IG-Maths
0580

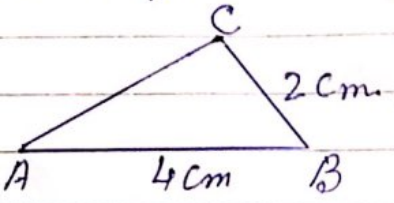
Vectors - Notes

IG-Maths 0580

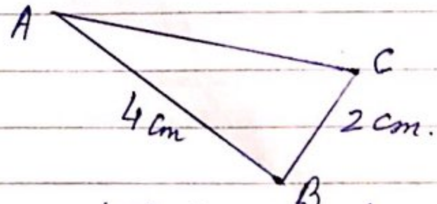
Vectors Notes

Scalar Quantities: A scalar quantity has magnitude (unit and a real number), length, mass, distance, speed, density are all scalar quantities.

Example:



Length of side AB = 4 cm.
Length of side BC = 2 cm.



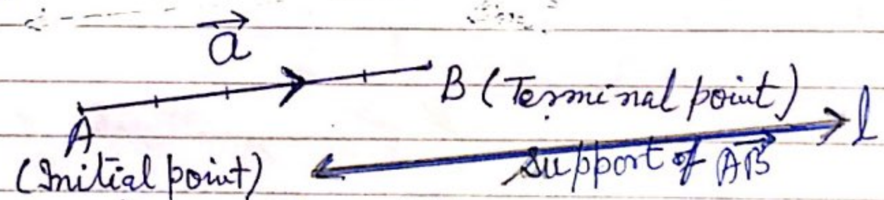
(Note: Here direction of the sides) is not considered)

Vectors: A vector quantity has magnitude (unit and real no) and direction (support and sense).
Velocity, displacement, force are vector quatities.

Geometric representation of a Vector:

Vectors are represented by directed line segments.

Example: A force of 5 newtons is applied, it is denoted by "**a** — (bold face letter) or $\vec{a}$ or $\underset{\sim}{a}$ or **AB** or $\overrightarrow{AB}$"



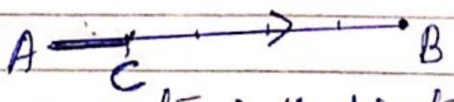
Magnitude of Vector $\overrightarrow{AB} = |\overrightarrow{AB}|$ or $|\vec{a}|$ = 5 Newton

{In this particular example).
Unit - Newton
Real no - 5

Direction of Vector $\overrightarrow{AB}$:

support AB // line l or // line AB (on $\overleftrightarrow{AB}$)
sense: A to B. (Shown by arrow)

Unit Vector:

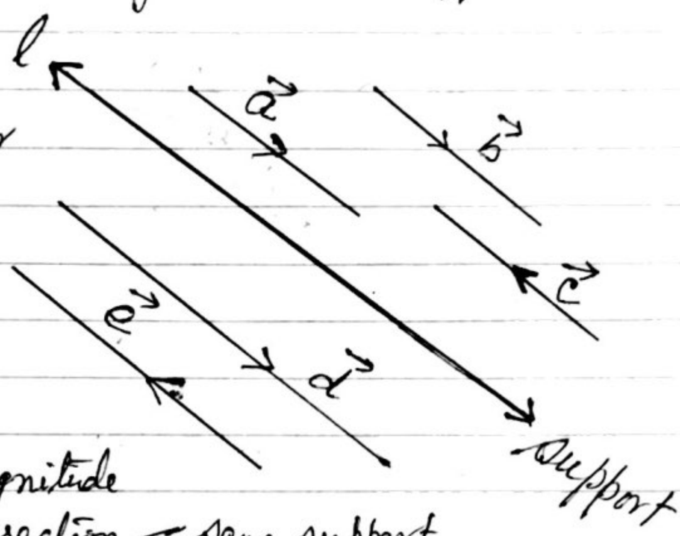


Given a Vector $\overrightarrow{AB}$, then a vector in the direction of $\overrightarrow{AB}$ and magnitude = 1

Unit Vector $\overrightarrow{AC} = \dfrac{\overrightarrow{AB}}{|\overrightarrow{AB}|}$

Collinear Vectors: Vectors having the same support.

Vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ and $\vec{e}$ are all collinear vectors as they have the same support (line l)



(a) Equal Vectors: $\vec{a} = \vec{b}$

(i) same magnitude

and (ii) same direction ← same support, same sense

(b) Parallel Vectors: $\vec{m} = k\vec{n}$ $k \neq 0$ real no.

all the vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}, \vec{e}$ are parallel vectors

(i) if $k > 0$ same direction ; $\vec{d} = 2\vec{a}$

Vectors $\vec{a}, \vec{b}, \vec{d}$ are in the same direction.

and $\vec{c}$ and $\vec{e}$ are also in the same direction.

(ii) If $k < 0$

Vectors $\vec{a}$ and $\vec{e}$ are in opposite directions

as $\vec{e} = -\frac{3}{2}\vec{a}$

and $\vec{d} = -2\vec{c}$

(c) Opposite Vectors:

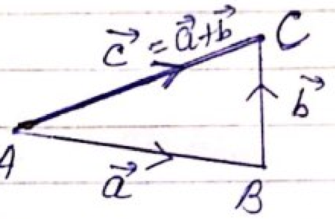
$\vec{b} = -\vec{c}$

(i) same magnitude but opposite direction (sense) (same magnitude and same support but opp.)

Vectors Notes

Addition of Vectors (Triangle Law of Vector addition):

$$\vec{AB} + \vec{BC} = \vec{AC} \quad [\vec{a}+\vec{b}=\vec{c}]$$

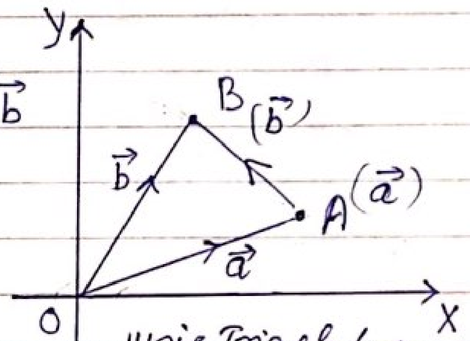


If two vectors are denoted by two sides of a triangle with their magnitude and directions taken in same order then the third side denotes their vector sum by magnitude and direction in opposite order.

[or The head of vector $\vec{a}$ connects to the tail of vector $\vec{b}$, Then the vector obtained by joining the tail of $\vec{a}$ to the head of vector $\vec{b}$ gives their vector sum)

Position Vector of a point:

Let $\vec{OA}=\vec{a}$ and $\vec{OB}=\vec{b}$



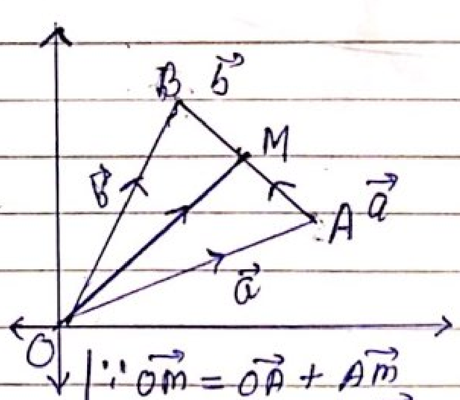
Let the reference point is origin O.
Then the position Vector of point $A = \vec{a}$
and the position Vector of point $B = \vec{b}$

Note: $\vec{AB} = \vec{b} - \vec{a}$
(Position Vector Terminal point) − (Position vector of Initial point)

$$\boxed{\vec{AB} = \vec{b} - \vec{a}}$$

Using Triangle law;
$\because$ In ΔOAB
$\vec{OA} + \vec{AB} = \vec{OB}$
$\therefore \vec{AB} = \vec{OB} - \vec{OA}$
$= \vec{b} - \vec{a}$

Position Vector of Mid point:



Given the position Vectors of points A and B (with reference to origin O) are $\vec{a}$ and $\vec{b}$. Let M is the mid point of segment AB.
Then the position Vector of the mid point,

$$\vec{OM} = \left(\frac{\vec{a}+\vec{b}}{2}\right)$$

$\because \vec{OM} = \vec{OA} + \vec{AM}$
$= \vec{a} + \frac{1}{2}\vec{AB}$
$= \vec{a} + \frac{1}{2}(\vec{b}-\vec{a})$
$= \frac{(\vec{a}+\vec{b})}{2}$

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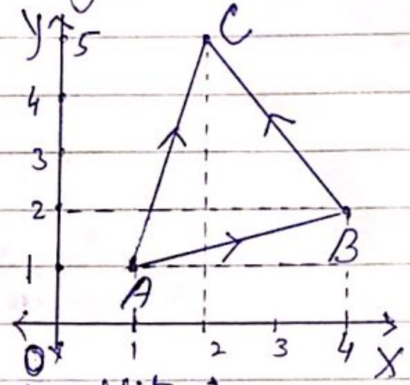
Vectors (In two Dimensions) Notes

§ **Column Vector:** A column vector indicates a **Translation** (or a shift) in the direction of x-axis and along y-axis.

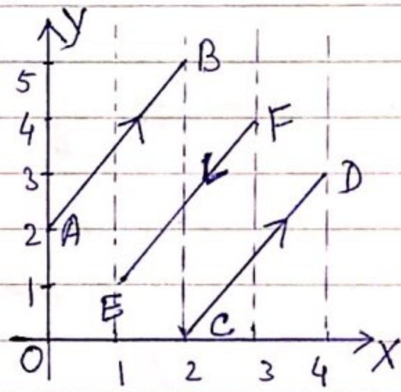
(i) $\vec{AB} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$; $\vec{BC} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$; $\vec{AC} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$



Now $\begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

$\therefore \quad \vec{AB} + \vec{BC} = \vec{AC}$ [Triangle Law of Vector addition]

(ii) $\vec{AB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$; $\vec{CD} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\vec{FE} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$



$\therefore \vec{AB} = \vec{CD}$ Equal Vectors

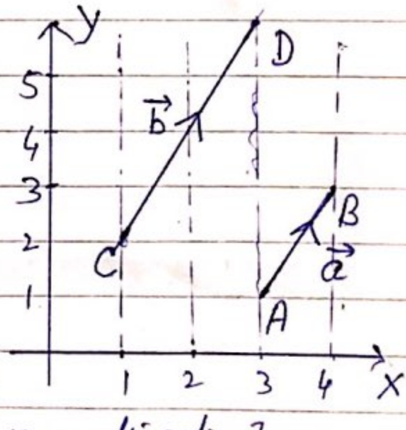
$\vec{FE} = \begin{pmatrix} -2 \\ -3 \end{pmatrix} = -1\begin{pmatrix} 2 \\ 3 \end{pmatrix} = -\vec{AB} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$

or $\vec{AB} = -\vec{FE}$

$\therefore \vec{AB}$ and $\vec{FE}$ are opposite vectors.

§ **Scalar Multiplication:**

$\vec{AB} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$; $\vec{CD} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$



$2 \cdot \vec{AB} = 2\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \vec{CD}$

$\vec{AB}$ & $\vec{CD}$ are two vectors in the same direction.

[$\vec{a} = k\vec{b}$ and $k > 0$ $\vec{a}$ and $\vec{b}$ are in the same direction]

§ **Magnitude of a Column Vector:**

$\vec{AB} = \begin{pmatrix} x \\ y \end{pmatrix}$ Then magnitude of $\vec{AB} = |\vec{AB}| = \sqrt{x^2+y^2}$

Example: $\vec{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \Rightarrow |\vec{a}| = \sqrt{1^2+2^2} = \sqrt{5}$

$\vec{b} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \Rightarrow |\vec{b}| = \sqrt{3^2+4^2} = 5$

Vectors in two dimensions Notes

Unit Vector: A vector whose magnitude is one unit.

Given a vector $\vec{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

(i) Find a unit vector in the direction of $\vec{a}$

(ii) Find a vector $\vec{b}$ in the direction of $\vec{a}$ and $|\vec{b}| = 7$

Solution (i) $\vec{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \Rightarrow |\vec{a}| = \sqrt{3^2+4^2} = 5$

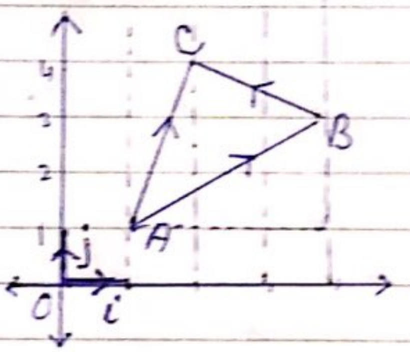
$\therefore$ Unit Vector in the direction of $\vec{a} = \dfrac{\vec{a}}{|\vec{a}|} = \frac{1}{5}\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

(ii) $\vec{b} = 7 \times$ Unit Vector along $\vec{a}$

$= 7 \times \frac{1}{5}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{21}{5} \\ \frac{28}{5} \end{pmatrix}$

Basic Unit Vectors (in two dimensions)

$i = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $j = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$



$\vec{AB} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} = 3i + 2j$

$\vec{BC} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} = -2i + j$

$\vec{AC} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$\vec{AC} = (i + 3j)$

(i) Now $\vec{AB} + \vec{BC}$

$= (3i+2j) + (-2i+j) = (3-2)i + (2+1)j$

$= i + 3j = \vec{AC}$

$\boxed{\vec{AB} + \vec{BC} = \vec{AC}}$ (Verifies triangle law of vector addition)

(ii) $\vec{AB} = 3i + 2j$

$\therefore |\vec{AB}| = \sqrt{3^2+2^2} = \sqrt{13}$

Unit Vector along $\vec{AB} = \frac{1}{\sqrt{13}}\vec{AB} = \frac{1}{\sqrt{13}}(3i+2j)$ ✓

(iii) $3\vec{AB} + 2\vec{BC} = 3(3i+2j) + 2(-2i+j)$

$= (9i+6j) + (-4i+2j) = (5i+8j)$

(iv) Position Vector B $= \vec{OB} = (4i + 3j)$

(v) Position Vector A $= \vec{OA} = (i + j)$

$\vec{AB} = \vec{OB} - \vec{OA}$

$= (4i+3j) - (i+j)$

$= (3i+2j)$ ✓

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Vectors

Example 1 (a) $\overrightarrow{GH} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$

Find (i) $5\overrightarrow{GH}$ ---[1]

(ii) $\overrightarrow{HG}$ ---[1]

(b) $\begin{pmatrix} 6 \\ 7 \end{pmatrix} + \begin{pmatrix} 2 \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 3 \end{pmatrix}$ find the value of y. ---[1]

S-17/21/Q18

Solution (a)(i) $\overrightarrow{GH} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$

$\therefore\ 5\overrightarrow{GH} = 5\begin{pmatrix} 6 \\ -4 \end{pmatrix} = \begin{pmatrix} 30 \\ -20 \end{pmatrix}$ ✓

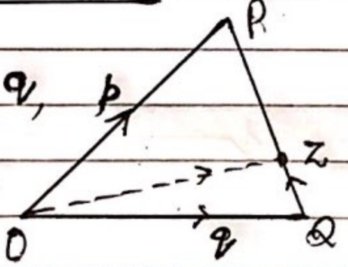
(ii) $\overrightarrow{HG} = -\overrightarrow{GH} = -\begin{pmatrix} 6 \\ -4 \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \end{pmatrix}$ ✓

(b) $\begin{pmatrix} 6 \\ 7 \end{pmatrix} + \begin{pmatrix} 2 \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 3 \end{pmatrix}$

or $\begin{pmatrix} 6+2 \\ 7+y \end{pmatrix} = \begin{pmatrix} 8 \\ 3 \end{pmatrix}$

$\therefore\ y+7=3$ or $y=-4$ ✓

Example 2: O is the origin, $\overrightarrow{OP} = p$ and $\overrightarrow{OQ} = q$,
Z is a point on PQ such that,
$PZ : ZQ = 5 : 2$



Work out, in term of p and q, the position vector of Z, give your answer in its simplest form. ---[3]

S-17/23/Q17

Solution: Position vector of Z $= \overrightarrow{OZ} = \overrightarrow{OQ} + \overrightarrow{QZ}$ --(i) (using Triangle law of addition)

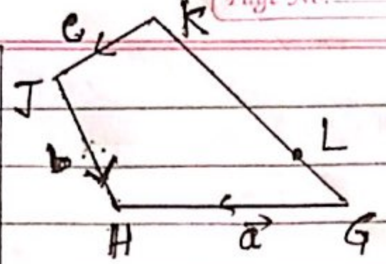
Now $\overrightarrow{QZ} = \frac{2}{7}\overrightarrow{QP} = \frac{2}{7}(p-q)$ --(ii) $\left[\because PZ : ZQ = 5:2,\ ZQ = \frac{2}{7}QP\right]$

from (i) and (ii) $\overrightarrow{OZ} = \overrightarrow{OQ} + \overrightarrow{QZ}$

$= \vec{q} + \frac{2}{7}(p-q)$

$\therefore$ Position Vector of Z $= \frac{2}{7}p + \frac{5}{7}q$ ✓

Example 3: GHJK is a quadrilateral, $\vec{GH} = \mathbf{a}$, $\vec{JH} = \mathbf{b}$ and $\vec{KJ} = \mathbf{c}$, L lies on GK so that $LK = 3GL$. Find an expression, in terms of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ for GL. ——— [2]



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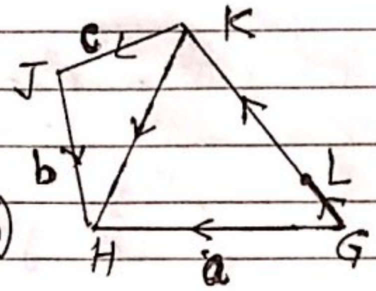
Solution:

$$\vec{GL} = \frac{1}{4}\vec{GK} \quad [\because KL = 3GL] \text{ —(i)}$$



Join KH, $\vec{KH} = \mathbf{c} + \mathbf{b}$ —(ii)
(Triangle Law)

Now In Triangle GKH,

$$\vec{GK} + \vec{KH} = \vec{GH}$$

$$\therefore \vec{GK} = \vec{GH} - \vec{KH}$$

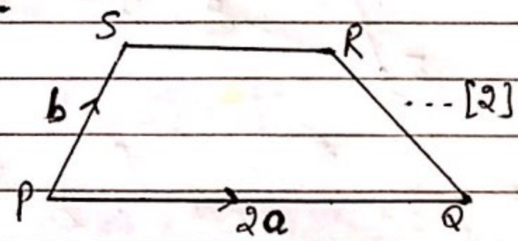
$$\text{or } \vec{GK} = \mathbf{a} - (\mathbf{c}+\mathbf{b}) \quad \text{frm (ii)} \text{ —(iii)}$$

frm (i)

$$\vec{GL} = \frac{1}{4}\vec{GK} = \frac{1}{4}(\mathbf{a} - \mathbf{b} - \mathbf{c}) \quad \text{frm (iii)}$$

$$\text{or } \vec{GL} = \frac{1}{4}\mathbf{a} - \frac{1}{4}\mathbf{b} - \frac{1}{4}\mathbf{c} \checkmark$$

Example 4 (a) PQRS is a trapezium with $PQ = 2SR$; $\vec{PQ} = 2\mathbf{a}$, $\vec{PS} = \mathbf{b}$
Find $\vec{QR}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. ... [2]



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Solution: Required vector:

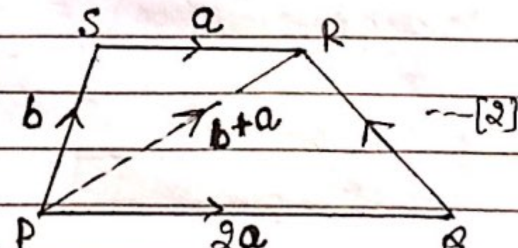
$$\vec{QR} = \vec{PR} - \vec{PQ} \text{ -- (i)}$$

In Triangle PSR, $\vec{PR} = \vec{PS} + \vec{SR}$ ⊛
(Triangle Law of addition)

or

$$\vec{PR} = \mathbf{b} + \mathbf{a} \text{ --(ii)}$$

Given $\vec{PQ} = 2\mathbf{a}$



--- [2]

frm (i) $\vec{QR} = \vec{PR} - \vec{PQ}$

$$= (\mathbf{b}+\mathbf{a}) - 2\mathbf{a}$$

$$= \mathbf{b} - \mathbf{a} \checkmark$$

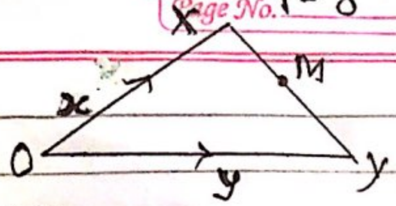
$\because \vec{PQ} = 2\vec{SR}$

$2\mathbf{a} = 2\vec{SR}$

$\therefore$ ⊛ $\vec{SR} = \mathbf{a}$

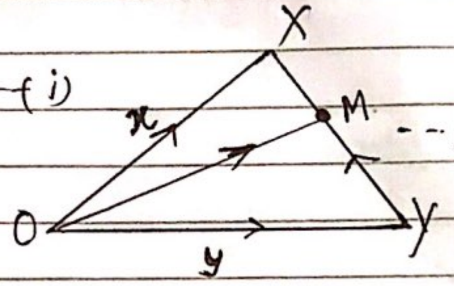
(continued →)

(Continued→)



Example 4(b). $\vec{OX} = x$ and $\vec{OY} = y$
M is a point on XY such that XM : MY = 3 : 5
Find $\vec{OM}$ in terms of x and y.

Solution: Join OM.
Required vector $\vec{OM} = \vec{OY} + \vec{YM}$ —(i)



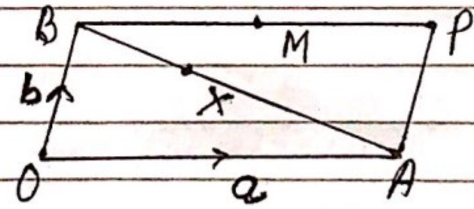
--- [2]

Given XM : MY = 3 : 5
$\therefore$ MY $= \frac{5}{8}$ XY
or $\vec{YM} = \frac{5}{8}\vec{YX}$ —(ii)

In triangle OXY, $\vec{YX} = x - y$ —(iii)

from (ii) & (iii) $\vec{YM} = \frac{5}{8}(x-y)$ —(iv)

from (i) & (iv) $\vec{OM} = \vec{OY} + \vec{YM}$
$= y + \frac{5}{8}(x-y)$
$= \frac{3}{8}y + \frac{5}{8}x$ ✓



Example 5(a) OAPB is a parallelogram,
O is the origin, $\vec{OA} = a$ and $\vec{OB} = b$
M is the mid point of BP.
(a) Find in terms of of a and b,
(i) $\vec{BA}$ --- [1]
(ii) the position Vector of M. --- [1]

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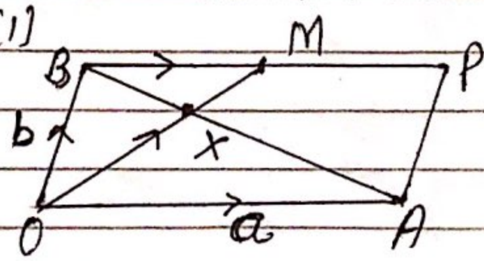


Solution: Join OM, (i) $\vec{BA} = a - b$ ✓ ---(i)
(ii) Position vector of M = $\vec{OM}$
$\vec{OM} = \vec{OB} + \vec{BM}$
$= b + \frac{1}{2}a$ ✓
$= \frac{1}{2}(2b + a)$ ✓ --- (ii)

[∵ $\vec{BM} = \frac{1}{2}\vec{BP} = \frac{1}{2}a$
as BP ∥ OA and equal

(Continued)

(Continued→)

Example 5(b) X is on BA, so that BX : XA = 1:2
Show that X lies on OM. - - - [4]

Solution: $\vec{OX} = \vec{OB} + \vec{BX}$

$= \vec{b} + \frac{1}{3}\vec{BA}$ (BX : XA = 1:2 ∴ $BX = \frac{1}{3} BA$)

$= \vec{b} + \frac{1}{3}(a - b)$ (∵ from (i) $\vec{BA} = a - b$)

$= \frac{1}{3}(2b + a)$

$= \frac{1}{3} \times 2\vec{OM}$ ∵ from (ii) $\vec{OM} = \frac{1}{2}(2b + a)$, $2b + a = 2\vec{OM}$

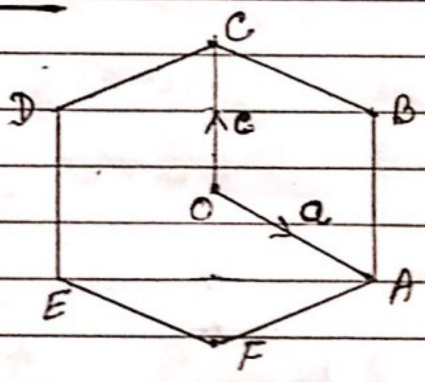
∴ $\vec{OX} = \frac{2}{3}\vec{OM}$

∴ OX ∥ OM but O is common point.

∴ O, X and M are collinear

or X lies on OM.

Example 6. O is origin; ABCDEF is a regular hexagon and O is the mid point of AD. $\vec{OA} = a$ and $\vec{OC} = c$. Find, in terms of a and c,



(i) $\vec{BE}$ - - - [2]
(ii) $\vec{DB}$ - - - [2]
(iii) the position vector of E. - - [2]

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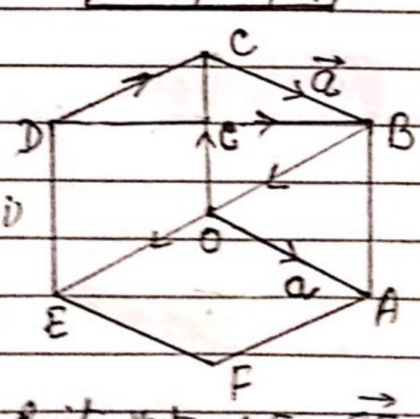
Solution:



(i) $\vec{CB} = \vec{OA} = a$

In triangle OBC, $\vec{OC} + \vec{CB} = \vec{OB}$

$c + a = \vec{OB}$

∴ $\vec{BO} = -\vec{OB} = -(c + a)$ - (i)

Now $\vec{BE} = 2\vec{BO} = 2 \times [-(c + a)]$

or $\vec{BE} = (-2a - 2c)$ ✓

(ii) $\vec{DB} = \vec{DC} + \vec{CB}$

$= \vec{OB} + \vec{CB} = (c + a) + a$

$= c + 2a$ ✓

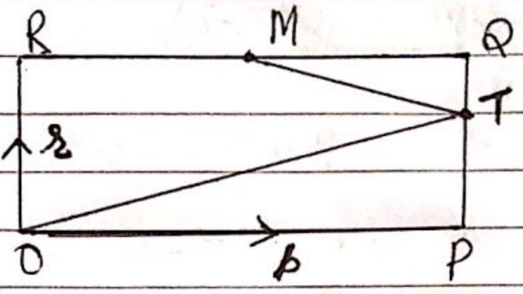
(iii) Position vector of E $= \vec{OE}$

$= \vec{BO}$

$= -c - a$ ✓ from (i)

Example 7: OPQR is a rectangle and M is the mid point of RQ, and PT : TQ = 2 : 1

$\vec{OP} = p$ and $\vec{OR} = r$



(a) Find in terms of p and/or r,

(i) $\vec{MQ}$ --- [1]

(ii) $\vec{MT}$ --- [1]

(iii) $\vec{OT}$ --- [1]

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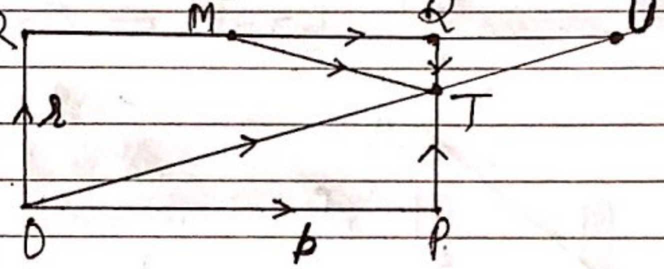
SP-20/04/Q6

Solution:



(i) $\vec{MQ} = \frac{1}{2}\vec{RQ}$
$= \frac{1}{2}\vec{OP}$
$= \frac{1}{2}p$ ✓ --- (i)

(ii) $\vec{MT} = \vec{MQ} + \vec{QT}$
$= \frac{1}{2}p - \frac{1}{3}r$ ✓

$\vec{QT} = \frac{1}{3}\vec{QP}$
$= \frac{1}{3}\vec{RO}$
$= \frac{1}{3}(-r)$

(iii) $\vec{OT} = \vec{OP} + \vec{PT}$
$= p + \frac{2}{3}r$

$\vec{PT} = \frac{2}{3}\vec{PQ}$
$= \frac{2}{3}\vec{OR}$
$= \frac{2}{3}r$

(b) RQ and OT are extended to meet at U. Find the position vector of U in terms of p and r.

Solution: Position Vector of $U = \vec{OU}$

$\vec{OU} = \vec{OR} + \vec{RU}$ —— (i)

$\Delta QTU \sim \Delta PTO$

$\frac{QU}{OP} = \frac{QT}{PT} = \frac{1}{2}$

$\therefore \vec{QU} = \frac{1}{2}\vec{OP} = \frac{1}{2}p$ - (ii)

fm (i)

$\vec{OU} = \vec{OR} + \vec{RU}$
$= \vec{OR} + (\vec{RQ} + \vec{QU})$
$= r + (\vec{OP} + \vec{QU})$
$= r + p + \frac{1}{2}p$ (fm (ii)
$= r + \frac{3}{2}p$ ✓

(c) $\vec{MT} = \begin{pmatrix} 2k \\ -k \end{pmatrix}$; $|\vec{MT}| = \sqrt{180}$

find the Value of k --- [3]

Solution: $\vec{MT} = \begin{pmatrix} 2k \\ -k \end{pmatrix}$

$\therefore |\vec{MT}| = \sqrt{(2k)^2 + (-k)^2}$
$= \sqrt{5k^2} = \sqrt{180}$ given

$\therefore 5k^2 = 180$
or $k^2 = 36$
$\therefore k = 6$ ✓

Example 8 (a) $\overrightarrow{PQ} = \begin{pmatrix} 5 \\ -8 \end{pmatrix}$ [S-15/42/Q10(a)(b)]

(i) Find the value of $|\overrightarrow{PQ}|$ ---[2]

(ii) Q is the point $(2,-3)$, find the coordinates of P. ---[1]

Solution: (i) $\overrightarrow{PQ} = \begin{pmatrix} 5 \\ -8 \end{pmatrix}$

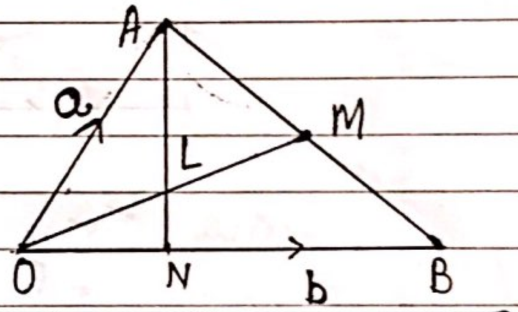
$\therefore |\overrightarrow{PQ}| = \sqrt{(5)^2 + (-8)^2} = \sqrt{89} = 9.43$ ✓

(ii) $Q(2,-3)$, let $P(x,y)$

$\therefore \overrightarrow{PQ} = \begin{pmatrix} 2-x \\ -3-y \end{pmatrix} = \begin{pmatrix} 5 \\ -8 \end{pmatrix}$ given

$\Rightarrow \begin{cases} 2-x=5 \\ -3-y=-8 \end{cases} \Rightarrow \begin{cases} x=-3 \\ y=5 \end{cases}$ $\therefore P(-3,5)$ ✓

(b) In the diagram, M is the mid point of AB and L is the mid point of OM. The lines OM and AN intersect at L and $ON = \frac{1}{3}OB$, $\overrightarrow{OA} = a$, $\overrightarrow{OB} = b$

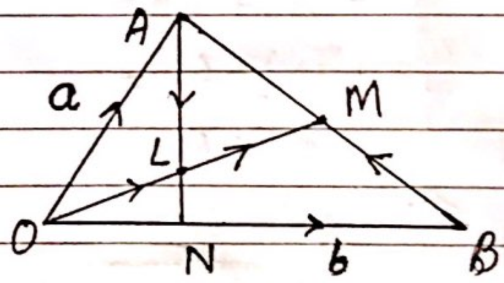


(i) Find in terms of a and b,

(a) $\overrightarrow{OM}$ ---[2]

(b) $\overrightarrow{OL}$ ---[1]

(c) $\overrightarrow{AL}$ ---[2]



(ii) Find the ratio AL : AN in its simplest form. --[3]

Solution: (i) (a) $\overrightarrow{OM} = \overrightarrow{OB} + \overrightarrow{BM}$ --(i)

$\overrightarrow{BM} = \frac{1}{2}\overrightarrow{BA} = \frac{1}{2}(a-b)$

$\therefore$ from (i) $\overrightarrow{OM} = \overrightarrow{OB} + \overrightarrow{BM}$

$= b + \frac{1}{2}(a-b)$

$= \frac{1}{2}a + \frac{1}{2}b$ ✓

(b) $\overrightarrow{OL} = \frac{1}{2}\overrightarrow{OM}$ ($\because$ L is mid point of OM)

$= \frac{1}{2}\left[\frac{1}{2}a + \frac{1}{2}b\right]$

$= \frac{1}{4}a + \frac{1}{4}b$ ✓

(c) $\overrightarrow{AL} = \overrightarrow{OL} - \overrightarrow{OA}$

$= \left(\frac{1}{4}a + \frac{1}{4}b\right) - a$

$= \frac{1}{4}b - \frac{3}{4}a$ ✓

(ii) $\overrightarrow{ON} = \frac{1}{3}\overrightarrow{OB} = \frac{1}{3}b$

$\therefore \overrightarrow{AN} = \overrightarrow{ON} - \overrightarrow{OA}$

$= \frac{1}{3}b - a$ ✓

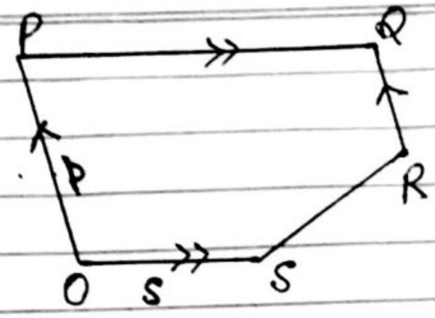
$= \frac{1}{3}(b-3a)$

Now $\frac{\overrightarrow{AL}}{\overrightarrow{AN}} = \frac{\frac{1}{4}(b-3a)}{\frac{1}{3}(b-3a)} = \frac{3}{4}$

$\therefore AL : AN = 3:4$ ✓

Example 9(b) In the pentagon OPQRS, OP is parallel to RQ and OS is parallel to PQ.
PQ = 2OS and OP = 2RQ.
O is the origin, $\vec{OP} = p$ and $\vec{OS} = s$. Find in terms of p and s,
(i) the position vector of Q. --- [2]
(ii) $\vec{SR}$ --- [2]

(c) Explain what your answers in part (b) tell you about OQ and SR. --- [1]



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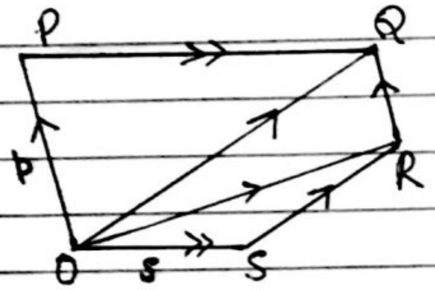
Solution:

(b)(i) Position vector of Q.



In ΔOPQ, $\vec{OQ} = \vec{OP} + \vec{PQ}$

$= \vec{OP} + 2\cdot\vec{OS}$

$= p + 2s$ ✓ ---(i)

(ii) $\vec{SR} = \vec{OR} - \vec{OS}$ ——— (ii)

In ΔORQ, $\vec{OR} + \vec{RQ} = \vec{OQ}$

or $\vec{OR} = \vec{OQ} - \vec{RQ}$

$= (p+2s) - \frac{1}{2}p$ [$\because OP = 2RQ$, $OP \parallel RQ$]

$= \frac{p}{2} + 2s$ ---(iii)

fr(i) $\vec{SR} = \vec{OR} - \vec{OS}$

$= \left(\frac{p}{2} + 2s\right) - s$ fr iii)

$= p/2 + s$ ✓

(c) $\vec{SR} = \frac{p}{2} + s = \frac{1}{2}(p + 2s)$

$= \frac{1}{2}\vec{OQ}$ [$\because$ fm(i) $\vec{OQ} = p + 2s$]

i.e. $\vec{SR} = \frac{1}{2}\vec{OQ}$

$\Rightarrow \vec{SR} \parallel \vec{OQ}$ ✓ and $OQ = 2SR$ ✓