§ Transformation:

With the application of a transformation a **shape** changes its **position** or **size** or both.
The original shape is called an **object** and the new shape is called its **image**.

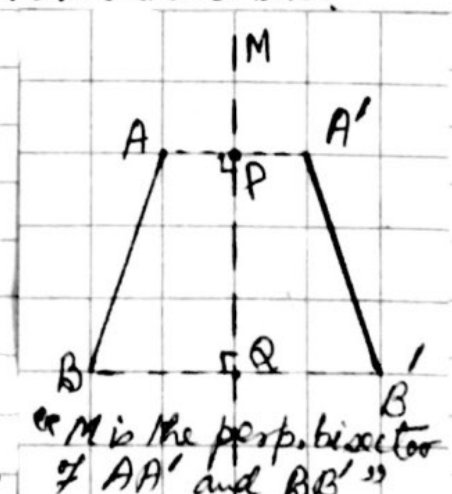
§ Types of Transformations:

1. Reflection
2. Rotation
3. Translation
4. Enlargement.

Note: In the first three transformations, Reflection, Rotation and Translation the only **changes** its **position** and **image** remains congruent the object.

whereas if Enlargement is used then the **size** of the object **changes** but image remains similar to the object, size may be larger or smaller then the object.

M is Mirror line.



"M is the perp. bisector of AA' and BB'"

1. Reflection transformation:

Reflection is the Flip of a shape over the line of reflection, called **mirror line**.
Any point on the object and the corresponding point on the image are equidistant from the mirror line along a perpendicular line.
'M' is the mirror line.
AB is an Object and A'B' is image.

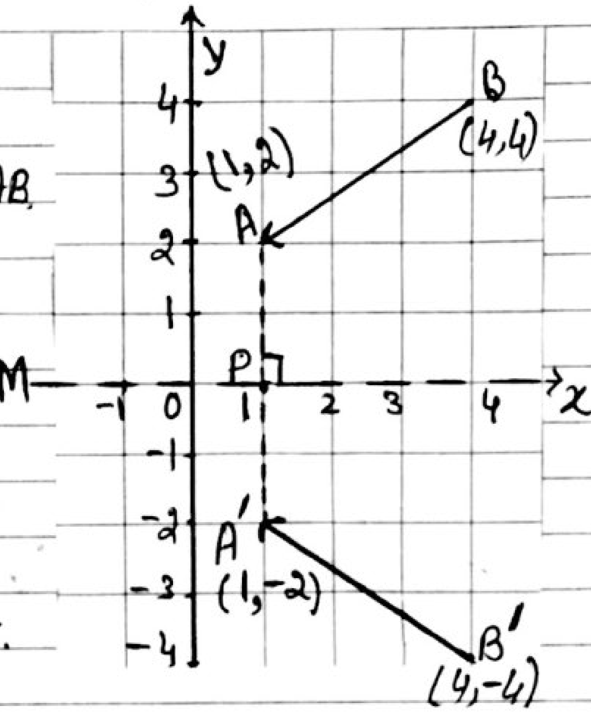
$AA' \perp M$, $BB' \perp M$,
$AP = PA'$ and $BQ = QB'$

IG-Maths 0580

Transformations

Reflection.

§1(a) Mirror line is along X-axis (or $y=0$)



AB is an Object.
X-axis (or $y=0$) is Mirror line
$A'B'$ is the reflection image of AB.

$AA' \perp$ X-axis and $AP = A'P$

Image of $A(1,2)$ is $A'(1,-2)$

In general the position of

$$A(x,y) \xrightarrow{\text{Ref. in X-axis}} A'(x,-y)$$

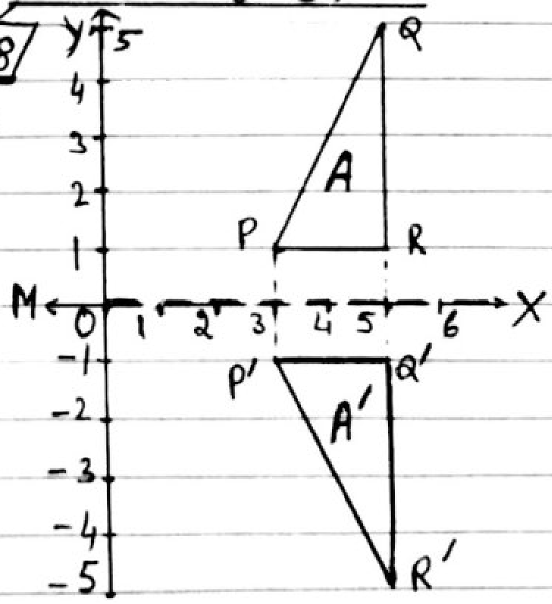
If we write as a column vector.

$$A\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow A'\begin{pmatrix} x \\ -y \end{pmatrix}$$

Note: The Matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ is the reflection in X-axis.

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1\times x + 0 \\ 0\times x + (-1)y \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix}$$

Example 1 Draw the image of triangle A after the transformation represented by $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ ---[3] W-16/22/Q18



Solution. Triangle A → $P(3,1)$, $Q(5,5)$, $R(5,1)$

Image of $P\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} P'$

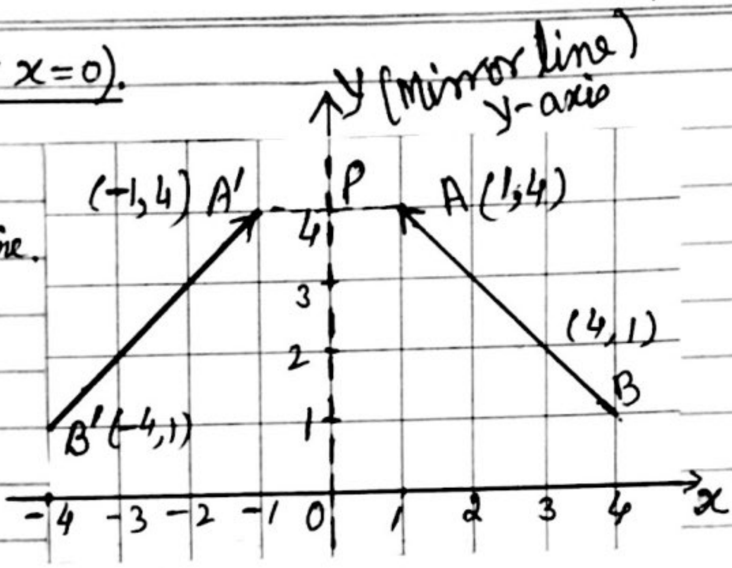
Image of $Q\begin{pmatrix} 5 \\ 5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 5 \\ 5 \end{pmatrix} = \begin{pmatrix} 5 \\ -5 \end{pmatrix} Q'$

Image of $R\begin{pmatrix} 5 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 5 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \end{pmatrix} R'$

Transformations

Reflection

§1(b) Reflection in y-axis (or $x=0$).



AB is an Object.
y-axis (or $x=0$) is the Mirror line.
A′B′ is the image of AB under Reflection in y-axis.
AA′ ⊥ y-axis
P is the mid point of AA′.

Image of A(1,4) is A′(−1,4)

In general the position of

$$A(x,y) \xrightarrow{\text{Reflection in y-axis}} A'(-x,y)$$

If we write the point point A as column matrix.

$$A\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow A'\begin{pmatrix} -x \\ y \end{pmatrix}$$

Note: The matrix $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ represents the reflection in y-axis

i.e. $$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ y \end{pmatrix}$$

Example 2. Describe fully the single transformation represented by $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ [W-17/41/Q11(b)] - - - [2]

Solution: Reflection in y-axis

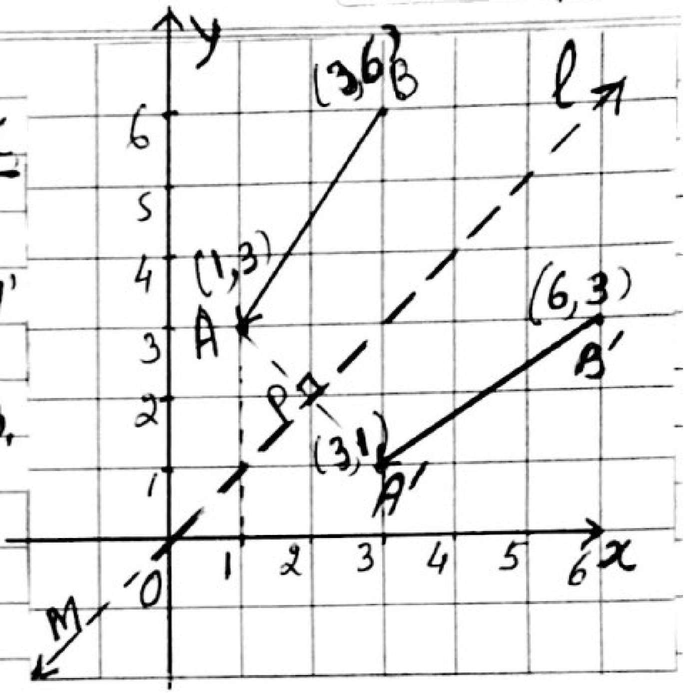
Example 3. Describe fully the single transformation represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ - - - [2] [M-15/42/Q7(d)]

Ans: Reflection in x-axis.

IG-Maths 0580

Transformations

Reflection



§1(c) Reflection in line $y=x$

AB is an object,
$y=x$ is line 'l' is the Mirror 'M'

A'B' is the Reflection image of AB.

Image of $A(1,3)$ is $A'(3,1)$

In general the Reflection of

$A(x,y)$ in line $y=x$ is $A'(y,x)$
we may write as a column matrix

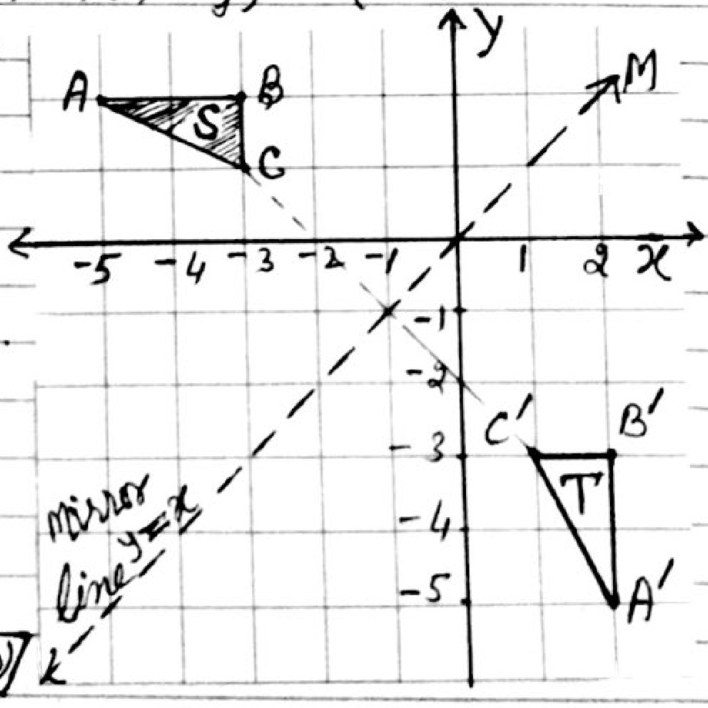
$$A\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow A'\begin{pmatrix} y \\ x \end{pmatrix}$$

Note: The matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ represents the reflection in line $y=x$;

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x \end{pmatrix}$$

Example 4 (i) Draw the reflection of shape S in the line $y=x$ --- [2]

(ii) Write down the matrix that represents the transformation in part (i) -- [2].



Shape S → Shape T

Ans (i)

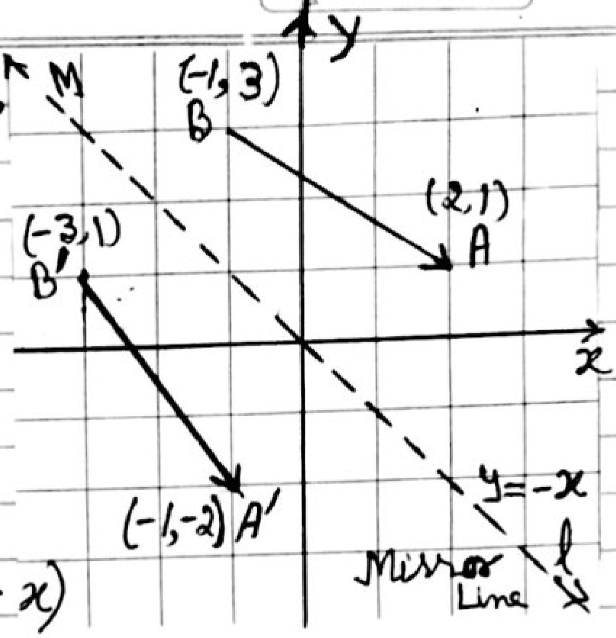
Mirror $y=x$:
$A(-5,2) \longrightarrow A'(2,-5)$
$B(-3,2) \longrightarrow B'(2,-3)$
$C(-3,1) \longrightarrow C'(1,-3)$

(ii) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\boxed{M\text{-}17/42/Q2(b)}$

Transformations

Reflection

§1(d) Reflection in line $y=-x$,



AB is an object,
$y=-x$ is the Mirror line 'l'

$A'B'$ is the Reflection of AB.

Image of $A(-1,3)$ is $A'(-3,1)$

In general the reflection of $A(x,y)$ in line $y=-x$ is $A'(-y,-x)$

If we write A is a column matrix,

$$A\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow A'\begin{pmatrix} -y \\ -x \end{pmatrix}$$

Note: The matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ represents the reflection in $y=-x$.

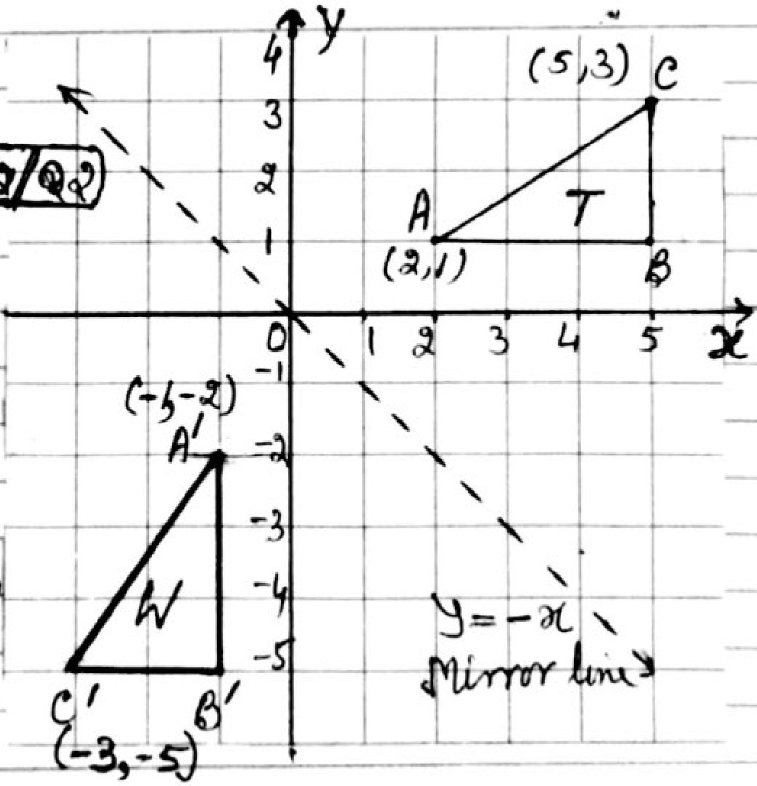
or $$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ -x \end{pmatrix}$$

Example 5.
Describ fully the single transformation represented by the matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ --- [2] $\boxed{\text{S-17} \mid 42 \mid \text{Q2}}$

Ans: Reflection in $y=-x$.

Example 6 (i) Describe fully the single transformation that maps Triangle T onto triangle W. --- [2]
(ii) Find the 2×2 matrix that represents the transformation in part (i) --- [2]

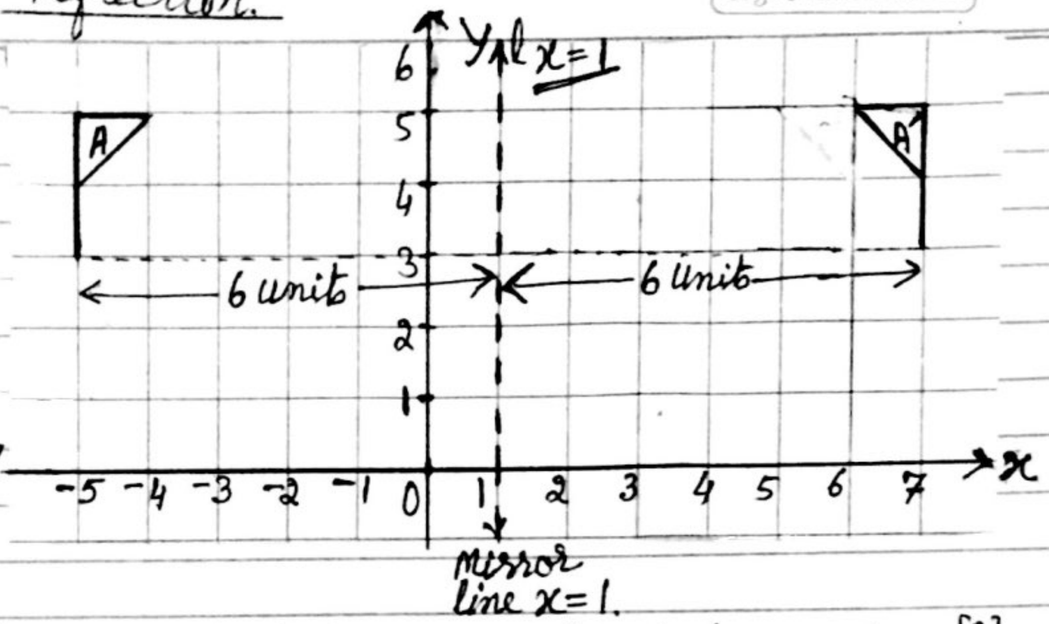
Ans.(i) Reflection in line $y=-x$.
(ii) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$



Transformations

Reflection.

Example 7



M-15/42/Q C,d

(i) Draw the image of Flag A after a reflection in line $x=1$. ---[2]

Answer: Image of Flag A in line $x=1$ 'l'. is A' (Reflection)

(ii) Describ fully the single transformation represented by the ---[2]

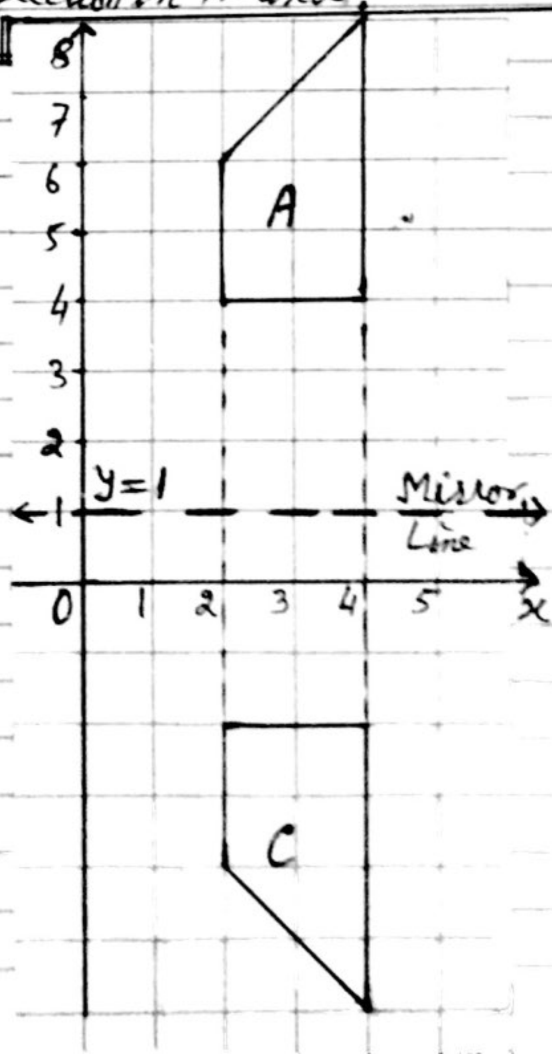
matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ - Answer - Reflection in X-axis.

Example 8. Describe fully the single transformation that maps:

Shape A onto Shape C. ----[2]

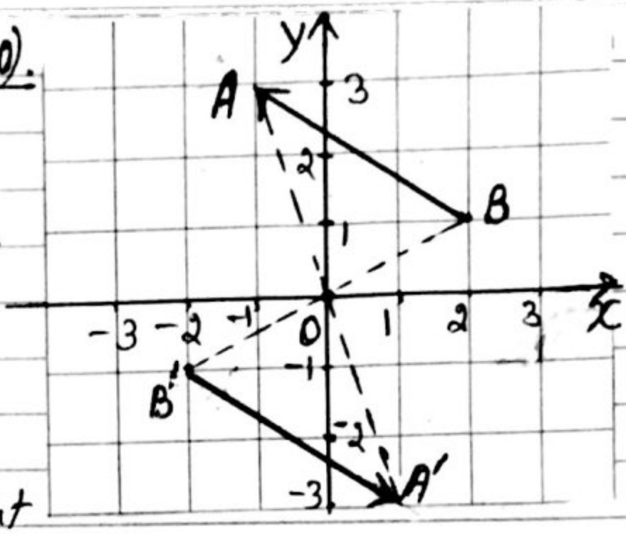
Answer: Reflection in $y=1$.

W-15/42/Q7(a)(ii)



Reflection

5(e). Reflection in Origin (0,0).



AB is an object.

Origin O(0,0) in the point Mirror.

A'B' is the reflection of AB in O.

Image of A (-1,3) is A' (1,-3).

In general the reflection of point A(x, y) in origin O is A'(-x,-y).

If we write A as a column matrix.

$$A\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow A'\begin{pmatrix} -x \\ -y \end{pmatrix}$$

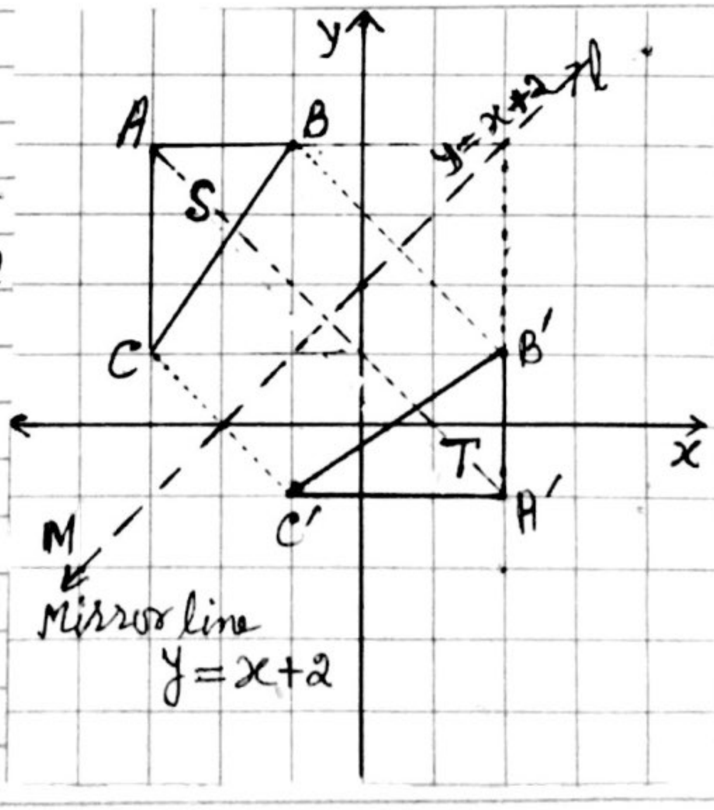
Note: The matrix $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ represent the Reflection in Origin

i.e. $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ -y \end{pmatrix}$

Example 9.

Draw the Reflection of figure 'S' in line $y = x+2$

Answer: T is the reflection of 'S' in line $y = x+2$.



Transformations

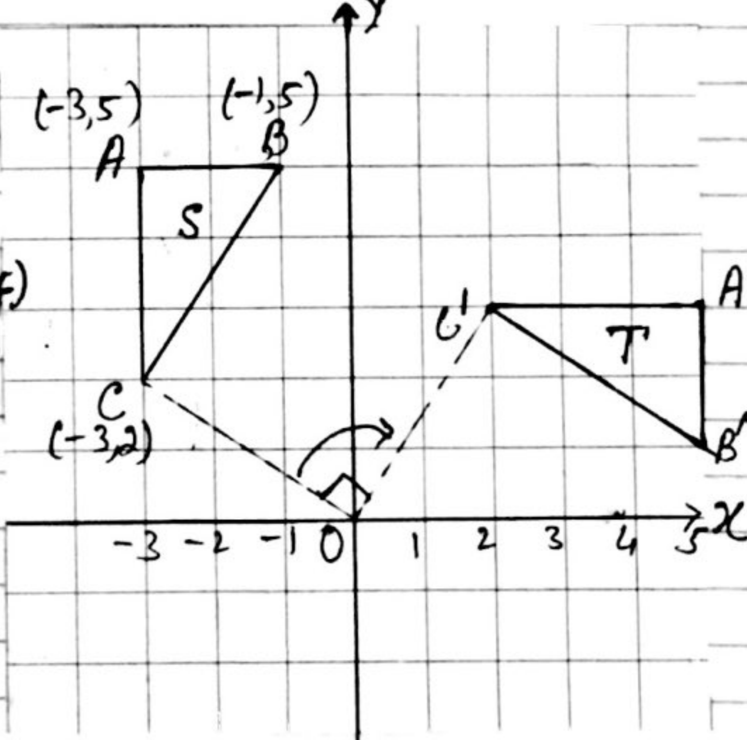
Rotation

§2. A rotation is a transformation in which the object is rotated about a fixed point (centre of rotation) by an angle in a direction (clockwise/anticlockwise).

The three terms involved are:

(i) Centre of rotation
(ii) Angle of rotation.
(iii) Direction of rotation (Clockwise/anticlockwise).

§2(a). Rotation through 90° clockwise, Centre of rotation is origin:



S is object triangle ABC. rotation about origin through 90° clockwise image is T.

In General $P(x,y) \to P'(y,-x)$

or $P\begin{pmatrix} x \\ y \end{pmatrix} \to P'\begin{pmatrix} y \\ -x \end{pmatrix}$

$A\begin{pmatrix} -3 \\ 5 \end{pmatrix} \to A'\begin{pmatrix} 5 \\ +3 \end{pmatrix}$

$B\begin{pmatrix} -1 \\ 5 \end{pmatrix} \to B'\begin{pmatrix} 5 \\ +1 \end{pmatrix}$

$C\begin{pmatrix} -3 \\ 2 \end{pmatrix} \to \begin{pmatrix} 2 \\ +3 \end{pmatrix}$

Note: The Matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ represents rotation about origin by angle 90° in the clockwise direction.

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ -x \end{pmatrix} \checkmark$$

Alternate: Matrix Method:

$$\underset{2\times 2}{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}}\underset{2\times 3}{\overset{\begin{matrix} A & B & C \end{matrix}}{\begin{pmatrix} -3 & -1 & -3 \\ 5 & 5 & 2 \end{pmatrix}}} = \underset{2\times 3}{\overset{\begin{matrix} A' & B' & C' \end{matrix}}{\begin{pmatrix} 5 & 5 & 2 \\ 3 & 1 & 3 \end{pmatrix}}}$$

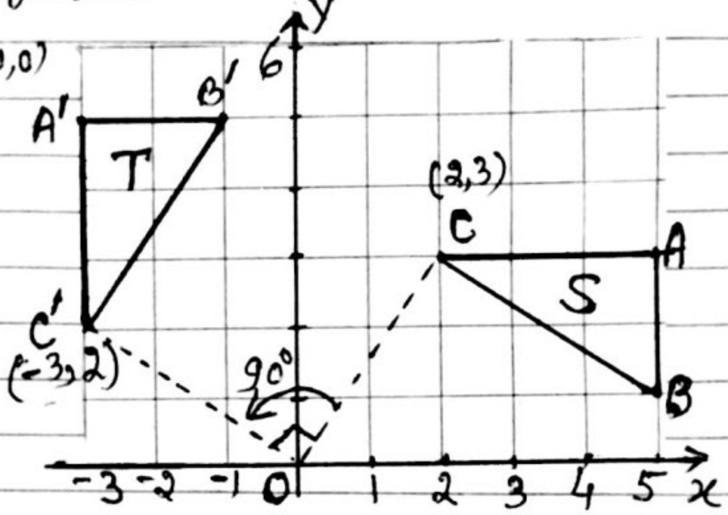
IG-Maths 0580

Transformations

Rotation

§ 2(b) **Rotation through 90° in the anticlockwise direction, Centre of rotation is origin (0,0).**

Object 'S' is triangle ABC.
Centre of rotation is origin (0,0)
'T' Triangle A'B'C' is the image after a rotation through 90° in anticlockwise direction.



In general. $A(x,y) \to A'(-y,x)$

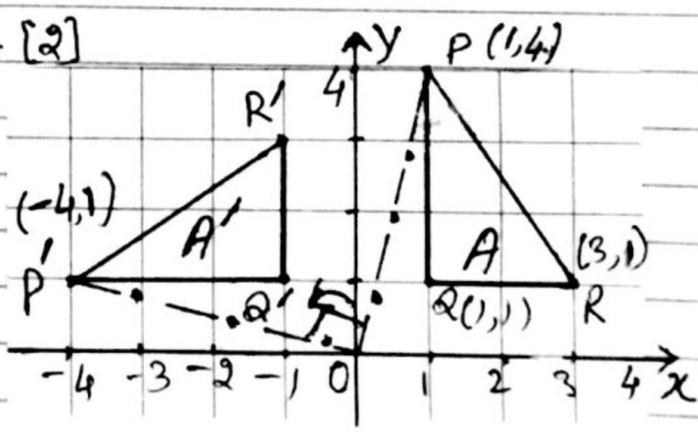
or $A\begin{pmatrix} x \\ y \end{pmatrix} \to A'\begin{pmatrix} -y \\ x \end{pmatrix}$

$C\begin{pmatrix} 2 \\ 3 \end{pmatrix} \longrightarrow C'\begin{pmatrix} -3 \\ 2 \end{pmatrix}$

Note: The matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ represents rotion about origin in the anticlockwise direction through 90°.

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ x \end{pmatrix}$$ ✓

Example 10: Draw the image of triangle A after rotation of 90° anticlockwise about (0,0). --- [2]



The Object is A and the image is shape A'

$\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} -y \\ x \end{pmatrix}$

$P\begin{pmatrix} 1 \\ 4 \end{pmatrix} \longrightarrow P'\begin{pmatrix} -4 \\ 1 \end{pmatrix}$

S-17/41/Q3(a)(ii)

or
$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{array}{c} \begin{array}{ccc} P & Q & R \end{array} \\ \begin{pmatrix} 1 & 1 & 3 \\ 4 & 1 & 1 \end{pmatrix} \end{array} = \begin{array}{c} \begin{pmatrix} -4 & -1 & -1 \\ 1 & 1 & 3 \end{pmatrix} \\ \begin{array}{ccc} P' & Q' & R' \end{array} \end{array}$$

Transformations

Rotation

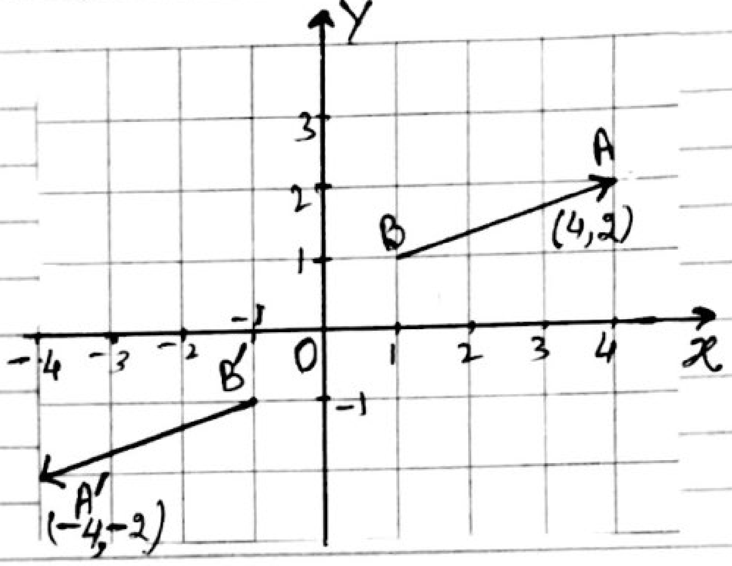
§ 2(C) Rotation through 180°, Centre of rotation is Origin (0,0)

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -x \\ -y \end{pmatrix}$$

$$A\begin{pmatrix} 4 \\ 2 \end{pmatrix} \rightarrow A'\begin{pmatrix} -4 \\ -2 \end{pmatrix}$$

Matrix $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ -y \end{pmatrix}$$



Note: It is same as reflection in origin (0,0)

§§ 2(d) Rotation through 90° in the anticlockwise direction with centre (h, k)

$$\boxed{\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} -(y-k)+h \\ (x-h)+k \end{pmatrix}}$$

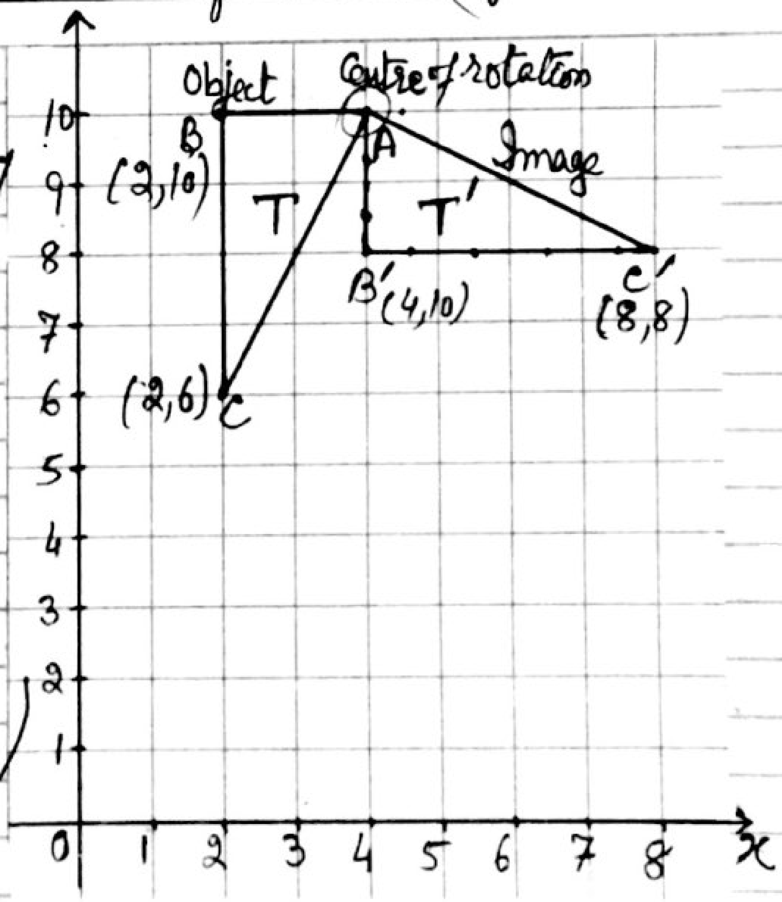
Example 11 Draw the image of triangle T after rotation through 90° anticlockwise with centre (4,10).

Solution $A\begin{pmatrix} 4 \\ 10 \end{pmatrix} \rightarrow A'\begin{pmatrix} 4 \\ 10 \end{pmatrix}$

$$B\begin{pmatrix} 2 \\ 10 \end{pmatrix} \rightarrow \begin{pmatrix} -(10-10)+4 \\ (2-4)+10 \end{pmatrix} = B'\begin{pmatrix} 4 \\ 8 \end{pmatrix}$$

$$C\begin{pmatrix} 2 \\ 6 \end{pmatrix} \rightarrow \begin{pmatrix} -(6-10)+4 \\ (2-4)+10 \end{pmatrix}$$

$$= C'(8,8)$$



-- [2] S-17/42/Q4(a)(ii)

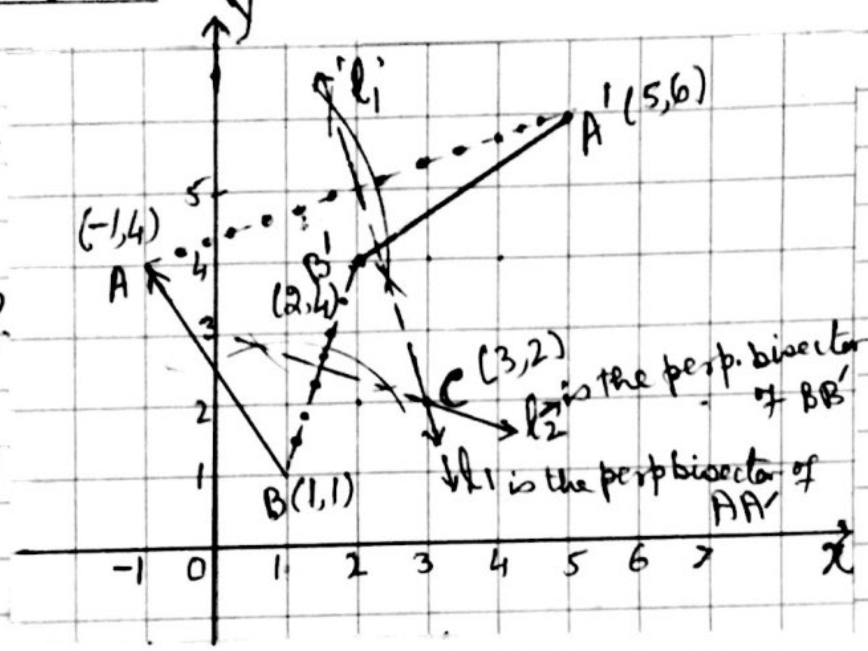
Note: Rotation through 90° anticlockwise about origin $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -y \\ x \end{pmatrix}$

Rotation

§ To find the centre of Rotation, given the image and object under rotation:

Given AB is an Object and A'B' is its image under rotation through 90° clockwise.
To find the centre of rotation?

Join AA' and BB'.
Draw the perp. bisector 'l₁' of segment AA' and perp. bisector of BB' as l_2,

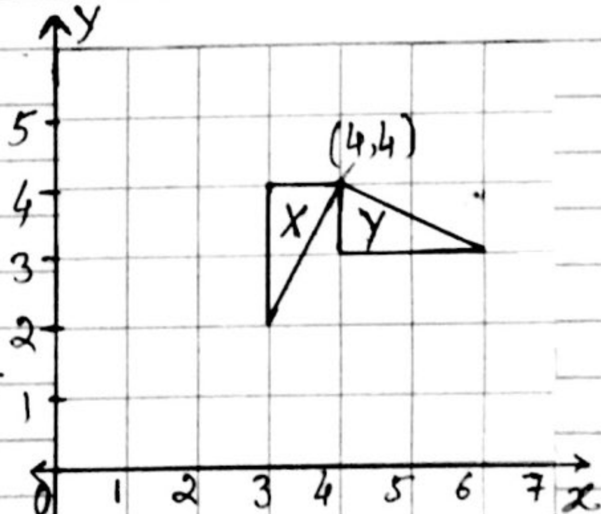


Let 'l₁' and 'l' intersect at point C(3,2). Then C(3,2) is the centre

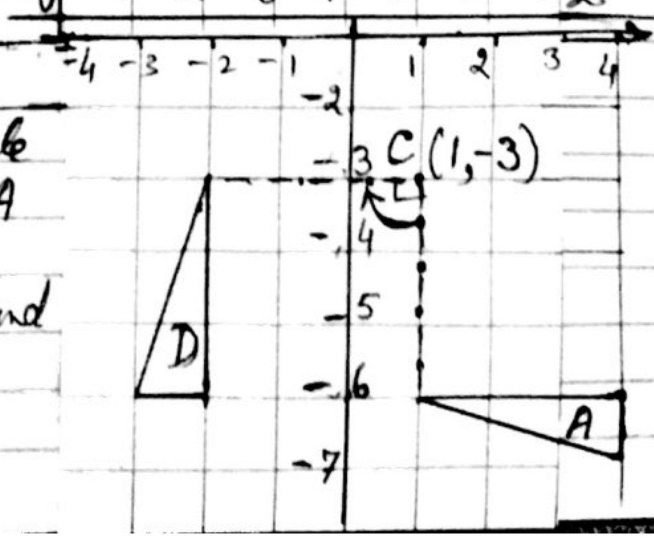
Example 12: Describe fully the **single** transformation that maps --- [3]
(i) Triangle X onto triangle Y.

Answer: Rotation by 90° anticlockwise about (4,4).

M-16 | 42 | Q6(a)(i)



Example 13 Describe fully the single transformation which maps triangle A to triangle D.
Ans Rotation by 90° clockwise around C(-1,3).



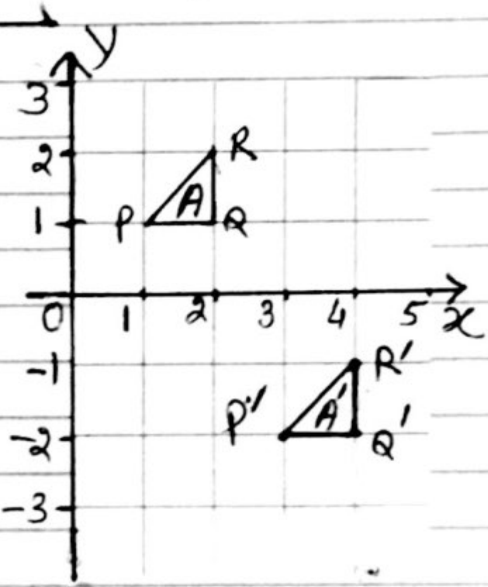
§3. Translation Transformation: (Slide)

It moves every point of of figure by the same distance in a given direction. The size, shape of the image remains same as that of the object.

Translation can be represented by a column vector $\begin{pmatrix} h \\ k \end{pmatrix}$

Object. $A\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow A'\begin{pmatrix} x+h \\ y+k \end{pmatrix}$ Image.

Example 13. Draw the image of shape A, after a translation $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$



Ans $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x+2 \\ y-3 \end{pmatrix}$

$P\begin{pmatrix} 1 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1+2 \\ 1-3 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} P'$

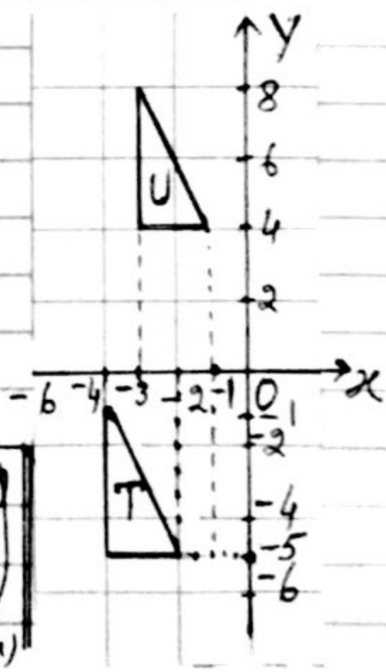
$Q\begin{pmatrix} 2 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 2+2 \\ 1-3 \end{pmatrix} = Q'\begin{pmatrix} 4 \\ -2 \end{pmatrix}$

$R\begin{pmatrix} 2 \\ 2 \end{pmatrix} \rightarrow \begin{pmatrix} 2+2 \\ 2-3 \end{pmatrix} = R'\begin{pmatrix} 4 \\ -1 \end{pmatrix}$

The image of A is A' ---[2] W-15/21/Q3

Example 14: Describe fully the single transformation that maps triangle T onto triangle U. ---[2]

Answer: Translation by $\begin{pmatrix} 1 \\ 9 \end{pmatrix}$



S-16/43/Q6(a)(iii)

Note: Not in the Syllabus

Image → $\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{pmatrix} \underbrace{\begin{pmatrix} x \\ y \\ 1 \end{pmatrix}}_{3\times1} = \underbrace{\begin{pmatrix} x+h \\ y+k \\ 1 \end{pmatrix}}_{3\times1}$

Enlargement

§4. Enlargement transformation:

In the enlargement transformation the Object is enlarged, the image is geometrically similar to the object. (may be larger or smaller in size).

For "Enlargement transformation" we are given the position of the **centre of enlargement** and the **Scale factor**.

§ **Given** the centre of enlargement origin (0,0) and scale factor $K>0$, then object and image are on the same side of the centre.

$$\text{Object } P\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow P'\begin{pmatrix} kx \\ ky \end{pmatrix} \text{ Image.}$$

§ The Matrix is $\begin{pmatrix} K & 0 \\ 0 & K \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} kx \\ ky \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}$

Example 15:

Describe fully the **single transformation** that maps triangle A onto triangle B. ...[3]

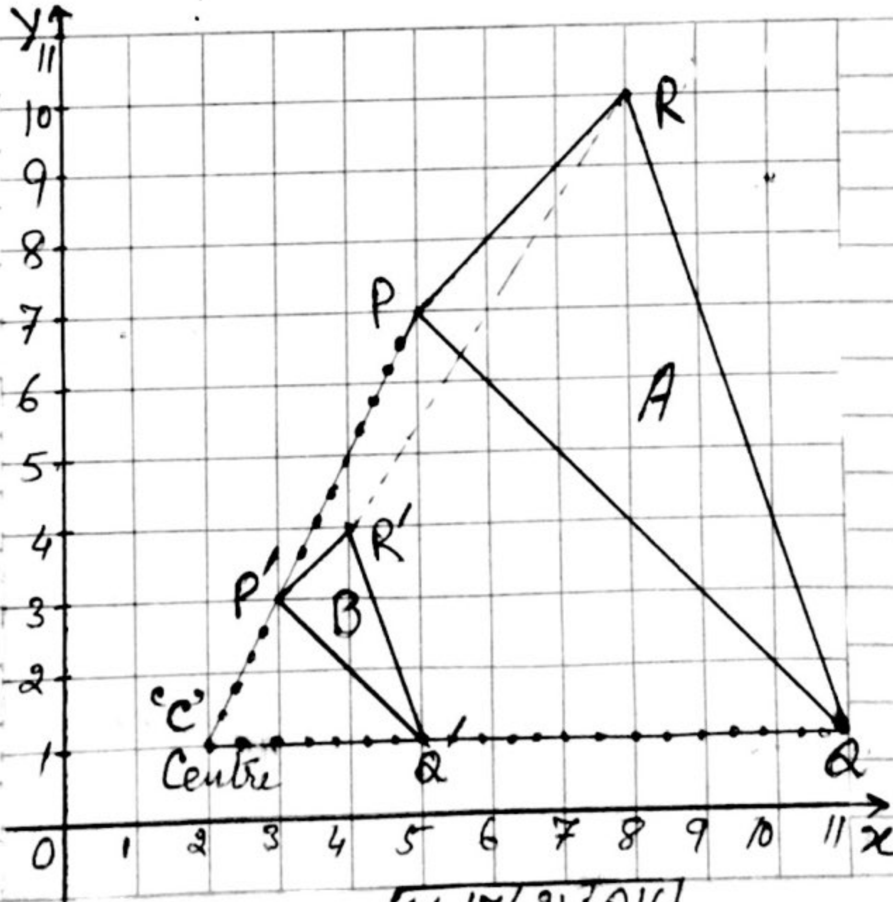
Ans: **Enlarment** with centre at (2,1) and scale factor of enlargement $1/3$.

§ To find the centre join PP′ and QQ′ extended meet at C

✓ C(2,1) is Centre.

and $\frac{CQ'}{CQ} = \frac{3}{9} = \frac{1}{3}$ ✓

$\therefore$ S.f = $1/3$



W-17/21/Q16

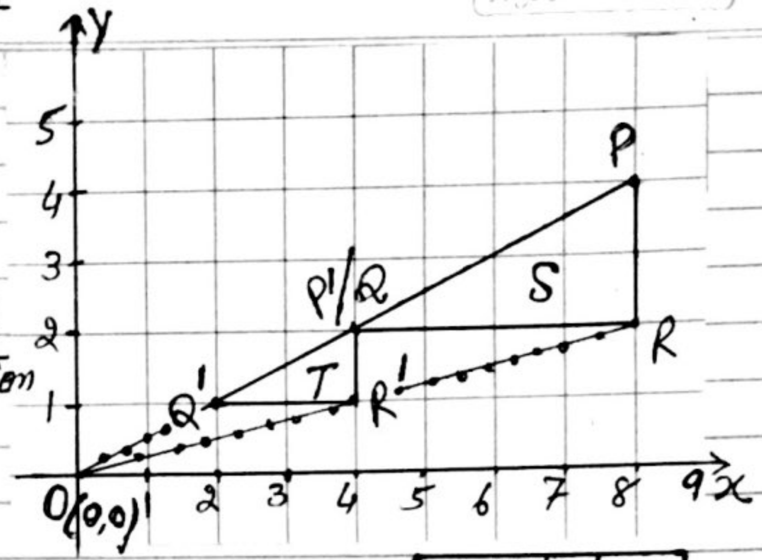
IG_Maths 0580

Transformations

Enlargement.

Example 16: (a) Describe fully the **single** transformation that maps triangle S onto triangle T --- [3]

(b) Find the matrix which represent the tranformation that maps triangle S onto triangle T. --- [2]



[W-15/21/Q17]

Solution (a) Join PP' and produce, also Join RR' and produce the two intersect at origin. ∴ Centre of Enlargment is (0,0). and $\frac{P'R'}{PR} = \frac{1}{2}$ (s.f)

∴ It is an enlargement with centre-origin and scale factor $\frac{1}{2}$.

(b) Matrix $\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$ $\left[\because \begin{pmatrix} K & 0 \\ 0 & K \end{pmatrix}; K = \frac{1}{2} \right]$

Example 17. Describe full the **single** transformation represented by the matrix $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ --- [3]

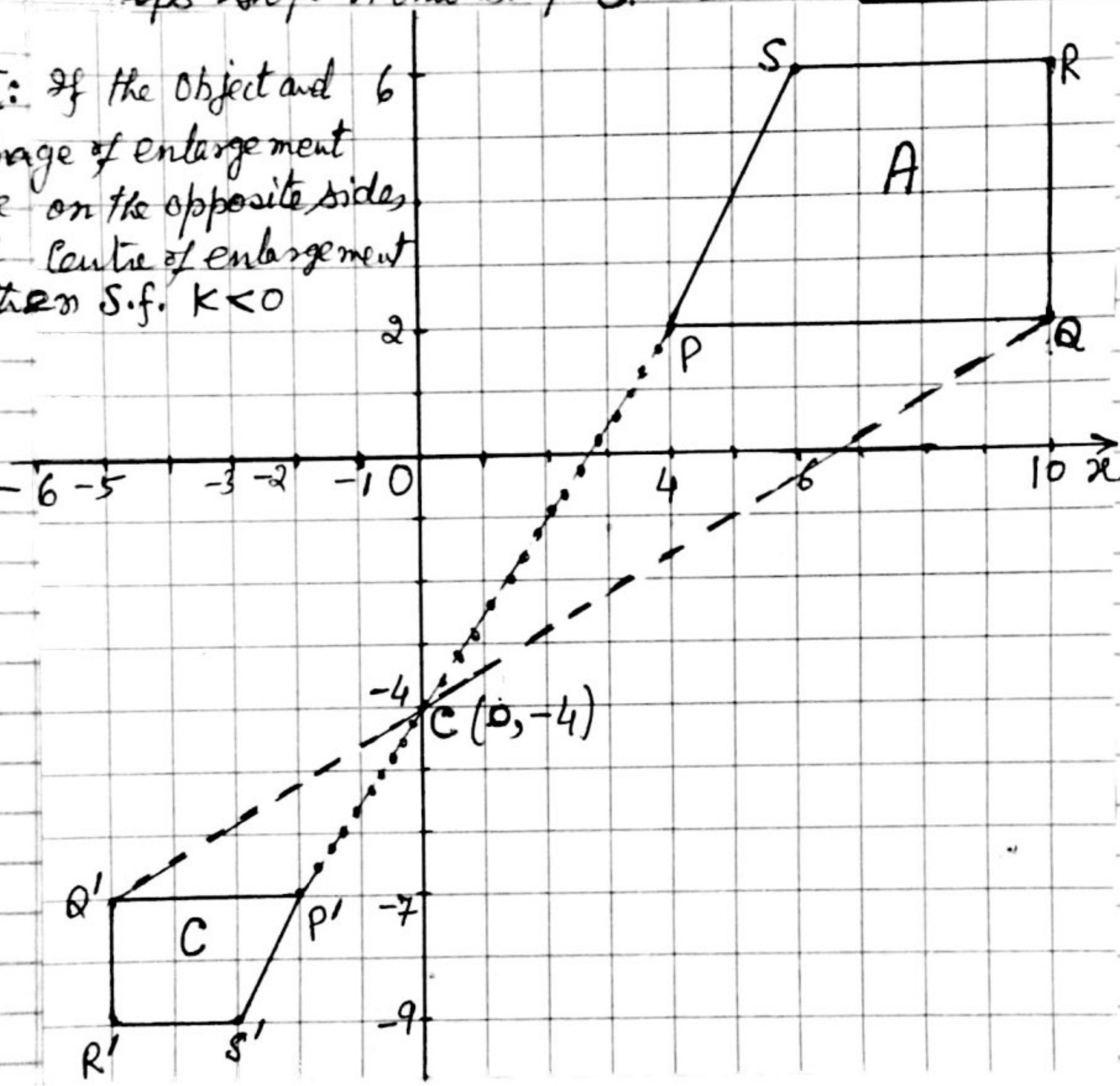
Ans: Enlargement, s.f = 3 and centre of enlargement at origin (0,0).

[S-17/43/Q6(d)]

Enlargement

Example 18: Describe fully the **single** transformation that maps shape A onto shape C. ---[3] [S-17/43 Q6a(ii)]

Note: If the object and image of enlargement are on the opposite sides of centre of enlargement then S.f. $K<0$



Join PP' and QQ' the corresponding points on the object and image. PP' and QQ' intersect at C(0,−4)

also $\frac{P'Q'}{PQ} = \frac{3}{6} = \frac{1}{2}$ but object and image are on the opposite sides of centre. $S.f = -\frac{1}{2}$

∴ Enlargement $S.f = -\frac{1}{2}$ and centre of Enlargement is C(0,−4).

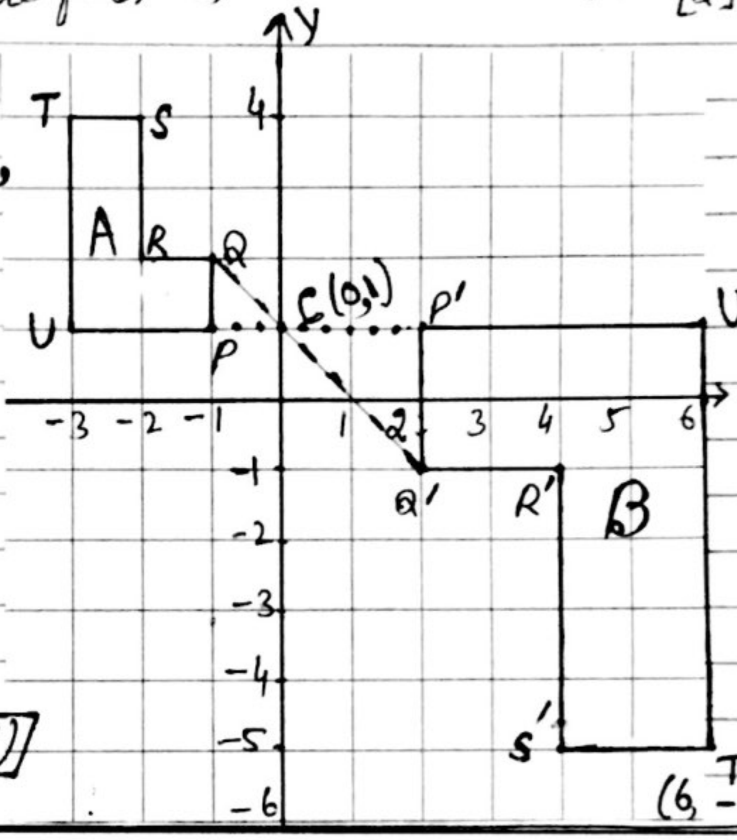
IG-Maths 0580

Enlargement   Transformations

Example 19: On the grid draw the image of shape A after an enlargement with scale factor −2, and centre (0,1). ... [2]

Object: "PQRSTU"
Image is "P'Q'R'S'T'U'"

$\frac{P'C}{PC} = \frac{Q'C}{QC} = \frac{U'C}{UC} = 2$ ✓



S-16 | 42 | Q3(a)(ii)

§ Given a point $A(x, y)$ on the grid.
The image of $A \longrightarrow A'(x', y')$ after an enlargement with scale factor k, and Centre (x_1, y_1) is given by.

$$\boxed{A'\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} k(x-x_1)+x_1 \\ k(y-y_1)+y_1 \end{pmatrix} = \begin{pmatrix} kx + x_1(1-k) \\ ky + y_1(1-k) \end{pmatrix}} = \begin{pmatrix} x' \\ y' \end{pmatrix}$$

In Example 19 above, s.f., $k = -2$, Centre $(0,1)$

$$\therefore A'\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -2x + 0(1-(-2)) \\ -2y + 1(1-(-2)) \end{pmatrix} = \begin{pmatrix} -2x \\ -2y+3 \end{pmatrix}$$

$\therefore$ Image of point $T(-3,4)$,

$$T'\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -2\times(-3) \\ -2\times 4 + 3 \end{pmatrix} = \begin{pmatrix} 6 \\ -5 \end{pmatrix} T'$$ ✓

"See the figure above"

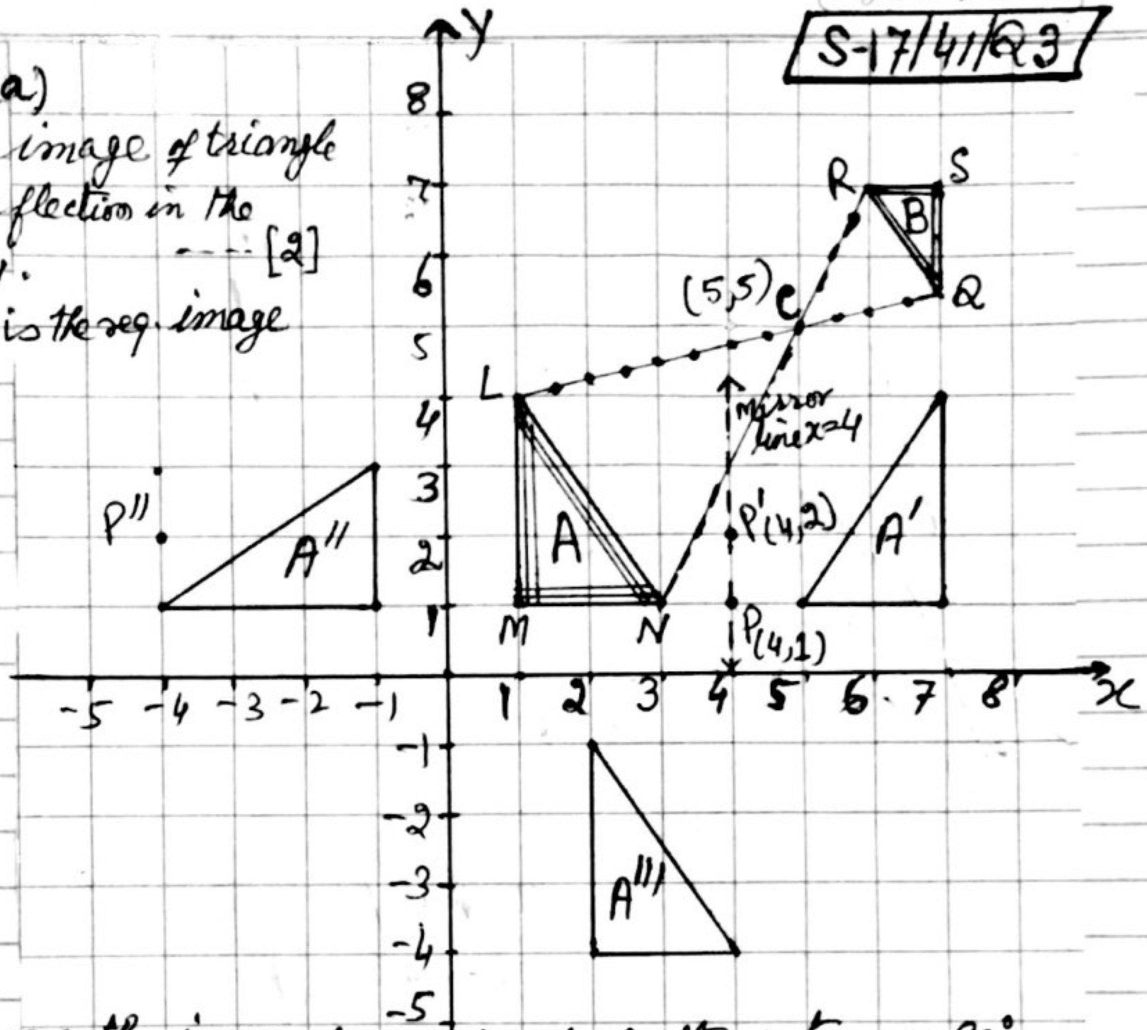
Transformations

S-17/41/Q3

Example 20 (a)

(i) Draw the image of triangle A after reflection in the line $x=4$. --- [2]

Ans: A' is the req. image



(a)(ii) Draw the image of of triangle A after rotion of 90° anticlockwise about (0,0) --- [2]

Ans: A'' is the required image such that $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -y \\ x \end{pmatrix}$

(a)(iii) Draw the image of triangle A after translation by the Vector $\begin{pmatrix} 1 \\ -5 \end{pmatrix}$. --- [2]

Ans: A''' is the image. Use $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x+1 \\ y-5 \end{pmatrix}$

(b) Describe fully the single transformation that maps triangle A onto triangle B. --- [3]

Ans: Enlargement $S.f = -\frac{1}{2}$, Centre (5,5),

Segment LQ and NR joining the corresponding point of shape A and B intersect at a point C(5,5) is Centre

and $\frac{RS}{MN} = \frac{1}{2}$ but are the opposite side of Centre of Enlargement $\therefore$ $S.f = -\frac{1}{2}$.

(Continued)

Transformations

(Continued→)

Example 20(C). Find the matix that represent the transformation in part (a) (ii) --[2]

Ans: Rotation of 90° in anticlockwise about (0,0), $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} -y \\ x \end{pmatrix}$

$\therefore$ Required Matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ ✓

(d) Point P has coordinates (4,1), $F = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ and $G = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ Represent the transformations:

(i) Find; $G(P) = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ P' ✓ --[2]

(ii) $GF(P) = \left(\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\right)\begin{pmatrix} 4 \\ 1 \end{pmatrix}$ ---[3]

Find GF(P):

$= \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 4 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} -4 \\ 2 \end{pmatrix}$ P'' ✓

(iii) Find the matrix Q such $GQ(P) = P$

Ans: $GQ(P) = P$

$\therefore Q = G^{-1}$

$= \frac{1}{2}\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ ✓

$\left(\because GG^{-1}(P) = P\right)$

$\because |G| = \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 2$

$\because G = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$G^{-1} = \frac{1}{|G|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

S-17/41/Q3