DATE 08/12/20

0580

IGCSE Maths

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Trigonometry Revision

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IG 0580

Three figures bearings and Sine rule/Cosine rule

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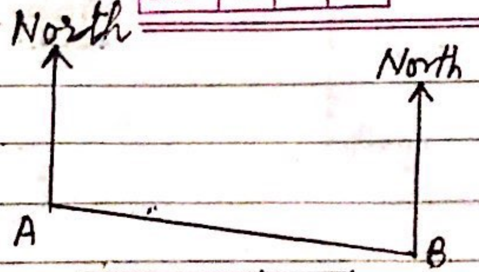
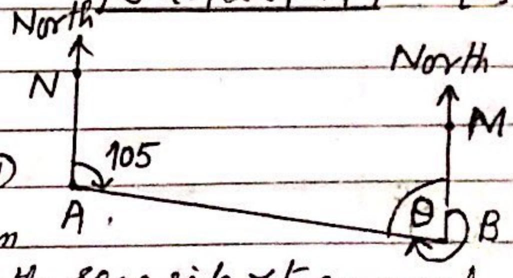
Date

1 The bearing of B from A is 105°
Find the bearing of A from B.



Solution: Bearing of B from A is
angle $NAB = 105°$ (from North clockwise)

/S-20/22/Q9/ ---[2]



Now Bearing of A from B = angle MBA

$= 360 - \theta$ —①

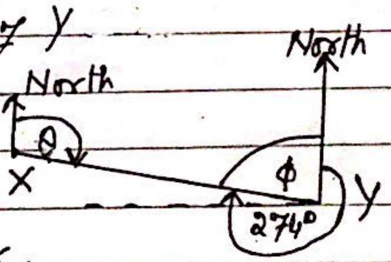
$\theta + 105 = 180°$ (Interior angle between two parallel lines on the same side of transversal are supplementary)

$\Rightarrow \theta = 180 - 105 = 75°$

∴ from ① The Bearing of the point A from B $= 360 - 75 = \underline{285°}$ ✓

2. The bearing of X from Y is 274° ---[2]
Calculate the bearing of Y

/S-20/23/Q8/



Solution: Given Bearing of X from Y = 274°.
The bearing of Y from X.

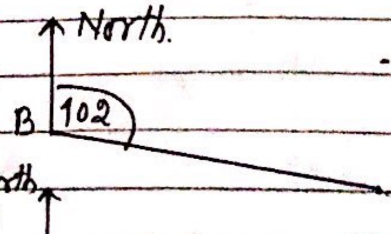
$\theta = 180 - \phi$ —① (∵ θ and φ are Co-Interior angles)

Now $\phi = 360° - 274 = 86°$

from ① $\theta = 180 - 86 = 94°$

∴ The bearing of Y from X = $\underline{094°}$ ✓

3. The bearing of P from B is 102° ---[2]
Find the bearing of B from P.



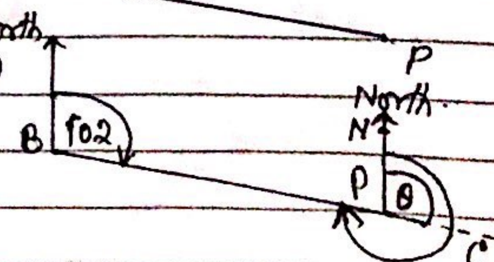
Solution: Let angle $NPC = \theta$ (BP is extended)

$\theta = 102°$ (Corresponding angles) —①

∴ Bearing of B from P $= \theta + 180°$

$= 102 + 180°$

$= \underline{282°}$ ✓



/W-19/22/Q8/

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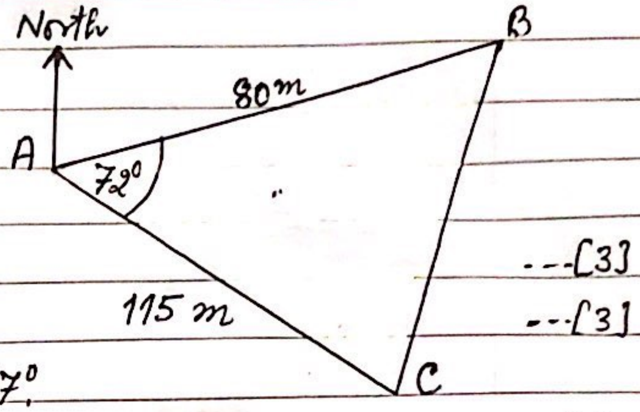
4. The diagram shows the positions of three points A, B, and C in a field.



(a) Shows that BC is 118.1 m correct to 1d.p. ---[3]

(b) Calculate angle ABC. ---[3]

(c) The bearing of C from A is 147°.
Find the bearing of (i) A from B ---[3]
(ii) B from C ---[2]

S-20/41/Q7

(d) Mitchell takes 35 seconds to run from A to C.
Calculate the average running speed in kilometres per hour. ---[3]

(e) Calculate the shortest distance from B to AC. --[3]

Solution: (a) $BC^2 = AB^2 + AC^2 - 2AB\cdot AC\cos BAC$ (Using cosine rule

$= 80^2 + 115^2 - 2\times80\times115\times\cos72°$ ($a^2 = b^2 + c^2 - 2bc\cos A$)

$= 6400 + 13225 - 18400\times0.309 = 19625 - 5685.6 = 13939.4$

$BC = \sqrt{13939.4} = \underline{118.06}$ ✓

(b) In ΔABC using Sine Rule $\left(\frac{a}{\sin A} = \frac{b}{\sin B}\right)$

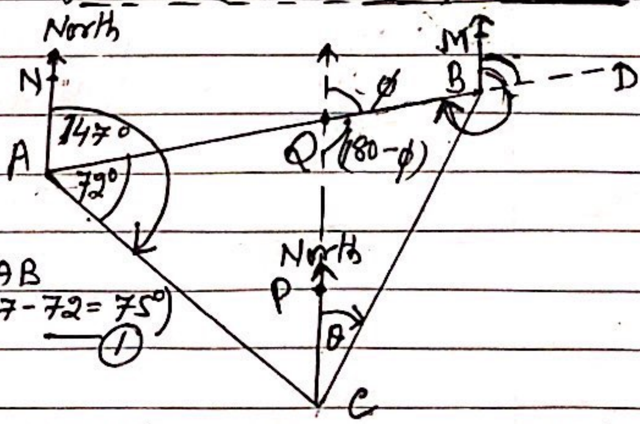
$\frac{BC}{\sin 72} = \frac{AC}{\sin ABC} \Rightarrow \frac{118.06}{\sin 72} = \frac{115}{\sin ABC} \Rightarrow \sin ABC = \frac{115\times\sin 72°}{118.06}$

$= 0.9264$

$\therefore$ Angle $ABC = \sin^{-1}(0.9664) = \underline{67.88°}$ ✓

(C) Bearing of C from A = 147°

$\angle NAC = 147°$



(i) Bearing of A from B.

$= 180° +$ angle MBD

$= 180 + 75°$ ($\because \angle MBD = \angle NAB = 147 - 72 = 75°$) ①

$= \underline{225°}$ ✓

(ii) Bearing of B from C = θ let

In ΔQCB, $\theta + (180-\phi) + \angle ABC = 180°$ ($\because \phi = \angle MBD = 75°$ from ①)

$\theta + 180 - 75 + 67.88 = 180°$

$\theta = 75 - 67.88 = \underline{7.12°}$ ✓

(Continued→)

(continued→)

4(d) average speed to run from A to C (115m) in 35 second

$= \frac{115}{35} = 3.2857$ m/s

$= 3.2857 \times 3.6$ Km/H

$= \underline{11.82}$ Km/H. ✓ ($\because$ 1m/s = 3.6 Km/H)

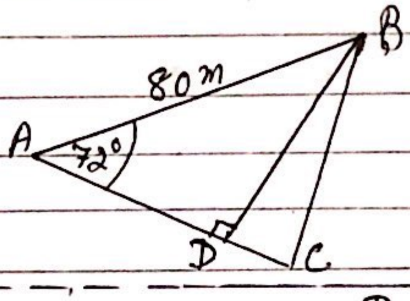
e. draw BD perpendicular to BC.

The shortest distance of B from AC = BD.

In ΔBAD, $\frac{BD}{AB} = \sin 72$

$\Rightarrow BD = 80 \times \sin 72$

$= \underline{76.08m}$ ✓



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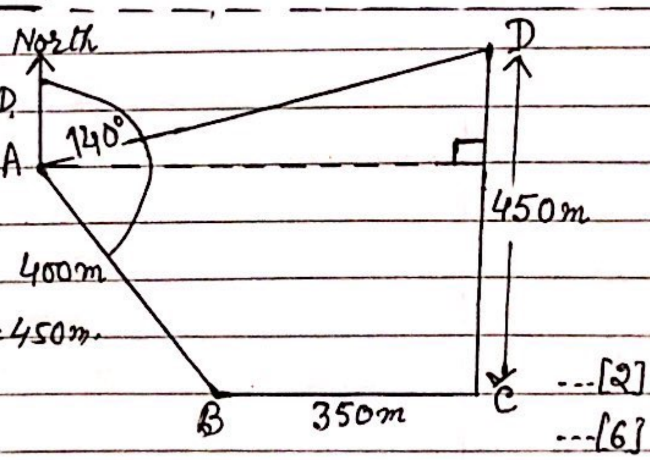
5. The diagram shows a field ABCD.

The bearing of B from A is 140°

C is due east of B and D is due north of C.

AB = 400 m, BC = 350m and CD = 450m.



(a) Find the bearing of D from B. ---[2]

(b) Calculate the distance from D to A. ---[6]

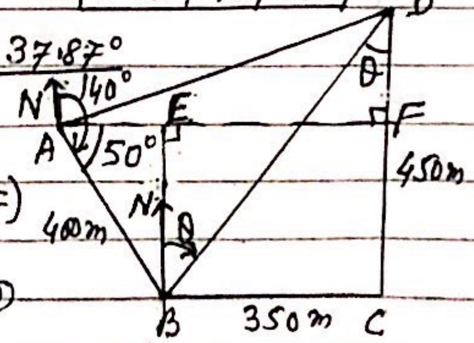
(C) Jono runs around the field from A to B, B to C, C to D and D to A.

He runs at a speed of 3m/s.

Calculate the total time Jono takes to run around the field. ---[4]

Give your answer in minutes and seconds, correct to nearest second.

[S-20/42/Q5]



Solution (a) Let $\angle BDC = \theta \Rightarrow \tan\theta = \frac{350}{450} \Rightarrow \theta = \underline{37.87°}$

$\therefore$ Bearing of D from B = θ = 37.87° ✓

(b) $BE = 400 \sin 50 = 306.4m$ —— (1) ($BE \perp AF$)

$\Rightarrow DF = 450 - FC = 450 - BE = 450 - 306.4$

$= 143.6$ —— (2)

Now $AE = 400 \cos 50° = 257$ —— (3)

$\Rightarrow AF = AE + EF = AE + BC = 257 + 350 = 607$ —— (4)

$\therefore$ In rt ΔAFD, $AD = \sqrt{AF^2 + DF^2} = \sqrt{(143.6)^2 + (607)^2}$

$= \sqrt{389069.96} = 623.8m$

$\therefore \underline{AD = 624\ m}$ ✓

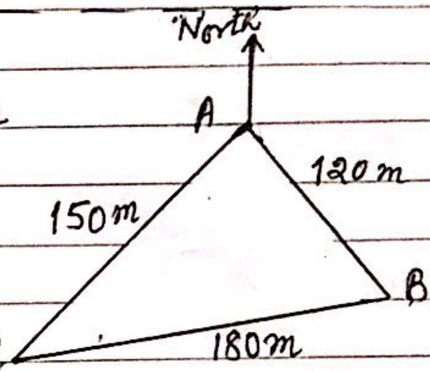
(continued→)

(continued→)

5 (C) $AB+BC+CD+DA = 400+350+450+624 = 1824m$

Speed $= 3m/s$

$\therefore$ Time $= \dfrac{\text{distance}}{\text{speed}} = \dfrac{1824}{3} = 608s = \underline{10m, 8s}$



6. The diagram shows a triangular field, ABC, on horizontal ground.

(a) Olav runs from A to B at a constant speed of 4m/s and then from B to C at a constant speed of 3m/s. He then runs at a constant speed from C to A. His average speed for the whole journey is 3.6 m/s. Calculate his speed when he runs from C to A. --[3]

(b) Use cosine rule to find angle BAC. --[4]

(C) The bearing of C from A is 210°

(i) Find the bearing of B from A. --[1]

(ii) Find the bearing of A from B. --[2]

(d) D is the point on AC that is nearest to B. --[2]

Calculate the distance from D to A. [W-19/41/Q5]

<u>Solution</u> (a) Let the speed from C to A $= x$ m/s

Total Time from A to B, B to C and C to A $= \left(\dfrac{120}{4} + \dfrac{180}{3} + \dfrac{150}{x}\right)$ second —①

also the average speed is 3.6m/s

Total time $\Rightarrow \dfrac{(120+180+150)}{3.6} = 125$ second —②

from ① & ② $\left(30+60+\dfrac{150}{x}\right) = 125 \Rightarrow \dfrac{150}{x} = 35 \Rightarrow x = \dfrac{150}{35} = 4.286 m/s$

$\therefore$ speed from C to A $= \underline{4.29 m/s}$

(b) $\cos BAC = \dfrac{150^2+120^2-180^2}{2\times150\times120} = \dfrac{4500}{36000} = 0.125$ $\left(\cos A = \dfrac{b^2+c^2-a^2}{2bc}\right)$

$\therefore$ angle $BAC = \cos^{-1} 0.125$

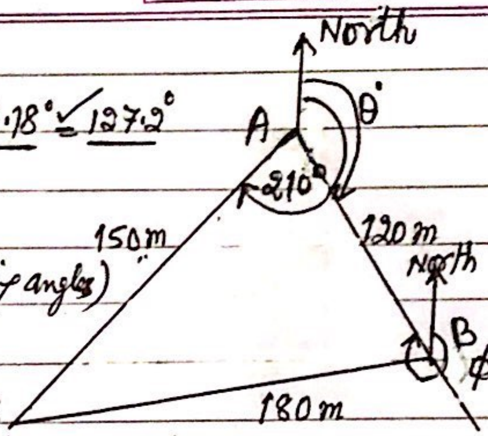
$= \underline{82.82^\circ}$

(continued→)

(continued→)

6 (C) (i) Bearing of B from A = 210 − angle BAC

$\theta = 210 - 82.82 = \underline{127.18^\circ} \simeq \underline{127.2^\circ}$



(ii) Bearing of A from B

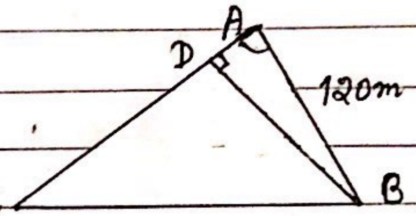
$= 180 + \phi$ ($\phi = \theta$ corresponding angles)

$= 180 + \theta$

$= 180 + 127.2 = \underline{307.2^\circ}$

(d) Draw $BD \perp AC$

D is the nearest point on AC from B.

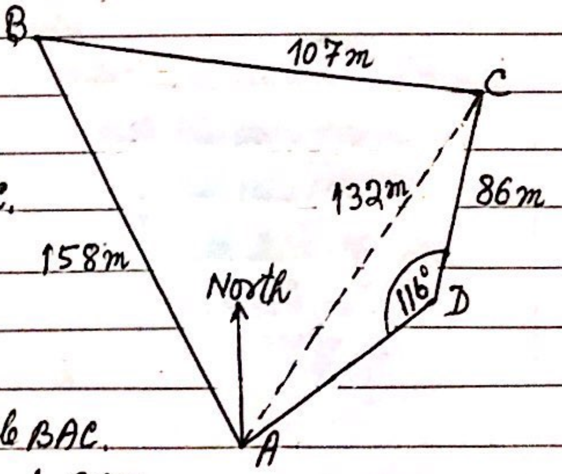


In $\triangle ADC$, $\frac{AD}{120} = \cos A$

$AD = 120 \times \cos 82.82^\circ$ (from part (b) angle $BAC = 82.82^\circ$)

$= \underline{15m}$

7. The diagram shows a field, ABCD, on a horizontal ground.



(a) There is a vertical post at C. From B, the angle of elivation of the top of the post is 19°. Find the height of the post. ---[2]

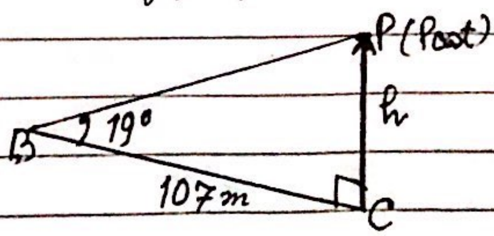
(b) Use the cosine rule to find angle BAC. ---[4]

(c) Use sine rule to find angle CAD. ---[3]

(d) Calculate the area of the field. ---[3]

[W-19 | 43 | Q4]

(e) The bearing of D from A is 070°. Find the bearing of A from C. ---[2]



Solution (a) In rt $\triangle PCB$, $\frac{h}{107} = \tan 19^\circ$

$\Rightarrow h = 107 \times \tan 19^\circ = 36.84m$

$\therefore h = \underline{36.84m}$

(b) $\cos BAC = \frac{158^2 + 132^2 - 107^2}{2 \times 158 \times 132} = 0.7417$ $\left(\because \cos A = \frac{b^2 + c^2 - a^2}{2bc}\right)$

angle $BAC = \cos^{-1} 0.7417 = \underline{42.12^\circ}$

(Continued→)

Bearings / Sine rule and Cosine rule.

(Continued→)

7 (c) $\frac{86}{\sin CAD} = \frac{132}{\sin 116^\circ}$ (Sine Rule $\frac{a}{\sin A} = \frac{b}{\sin B}$)

$\sin CAD = \frac{86 \cdot \sin 116}{132} = 0.5856$

$\Rightarrow$ Angle $CAD = \sin^{-1} 0.5856 = 35.84^\circ$

(d) Area ABCD = area ΔBAC + ar ACD — ①

area $\Delta BAC = \frac{1}{2} bc \sin A$

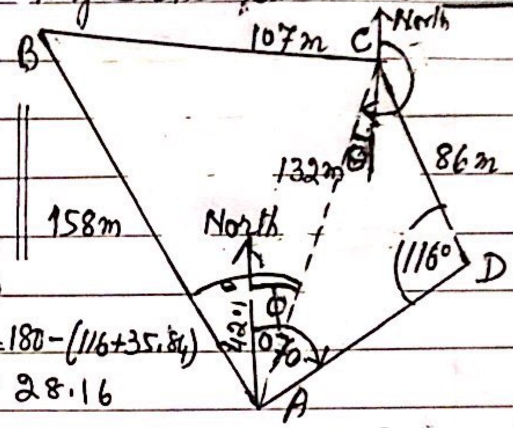
$= \frac{1}{2} \times 132 \times 158 \cdot \sin 42.1^\circ$

$= 6991.2$ — ②

area $\Delta ACD = \frac{1}{2} \times 132 \times 86 \cdot \sin ACD$

$= \frac{1}{2} \times 132 \cdot 86 \cdot \sin 28.16^\circ$ [angle $ACD = 180 - (116 + 35.84)$ $= 28.16$

$= 2678.7$ — ③



from ② & ③ in ① ⇒ Area ABCD = $6991.2 + 2678.7 = \underline{9669.9}$ ✓

(e) Bearing of D from A = 070°

Bearing of A from C = $180 + \theta = 180 + \phi$ ($\theta = \phi$ Alternate angles)

$= 180 + (70 - \text{angle } CAD)$

$= 180 + (70 - 35.84)$

$= 214.16$

$= \underline{214.2^\circ}$

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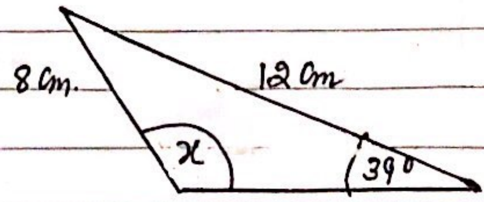
8. Calculate the Obtuse angle x in the triangle. ---[3]



Solution:

using sine rule:

$\frac{12}{\sin x} = \frac{8}{\sin 39}$ $\left(\frac{a}{\sin A} = \frac{b}{\sin B}\right)$

$\Rightarrow \sin x^\circ = \frac{12 \sin 39}{8} = 0.944$ | W-19 | 21 | Q16 |

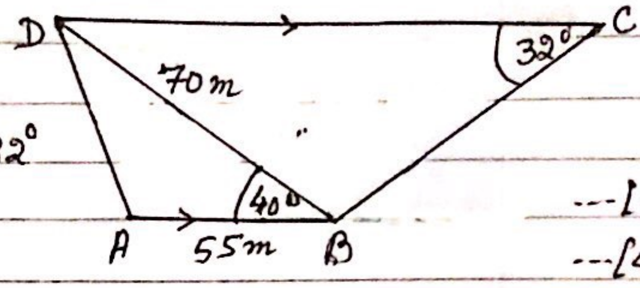
$x = \sin^{-1} 0.944$

$= 70.7$ or $(180 - 70.7)$

$= 70.3$ ✗ 109.3° ✓ (obtuse angle)

$\therefore x = \underline{109.3^\circ}$

9. The diagram shows a trapezium ABCD.
AB is parallel to DC.
$AB = 55m$, $BD = 70m$,
angle $ABD = 40^\circ$ and angle $BCD = 32^\circ$



(a) Calculate AD ---[4]
(b) Calculate BC ---[4]
(c) Calculate area of ABCD. --[3]
(d) Calculate the shortest distance from A to BD. --[2]

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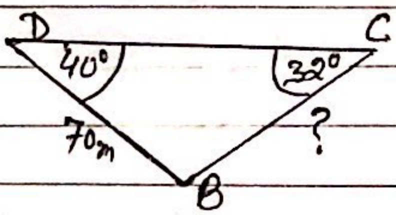
Solution: In Δ ABD

(a) $AD^2 = 70^2 + 55^2 - 2 \times 70 \times 55 \times \cos 40^\circ$ (Cosine rule, $a^2 = b^2 + c^2 - 2bc \cos A$)

$= 2026.5$

$\Rightarrow AD = \sqrt{2026.5} = \underline{45.02}\ m$ ✓

(b) Angle $BDC = \angle ABD = 40^\circ$ (Alternate angle)



$\frac{BC}{\sin 40} = \frac{70}{\sin 32^\circ}$ (Sine Rule $\frac{a}{\sin A} = \frac{b}{\sin B}$)

$BC = \frac{70 \times \sin 40^\circ}{\sin 32^\circ} = 84.9$

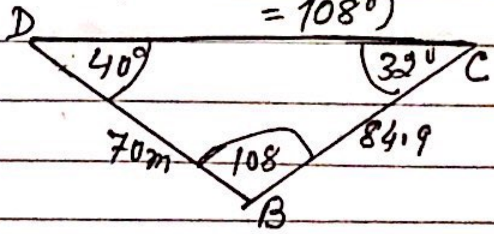
$\Rightarrow BC = \underline{84.9\ m}$ ✓

(c) area $ABCD$ = ar ΔABD + ar ΔBCD —— (1)

area $\Delta ABD = \frac{1}{2} \cdot 70 \times 55 \cdot \sin 40^\circ = 1237.36$ (ar $\Delta = \frac{1}{2} ab \sin C$)

area $\Delta BCD = \frac{1}{2} \times 70 \times 84.9 \sin 108^\circ$ $\because (\angle DBC = 180 - (40+32) = 108^\circ)$

$= 2826.06$



$\therefore$ area $ABCD = 1237.36 + 2826.06$

$= \underline{4063.42}$ sq m. ✓

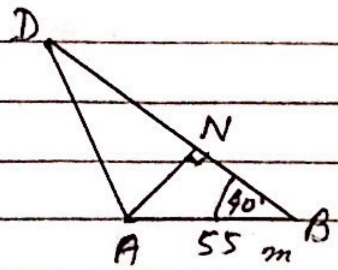
(d) Draw AN perp. to BD, AN is the required {Shortest dis.

In rt Δ ANB

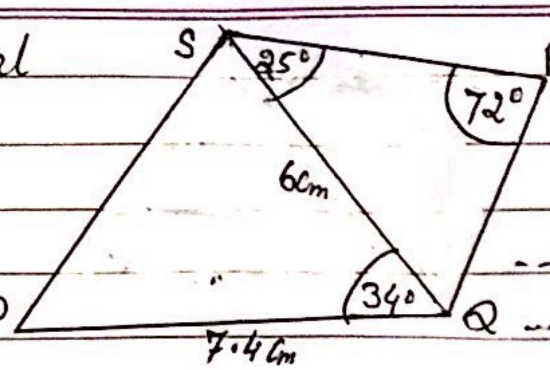
$\frac{AN}{55} = \sin 40^\circ \Rightarrow AN = 55 \cdot \sin 40^\circ$

$= 35.35\ m$

$\therefore$ Shortest dis $= \underline{35.35\ m}$ ✓



10. The diagram shows a quadrilateral PQRS formed from two triangles, PQS and QRS. Calculate:



(i) QR ---[3]

(ii) PS ---[3]

(iii) Area of quadrilateral PQRS. --[4]

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Solution (i) In ΔQRS

$$\frac{QR}{\sin 25} = \frac{6}{\sin 72} \quad \left(\text{Using Sine Rule. } \frac{a}{\sin A} = \frac{b}{\sin B}\right)$$

$$\Rightarrow \quad QR = \frac{6 \sin 25^\circ}{\sin 72^\circ} = \underline{2.67 \text{ cm}} \checkmark$$

(ii) In ΔPQS.

$$\left\{ \begin{array}{l} \text{Cosine Rule} \\ a^2 = b^2 + c^2 - 2bc \cos A \end{array} \right.$$

$$PS^2 = 6^2 + 7.4^2 - 2 \times 6 \times 7.4 . \cos 34^\circ$$
$$= 90.76 - 73.61 = 17.14$$
$$\therefore PS = \sqrt{17.41} = \underline{4.14 \text{ cm.}} \checkmark$$

(iii) area PQRS = ar ΔPQS + ar ΔQRS —— ①

area $\Delta PQS = \frac{1}{2} \times 7.4 \times 6 \times \sin 34^\circ$ $\quad$ (ar $\Delta = \frac{1}{2} ab \sin C$)
$= 12.414$ —— ②

ar $\Delta QRS = \frac{1}{2} \times QR \times SQ \times \sin SQR$ $\quad$ (angle SQR = 180 − (72+25)
$= \frac{1}{2} \times 2.67 \times 6 \times \sin 83$ $\quad$ = 83°
$= 7.95$ —— ③ $\quad$ QR = 2.67 Part (i))

from ② & ③ in ①

Area of Quad. PQRS = 12.41 + 7.95 = $\underline{20.36 \text{ cm}^2}$ ✓

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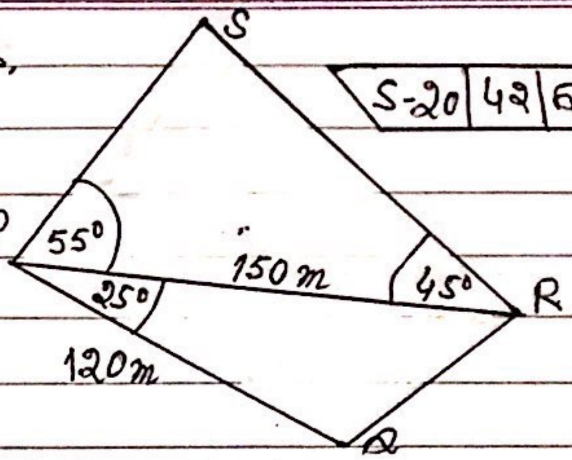
11. The diagram shows two triangles.
(a) Calculate QR. ---[3]
(b) Calculate RS ---[4]
(c) Calculate the total area of the two triangles. ---[3]



Solution: In Δ PQR (Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$)

(a) $QR^2 = 120^2 + 150^2 - 2 \times 120 \times 150 \times \cos 25$

$= 36900 - 32627 = 4273$

$QR = \sqrt{4273} = \underline{65.37}$ ✓

(b) angle $PSQ = 180 - (55 + 45°) = 80°$

Using sine rule in Δ PSQ, $\left(\frac{a}{\sin A} = \frac{b}{\sin B}\right)$

$\frac{RS}{\sin 55°} = \frac{150}{\sin 80°}$

$\Rightarrow RS = \frac{150 \times \sin 55}{\sin 80°} = \frac{122.87}{0.9845} = \underline{124.80 m}$ ✓

(c) Total area = ar Δ PQR + ar Δ PRS ——— ①

ar $\Delta QPR = \frac{1}{2} \times 120 \times 150 \times \sin 25$ (ar $\Delta = \frac{1}{2} ab \sin C$)

$= 3803.56$ ——— ②

ar $\Delta PRS = \frac{1}{2} \times PR \times RS \times \sin 45°$ Part (ii)

$= \frac{1}{2} \times 150 \times 124.8 \times \sin 45°$ [$RS = 124.8$]

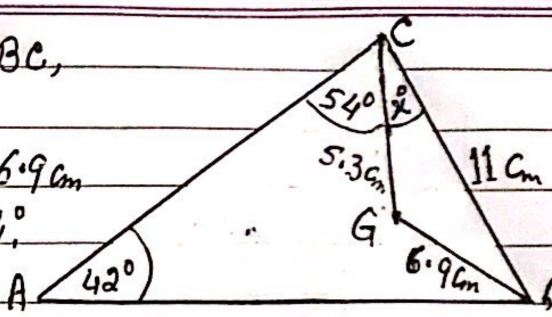
$= 6618.52$ ——— ③

from ② & ③ in ①

Total area $= 3803.56 + 6618.52$

$= \underline{10422}$ ✓ m^2

12. The diagram shows triangle ABC, with point G inside.
CB = 11 cm, CG = 5.3 cm and BG = 6.9 cm
Angle CAB = 42° and angle ACG = 54°.
(i) Calculate the value of x. ---[4]
(ii) Calculate AC. ---[4]



[S-20/43/Q6(a)]

Solution: (i) $\cos x° = \dfrac{11^2 + 5.3^2 - 6.9^2}{2 \times 11 \times 5.3}$ [Cosine Rule, $\cos C = \dfrac{a^2+b^2-c^2}{2ab}$]

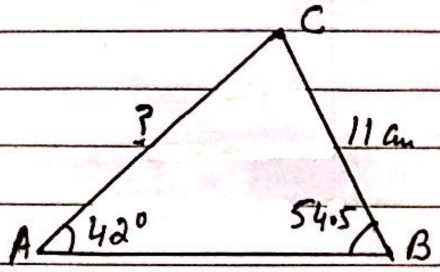
$= \dfrac{101.48}{116.6} = 0.87$

$\Rightarrow x = \cos^{-1}(0.87) = \underline{29.5°}$ ✓

(ii) angle ABC $= 180 - \{42 + (54 + 29.5)\} = 54.5$

Using sine rule in Δ ABC.

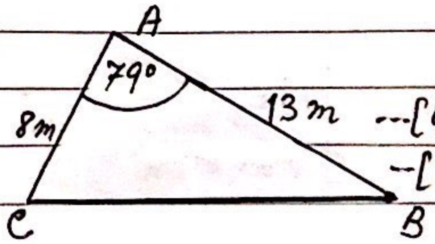
$\dfrac{AC}{\sin 54.5} = \dfrac{11}{\sin 42}$



$\Rightarrow AC = \dfrac{11 \times \sin 54.5}{\sin 42°}$

$= 13.38$ or $\underline{AC = 13.4\,cm}$ ✓

13. The diagram shows triangle ABC.
(i) Use cosine rule to calculate BC. ---[4]
(ii) Use sine rule to calculate angle ACB. ---[3]



Solution (i) In Δ ABC.

$BC^2 = 8^2 + 13^2 - 2 \times 8 \times 13 \times \cos 79°$ (Cosine Rule, $a^2 = b^2 + c^2 - 2bc\cos A$)

$= 64 + 169 - 39.69 = 193.31 \Rightarrow BC = \sqrt{193.31} = \underline{13.9\ m}$ ✓

(ii) $\dfrac{13}{\sin C} = \dfrac{13.9}{\sin 79}$ ($\because$ Sine Rule $\dfrac{c}{\sin C} = \dfrac{a}{\sin A}$)

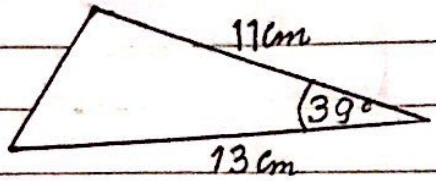
$\sin C = \dfrac{13 \times \sin 79}{13.9} = 0.918$

[W-19/42/Q6(a)]

$\therefore \angle ACB = \sin^{-1}(0.918) = \underline{66.64°}$ ✓

Area of Triangle.

14. Calculate the area of triangle.

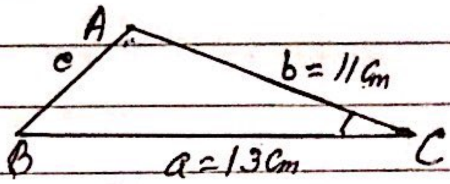


Solution:

Area of triangle $= \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 13 \times 11 \times \sin 39$

$= \underline{45\,cm^2}$ ✓



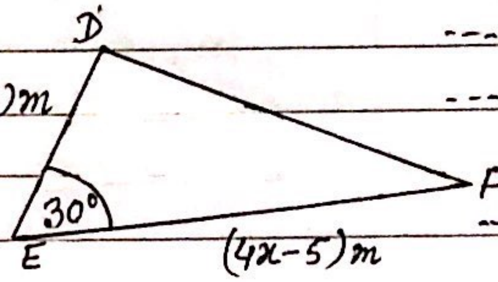
Area of $\Delta = \frac{1}{2} ab \sin C$.

15. The area of triangle DEF is $70\,m^2$

(i) Show that $4x^2+11x-300=0$ ---[4]

(ii) Use the quadratic formula to solve $4x^2+11x-300=0$ ---[4]

(iii) Find the length of DE. ---[1]



| W-19 | 42 | Q6(b) |

Solution:

(i) Area of triangle DEF $= \frac{1}{2}(x+4)(4x-5)\sin 30 = 70$ (Given)

$\frac{1}{2} \times (4x^2+11x-20) \times \frac{1}{2} = 70$

$\Rightarrow 4x^2+11x-20 = 280$

$\Rightarrow 4x^2+11x-300=0$ ✓

(ii) $4x^2+11x-300=0$

$a=4,\ b=11,\ c=-300$

$b^2-4ac = 11^2-4\times4\times(-300)$

$= 4921$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-11 \pm \sqrt{4921}}{2\times4} = \frac{-11 \pm 70.15}{8} = \frac{-81.15}{8}, \frac{59.15}{8}$

$\therefore x = \underline{-10.14}$ or $x = \underline{7.39}$ ✓

(iii) $DE = x+4$

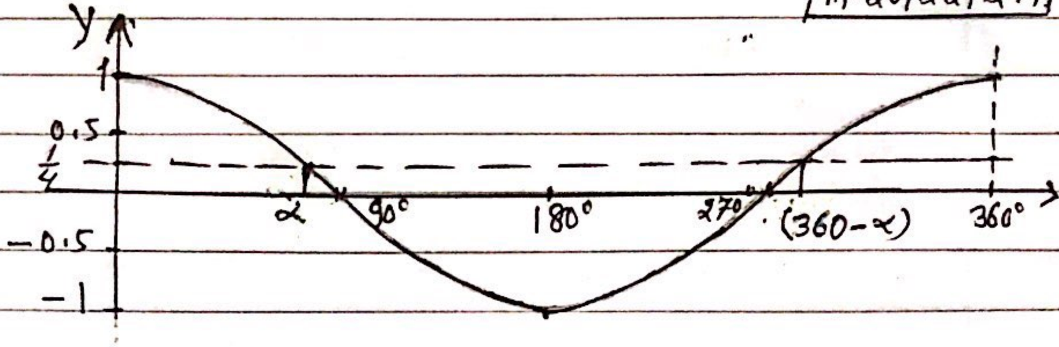
$= 7.4+4$ ($x=7.4$ +ve value)

$= \underline{11.4\,m}$ ✓

16. (a) On the diagram, sketch the graph of $y = \cos x$ for $0 \le x \le 360°$ ---[2]
(b) Solve the equation $4\cos x + 2 = 3$ for $0 \le x \le 360°$ ---[3]

M-20/22/Q19

Solution:



(b) $4\cos x + 2 = 3 \Rightarrow \cos x = \frac{1}{4} = \cos\alpha$ (let)

$\alpha = \cos^{-1}\frac{1}{4} = 75.5°$

$\therefore x = 75.5°$ and $360 - 75.5°$

$x = \underline{75.5°}$ and $\underline{284.5}$ ✓

17. When $\sin x° = 0.36$, find
(a) the acute angle $x°$, ---[1]
(b) the obtuse angle $x°$ ---[1]

Solution (a) $\sin x° = 0.36$

$\Rightarrow x° = \sin^{-1} 0.36 = \underline{21.1°}$ ✓

(b) Obtuse angle $= (180 - 21.1)$

$= \underline{158.9°}$ ✓

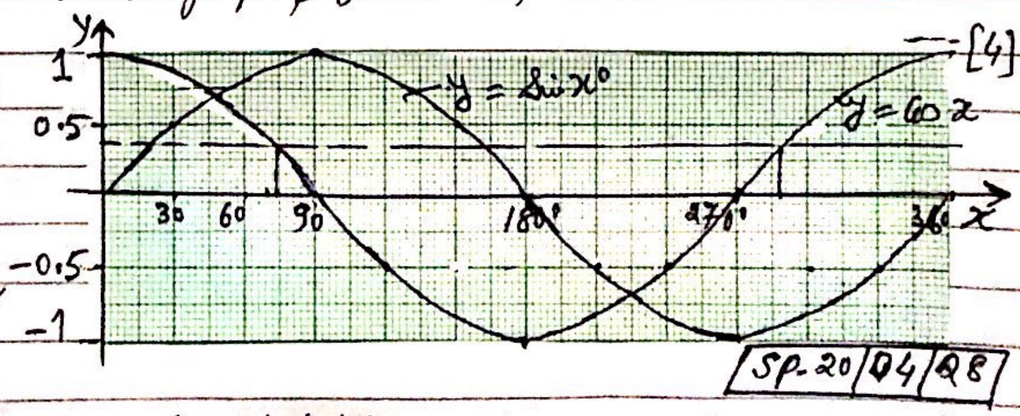
18. The grid shows the graph of $y = \cos x$ for $0° \le x \le 360°$

(a) Solve the equation $3\cos x = 1$ for $0 \le x \le 360°$ ---[4]

Ans $\cos x = \frac{1}{3}$

$x = 70.5°$ ✓

or $360 - 70.5 = 289.5$ ✓



SP-20/04/Q8

(b) On the same grid, sketch the graph of $y = \sin x$ for $0 \le x \le 360°$ --[2]

19.(a) (i) On the axis, sketch the graph of $y = \sin x$ for $0 \leq x \leq 360°$ ---[2]

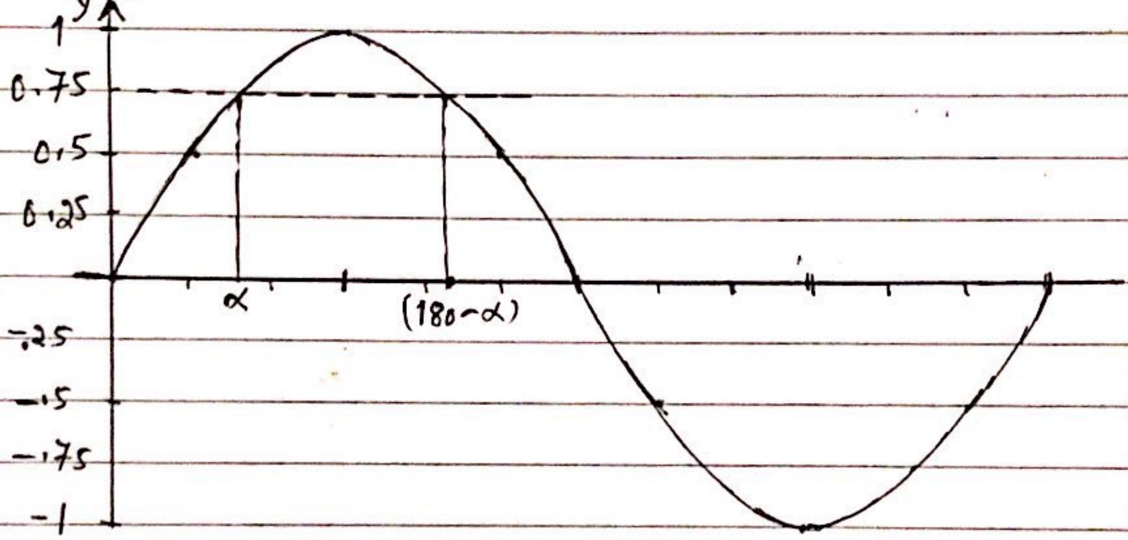
(ii) Describe fully the symmetry of the graph of $y = \sin x; 0 \leq x \leq 360$ -- [2]

(b) Solve $4 \sin x - 1 = 2$ for $0 \leq x \leq 360°$ --[3]

S-20 | 41 | Q8(a)(b)

Solution: $y = \sin x \qquad 0 \leq x \leq 360°$

a(i)



(ii) Rotational symmetry of order 2, centre (180; 0)

(iii) Solve: $4 \sin x - 1 = 2 \qquad 0° \leq x \leq 360°$

$\sin x = \frac{3}{4} = 0.75$

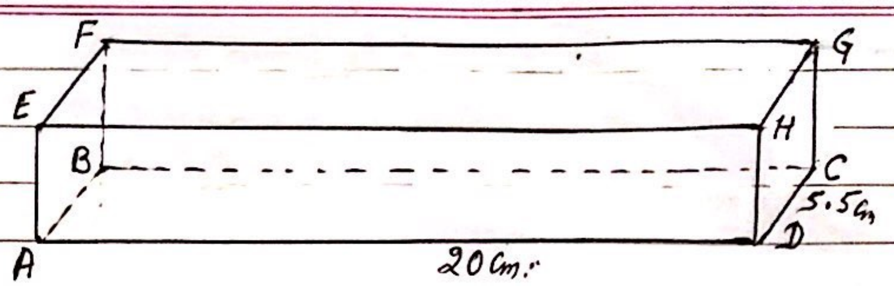
$x = \sin^{-1} 0.75 = 48.6°$

$\therefore x = 48.6° \;;\; 180 - 48.6°$

$x = \underline{48.6} \;;\; x = \underline{131.4°}$

Three dimensions problems.

20.



The diagram shows cuboid ABCDEFGH of length 20 cm and width 5.5 cm. The volume of the cuboid is 495 cm^3.
Find the angle between the line AG and the base of the cuboid ABCD.

[S-20 | 21 | Q 19]

Solution: Volume of cuboid $= l \times b \times h$ [Let $GC = h$ cm]

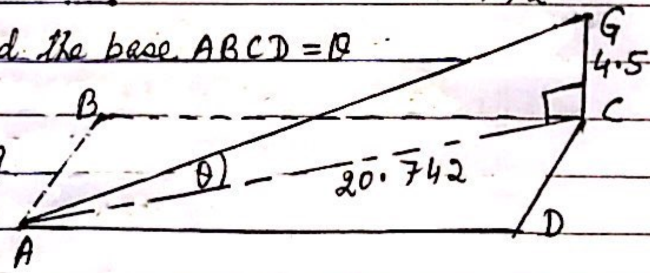
$$= 20 \times 5.5 \times h = 495 \quad \text{(Given)}$$

$$\Rightarrow h = \frac{495}{20 \times 5.5} = 4.5 \text{ cm.} \quad \text{---} (1)$$

Now In rectangular base ABCD, $AC^2 = 20^2 + 5.5^2 = 430.25$

$$AC = \sqrt{430.25} = 20.742$$

Let the angle between AG and the base ABCD $= \theta$



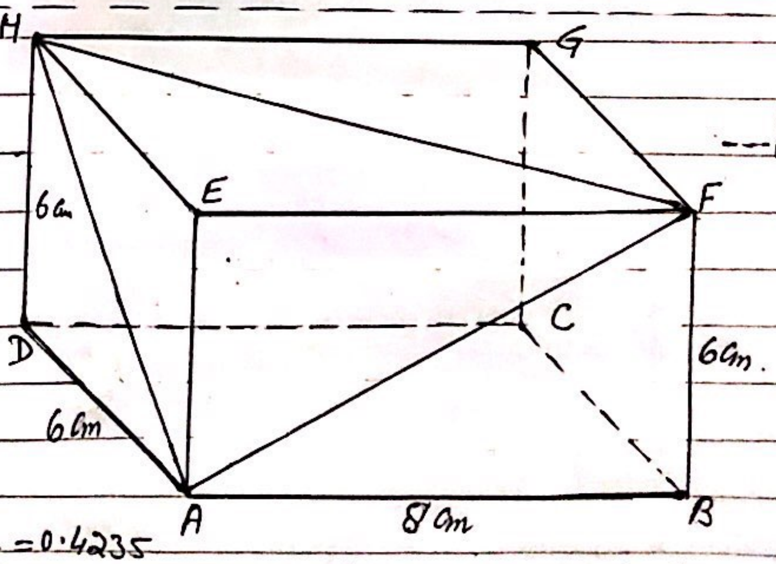
$$\tan\theta = \frac{GC}{AC} = \frac{4.5}{20.742} = 0.2169$$

$$\therefore \theta = \tan^{-1} 0.2169$$

Angle GAC $= \underline{12.24°}$ ✓

---

21. The diagram shows a cuboid, AB = 8 cm. AD = 6 cm, BE = 6 cm. Calculate angle HAF. --- [6]



Solution: Using Cosine Rule:

$AF = \sqrt{8^2 + 6^2} = 10$

$FH = \sqrt{8^2 + 6^2} = 10$

$AH = \sqrt{6^2 + 6^2} = 6\sqrt{2}$

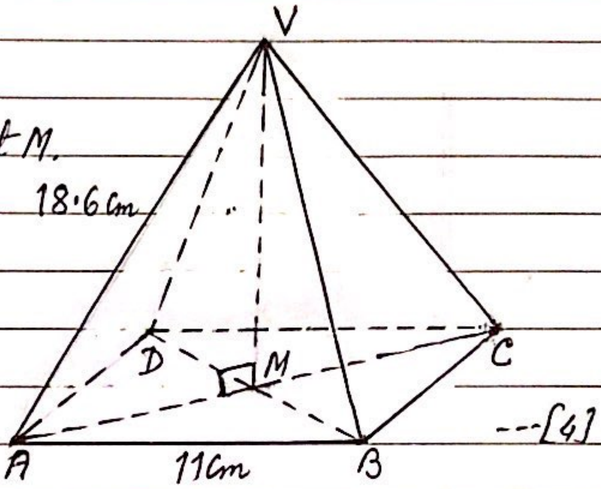
$$\cos HAF = \frac{100 + 72 - 100}{2 \times 10 \times 6\sqrt{2}} = \frac{72}{170} = 0.4235$$

$$\left[\because \cos HAF = \frac{AF^2 + AH^2 - FH^2}{2 \times AF \times AH}\right]$$

$\Rightarrow$ Angle HAF $= \cos^{-1}(0.4235) = \underline{64.9°}$ ✓

[S-20 | 22 | Q 27]

22. The diagram shows a pyramid with a square base ABCD.
The diagonals AC and BD intersect at M.
The vertex V is vertically above M.
AB = 11 cm and AV = 18.6 cm.
Calculate the angle that AV makes with the base.



---[4]

[S-20/23/Q20]

Solution: $AC = \sqrt{11^2+11^2} = 15.556$

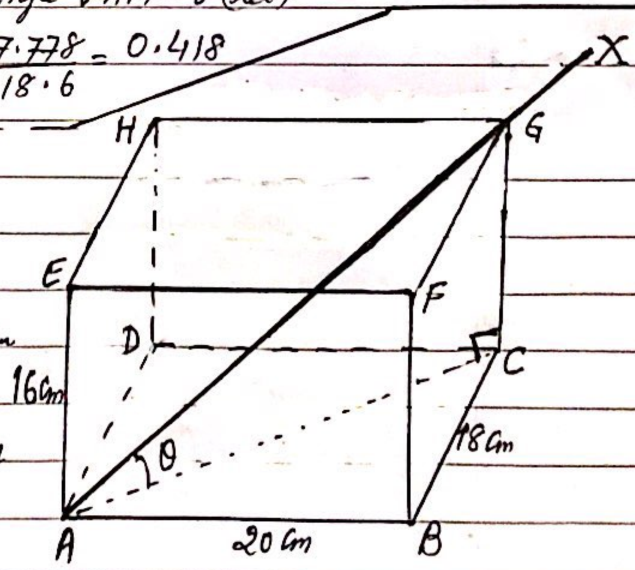
$AM = \frac{1}{2} AC = \frac{1}{2} \cdot 15.556 = 7.778$ cm

Angle between AV and base = angle VAM = θ (let)

$\cos\theta = \frac{AM}{VA} = \frac{7.778}{18.6} = 0.418$

$\Rightarrow \theta = \cos^{-1} 0.418 = \underline{65.28°}$ ✓

23. The diagram shows an open box ABCDEFGH in the shape of a cuboid. AB = 20 cm, BC = 18 cm and AE = 16 cm.
A thin rod AGX rests partly in the box as shown.
The rod is 40 cm long.



(i) Calculate GX, the length of the rod which is outside the box. ---[4]
(ii) Calculate the angle the rod makes with the base of the box. ---[3]

[M-20/42/Q8(b)]

Solution:
$AG = \sqrt{AC^2+GC^2} = \sqrt{(AB^2+BC^2)+GC^2} = \sqrt{20^2+18^2+16^2} = 31.3$

(i) $\therefore GX = AX - AG = 40 - 31.3 = \underline{8.7}$ ✓

(ii) Let the angle between the rod AG and base = $\angle GAC = \theta$ (let)

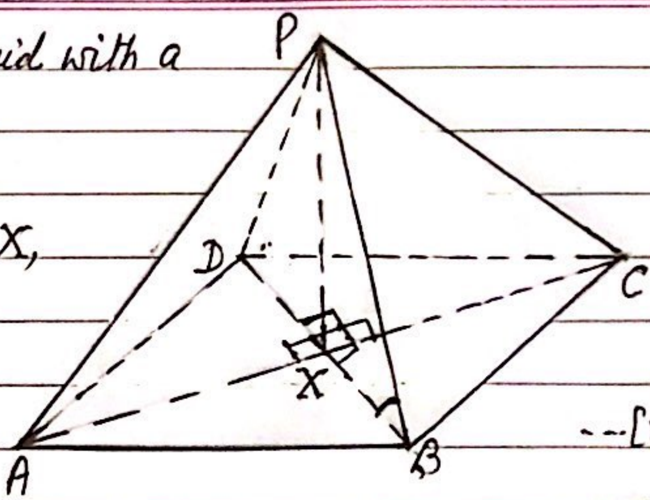
In rt Δ ACG: $\theta = \frac{GC}{AG} = \frac{16}{31.6} = 0.5112$

$\therefore \theta = \sin^{-1} 0.5112 = 30.74°$

$\therefore \underline{\theta = 30.7°}$ ✓

[M-20/42/Q8(b)]

22. The diagram shows a pyramid with a square base ABCD.
DB = 8 cm.
P is vertically above the centre X, of the base and PX = 5 cm.
Calculate the angle between PB and the base ABCD.



---[3]

[W-19/41/Q4(]

Solution: $XB = \frac{1}{2} BD = \frac{1}{2} \times 8 = 4$ cm
$PX = 5$ cm (Given)
In rt Δ PXB, (angle PXB = 90°)
$\tan\theta = \frac{PX}{BX} = \frac{5}{4} = 1.25$ (Let angle PBX = θ)

$\therefore \theta = \tan^{-1} 1.25 = 51.34^\circ$
$\therefore$ angle between PB and the base ABCD = 51.34° ✓