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(0580)

IGCSE Maths

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Vectors and Transformations Revision

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Vectors

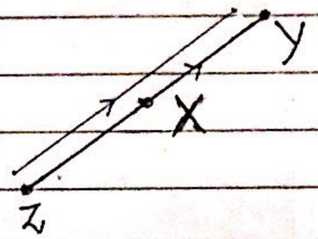
1 Point A has coordinates (6,4) and point B has cordinate (2,7)
Write $\overrightarrow{AB}$ as a column vector. [M-20/22/Q3] ---[1]

Solution: $\overrightarrow{AB} = \begin{pmatrix} 2-6 \\ 7-4 \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$ ✓

$\because$ $A(x_1, y_1)$, $B(x_2, y_2)$
$\overrightarrow{AB} = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix}$

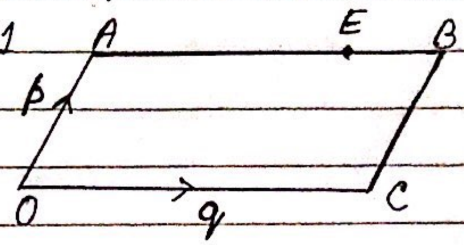
2. $\overrightarrow{XY} = 3a + 2b$ and $\overrightarrow{ZY} = 6a + 4b$
Write down two statements about the relationship between the points X, Y and Z. [M-20/22/Q21] ---[2]

Solution: $\overrightarrow{ZY} = 6a + 4b$
$= 2(3a + 2b)$
$= 2 \cdot \overrightarrow{XY}$



(i) X, Y and Z are collinear.
(ii) ZY is twice the magnitude of XY
or X is the mid point of ZY.

3(a)(i) $m = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$, find $3m$ ---[1]
(ii) $\overrightarrow{VW} = \begin{pmatrix} 10 \\ -24 \end{pmatrix}$, find $|\overrightarrow{VW}|$ --[2]
(b) OABC is a parallelogram. $\overrightarrow{OA} = p$ and $\overrightarrow{OC} = q$ [S-20/21/Q17]
E is a point on AB, such that AE : EB = 3 : 1
Find $\overrightarrow{OE}$, in terms of p and q.

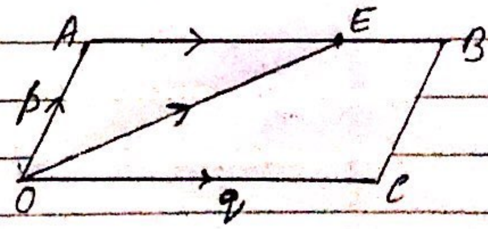


Solution: (a)(i) $m = \begin{pmatrix} 5 \\ 7 \end{pmatrix} \Rightarrow 3m = \begin{pmatrix} 5 \times 3 \\ 7 \times 3 \end{pmatrix} = \begin{pmatrix} 15 \\ 21 \end{pmatrix}$ ✓

(a)(ii) $\overrightarrow{VW} = \begin{pmatrix} 10 \\ -24 \end{pmatrix} \Rightarrow |\overrightarrow{VW}| = \sqrt{(10)^2 + (-24)^2} = \sqrt{676} = 26$ ✓

(b) $AE : EB = 3:1 \Rightarrow AE = \frac{3}{4} AB \Rightarrow \overrightarrow{AE} = \frac{3}{4} \overrightarrow{OC} = \frac{3}{4} q$ —① ($\because AB \parallel OC$)

In ΔOAE, $\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE}$
$= p + \frac{3}{4} q$ ✓



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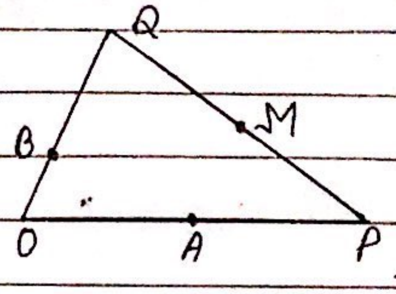
4. O is the origin, $\vec{OP} = 2\vec{OA}$, $\vec{OQ} = 3\vec{OB}$ and $\vec{PM} = \vec{MQ}$

$\vec{OP} = p$ and $\vec{OQ} = q$,

Find, in terms of p and q, in its simplest form.



(a) $\vec{BA}$ ...[2]

(b) the position vector of M

[W-19/21/Q25] ...[2]

Solution: $3\cdot\vec{OB} = \vec{OQ} = q \Rightarrow \vec{OB} = \frac{1}{3}q$ —— ①

(a) and $2\cdot\vec{OA} = \vec{OP} = p \Rightarrow \vec{OA} = \frac{1}{2}p$ —— ②

Now $\vec{BA} = \vec{OA} - \vec{OB}$

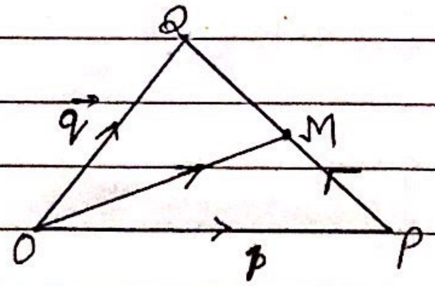
$= \frac{1}{2}p - \frac{1}{3}q$

(b) $\vec{OM} = \vec{OP} + \vec{PM}$ —— ③

Now $\vec{PM} = \frac{1}{2}\vec{PQ}$ (as $\vec{PM} = \vec{MQ}$)

$\vec{PM} = \frac{1}{2}(q - p)$ ($\because \vec{PQ} = q - p$)

Put in ③



$\vec{OM} = p + \frac{1}{2}(q - p)$

$= p - \frac{1}{2}p + \frac{1}{2}q$

$= \frac{1}{2}p + \frac{1}{2}q$

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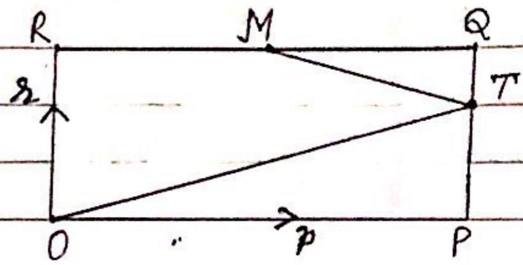
5. OPQR is a rectangle and O is the origin.
M is the mid point of RQ and
$PT : TQ = 2:1$
$\vec{OP} = p$ and $\vec{OR} = r$



(a) Find in terms of p and/or r, in its simplest form.

(i) $\vec{MQ}$ --[1]

(ii) $\vec{MT}$ --[1]

(iii) $\vec{OT}$ --[1]

(b) RQ and OT are extended and meet at U.
Find the position vector of U in terms of p and r. --[2]

(c) $\vec{MT} = \begin{pmatrix} 2k \\ -k \end{pmatrix}$ and $|\vec{MT}| = \sqrt{180}$
Find the positive value of k. $\boxed{SP\text{-}20/04/Q6}$ --[3]

Solution (a) (i) M is the mid point of RQ $\Rightarrow \vec{MQ} = \frac{1}{2}\vec{RQ} = \frac{1}{2}\vec{OP} = \underline{\frac{1}{2}p}$ ✓

(ii) $\vec{MT} = \vec{MQ} + \vec{QT} = \vec{MQ} + \frac{1}{3}\vec{QP}$
$= \underline{\frac{1}{2}p - \frac{1}{3}r}$ ✓

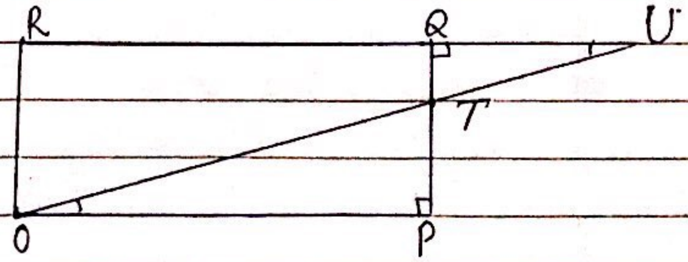
$\left(\begin{array}{l} PT : TQ = 2:1. \\ \Rightarrow \vec{QT} = \frac{1}{3}\vec{QP} = \frac{1}{3}(-\vec{PQ}) \\ = -\frac{1}{3}\vec{OR} = -\frac{1}{3}r \end{array} \right)$

(iii) $\vec{OT} = \vec{OP} + \vec{PT} = \vec{OP} + \frac{2}{3}\vec{PQ} = \vec{OP} + \frac{2}{3}\vec{OR}$
$= \underline{p + \frac{2}{3}r}$ ✓ —— (1)

(b) $\Delta QUT \overset{\text{Similar}}{\sim} \Delta POT$

$\frac{TU}{OT} = \frac{QT}{PT} = \frac{1}{2}$

$TU = \frac{1}{2}OT$ —— (2)



Now Position vector of U is $\vec{OU} = \vec{OT} + \vec{TU}$
$= \vec{OT} + \frac{1}{2}\vec{OT}$ (from (2))
$= \frac{3}{2}\vec{OT}$
$= \frac{3}{2}(p + \frac{2}{3}r)$
$= \underline{\frac{3}{2}p + r}$ ✓

(c) $\vec{MT} = \begin{pmatrix} 2k \\ -k \end{pmatrix}$

$\Rightarrow |\vec{MT}| = \sqrt{(2k)^2 + (-k^2)}$
$= \sqrt{4k^2 + k^2} = \sqrt{180}$ given
$= 5k^2 = 180$
$\Rightarrow k^2 = 36$
$k = \sqrt{36} = \underline{6}$ ✓ (Only +)

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6(a) $p = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ $q = \begin{pmatrix} -2 \\ 7 \end{pmatrix}$ [S-20/42/Q2(a),(b),(d)]

(i) find $2p+q$ ---[2]

(ii) find $|p|$ ---[2]

Solution: (i) $2p+q = 2\cdot\begin{pmatrix} 4 \\ 5 \end{pmatrix} + \begin{pmatrix} -2 \\ 7 \end{pmatrix}$

$= \begin{pmatrix} 8 \\ 10 \end{pmatrix} + \begin{pmatrix} -2 \\ 7 \end{pmatrix} = \begin{pmatrix} 8-2 \\ 10+7 \end{pmatrix} = \underline{\begin{pmatrix} 6 \\ 17 \end{pmatrix}}$ ✓

(ii) $|p| = \sqrt{4^2+5^2} = \sqrt{16+25}$

$= \sqrt{41} = \underline{6{\cdot}403}$ ✓

(b) A is the point (4,1) and $\vec{AB} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$

Find the coordinates of B. ---[1]

Solution: Let the coordinates of B are (x, y), $A(4,1)$

$\Rightarrow \vec{AB} = \begin{pmatrix} x-4 \\ y-1 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ Given

$\Rightarrow x-4 = -3 \Rightarrow x = 1$

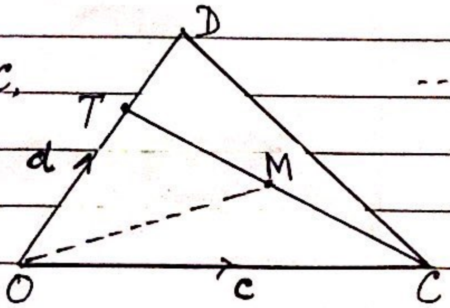
$y-1 = 1 \Rightarrow y = 2$

$\therefore \underline{B(1,2)}$ ✓

(c) In the diagram, O is the origin, $OT = 2TD$ and M is the mid point of TC. ---[3]

$\vec{OC} = \mathbf{c}$ and $\vec{OD} = \mathbf{d}$

Find the position vector of M. Give your answer in terms of **c** and **d**.



Solution: Join OM, Position vector of $M = \vec{OM}$

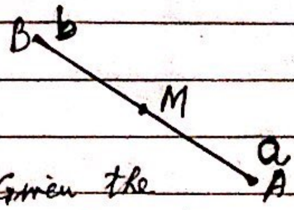
$OT = 2TD \Rightarrow OT = \frac{2}{3}OD \Rightarrow \vec{OT} = \frac{2}{3}\vec{OD} = \frac{2}{3}\vec{d}$ —— (1)

and position vector of $C = \vec{OC} = \vec{c}$ —— (2)

$\therefore$ Position vector of the mid point of TC

$\vec{OM} = \frac{\left(\vec{c} + \frac{2}{3}d\right)}{2} = \frac{1}{2}\left(c + \frac{2}{3}d\right)$

$= \frac{1}{2}\mathbf{c} + \frac{1}{3}\mathbf{d}$



Given the position vectors of A and B are **a** & **b**

P.V. of $M\left(\frac{a+b}{2}\right)$

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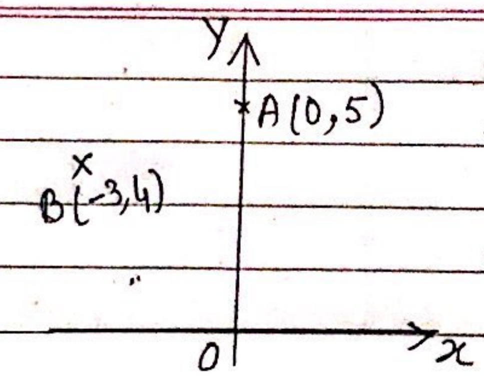
7.(a) (i) Write $\vec{OA}$ as a column vector. --[1]

(ii) Write $\vec{AB}$ as a column vector. --[1]

(iii) A and B lie on a circle, centre O.

Calculate the length of arc AB. --[6]



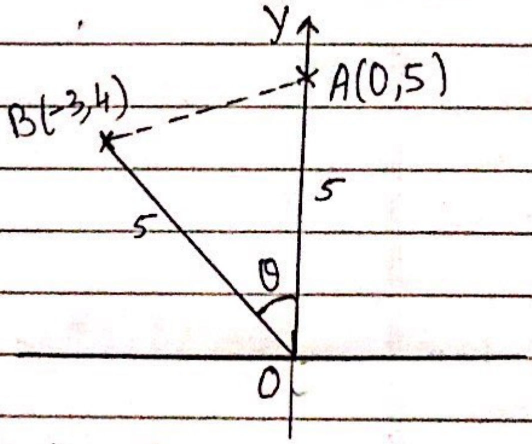
Solution (i) $\vec{OA} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$ $[A(0,5)]$

(ii) $\vec{OB} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} \Rightarrow \vec{AB} = \vec{OB} - \vec{OA}$

$= \begin{pmatrix} -3 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 5 \end{pmatrix} = \begin{pmatrix} -3-0 \\ 4-5 \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$ ✓

(iii) $OA = 5$

$OB = \sqrt{(-3)^2+4^2} = \sqrt{9+16} = 5$



$AB = |\vec{AB}| = \sqrt{(-3)^2+(-1)^2} = \sqrt{10}$

Using cosine rule; Let $\angle AOB = \theta$

$= \dfrac{OA^2+OB^2-AB^2}{2\times OA\times OB}$

$= \dfrac{5^2+5^2-(\sqrt{10})^2}{2\times5\times5} = \dfrac{25+25-10}{50} = \dfrac{4}{5}$

$\therefore$ angle $AOB = \theta = \cos^{-1}\left(\frac{4}{5}\right) = 36.87°$ ✓

Circle with centre at O, passes through AB,

$OA = OB =$ radius $r = 5$ ✓

$\therefore$ length of arc $\widehat{AB} = \dfrac{\theta}{360} \times 2\pi r$

$= \dfrac{36.87}{360} \times 2\pi \times 5$

$= 3.217$ units ✓

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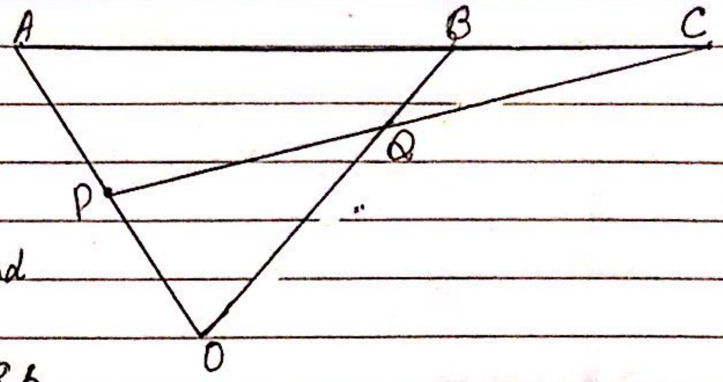
8. OAB is a triangle and ABC and PQC are straight lines.
P is the mid point of OA,
Q is the mid point of PC and
$OQ : QB = 3 : 1$.
$\vec{OA} = 4a$ and $\vec{OB} = 8b$.



W-19/43/Q11

(a) Find in terms of **a** and/or b.
(i) $\vec{AB}$ ---[1]
(ii) $\vec{OQ}$ ---[1]
(iii) $\vec{PQ}$ ---[1]
(b) By using vectors, find the ratio AB : BC. --[3]

Solution (a)(i) $\vec{OA} = 4a$ and $\vec{OB} = 8b \Rightarrow \vec{AB} = \vec{OB} - \vec{OA}$
$= \underline{8b - 4a}$ ✓

(ii) $OQ : QB = 3:1 \Rightarrow \vec{OQ} = \frac{3}{4}\vec{OB} = \frac{3}{4} \times 8b = \underline{6b}$ ✓ — (1)

(iii) $\vec{OP} = \frac{1}{2}\vec{OA}$ (Since P is the mid point of OA)
$= \frac{1}{2} \times 4a = 2a$ — (2)

$\therefore \vec{PQ} = \vec{OQ} - \vec{OP} = \underline{6b - 2a}$ ✓ (from (1) & (2))

(b) Q is the mid point of PC $\Rightarrow \vec{QC} = \vec{PQ} = 6b - 2a$
$\therefore \vec{QC} = 6b - 2a$ — (3)

In ΔQBC, $\vec{QB} + \vec{BC} = \vec{QC}$
$\therefore \vec{BC} = \vec{QC} - \vec{QB}$ $\left[\vec{QB} = \frac{1}{4}\vec{OB} = \frac{1}{4} \times 8b = 2b\right]$ — (4)
$= (6b - 2a) - 2b$
$= 4b - 2a$ — (5)

from Part (a)(i) $\vec{AB} = 8b - 4a$
$= 2(4b - 2a)$
$\vec{AB} = 2\vec{BC}$ (from (5))

$\therefore \frac{\vec{AB}}{\vec{BC}} = \frac{2}{1}$

or $AB : BC = 2:1$ ✓

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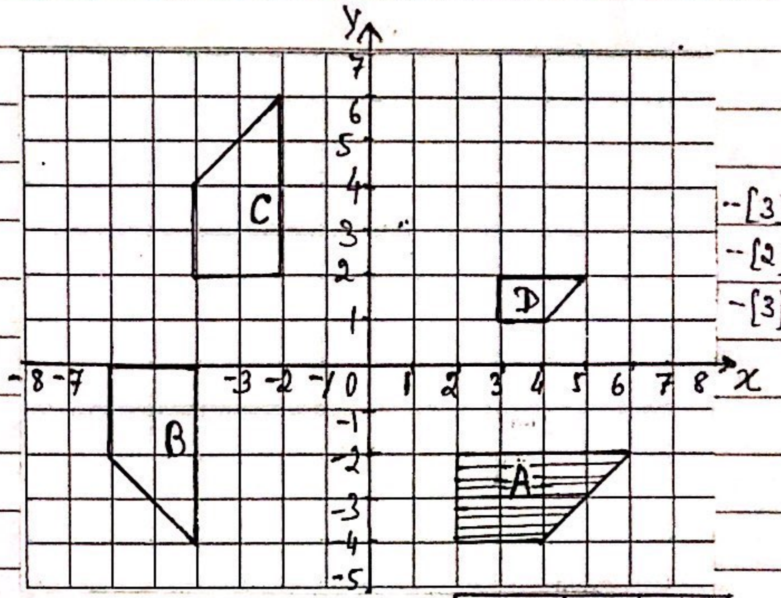
Transformations

9. Describe fully the single transformation that maps.

(a) Shape A onto shape B --[3]

(b) Shape A onto shape C --[2]

(c) Shape A onto shape D --[3]



[M-20 | 22 | Q11]

Solution:

(a) Rotation 90° clockwise, (0,2).

(b) Reflection. $y = x$

(c) Enlargement (sf) $\frac{1}{2}$, (4,6).

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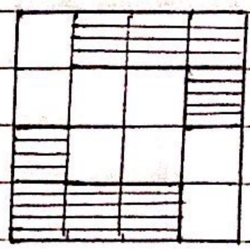
10. Write down the order of rotational symmetry of the diagram. [S-20 | 22 | Q1] ---[1]



Answer: Order of rotational symmetry = 2.

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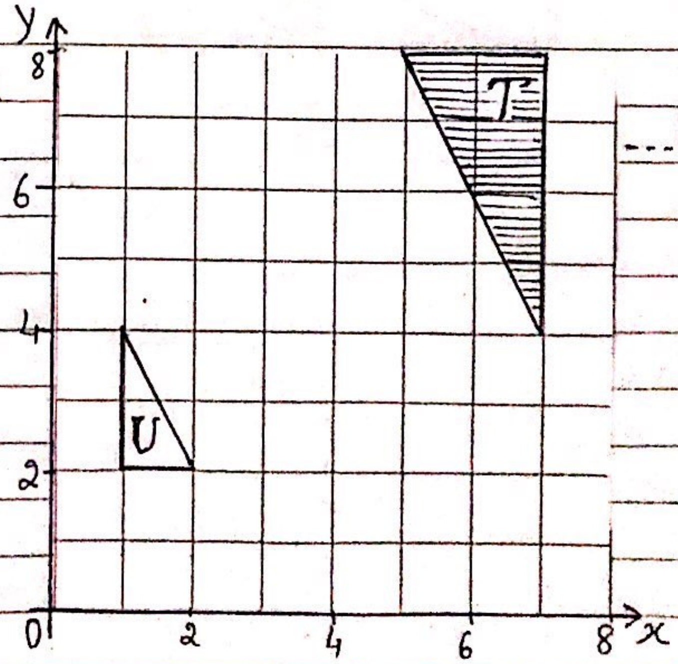
11. Describe fully the single transformation that maps triangle T onto triangle U. ---[3]



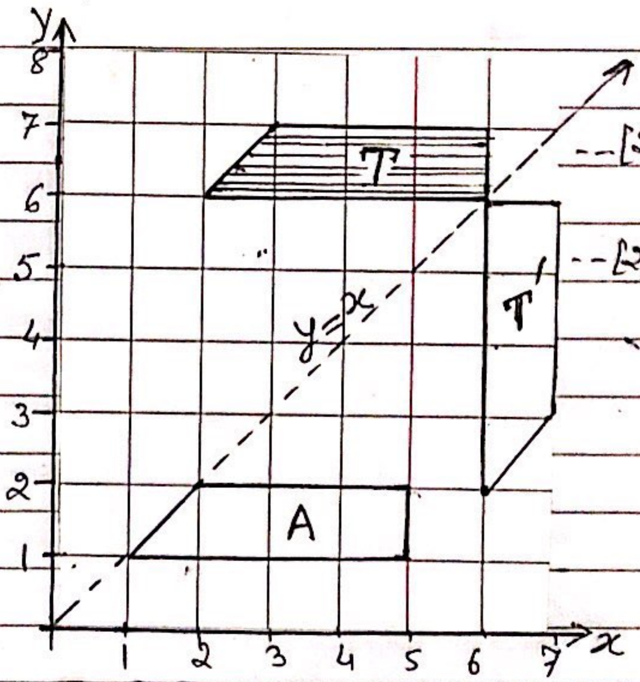
Answer: Enlargement
scale factor $-\frac{1}{2}$
Centre (3,4)

[S-20 | 22 | Q18]

12.(a) Describe fully the single transformation that maps shape T onto shape A. --[2]

(b) on the grid reflect T in the line y=x --[2]

W-19/21/Q20



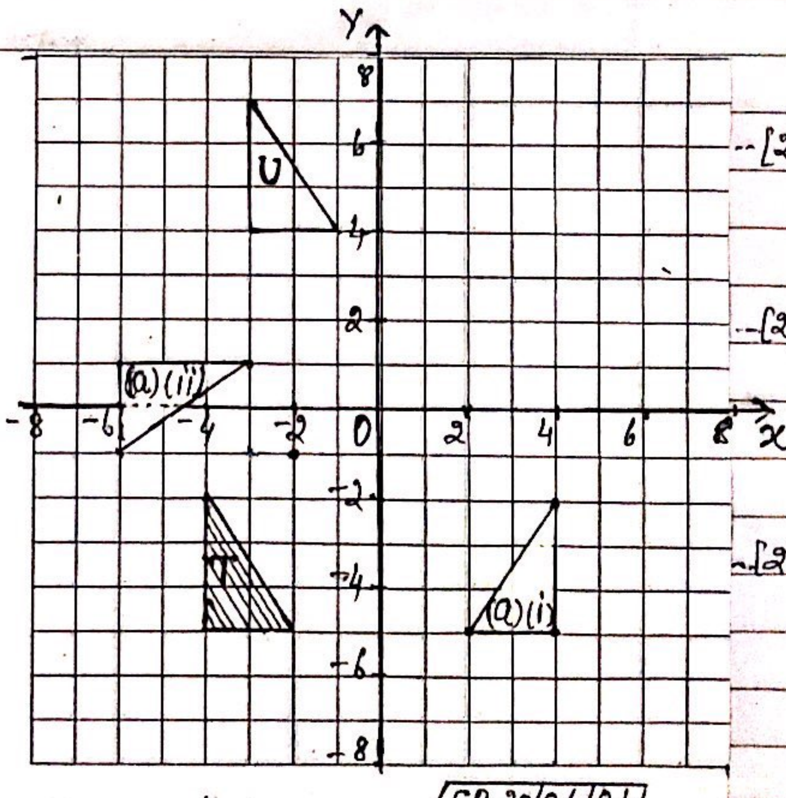
Solution(a) Translatation $\begin{pmatrix}-1\\-5\end{pmatrix}$

(b) Correct reflection at, T'
(6,2), (6,6), (7,6), (7,3)

13.(a)(i) Draw the reflection of triangle T in the line x=0 --[2]

(a)(ii) draw the rotation of Triangle T about (-2,-1) through 90°, clockwise. --[2]

(b) describe fully the single transformation that maps triangle T onto triangle U. --[2]



SP-20/04/04

Solution (i) Image at (2,-5), (4,-5), (4,-2)

(a)(ii) Image at (-3,1), (-6,1), (-6,-1)

(b) Translation $\begin{pmatrix}1\\9\end{pmatrix}$

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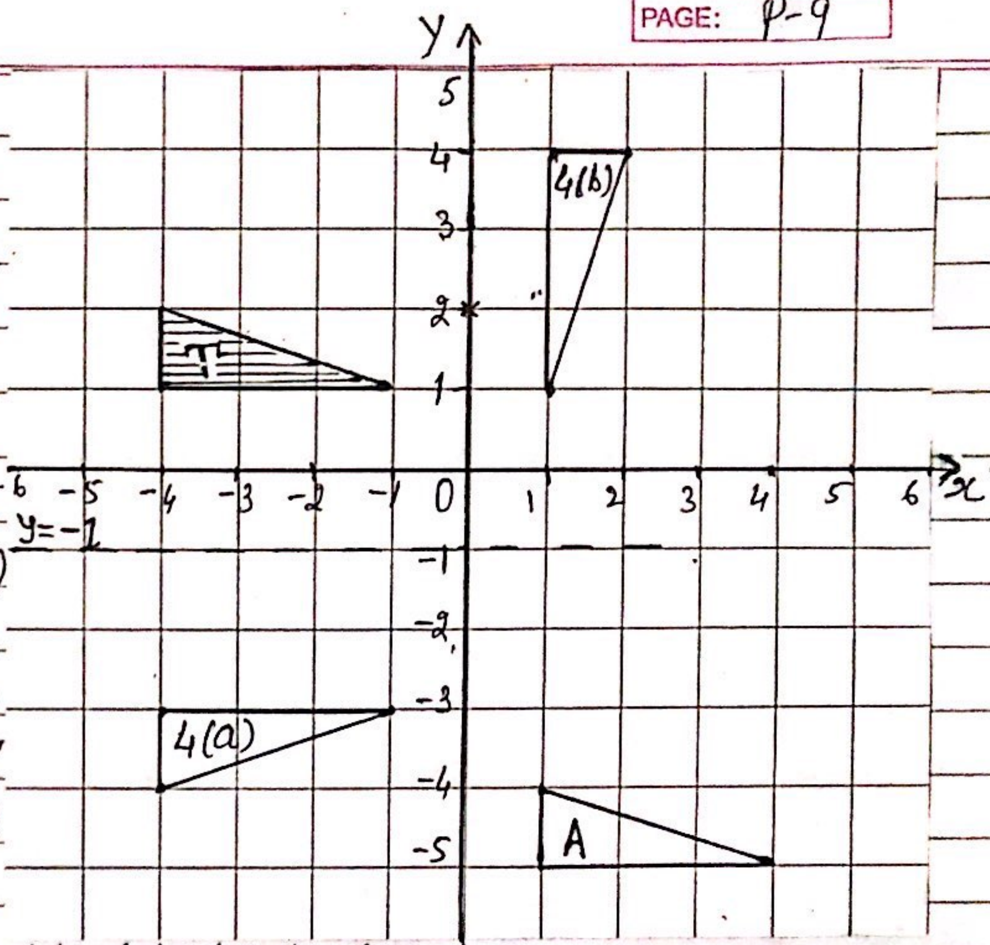
14 (a) Draw the image of triangle T after a reflection in the line $y=-1$ ---[2]

(b) Draw the image of Triangle T after a rotation through 90° clockwise about (0,2)

(c) Describe fully the single transformation that maps triangle T onto triangle A. --[2]



Solution (a) Triangle at (-4,-4), (-1,-3), (-4,-3)

(b) Triangle at (1,1), (1,4), (2,4)

(c) Translation $\begin{pmatrix}5\\-6\end{pmatrix}$

S-20/41/Q4

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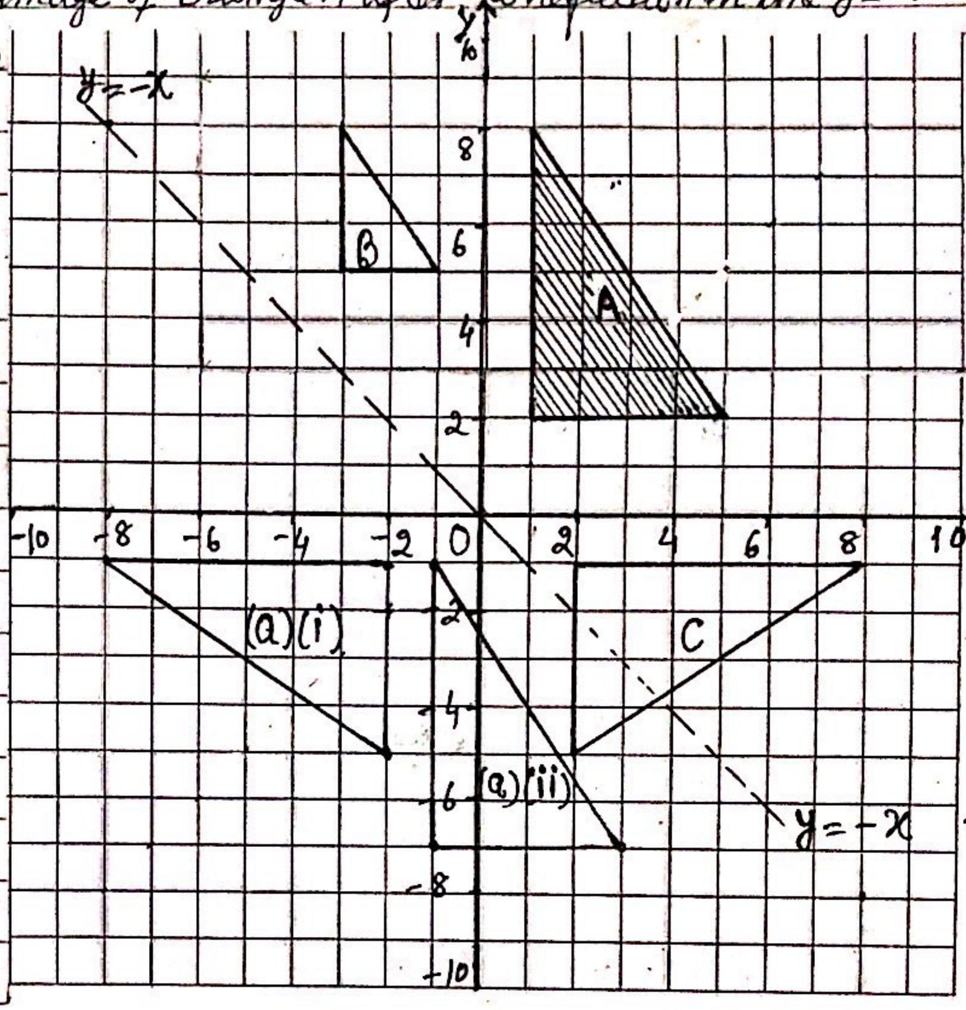
15 (a)(i) Draw the image of triangle A after a reflection in line $y = -x$ ---[2]

(ii) Draw the image of triangle A after a translation by the vector $\begin{pmatrix} -2 \\ -9 \end{pmatrix}$ --[2]

(b) describe fully the single transformation that maps:

(i) triangle A onto triangle B. --[3]

(ii) triangle A onto triangle C. --[3]



S-20/43/Q2

Solution:

(a)(i) triangle with vertices $(-2,-1)$, $(-8,-1)$, $(-2,-5)$

(ii) triangle with vertices: $(-1,-1)$, $(-1,-7)$, $(3,-7)$.

b(i) Enlargement, centre $(-7, 8)$, sf $-\frac{1}{2}$

(ii) Rotation, centre $(0,0)$, 90° clockwise.

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16 A line joins A(1,3) to B(5,8)

(a) The line is transformed to line PQ.
Find the coordinates of P and the coordinates of Q after AB is transformed by:

(i) a translation by the vector $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$ ---[2]

(ii) a rotation through 90° anticlockwise about the origin ---[2]

(iii) a reflection in the line $x=2$

(b) Describe fully the single transformation that maps the line AB onto the line PQ where P is the point (-2,-6) and Q(-10,-16). --- [3]

W-19/42/Q3(b)(C)

Solution (a)(i) (6,1), (10,6)

(ii) (-3,1), (-8,5)

(iii) (3,3), (-1,8)

(b) Enlargement, s.f. -2, Centre (0,0)

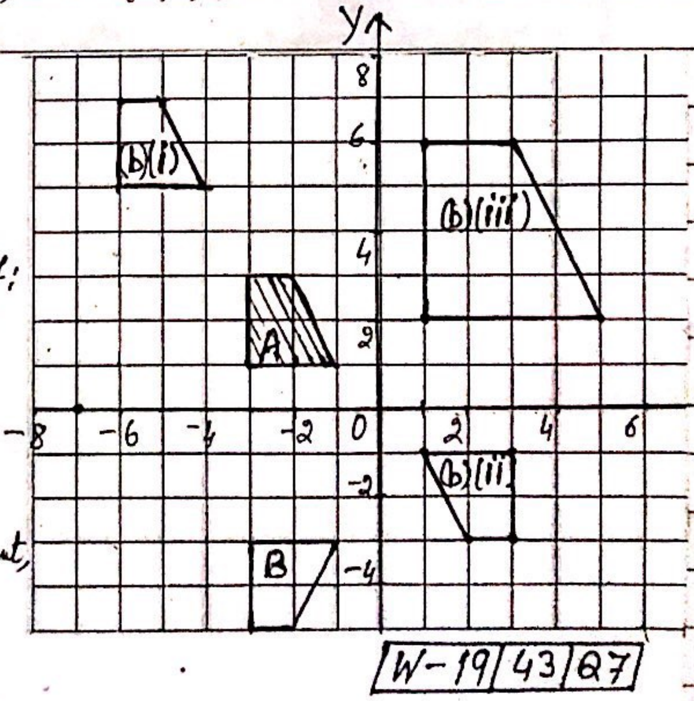
17(a) Describe fully the single transformation that maps shape A onto shape B. --[2]

(b) On the grid, draw the image of:

(i) Shape A after translation by the vector $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$. --[2]

(ii) Shape A after a rotation through 180° about (0,0)

(iii) Shape A after an enlargement, scale factor 2, Centre (-7,0)



W-19/43/Q7

Solution (a) Reflection on $y=-1$

(b)(i) Image at (-6,5), (-6,7), (-5,7), (-4,5)

(ii) Image at (1,-1), (3,-1), (3,-3), (2,-3)

(iii) Image at (1,2), (1,6), (3,6), (5,2)

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