio,

economy : business : first class = 2 : 5 : 9

(i) The price of business ticket is $2350.
Calculate the price of a first class ticket --- [2]

(ii) Work out the price of an economy ticket as a percentage of the price of a first class ticket. --- [1]

(C) The price of a business ticket for the same journey with another airline is $2240.

(i) The price of first class ticket is 70% more than a business ticket. Calculate the price of this first class ticket. --- [2]

(ii) The price of business ticket is 180% more than an economy ticket. Calculate the price of this economy ticket. --- [3]

S-16/42/Q1(b)(c)

Solution (b) (i) economy : business : first class = 2 : 5 : 9 [Total Parts 2+5+9=16

Let the total price of all three classes = $\$x$

$\therefore$ Business ticket $= \frac{5}{16}x = 2350$

$\Rightarrow x = 2350 \times \frac{16}{5} = 470 \times 16$

$\therefore$ first class $= \frac{9}{16} \times 470 \times 16 = \$4230$ ✓

(i) Alternate method
first class $= \frac{9}{5} \times 2350$
$= \$4230$

(ii) Economy : first class = 2 : 9

$\therefore$ Economy $= \frac{2}{9}$ % of first class = 22.2% ✓

(continued→)

(Continued→)

Percentage

Example 6:

Solution(c)(i) Now New business ticket = $ 2240

first class is 70% more than business.

$\therefore$ first class $= \left(\frac{100+70}{100}\right) \times 2240$

$= \frac{170}{100} \times 2240 = \$3808$ ✓

(ii) Business class is 180% more than Economy

$\therefore$ Business class $= \left(\frac{100+180}{100}\right) \times Eco = 2240$ (Given)

$\Rightarrow$ Eco. class $= \frac{2240 \times 100}{280} = \$800$ ✓

*Example 7(a) Last year a golf club charged $ 1650 for a family membership. This year the cost is increased by 12%.

Calculate the cost of a family membership this year. ---[2]

(b) The golf club runs a competition. The total prize money is shared in the ratio, 1st prize : 2nd prize = 9 : 5

The first prize is $ 500 more than the 2nd prize.

(i) Calculate the total prize money for the competition. ---[2]

(ii) What percentage of the total prize money is given as the the 1_{st} prize. ---[1]

(c) For the members of the golf club the ratio men : children = 11 : 2

the ratio women : children = 10 : 3

(i) Find the ratio men : women. ---[2]

(ii) The golf club has 24 members who are childrem.

Find the total number of members. ---[3]

(d) The club shop sold a box of golf balls for $20·40

The shop made a profit of 20% on the cost price.

Calculate the cost price of the golf balls. $\boxed{S\text{-}15/42/Q1}$ ---[3]

Solution: (a) Last year charges = $ 1650

Increase = 12%

$\therefore$ This year charge $= \left(\frac{100+12}{100}\right) \times 1650 = \frac{112}{100} \times 1650$

$= \$1848$ ✓

(Continued→)

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Number

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Percentage

(Continued Example 7)

Solution (b). Given 1^{st} Prize : II^{nd} Prize = 9:5 [Total part = 9+5 = 14

(i) Let the total prize money is $\$x$ [diffence of parts [9−5 = 4

∴ Diffence of I_{st} and 2nd prize = 500

$$\Rightarrow \frac{4}{14} \times x = 500$$

$$\Rightarrow x = 500 \times \frac{14}{4}$$

∴ Total prize Money. $x = \$1750$ ✓

(ii) Ist Prize : 2nd prize = 9:5

∴ Ist prize is % of Total $= \frac{9}{14} \times 100 = 64.285$

$= 64.3\%$ ✓

(c) (i) men : women : children

11 : – : 2 ×3 L.C.M of 2&3 = 6

10 : 3 ×2

33 : – : 6

20 : 6

∴ men : women : children

33 : 20 : 6

∴ Ratio men : women = 33 : 20 ✓

[Total Parts 33+20+6 = 59

(ii) men : women : child = 33 + 20 + 6 ⟹ = 59

Let total number of members = x

∴ Number of Children $= \frac{6}{59} \times x = 24$ (given)

$$\Rightarrow x = 24 \times \frac{59}{6} = 236$$

∴ Total = 236 ✓

(d) S.P = \$ 20.40

Profit % = 20%

Let the C.P = $\$x$

$$S.P = \frac{(100+20)}{100} \times x = 20.40$$

$$\Rightarrow x = 20.40 \times \frac{100}{120} = \$17$$

∴ Cost Price = \$17 ✓

Interest

Simple Interest:

§ Some terms:

1. Principal 'P': money invested (or borrowed)
2. Rate of Interest: r% (of principal)
3. Time 't' for which money is invested (or borrowed)
4. Interest 'I' received (or to be paid)
5. Amount 'A' = P+I

If Principal $'P' is invested for 't' years at the rate of interest 'r'% per year, then

$$\begin{cases} \text{Simple Interest 'SI'} = \$\dfrac{P \times r \times t}{100} \\ \therefore \ \text{Amount } A = P + I \end{cases}$$

Example 1 Find the simple interest on \$500 invested for 3 years at 6% per year.

Solution: $P = \$500$, $t = 3$ years, $r = 6\%$

$$\therefore SI = \$\frac{500 \times 6 \times 3}{100} \qquad \left[\because S.I = \frac{P \cdot r \cdot t}{100}\right.$$

$\therefore$ Simple Interest = \$ 90 ✓

Example 2: Marcel invested \$350 at 4% and received \$42 as S.I, find the duration for which money was invested.

Solution: $P = \$350$, $t = ?$, $r = 4\%$ $SI = \$42$

$$42 = \frac{350 \times 4 \times t}{100} = 14t \qquad \left[\because S.I = \frac{P \times r \times t}{100}\right.$$

$$\Rightarrow t = 42/14 = 3 \text{ year.}$$

IG-Maths 0580

Number

Interest

Simple Interest

Example 3: Omar received \$50 as S.I by investing \$P for 4 years at the rate 5% p.a. Find the value of P.

Solution S.I = \$50, P = ?, r = 5%, t = 4

$$\because S.I = \frac{Prt}{100}$$

$$\therefore 50 = \frac{P \times 5 \times 4}{100}$$

$$\Rightarrow P = \frac{50 \times 100}{5 \times 4} = 250$$

$\therefore$ Investment = **\$250**

Note: p.a means per annum or per year

Example 4: A sum of money doubles itself in 10 years. Find the rate of interest.

Solution: r = ?, t = 10 years. Let Principal = \$P and Amount = \$2P

$\therefore$ S.I = Amount − Principal = 2P − P = \$P.

$$S.I = \frac{Prt}{100}$$

$$\therefore P = \frac{P \times r \times 10}{100} \Rightarrow r = \frac{P \times 100}{P \times 10} = 10$$

$\therefore$ rate of interest = 10% per year.

Example 5: Kavita invested \$150 at the rate of 6% p.a, Calculate in how many years the money will be doubled.

Solution P = \$150, r = 6% and time t = ?

A = \$300

$\therefore$ S.I = A − P = 300 − 150 = \$150.

$$150 = \frac{150 \times 6 \times t}{100} \qquad \left[S.I = \frac{Prt}{100} \right.$$

$$\Rightarrow t = \frac{100}{6} = 16.66\ldots = \underline{16.7 \text{ year}} \text{ (approx)}$$

Number

Interest

§ Compound Interest

When we calculate the simple interest (S.I) over the period of time the Principal 'P' remain the same year after year (or may be any specified time interval)

But if the interest earned is added to the 'Principal' after one year (or specified), and then the interest is calculate for the next year and so on.

For Example 6:

$P = \$10{,}000$, $r = 10\%$ and $t = 3$ year

Then S.I for 1st year $= \dfrac{10{,}000 \times 10 \times 1}{100}$ $\left[I = \dfrac{Prt}{100} \right.$

$= \$1000$

$\therefore$ Amount after one year $= 10{,}000 + 1000$

$= \$11{,}000$ ✓

Now for second year $P = 11{,}000$

$\therefore$ for second year $S.I = \dfrac{11000 \times 10 \times 1}{100} = \$1100$

$\therefore$ Amount after 2 year $= 11{,}000 + 1100$

$= \$12{,}100$ ✓

Finally for third year $P = \$12{,}100$

$\therefore$ Interest for third year $= \dfrac{12{,}100 \times 10 \times 1}{100} = 1210$

$\therefore$ Amount after 3 years $= 12{,}100 + 1210$

$= \$13{,}310$ ✓

$\therefore$ Compound Interest $= 13{,}310 - 10{,}000$
(Amount − P) $= \underline{\$3310}$ ✓

{ whereas if calculate the S.I for 3 years $= \dfrac{10{,}000 \times 10 \times 3}{100}$
$\$3000/=$

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Interest

Number

§ **Compound Interest:** rate of int. r% per year, Time n year

Formula: $A = P\left(1+\frac{r}{100}\right)^n$

Example 6(i) (Again)

$P = \$10{,}000$, $r = 10\%$ annually, $t = 3$ years

to find the compound Interest. (Compounded annually)

$$A = 10000\left(1+\frac{10}{100}\right)^3$$

$$= 10{,}000 \times (1.1)^3$$

$$= 10{,}000 \times 1.331$$

$$A = \$13{,}310$$

∴ Compound Interest C.I = $A - P$

$$= 13{,}310 - 10{,}000$$

$$= \$3310$$

Example 6(ii) $P = \$10{,}000$, annual rate of int $r = 10\%$, $t = 3$ year

but interest is calculated half yearly (compounded half yearly)

Solution: $P = \$10{,}000$, half yearly rate $= \frac{10}{2} = 5\%$

no of conversion periods $n = 3 \times 2 = 6$ (six half years)

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$= 10{,}000\left(1+\frac{5}{100}\right)^6 = 10{,}000(1.05)^6$$

$$= 10{,}000 \times 1.340095\ldots$$

$$= 13400.95$$

$$= \$13401$$

∴ C.I $= 13401 - 10{,}000 = \$3401$

§ Compound Interest (or Population Growth + Exponentially)

*Example 7: Marcel invests $2500 for 3 years at a rate of 1.6% per year simple interest. Jacques invests $2000 for 3 years at a rate of x% per year compound interest. At the end of the 3 years Marcel and Jacques receive the same amount of interest. Calculate the value of x correct to 3 significant figures. [5]

S-17/23/Q24

Solution for Marcel: $P = \$2500$, $r = 1.6\%$, $t = 3$ year

$$S.I = \frac{Prt}{100} = \frac{2500 \times 1.6 \times 3}{100} = \$120$$

for Jacques C.I = Same as S.I for Marcel = $120

$P = 2000$

$\therefore$ Amount $= 2000 + 120 = \$2120$ ——— (1)

also. $P = 2000$, rate of compound int $r = x\% = ?$; $t = 3$

$$\therefore \quad A = P\left(1 + \frac{r}{100}\right)^n$$

from (1) $$2120 = 2000\left(1 + \frac{x}{100}\right)^3$$

$$\Rightarrow (1 + .01x)^3 = \frac{2120}{2000} = 1.06$$

$$\Rightarrow 1 + .01x = \sqrt[3]{1.06} = 1.019612$$

$$\Rightarrow 0.01x = 0.019612$$

$$\Rightarrow x = \frac{0.019612}{0.01} = 1.9612$$

$$\therefore x = 1.96 \checkmark \quad (3\ s.f)$$

§ Compound Interest (or Population Growth Exponentially)

Example 8: The population of the world grows exponentially at a rate of 1.1 per year. Find the number of years it takes for the population from 7 billion to 7.31 billion. Give your answer to nearest whole number. [M-17/22/Q4] ---[2]

$P = 7$, $A = 7.31$, $r = 1.1\%$, $t = ?$

Solution: $7.31 = 7\left(1 + \frac{1.1}{100}\right)^t$ $\qquad \left[A = P\left(1 + \frac{r}{100}\right)^n\right.$

$\Rightarrow (1.011)^t = \frac{7.31}{7}$

$\Rightarrow (1.011)^t = 1.044255$ $\qquad \left[\because (1.011)^3 = 1.03336\right.$ and $(1.011)^4 = 1.04473$ ✓

$\Rightarrow t = 4$ years ✓

Example 9: At the start of an experiment there are 20000 bacteria. The number of bacteria increases at a rate of 30% per hour

(a) Work out the number of bacteria after 4 hours. ---[2]

(b) After how many whole hours, from the start of the experiment, will the number of bacteria be greater than one million? [S-16/23/Q19] --[2]

Solution: (a) $P = 20{,}000$, $r = 30\%$ per hr., $n = 4$ hr $\left[A = P\left(1 + \frac{r}{100}\right)^n\right.$

$\therefore A = 20{,}000\left(1 + \frac{30}{100}\right)^4$

$= 20{,}000 \times 1.3^4$

$= 20{,}000 \times 2.8561 = 57122$ ✓

$\therefore$ Number of bacteria after 4 hours $= 57{,}122$ ✓

(b) $P = 20{,}000$, $A > 10{,}00000$, $r = 30\%$, $n = ?$

$A = 20000\left[1 + \frac{30}{100}\right]^n > 10{,}00000$ $\qquad \left[A = P\left(1 + \frac{r}{100}\right)^n\right.$

$(1.3)^n > 50$

$n = 15$ ✓ $\qquad \because \left|\begin{array}{l}(1.3)^{14} = 39.37 \\ (1.3)^{15} = 51.185\end{array}\right.$

Number

Simple Interest and Compound Interest

Example 9(b) Boris invests \$ 550 at a rate of 2% per year simple interest. Calculate amount Boris has after 10 years. --[3]

(c) Marlene invests \$550 at a rate of 1.9% per year compound interest, Calculate the amount Marlene has after 10 years. --[2]

*(d) Hans invests \$550 at rate of x% per year compound interest. At the end of 10 years he has a total amount of \$638.30, correct to the nearest cent. Find the value of x. 5-16/41/Q1 --[3]

Solution (b) $P = \$550$, $t = 10$ year, $r = 2\%$

$$S.I = \frac{Prt}{100} = \frac{550 \times 2 \times 10}{100} = \$110$$

$$\therefore \text{Amount} = P + I = 550 + 110 = \$\underline{660}$$

(c) $P = 550$, $r = 1.9\%$ compounded per year, $t = 10$ year

$$A = 550 \cdot \left(1 + \frac{1.9}{100}\right)^{10} \qquad \left[A = P\left(1 + \frac{r}{100}\right)^n\right.$$

$$= 550 \cdot (1.019)^{10}$$

$$A = 550 \times 1.2071 = \$\underline{663.9}$$

*(d) $P = \$550$, $A = \$638.30$, $t = 10$ year, $r = x\%$ per year? Compound Interest.

$$638.30 = 550\left(1 + \frac{x}{100}\right)^{10} \qquad \left[A = P\left(1 + \frac{r}{100}\right)^n\right.$$

$$\Rightarrow \left(1 + \frac{x}{100}\right)^{10} = \frac{638.30}{550}$$

$$= 1.16045$$

$$\Rightarrow 1 + \frac{x}{100} = \sqrt[10]{1.1604545} = 1.01499$$

$$\Rightarrow 1 + \frac{x}{100} = 1.015 \Rightarrow \frac{x}{100} = 0.015 \Rightarrow x = 1.5$$

$$\therefore x = 1.5$$

IG- Maths 0580

Classmate P-49

Number

Compound Interest / Depreciaciation.

Example 10 (i) Each year the value of a car decreases by 15% of its value at the beginning of that year. Alberto buys a car for \$18000. Calculate the value of Alberto's car after 3 years. --[2]

(ii) Belinda bought a car one year ago, the value of this car has decreased by 15% to \$14025. Calculate how much Belinda paid for the car. W-16/42/Q1(a) --[3]

Solution

(i) Rate of depreciation $x = 15\%$ per year

Initial Value $P = \$18000$

Time $t = 3$ years

Initial Value = P
Rate of deprecation = $x\%$ yearly
Time = t years
Value after 't' years A
$$A = P\left(1 - \frac{x}{100}\right)^t$$

Final Value $A = 18000\left(1 - \frac{15}{100}\right)^3$

$= 18000 \times (.85)^3$

$= 18000 \times 0.614125 = \$\ 11054.25$ ✓

(ii)

Let One year ago car is bought for \$P.

rate of depreciation $x = 15\%$

$A =$ Now the decreased Value = \$14025

$t = 1$ year.

$$A = P\left(1 - \frac{x}{100}\right)^n$$

$$14025 = P\left(1 - \frac{15}{100}\right)^1$$

or $14025 = P \times 0.85 \Rightarrow P = \frac{14025}{0.85} = \$16500$ ✓

Q11 The number, P, of penguin in a colony, t years after 2000, is given by: $P = 2500 \times 1.02^t$

(a) (i) How many penguins were in the colony in the year 2000? --[1]

(ii) What information is given by 1.02 in the formula. --------[1]

S-15/04/Q9(a)

Solution (i) In 2000, time passed $t = 0 \Rightarrow P = 2500 \times (1.02)^0 = 2500 \times 1 = 2500$ ✓

(ii) 1.02 means increase of 2% ✓ [∵ Increase of 2% $= \left(1 + \frac{2}{100}\right) = (1 + .01 \times 2)$

here $1.02 = (1 + .01 \times 2)$

Average Speed:

§

Distance = D metres
Time = T second
Speed $= \frac{D}{T}$ m/s

Speed: $1\ Km/h = \frac{5}{18}\ m/s$

$\because 1\ Km/h = \frac{1000}{60 \times 60}$
$= \frac{5}{18}\ m/s$

Example 1. A car of length 4.3 m is travelling at 105 Km/h. It passes a bridge of length 36 m. Calculate the time, in seconds, it takes to pass over the bridge completely. $\boxed{S\text{-}16/21/Q18}$ ---[3]

Solution

Total Distance $= 36 + 4.3 = 40.3\ m$

Speed $= 105\ Km/h = 105 \times \frac{5}{18}\ m/s = 29.17\ m/s$

$\therefore$ Time $= \frac{D}{} = \frac{40.3}{29.17} = \underline{1.38\ s}$ ✓

Example 2: Every Monday, Sima travels by car to the library. The distance is 20 Km and the journey takes 23 minutes.

(i) Calculate the average speed for the journey in Km/h. ---[2]

(ii) One Monday, she delayed and her average speed is reduced by 32 Km/h. Calculate the percentage increase in the journey time. --- [5]

$\boxed{W\text{-}17/41/Q1(C)}$

Solution (i) Distance = 20 Km, time = 23 min. $= \frac{23}{60}$ h.

$\therefore$ Average speed $= \frac{D}{T} = 20 / \frac{23}{60} = \frac{20 \times 60}{23} = \underline{52.17}$ Km/h

(ii) New Speed = 32 Km/h.

distance = 20 Km

New Time $= \frac{D}{S} = \frac{20}{32}\ h = \frac{20}{32} \times 60\ m = 37.5\ m$ ✓

Previous time = 23 m

$\therefore$ Increase in time $= 37.5 - 23$
$= 14.5$ min.

Percentage increase time $= \frac{14.5}{23} \times 100 = 63$

$= 63\%$ Increase