IG-Maths
0580

Probability Notes

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IG-Maths 0580

Sample space and Event

Probability (or Chance) P-1

§ **Sample Space:**

When we perform an experiment, the set of all possible out comes (elementary event) is called sample space, denoted by 'S'. and the number of outcomes is denoted by $n(S)$.

Example:

[1] (i) A coin is tossed,

$S = \{H, T\}$; $n(S) = 2$

(ii) Two coins are are tossed simultaneously: (or a coins is tossed two times). $S = \{HH, HT, TH, TT\}$

$n(S) = 4$

[2] (i) A cubical die (with six faces, marked 1, 2, -- 6} is rolled;

$S = \{1, 2, 3, 4, 5, 6\}$

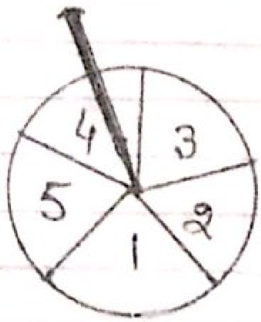
$n(S) = 6$

(ii) A pair of cubical dice are rolled:

$S = \{(1,1), (1,2) (1,3), (1,4), (1,5), (1,6)$
$(2,1) - - - - (2,6)$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad (6,6)\}$

$n(S) = 6 \times 6 = 36.$

[3] (i) A spinner have five sections numbered 1, 2, 3, 4, 5 is spun once.



$S = \{1, 2, 3, 4, 5\}$, $n(S) = 5$

(ii) If two such spinners are spun once,

$S = \{(1,1), (1,2), (1,3), (1,4), (1,5)$
$(2,1) - - - -$
$- - - - - - (5,5)\}$ $n(S) = 25$

[4] A bag contains 4 red and 3 white balls, and a ball is drawn at random, $S = \{r_1, r_2, r_3, r_4, w_1, w_2, w_3\}$

$n(S) = 7$

Probability (or Chance)

§ **Event:** It is a statement 'E' associated with a random experiment, so that one or more (may be none) of the outcomes of the experiment in its sample space makes it true.

Example 1: A pair of cubical dice are rolled,

$$S = \{(1,1), (1,2), (1,3) - - - (1,6) \\ - - - - - (6,6)\} \quad n(S) = 36$$

(i) Sum of the numbers on two dice is nine.

event → $E_1 = \{(3,6), (4,5), (5,4), (6,3)\}$

1, 2, 3, 4, 5, 6
6 5 4 3

$n(E_1) = 4$

(ii) Same number on both the dice (doublet).

event → $E_2 = \{(1,1)(2,2),(3,3),(4,4),(5,5),(6,6)\}$

$n(E_2) = 6$

Example 2: Two coins are tossed:

$S = \{HH, HT, TH, TT\}$

(i) Event - getting exact one head.

$E_1 = \{HT, TH\}$

(ii) getting atleast one tail:

$E_2 = \{HT, TH, TT\}$.

§ **Equally Likely:** The outcomes of a random experiment are said to be equally likely if the chances of appearing each outcome (elementary event) is same.

Example; (i) Tossing unbaised coin

(ii) Fair dice

(iii) Fair spinner

(iv) drawing balls from a bag with replacement.

Note: If the balls are drawn without replacement (at random) then the probability changes.

IG-Maths 0580

§ Probability :

If a ramdom experiment is performed, such that its sample space has 'n' equally likely outcomes (elementary events), and 'm' of them are favourable to an event 'E' associated with it, then probability of happening the event 'E' denoted by P(E) is:

$$P(E) = \frac{\text{No of favourable outcomes}}{\text{Total number of out comes}}$$

may be denoted by 'p' in short.

Note: $0 \leq p \leq 1$

(i) The prob. of an impossible event = 0

(ii) The prob of a sure event = 1

Expectation: Given the prob. of happening of an event = p and the number of times 'N' the experiment is repeated, such that the prob. remains same in each trial. Then the expection of the event = Np

Example 3 : $\boxed{S}$ $\boxed{P}$ $\boxed{A}$ $\boxed{C}$ $\boxed{E}$ $\boxed{S}$

One of the six letters in taken at random.

(a) Write down the prob, the letter is S. --- [1]

(b) The letter is replaced and again a letter is taken at random, This is repeated 600 times. [W-13/21/Q6]

How many times would you expect the letter to be S --- [1]

Solution: (a) Total number of letter $n = 6$

Number of times letter 'S' comes $m = 2$

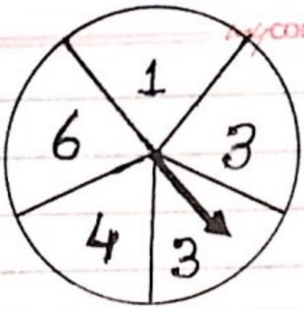
$\therefore P(S) = \frac{m}{n} = 2/6 = \frac{1}{3}$ ✓ $= p$ (let)

(b) As the letter is replaced, in every successive trial, the prob 'p' of drawing 'S' remains same. Given $N = 600$

$\therefore$ Expectation: Number of times we get 'S' $= Np = 600 \times \frac{1}{3} = 200$ ✓

Probability

Example 4: The diagram shows a fair spinner.
Anna spins it twice and add the scores.



(a) Complete the table for the total scores. ---------- [1]
(b) Write down the most likely score. --- [1]
(c) Find the probability that Anna scores.
(i) a total less than 6. ----- [2]
(ii) a total of 3. ---- [1]

S-17/21/Q20

		Score on first spin				
		1	3	3	4	6
Score on second spin	1	2	4	4	5	7
	3	4	6	6	7	9
	3	4	6	6	7	9
	4	-	-	-	-	-
	6	-	-	-	-	-

Solution:

(a)

		Score on first spin				
		1	3	3	4	6
Score on Second spin	1	2	4	4	5	7✓
	3	4	6	6	7✓	9
	3	4	6	6	7✓	9
	4	5	7✓	7✓	8	10
	6	7	9	9	10	12

(b) most likely score = 7✓ (repeats 5 times)

(c) (i) Total less than 6; $A = \{2, 4, 4, 4, 4, 5, 5\}$, $n(A) = 7$

Total No $= 5 \times 5 = 25$

$\therefore P(\text{Total less than } 6) = \frac{7}{25} = 0.28$✓

c (ii) Total Score 3, $B = \{\}$

$\because$ 1, 3, 3, 4, 6 on first spin and 1, 3, 3, 4, 6 on sec. spin
No pair makes total 3.

$\therefore P(\text{Total } 3) = \frac{0}{25} = 0$✓

§ $P(E) + P(\text{not } E) = 1$ or $P(\text{not } E) = 1 - P(E)$

Example 5: The probability that Stephanie wins her next tennis match is 0.85. Find the prob. that Stephanie does not win her next tennis match. --- [1] S-17/22/Q2

Solution: $P(\text{Stephanie does not win}) = 1 - (\text{Stephanie wins})$
$= 1 - 0.85 = 0.15$✓

Probability

§ Outcomes of a random experiment: $x_1, x_2, x_3 -- x_n$
and their corresponding Prob: $p_1, p_2, p_3 -- p_n$
(outcomes are not equally likely)

Then. $\sum p_i = p_1 + p_2 + -- p_n = 1$

Example 6: A biased 4-sided dice is rolled. The possible score are 1, 2, 3 or 4. The prob. of rolling 1, 3 or 4 are shown in the table: Complete the table. ---[2]

Score	1	2	3	4
Probability	0.15	—	0.3	0.35

S-15/22/Q5

Solution: Let the missing prob. $P(2) = p$.

Then $0.15 + p + 0.3 + 0.35 = 1$
or $p = 0.2$ ✓

§ Mutually Exclusive Events:

Two (or more) event A and B are said be mutually exclusive if there are no common outcomes.
(or A and B can not occur simutaneouly) (or $A \cap B = \phi$)

§ Addition law of Probability:
Given two (or more) mutually exclusive events:

$$P(A \text{ or } B) = P(A) + P(B).$$

§ Independent events: Two events A and B are said to be independent if the prob of happening (or non-happening) of A is not affected by the prob. of happening (or non-happening) of B and conversely.

§ Multiplication law of Prob:
Given two independent event A and B:

$$P(A \text{ and } B) = P(A) \times P(B)$$

Probability

Addition Law of Prob.

Example 7: A pair of fair dice are rolled, find the prob. of getting a total of 7 or a doublet (same number on both the dice).

Solution: A pair of dice is thrown:

Sample space $S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)$
$(2,1), (2,2) - - - - - - - (2,6)$
$(3,1), (3,2) - - - -$
$n(S) = 36$. (equally likely ordered pairs) $- - (6,6)\}$

Getting Total of 7, event $A = \{(1,6), (2,5), (3,6), (4,3), (5,2), (6,1)\}$ --- ①

first dice 1, 2, 3, 4, 5, 6
Sec. dice 6, 5, 4, 3, 2, 1

$n(A) = 6 \Rightarrow P(A) = \frac{6}{36}$

Also same number on both dice (doublet) $B = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$ --- ②

$n(B) = 6 \Rightarrow P(B) = 6/36$

Events A and B are mutually exclusive as we observe from ① & ② there is no common outcomes.

$\therefore P(A \text{ or } B) = P(A) + P(B)$

$= \frac{6}{36} + \frac{6}{36}$

$= \frac{12}{36}$

$\therefore P(\text{Total } 7 \text{ or doublet}) = \frac{1}{3}$ ✓

Multiplication Laws of Prob:

Example 8: A fair cubical die and an unbiased coin are thrown together, find the prob of getting an even prime number on the die and head on the coin.

Solution: Sample space of die $S_1 = \{1, 2, 3, 4, 5, 6\}$; $n(S_1) = 6$

Event getting an even prime number. $A = \{2\}$; $n(A) = 1$

$\therefore P(A) = P\{\text{getting an even prime number}\} = \frac{1}{6}$ --- (1)

for Coin; Sample Space $S_2 = \{H, T\}$; $n(S_2) = 2$

Event: Head on top $B = \{H\}$ $n(B) = 1$

$\therefore P(B) = P(\text{head}) = \frac{1}{2}$ ——— (2)

Now Prob getting 2 on the die does not affect the Prob of getting head on the coin.

$\therefore$ Events A and B are independent.

$\therefore P(A \text{ and } B) = P(A) \times P(B)$

$= \frac{1}{6} \times \frac{1}{2}$ from (1) & (2)

$= \frac{1}{12}$ ✓

Example 9:

A bag contain 4 red and 3 white balls. Two balls are drawn from the bag (or drawn successively without replacement) Find the prob. of getting both red balls.

Solution: Total number of balls $= 4 + 3 = 7$

number of red balls $= 4$

'A' denotes getting first red ball, $P(A) = \frac{4}{7}$ . . . (1)

Now as the ball drawn is not replaced. Now { total balls left $= 6$; red balls left $= 3$

B denoted getting second red ball. $P(B) = 3/6$

(Here A and B are independent events)

$\therefore P(\text{both red}) = P(A) \times P(B) = \frac{4}{7} \times \frac{3}{6} = \frac{12}{42} = \frac{2}{7}$ ✓ $= \frac{2}{7}$ ✓

IG - Maths
0580

Probability

Date P-8
Page No.

Drawn at random (without replacement)

Example 10: Samisa and Sonia each have a bag containing 20 sweets. In each bag there are 5 red, 6 green and 9 yellow sweets.

(a) Samisa chooses one sweet at random from her bag, write down the prob. that she chooses a yellow sweet. --- [1]

(b) Sonia chooses two sweet at random, without replacement, from her bag.

(i) Show that prob. that she chooses two green sweets is $\frac{3}{38}$. --- [2]

(ii) Calculate the prob. that the sweets she chooses are not both the same colour. --- [4] M-18/22/Q22

Solution: (a) Total number of balls $= 5+6+9 = 20$

No. of yellow sweets $= 9$

$\therefore$ P(one yellow sweet) $= \frac{9}{20}$ ✓

(b) (i) P(Ist green) $= \frac{6}{20}$ ---- ① [No. of green = 6]

Now when one green is drawn then { Total = 19 left, No green left = 5

$\therefore$ P(IInd green) $= \frac{5}{19}$ — ②

fr ① & ② P(Two green) = P(Ist green) × P(second green)

$= \frac{6}{20} \times \frac{5}{19} = \frac{3}{38}$ ✓

(ii)

P(Not both same colour) $= 1 -$ (both of same color) —— ③

fr b(i) P(both green) $= \frac{6}{20} \times \frac{5}{19} = \frac{30}{380}$ —— ④

Similarly P(both red) $= \frac{5}{20} \times \frac{4}{19} = \frac{20}{380}$ —— ⑤

and P(both yellow) $= \frac{9}{20} \times \frac{8}{19} = \frac{72}{380}$ —— ⑥

$\therefore$ P(both same colors) $= \frac{30}{380} + \frac{20}{380} + \frac{72}{380} = \frac{122}{380}$ —— ⑦

$\therefore$ fr ③ & ⑦ P(Not both same colour) $= 1 - \frac{122}{380} = \frac{258}{380}$ ✓

Probability

Prob. using tree diagrams

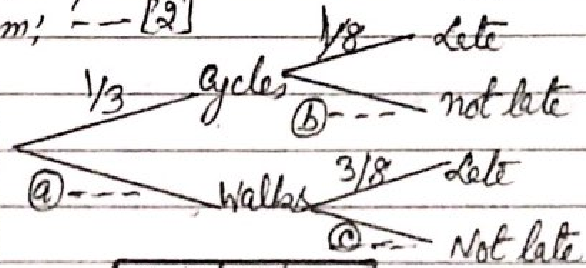
Using : Addition and Multiplication Laws of Prob.

Example 11: Dan either walks or cycles to school. The prob. that he cycles to school is $\frac{1}{3}$.

(a) Write down the prob. that Dan walks to school. --[1]

(b) When Dan cycles to school, the prob. that he is late is $1/8$.
When Dan walks to school, the prob. that he is late is $3/8$.
Complete the tree diagram: --[2]

(c) Calculate the prob.
(i) Dan cycles to school and is late --[2]
(ii) Dan is not late. ---[3]



M-16/22/Q21

Solution

(a) Walks and cycles are mutually exclusive and exhaustive events. $\therefore$ $P(\text{Walking}) + P(\text{Cycling}) = 1$

$$P(\text{Walking}) + \frac{1}{3} = 1$$

$$\therefore P(\text{Walking}) = 1 - \frac{1}{3} = \frac{2}{3} \checkmark$$

(b) On tree diagram, (a) --- $= 1 - \frac{1}{3} = 2/3 \checkmark$ from Part (a)

(b) -- $= 1 - \frac{1}{8} = \frac{7}{8} \checkmark$

(c) -- $= 1 - 3/8 = \frac{5}{8} \checkmark$

(c) (i) $P(\text{Dan cycles to School and late}) = \frac{1}{3} \times \frac{1}{8} = \frac{1}{24} \checkmark$

(ii)

Dan is not late $<$ Cycles and not late or Walkes and not late

$$\therefore P(\text{Dan not late}) = \frac{1}{3} \times \frac{7}{8} + \frac{2}{3} \times 5/8 = \frac{7+10}{24} = \frac{17}{24} \checkmark$$

IG-Maths 0580

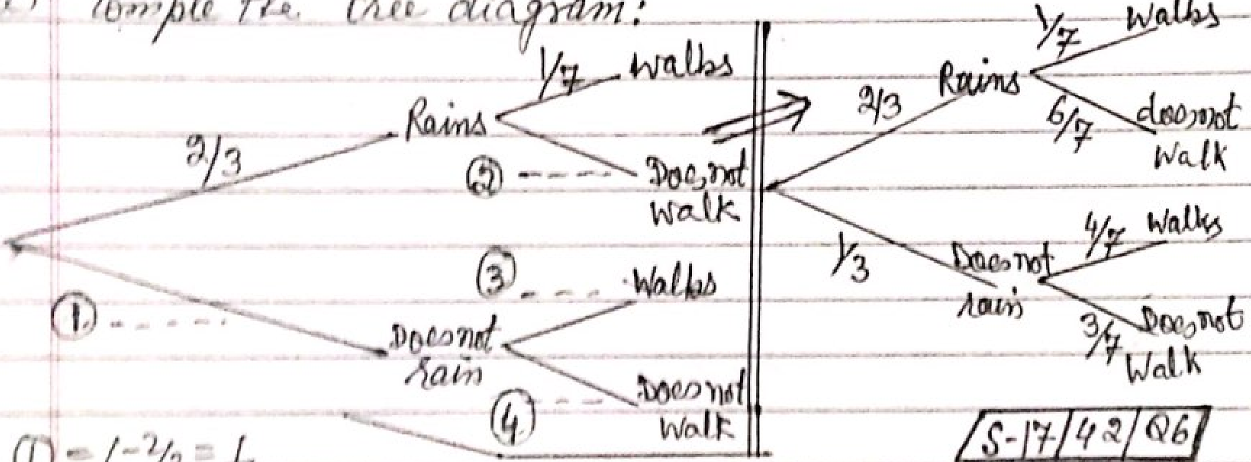
Probability

Date: P-10

Tree diagram / addition and multiplication Rule and "Prob. of exactly one success in n trials" ⊗

Example 12: Each morning the prob. that it rains is $2/3$.
If it rains, the prob. that Asha walk to school is $1/7$.
If it does not rain, the prob that Asha walks to school is $4/7$.

(a) Complete the tree diagram:



S-17/42/Q6 ---- [2]

(1) $= 1 - 2/3 = \frac{1}{3}$

(2) $= 1 - \frac{1}{7} = \frac{6}{7}$

(3) $= \frac{4}{7}$ (given) (4) $= 1 - \frac{4}{7} = \frac{3}{7}$

(b) Find the prob. that it rains and Asha walks to school. --- [2]

Solution P(rain & Asha walks) = P(rains) × P(walks when rains)

$= 2/3 \times \frac{1}{7} = \frac{2}{21}$ ✓

(c) (i) Find the prob. that Asha does not walk to school. --- [3]

Solution:

P(Asha does not walk to school)

= P(rains & does not walk) + P(does not rain and does not walk)

$= \frac{2}{3} \times \frac{6}{7} + \frac{1}{3} \times \frac{3}{7} = \frac{15}{21}$ ✓

(c)(ii) Find the expected number of days Asha does walk to school in a term of 70 days. --- [2]

Solution: Expectation = No. of Trials × Prob.(does not walk)

$= 70 \times \frac{15}{21} = 50$ ✓

(d) Find the prob. that it rains on exactly one morning in a school week of 5 days. ⊗ --- [2]

Solution R × N × N × N × N
or N × R × N × N × N
or N × N × R × N × N
or N × N × N × R × N
or N × N × N × N × R

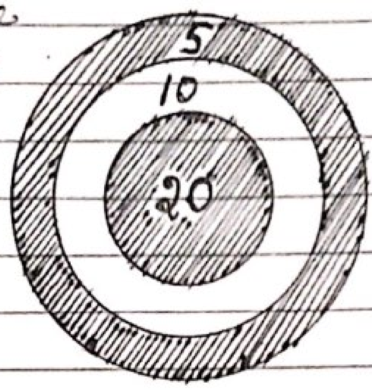
P(exactly one day rain)

$= 5 \times P(\text{Rain})\,[P(\text{does not rain})]^4$

$= 5 \times (2/3) \times (\frac{1}{3})^4 = \frac{10}{243}$ ✓

Note ⊗ If n trails of an experiment are repeated, such that the prob. of success of an event is p and failure is q in a trial. Then the $P(\text{exact. one success}) = n \times p^1 \times (q)^{n-1}$ ✓

Example 13: Kiah plays a game. The game involves throwing a coin onto a circular board. Points are scored for where the coin lands on the board.



If the coin lands on a part of a line or misses the board then '0' points are scored.

The table shows the probabilities of Kiah scoring points on the board with one throw.

Points Scored	20	10	5	0
Probability	x	0·2	0·3	0·45

S-16/42/Q5

(a) Find the value of x. --- [2]

Solution: $\Sigma p_i = 1$

$x + 0\cdot2 + 0\cdot3 + 0\cdot45 = 1$

$\therefore x = 0\cdot05$ ✓

(b) Kiah throws a coin fifty times. Work out the expected number of times she scores 5 points. --- [1]

Solution:

Expectation $= N \times p$

$= 50 \times P(5)$

$= 50 \times 0\cdot3 = 15$ ✓

(c) Kiah throws a coin two times. Calculate the prob that:

(i) She scores 5 or 0 in her first throw. --- [2]

Solution: $P(5 \text{ or } 0) = P(5) + P(0)$

$= 0\cdot3 + 0\cdot45 = 0\cdot75$ ✓

(ii) She scores 0 with her first throw and 5 with her second throw. --- [2]

Solution: P(0 first and 5 in second)

$= P(0) \times P(5) = 0\cdot45 \times 0\cdot3 = 0\cdot135$ ✓

(c)(iii) She scores a total of 15 points with her two throws. --- [3]

Solution:

P(15 points in two throws)

$= P(5) \times P(10) + P(10) \times P(5)$

$= 0\cdot3 \times 0\cdot2 + 0\cdot2 \times 0\cdot3$

$= 0\cdot12$ ✓

(d) Kiah throws a coin three times. Calculate the prob that she scores a total of 10 points with her three throws. --- [5]

Solution:

Case I 5, 5, 0

$P(5,5,0) = 3 \cdot P(5) \cdot P(5) \times P(0)$

$= 3 \times 0\cdot3 \times 0\cdot3 \times 0\cdot45$

$= 0\cdot1215$ —— (1)

Case II 10, 0, 0

$P(10,0,0) = 3 \times P(10) \times P(0) \times P(0)$

$= 3 \times 0\cdot2 \times 0\cdot45 \times 0\cdot45$

$= 0\cdot1215$ —— (2)

$\therefore P(\text{Total } 10) = 0\cdot1215 + 0\cdot1215$

$= 0\cdot243$ ✓

IG-Maths 0580

Probability

Drawing balls/beads from a bag with replacement / without replacement.

Example 14: A bag contains red beads and green beads. There are 80 beads altogether. The probability that a bead chosen at random is green is 0·35.

(i) Find the number of red beads in the bag. --- [2]

W-17/41/Q9

Solution: Let the number of red beads = r

Now the $P(\text{Green bead}) = 0.35$

$\therefore P(\text{red bead}) = 1 - 0.35 = 0.65$ —— ①

Also. $P(\text{red bead}) = \frac{r}{80} = 0.65$ (Given fr ①)

$\Rightarrow r = 0.65 \times 80 = \underline{52}$ ✓

(ii) Marcos choses a bead at random and replaces it in the bag. He does this 240 times.

Find the number of times he would expect to choose a green bead. --- [1]

Solution $P(\text{green bead}) = p = 0.35$

The prob. in each successive draw will be same as with replacement.

No. of time $N = 240$.

Expected Number $= N \cdot p$

$\therefore$ Expected Green beads $= 240 \times 0.35$

$= \underline{84}$ ✓

(b) Blue ; Yellow ; White
2 ; 3 ; 4

Total No = 9

(i) $P(1\text{ yellow}) = \frac{3}{9} = \frac{1}{3}$ --- (2)

as the beads are drawn with rep. prob in each trial will be same.

$\therefore P(3\text{ yellow}) = \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$ ✓

--- [3]

(ii) P(all three diff. colour)

$= 6 \cdot P(\text{Blue}) \times P(\text{Yellow})\, P(\text{wh.})$

$= 6 \times \frac{2}{9} \times \frac{3}{9} \times \frac{4}{9} = \frac{144}{729}$ ✓

(c) Another bag contains 2 green counters and 3 pink counters.

Teresa chooses three counter at random without replacement. --- [4]

Find the Prob. that she chooses more pink counters than green counters.

Solution Green – Pink | Total
2 – 3 | 5

Three drawn at random.

Case I all 3 Pink

P(3 Pink without rep)

$= \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} = \frac{6}{60}$ —— ②

Case II 2 Pink & 1 Green.

P(2 Pink & 1 Green without rep)

P × P × G

$= 3 \times \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} = \frac{36}{60}$ —— ③

fr ① & ②

P(more Pink than green) in Three beads

$= \frac{6}{60} + \frac{36}{60}$ fr ② & ③

$= \frac{42}{60}$ ✓

IG-Maths 0580

Probability

* **Prob. of atleast one success = 1 − P(no success)**

Example 15: Yeung and Ariven complete in a triathlon race. The prob. that Yeung finishes this race is $3/5$. The prob. that Ariven finishes this race is $2/3$. [W-14/43/Q4]

(a)(i) which of them is more likely to finish this race? Give a reason for your answer? --[1]

Solution: $P(\text{Yeung finishes}) = \frac{3}{5} = \frac{9}{15}$; $P(\text{Ariven finishes}) = \frac{2}{3} = \frac{10}{15}$

∴ Ariven is more likely to finish the race ($\because \frac{10}{15} > \frac{9}{15}$)

(a)(ii) Find the prob. that they both finish the race. --- [2]

$P(\text{both finish}) = P(\text{Yeung finish}) \times P(\text{Ariven finish the race})$

$= \frac{3}{5} \times \frac{2}{3} = \frac{6}{15}$ ✓

(iii) Find the Prob. that only one of them finishes the race. --- [3]

P(Only one of them finish)

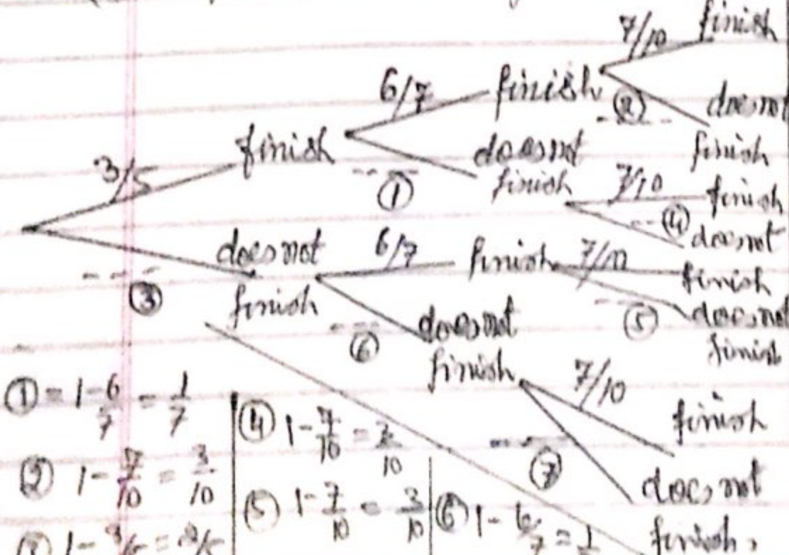
$= P(\text{Yeung finish}) \times P(\text{Ariven does not finish}) + P(\text{Yeung does not}) \times P(\text{Ariven does})$

$= \frac{3}{5} \times \left(1 - \frac{2}{3}\right) + \left(1 - \frac{3}{5}\right) \times 2/3$

$= \frac{3}{5} \times \frac{1}{3} + \frac{2}{5} \times \frac{2}{3} = \frac{7}{15}$ ✓

(b) After the first race, Yeung complete in two further triathlon races.

(i) Complete the tree diagram: -- [3]



① $= 1 - \frac{6}{7} = \frac{1}{7}$

② $1 - \frac{7}{10} = \frac{3}{10}$

③ $1 - 3/5 = 2/5$

④ $1 - \frac{7}{10} = \frac{3}{10}$

⑤ $1 - \frac{7}{10} = \frac{3}{10}$

⑥ $1 - \frac{6}{7} = \frac{1}{7}$

⑦ $1 - \frac{7}{10} = \frac{3}{10}$

(b)(ii) P(Yeung finish all three race)

$= P(\text{first}) \times P(\text{sec}) \times P(\text{third finish})$

$= \frac{3}{5} \times \frac{6}{7} \times \frac{7}{10} = \frac{126}{350}$ ✓

(iii) * P(Yeung finishes at least one of his races)

$= 1 - P(\text{Yeung does not finish any one of three})$

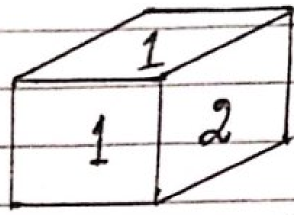
$= 1 - P(\text{not first}) \times P(\text{not sec}) \times P(\text{not third})$

$= 1 - \frac{2}{5} \times \frac{1}{7} \times \frac{3}{10}$

$= 1 - \frac{6}{350}$

$= \frac{344}{350}$ ✓

Example 16: The diagram a fair dice.
The numbers on the dice are 1,1,2,2,2,3.
When the dice is rolled, the score is the number on the top face.
Eva keeps rolling the dice until 1 is scored. Find the prob. this happens on the 5_{th} roll. ---[2]



W-17/42/Q7(c)

Solution: Sample space $S = \{1, 1, 2, 2, 2, 3\}$

$$P(\text{getting } 1) = \frac{2}{6} = \frac{1}{3}$$

$$P(\text{not } 1) = 1 - \frac{1}{3} = \frac{2}{3}$$

$$P(\text{scoring } 1 \text{ on } 5_{th} \text{ roll}) = P(\text{not } 1) \cdot P(\text{not } 1) \cdot P(\text{not } 1) \cdot P(\text{not } 1) \times P(1)$$

$$= \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{16}{243} \checkmark$$

Example 17: 200 students estimate the total area, $A\,m^2$, of the windows in the classroom. The table shows their results:

Area ($A\,m^2$)	$20 < A \leq 60$	$60 < A \leq 100$	$100 < A \leq 150$	$150 < A \leq 250$
Frequency	32	64	80	24

Two students are are chosen at random from those students that estimated the area of the windows to be more than $100\,m^2$. ---[3]
Find the Prob. that one of the two students estimates the area to be greater than $150\,m^2$ and the other student estimates the area to be $150\,m^2$ or less.

SP-2020/04/Q2(b)(iii)

Solution: Number of students who estimate the area more than $100\,m^2 = 124$
Number of students who estimate the area greater than $150\,m^2 = 24$
and Number of students who estimate the area to be $150\,m^2$ or less $= 80$ [but greater than $100\,m^2$]
Now two students are chosen at random (without replacement).
$\therefore$ P(one estimates more than $150\,m^2$ and other less than or equal to $150\,m^2$)

(order may be AB or BA) $= \frac{24}{124} \times \frac{80}{123} + \frac{80}{124} \times \frac{24}{123} = \frac{3840}{10712} = \frac{480}{1339} \checkmark$

Probability

Conditional Prob. (Using tree diagram)

[Syllabus-for 2020-2022 E-8.6/ Page 33]

§ Conditional Probability:

If A and B are two events, then the conditional prob. of happening of B, given that A has happen, is

$$P(B/A) = \frac{P(A \text{ and } B)}{P(A)} \text{ or } \frac{P(A \cap B)}{P(A)} \quad \text{—— (i)}$$

§ Multiplication Law of Prob:

$$P(A \text{ and } B) = P(A) \cdot P(B/A) \quad \text{from (i)}$$

§ Mutually Exhaustive Event:

Given events A, B, C, --- associated with a random expriment with sample space S,
Then two or more such events are said be mutually exhaustive if:

$$A \cup B \cup C \cup \text{---} = S.$$

§(i) Law of Total Probability:

Give two or more mutually exclusive and mutually exhaustive events, A, B, C, --- and 'E' is an event associated with them, then

$$P(E) = P(A) \cdot P(E/A) + P(B) \cdot P(E/B) + P(C) \cdot P(C/A) + \text{---} \quad \text{— (I)}$$

(ii) Bayes' Theorem:

$$P(A/E) = \frac{P(A) \cdot P(E/A)}{P(E)} \quad \text{fr (I)}$$

$$\text{or } P(B/E) = \frac{P(B) \cdot P(E/B)}{P(E)} \quad \text{fr (I)}$$

Conditional Probability

Example 18: Two dice are rolled. Syllabus-2020-22 / E 8.6 Example

Given that the total showing on the two dice is 7, Find the prob. that one of the dice shows the number 2.

Solution: Two dice are rolled,

Sample space $S = \{(1,1),(1,2),(1,3), ---- (1,6)$
$(2,1),(2,2) ---- (2,6)$
$(3,1) ---- (3,6)$
$---- (6,6)\}$

$n(S) = 36.$

Now let event 'Total 7 is denoted by 'A''

1 2 3 4 5 6
6 5 4 3 2 1

$A = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$

$n(A) = 6$

$P(A) = \frac{6}{36}$

Now let 'B' denote the event:
"One of dice shows number 2"

$B = \{(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)$
$(1,2), \cancel{(2,2)}, (3,2),(4,2),(5,2),(6,2)\}$

$A \cap B = \{(2,5),(5,2)\}$

$P(A \cap B) = 2/36$

$\therefore P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{2/36}{6/36} = \frac{2}{6} = \frac{1}{3}$ ✓

we can directly do it

$\frac{6}{36}$ Total 7 $\quad 2/6 \quad$ One dice show 2

$\therefore P(B/A) = P(\text{one dice show 2} / \text{Total 7}) = 2/6$ ✓

Conditional Prob. (Using Tree diagram).

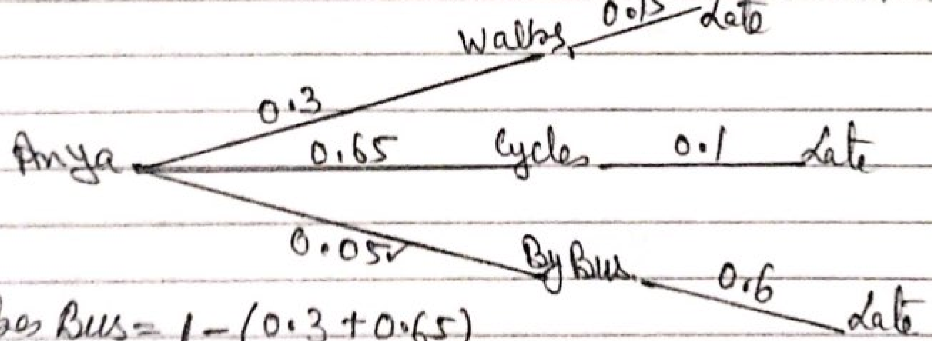
Example 19: When Anya goes to school, the probability that she walks is 0.3 and the prob. that she cycles is 0.65. If she does not walk or cycle, she takes the bus. When Anya walks the prob. that she is late is 0.15. When she cycles the prob. that is late is 0.1 and when she takes bus, the prob. that she is late is 0.6.

(i) Find the prob Anya reaches to school late. ---[3]

(ii) Given that Anya is late, find the prob. she cycles. ---[2]

[W-16/61/Q6]

Solution:



Prob. Anya takes Bus $= 1-(0.3+0.65)$
$= 0.05$

(i) $P(\text{Anya reaches late 'L'}) = P(W)\cdot P(L/W) + P(C)\cdot P(L/C) + P(B)\cdot P(L/B)$

$= 0.3\times0.15 + 0.65\times0.1 + 0.05\times0.6$

$\therefore P(L) = 0.14$

(ii) Now $P(C/L) = \dfrac{P(C\cap L)}{P(L)} = \dfrac{P(C)\cdot P(L/C)}{P(L)}$

$= \dfrac{0.65\times0.1}{0.14} = \underline{0.464}$

§ **Independent Events:**

Two events A and are said to be independent, iff. $P(A\cap B) = P(A)\cdot P(B)$

or
$P(B/A) = P(B)$
$P(A/B) = P(A)$

(Using Tree diagram)

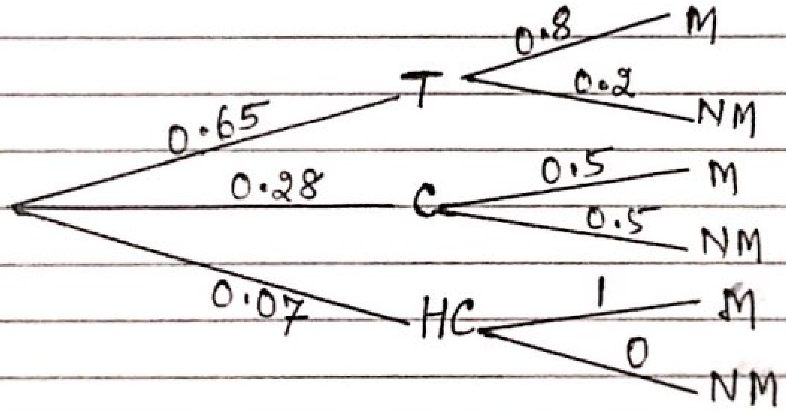
Example 20: Ayman's breakfast drink is tea, coffee or hotchocholate, with probabilities 0.65, 0.28, 0.07 respectively. When he drinks tea, the prob. that he has milk in it is 0.8. When he drinks coffee, the prob. that he has milk in it is 0.5, when he drinks hot-chocolate, he always has milk in it.

(i) Draw a fully labelled tree diagram to represent the information. ---[2]

(ii) Find the prob. that Ayman's breakfast drink is coffee, given that his dri has milk in it. --[3]

S-16/62/Q1

Solution: (i)



(ii) $P(C/\text{Milk}) = \dfrac{P(C \cap M)}{P(M)}$ — (1)

$P(C \cap M) = \qquad = 0.28 \times 0.5 = 0.140$

$P(M) = P(T) \cdot P(M/T) + P(C) \cdot P(M/C) + P(HC) \cdot P(M/HC)$

$= 0.65 \times 0.8 + 0.28 \times 0.5 + 0.07 \times 1$

$= 0.73$

$\therefore P(C/M) = \dfrac{P(C \cap M)}{P(M)} = \dfrac{0.14}{0.73} = 0.19178$

$= 0.192$