Date / /
Page No.

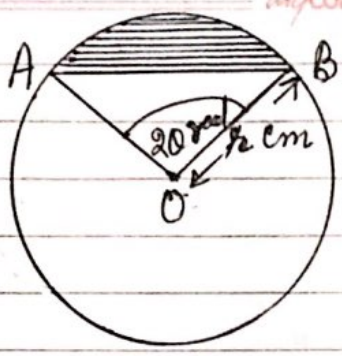
IG-0606
Additional Maths

Circular Measure
Exercise

Suresh Goel
(Director)
Alliance World School,
Noida, Delhi NCR. India.

Circular measure

Q1 The diagram shows a circle, centre O, radius r cm. The points A and B lie on the circle such that angle AOB = 2θ radians.



(a) Given that the perimeter of the shaded region is 20 cm, show that: $r = \frac{10}{\theta + \sin\theta}$ ---[3]

(b) Given that r and θ can vary, find the value of $\frac{dr}{d\theta}$ when $\theta = \frac{\pi}{6}$. ---[4]

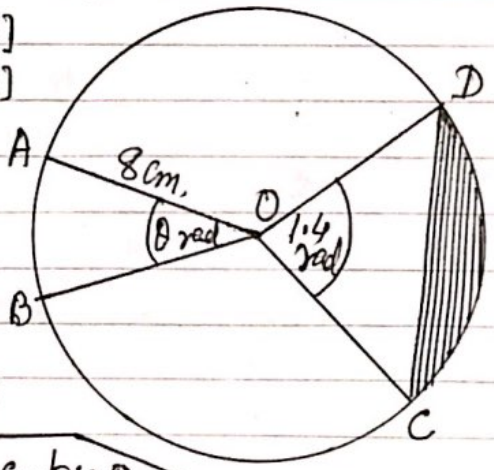
SP-20|02|Q8

Q2 The diagram shows a circle with centre O and radius 8 cm. The points A, B, C, D lie on the circumference of the circle, Angle AOB = θ radians and angle COD = 1.4 radians. The area of sector AOB is 20 cm².

(i) Find angle θ. ---[2]

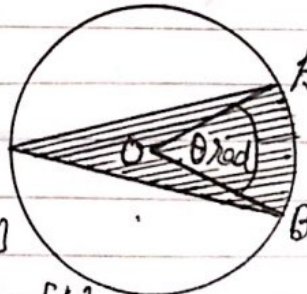
(ii) Find the length of arc AB. --[2]

(iii) Find the area of the shaded segment. ---[3]



M-18|22|Q7

Q3 The diagram shows a circle, centre O, radius 10 cm. Points A, B and C lie on the circumference of the circle such that AC = BC. The area of the minor sector AOB is 20π cm² and angle AOB is θ radians.



(i) Find the value of θ in terms of π. ----[2]

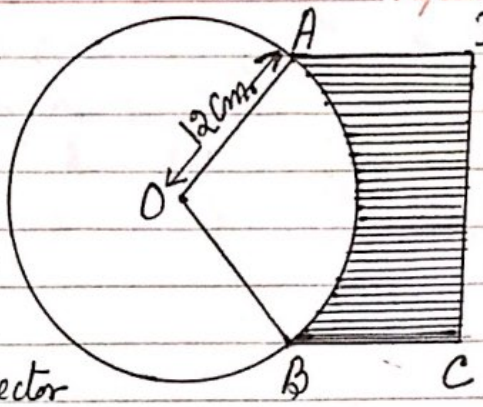
(ii) Find the perimeter of the shaded region. ---[4]

(iii) Find the area of the shaded area. ---[3]

M-17|12|Q9

Circular Measure

Q4. The diagram shows a circle, centre O, radius 12 cm. The points A and B lie on the circumference of the circle and form a rectangle with the points C and D. The length AD is 8 cm, and the area of the minor sector AOB is 150 cm².



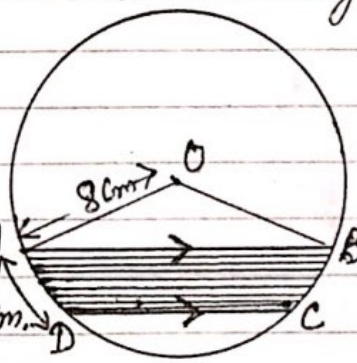
(i) Show that angle AOB is 2.08 radians, correct to 2 decimal places. ---[2]

(ii) Find the area of shaded region ADCB. ---[6]

(iii) Find the perimeter of the shaded region ADCB. ---[3]

S-17/11/Q6

Q5 The diagram shows a circle, centre O, radius 8 cm. The points A, B, C and D lie on the circumference of the circle such that AB is parallel to DC. The length of the arc AD is 4 cm, and the length of the chord AB is 15 cm.



(i) Find, in radians, angle AOD, ---[1]

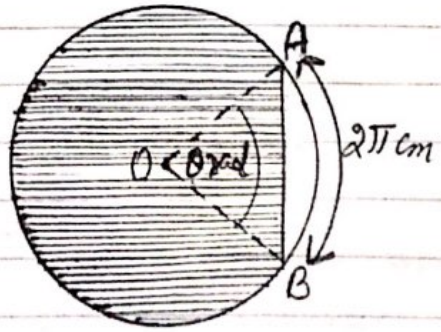
(ii) Hence show that angle DOC = 1.43 rad, correct to 2 decimal places. ----[3]

(iii) Find the perimeter of the shaded region. ---[3]

(iv) Find the area of the shaded region ----[4]

S-17/12/Q10

Q6 The diagram shows a circle, centre O of radius r cm, and a chord AB. Angle AOB = θ radians. The length of the major arc AB is 5 times the length of the minor arc AB. The minor arc AB has length 2π cm.



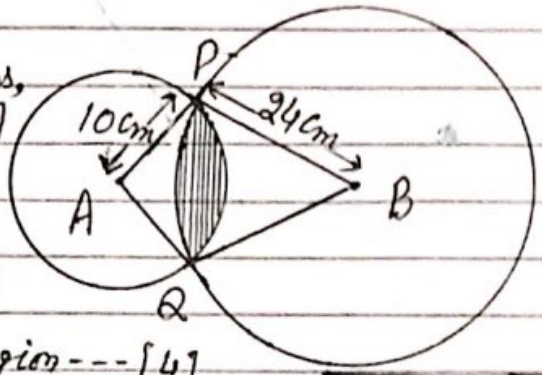
(i) Find the value of θ and of r. ---[2]

(ii) Calculate the exact perimeter of the shaded segment. ---[2]

(iii) Calculate the exact area of the shaded segment. ---[4]

S-17/23/Q8

Q7. The diagram shows a circle, centre A, radius 10cm, intersecting a circle, centre B, radius 24cm. The two circles intersect at the points P and Q. The radii AP and AQ are tangents to the circle with centre B. The radii BP and BQ are tangents to the circle with centre A.



(i) Show that angle PAQ is 2.35 radians, correct to 3 significant figures. --- [2]

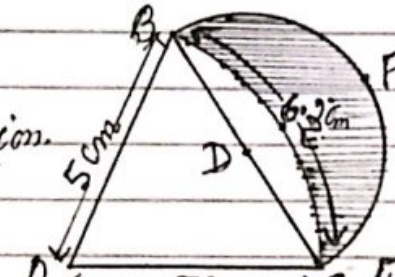
(ii) Find angle PBQ in Radians --- [1]

(iii) Find the perimeter of the shaded region. --- [3]

(iv) Find the area of the shaded region --- [4]

[W-17/11/Q9]

Q8 The diagram shows an isosceles triangle ABC, where AB=AC=5cm. The arc BEC is part of the circle with centre A and has length 6.2cm. The point D is the mid point of the line BC. The arc BFC is a semicircle centre D.



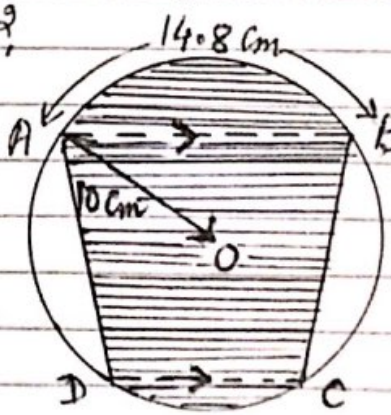
(i) Show that angle BAC is 1.24 radians. --- [1]

(ii) Find the perimeter of the shaded region. --- [3]

(iii) Find the area of the shaded region. --- [4]

[W-17/12/Q10]

Q9 The diagram shows a circle, centre O, radius 10cm. The points A, B, C and D lie on the circumference of the circle such that AB is parallel to DC. The length of minor arc AB is 14.8cm. The area of minor sector ODC is 21.8cm^2.



(i) Write down, in radians, angle AOB. --- [1]

(ii) Find, in radians, angle DOC. --- [2]

(iii) Find the perimeter of the shaded region. --- [4]

(iv) Find the area of the shaded region. --- [3]

[W-17/13/Q11]

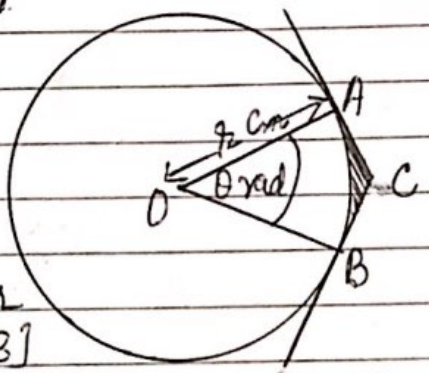
Q10 The diagram shows a circle, centre O, radius r cm. Points A, B and C are such that A and B lie on the circle and the tangents at A and B meet at C. Angle AOB = θ radians.

(i) Given the area of the major sector AOB is 7 times the area of the minor sector AOB, find the value of θ. ---[2]

(ii) Given also that the perimeter of the minor sector AOB is 20cm, show that the value of r, correct to two decimal places, is 7·18. ---[2]

(iii) Using the values of θ and r from part (i) and (ii), find the perimeter of the shaded region ABC. ---[3]

(iv) Find the area of the shaded region ABC. M-16|12|Q9 ---[3]



Q11 PQRS is a quadrilateral with PS parallel to QR. The perimeter of PQRS is 3m. The length of PQ is 1m and the length of PS is x m. The point T is on QR such that ST is parallel to PQ. Angle SRT is θ radians.

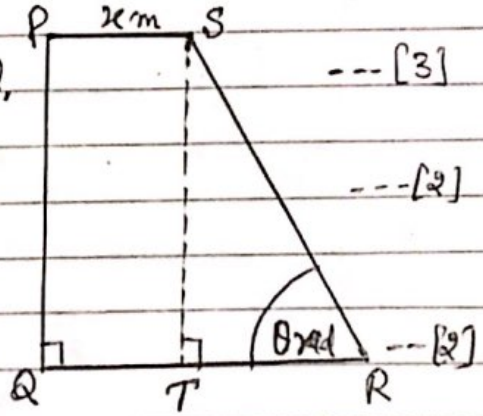
(i) Find an expression for x in terms of θ. ---[3]

(ii) Show that area, Am^2, of PQRS is given by,

$$A = 1 - \frac{\cos\theta}{2}$$ ---[2]

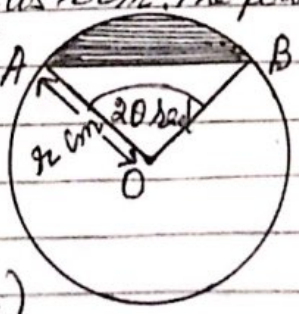
(iii) Hence find the exact value of θ, when $A = \left(1 - \frac{\sqrt{3}}{3}\right) m^2$ ---[2]

M-16|22|Q9



Q12 The diagram shows a circle, centre O, radius r cm. The points A and B lie on the circle such that angle AOB = 2θ radians.

(i) Find, in terms of r and θ, an expression for the length of the chord AB. ---[1]



(Continued →)

(Continued →)

Q12(ii) Given that the perimeter of the shaded region is 20cm, show that $r = \frac{10}{\theta + \sin\theta}$ ---[2]

(iii) Given that r and θ can vary, find the value of $\frac{dr}{d\theta}$ when $\theta = \frac{\pi}{6}$ ---[4]

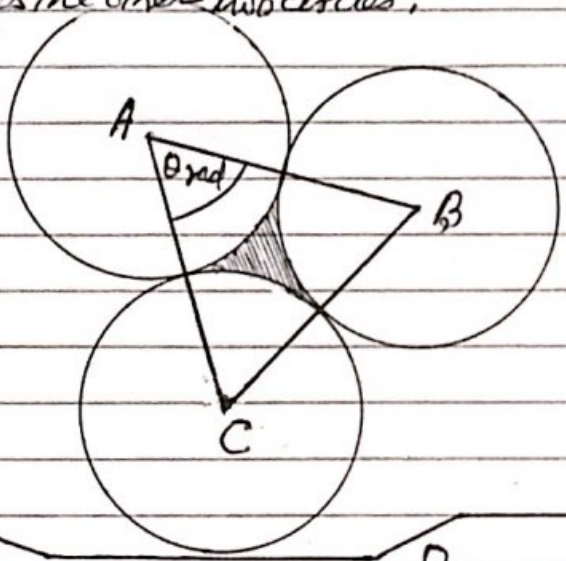
(iv) Given that r is increasing at the rate of $15\ ms^{-1}$, find the corresponding rate of change of θ when $\theta = \frac{\pi}{6}$ ---[3]

S-16 | II | Q11

Q13 The diagram shows 3 circles with centres A, B and C, each of radius 5cm. Each circle touches the other two circles. Angle BAC is θ radians.

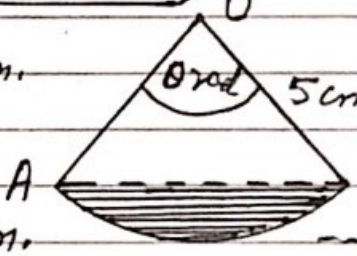
(i) Write down the value of θ, ---[1]

(ii) Find the area of the shaded region between the circles, ---[4]



S-16 | 21 | Q4

Q14 The diagram shows a sector of a circle with centre O and radius 5cm. The length of arc AB is 7cm, and angle AOB is θ radians.



(i) Explain why θ must be greater than 1 radian. ---[1]

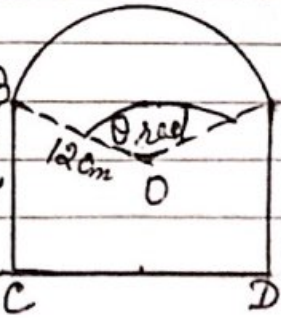
(ii) Find the value of θ, ---[2]

(iii) Calculate the area of the sector AOB, ---[2]

S-16 | 22 | Q6

(iv) Calculate the area of the shaded segment. ---[2]

Q15 The diagram shows a sector AOB of the circle, centre O, radius 12 cm, together with points C and D such that ABCD is a rectangle. The angle AOB is θ radians and perimeter of sector AOB is 47cm.



(Continued →)

Circular Measure

(→ continued)

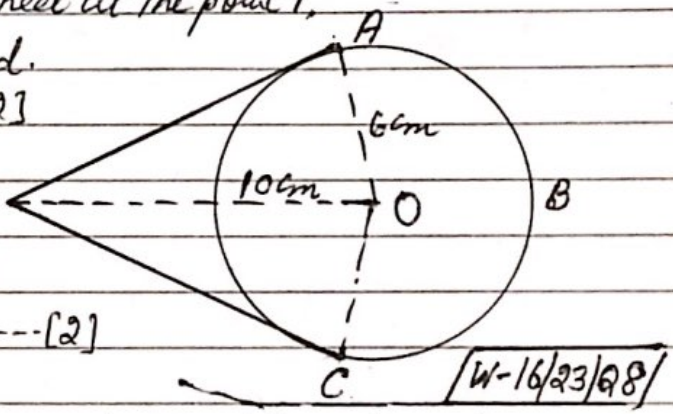
Q15 (i) Show that $\theta = 1.92$ radians correct to 2 decimal places. ---[2]

(ii) Find the length CD. ---[2]

(iii) Given that the total area of the shape is 425 cm^2, find the length of AD. ---[5]

[W-16/11/Q8]

Q16 The points A, B and C lie on a circle centre O, radius 6 cm. The tangents to the circle at A and C meet at the point T. The length of OT is 10 cm. Find.



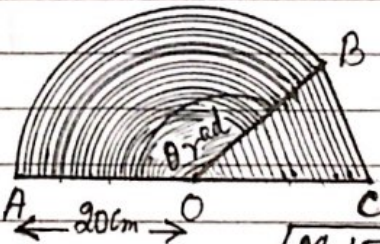
(i) the angle TOA in radians. ---[2]

(ii) the area of the region TABCT, ---[6]

(iii) the perimeter of the region TABCT. ---[2]

[W-16/23/Q8]

Q17 AOB is a sector of a circle with centre O and radius 20 cm. Angle AOB = θ radians. AOC is a straight line and triangle OBC is isosceles with OB = OC.

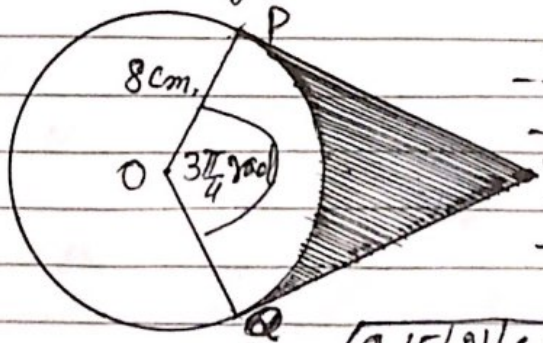


(i) Given that the length arc AB is 15π cm, Find the exact value of θ. ---[2]

(ii) Find the area of the shaded region. ---[4]

[M-15/22/Q6]

Q18 The diagram shows a circle, centre O, radius 8 cm. The points P and Q lie on the circle. The line PT and QT are tangents to the circle and angle POQ = $\frac{3\pi}{4}$ radians.



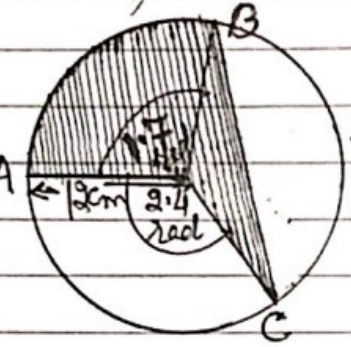
(i) Find the length of PT. ---[2]

(ii) Find the area of the shaded region. ---[3]

(iii) Find the perimeter of the shaded region. ---[2]

[S-15/21/4]

Q19 The diagram shows a circle, centre O, radius 12cm. The points A, B and C lie on the circumference of this circle such angle AOB = 1·7 radians and angle AOC is 2·4 radians.

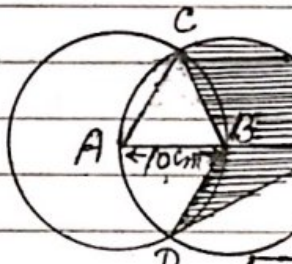


(i) Find the area of the shaded region. --[4]

(ii) Find the perimeter of the shaded region. --[5]

[S-15/12/Q4]

Q20 The diagram shows two circles, centres A and B, each of radius 10cm. The point B lies on the circumference of the circle with centre A. The two circles intersect at the points C and D. The point E lies on the circumference of the circle centre B such that ABE is a diameter.



(i) Explain why triangle ABC is equilateral. ---[1]

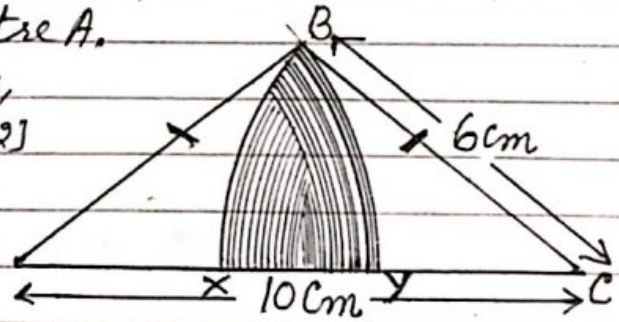
(ii) Write down in terms of π, angle CBE. ---[1]

(iii) Find the perimeter of the shaded region. ---[5]

(iv) Find the area of the shaded region. ---[3]

[W-15/11/Q10]

Q21 The diagram shows an isosceles triangle ABC such that AC = 10 cm and AB = BC = 6cm. BX is an arc of a circle, centre C, and BY is an arc of a circle, centre A.



(i) Show that angle ABC = 1·970 radians, correct to 3 decimal places. ---[2]

(ii) Find the perimeter of the shaded region. ---[4]

(iii) Find the area of the shaded region. ---[3]

[W-15/13/Q10]

Q22 A sector of a circle of radius r cm has an angle θ radians, where θ < π. The perimeter of the sector is 30cm.

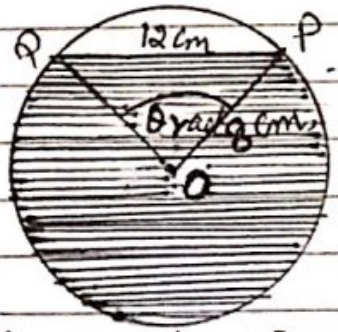
(i) Show that the area, A cm², of the sector is given by $A = 15r - r^2$ --[3]

(ii) Given r can vary, find the maximum area of the sector. --[3]

[S-14/21/Q8]

Circular Measure

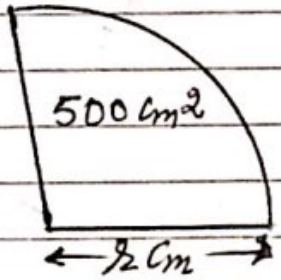
Q23 The diagram shows a circle, centre O, radius 8 cm. Points P and Q lies on the circle such that the chord PQ = 12 cm, and angle POQ = θ radians.



(i) Show that θ = 1.696, correct to 3 decimal places. ---[2]
(ii) Find the perimeter of the shaded region ---[3]
(iii) Find the area of the shaded region. ---[3]

[5-14/12/Q7]

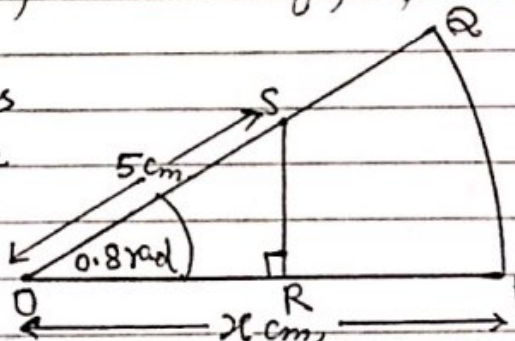
Q24 The diagram shows a sector of a circle of radius r cm, the angle of the sector is 1.6 radians and the area of the sector is 500 cm².



(i) Find the value of r. ---[2]
(ii) Hence find the perimeter of the sector. ---[2]

[5-14/23/Q1]

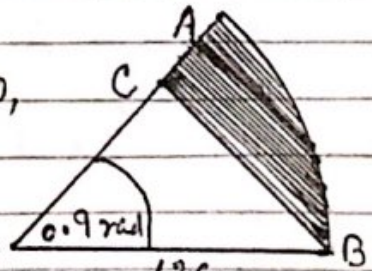
Q25 The diagram shows a sector OPQ of a circle with centre O and radius x cm. Angle POQ is 0.8 radians. The point S lies on OQ such that OS = 5 cm. The point R lies on OP such that angle ORS is a right angle. Given that the area of triangle ORS is one-fifth of the area of sector OPQ, find,



(i) the area of sector OPQ in terms of x and hence show that the value of x 8.837 correct to 4 significant figures. ---[5]
(ii) the perimeter of PQSR ---[3]
(iii) the area of PQSR ---[2]

[W-14/21/Q11]

Q26 The diagram shows a sector, AOB, of a circle centre O, radius 12 cm. Angle AOB = 0.9 radians. The point C lies on OA such that OC = CB.



(i) Show that OC = 9.65 cm, correct to 3 significant figures.

(continued →)

Circular Measure

(→Continued)

Q26(ii) Find the perimeter of the shaded region. ---[3]

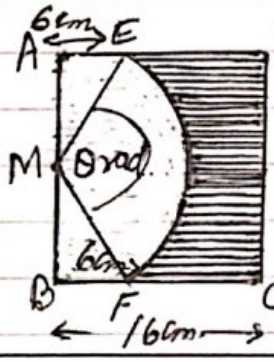
(iii) Find the area of the shaded region. W-14/13/Q6 ---[3]

Q27 The diagram shows a square ABCD of side 16cm. M is the mid point of AB. The points E and F are on AD and BC respectively such that AE = BF = 6 cm. EF is an arc of the circle centre M, such that angle EMF is θ radians.

(i) Show that θ = 1.855 radians, correct to 3 decimal places. ---[2]

(ii) Calculate the perimeter of the shaded region. ---[4]

(iii) Calculate the area of the shaded region. ---[3]

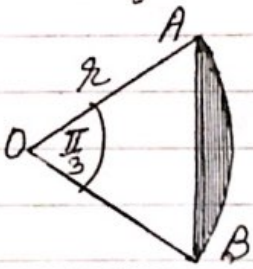


S-13/11/Q8

Q28 The shaded region in the diagram is a segment of a circle with centre O and radius r cm. Angle AOB = $\frac{\pi}{3}$ radians.

(i) Show that the perimeter of the segment is $r\left(\frac{3+\pi}{3}\right)$ cm. ---[2]

(ii) Given that the perimeter of the segment is 26cm, find the value of r and the area of the segment. ---[5]



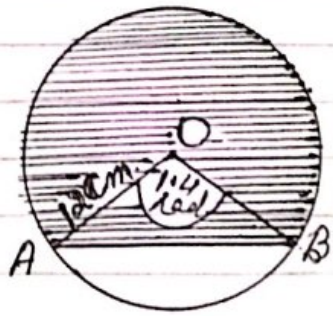
S-13/22/Q6

Q29 The diagram shows a circle with centre O and a chord AB. The radius of the circle is 12cm and angle AOB is 1.4 radians.

(i) Find the perimeter of the shaded region. ---[5]

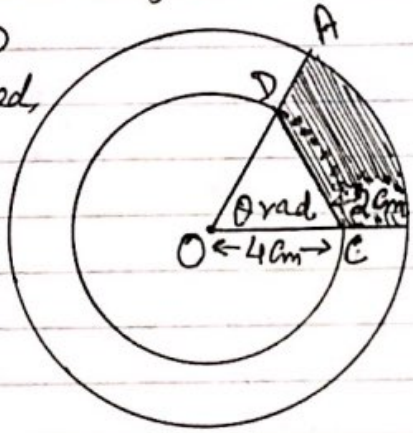
(ii) Find the area of the shaded region. ---[4]

W-13/21/Q10



Circular Measure.

Q 30 The diagram shows two concentric circles, centre O, radii 4 cm. and 6 cm. The points A and B lie on the larger circle and the points C and D lie on the smaller circle such that ODA and OCB are straight lines.



(i) Given that area of triangle OCD is 7.5 cm^2, show that $\theta = 1.215$ rad, to 3 decimal places. ---[2]

(ii) Find the perimeter of the shaded region. ---[4]

(iii) Find the area of the shaded region. ---[3]

W-13/13/Q8

Answers

Q1 (a) Perimeter of shaded. arc + chord

$r \cdot 2\theta + 2r\sin\left(\frac{2\theta}{2}\right) = 20$ given

$\Rightarrow r(\theta + \sin\theta) = 10$

$\Rightarrow r = \frac{10}{\theta + \sin\theta}$ ✓

(b) $\frac{dr}{d\theta} = \frac{-10(1+\cos\theta)}{(\theta+\sin\theta)^2}$

$\left(\frac{dr}{d\theta}\right)_{\theta=\frac{\pi}{6}} = -17.8$ ✓

Q2 (i) area $= \frac{1}{2}r^2\theta = \frac{1}{2}\times 8^2\times\theta = 20$ given ✓

$\Rightarrow \theta = \frac{5}{8} = 0.625$ rad

(ii) length of arc AB $= r\theta$

$= 8\times\frac{5}{8} = 5$ cm ✓

(iii) Area of shaded segment of circle

$= \frac{1}{2}r^2(\theta - \sin\theta)$

$= \frac{1}{2}\times 8^2(1.4 - \sin 1.4)$

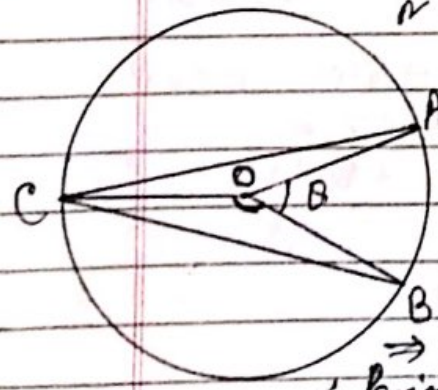
$= 32(1.4 - 0.985)$

$= 13.26$ ✓

Q3 (i) $\frac{1}{2}\times 10^2\times\theta = 20\pi \Rightarrow \theta = \frac{2\pi}{5}$ ✓

(ii) length of arc AB $= \theta\cdot r = 10\times\frac{2\pi}{5}$

$\therefore$ arc AB $= 4\pi$ ✓



$\angle BOC = \frac{4\pi}{5}$

In $\triangle BOC$

$\frac{BC}{\sin\frac{4\pi}{5}} = \frac{10}{\sin\frac{\pi}{10}}$

$\Rightarrow BC = 19.02$

$\therefore$ Perimeter $= 50.6$ ✓

(iii) Area $= 20\pi + 2\left[\frac{1}{2}\times 10\times 10\times\sin\frac{4\pi}{5}\right)$

$= 121.6$ ✓

Let angle AOB $= \theta$ rad

Q4 (i) $\frac{1}{2}\times 12^2\times\theta = 150$

$\Rightarrow \theta = 2.083$ rad.

(ii) area triangle AOB $= \frac{1}{2}\times 12^2\times\sin 2.08$

area segment $= 150 - \frac{1}{2}\times 12^2\times\sin 2.08$

$\frac{AB/2}{12} = \sin 1.04$

$\Rightarrow AB = 20.7$

Shaded area $= AB\times 8 -$ Area of segment

$= 78.5$ ✓

Q5 Let angle AOB $= \theta$

$AB = 2\times 8\times\sin\theta/2 = 15$

$\therefore \sin\theta/2 = \frac{15}{16}$

$\Rightarrow AOB = \theta = 2\cdot\sin^{-1}\left(\frac{15}{16}\right) = 2.43$ ⓐ

(i) angle AOD $= \frac{\text{arc AD}}{r} = \frac{4}{8} = 0.5$ ⓑ

(ii) angle DOC $= AOB - 2(AOD)$

$= 2.43 - 2\times 0.5$ [from ⓐ & ⓑ]

$= 1.43$ ✓

(iii) $DC = 2r\sin\frac{DOC}{2} = 2\times 8\times\sin\frac{1.43}{2}$

$= 16\sin 0.715 = 10.49$

$\therefore$ Perimeter $= 10.49 + 4 + 4 + 15 = 33.5$ ✓

(iv) $\frac{1}{2}\cdot 8^2(2.43 - \sin 2.43)$

$-\frac{1}{2}\times 8^2(1.43 - \sin 1.43)$

$= 42.8$ ✓

Q6 (i) Circumference $= 2\pi + 5\times 2\pi = 12\pi$

$\therefore 2\pi r = 12\pi \Rightarrow r = 6$ ✓

also length of arc $l = r\theta$

$2\pi = 6\times\theta \Rightarrow \theta = \frac{\pi}{3}$ ✓

(ii) $10 + 2\times 6\times\sin\frac{\pi}{6} = (10\pi + 6)$ ✓

(iii) $\frac{1}{2}\times 6^2\times\left(2\pi - \frac{\pi}{3}\right) + \frac{1}{2}6^2\cdot\sin\frac{\pi}{3}$

$= 30\pi + 9\sqrt{3}$ ✓

Circular Measure

Answers

Q7 (i) $\tan\frac{PAQ}{2} = 2.4$

$\Rightarrow PAQ = 2.35$ ✓

(ii) $PBQ = 0.790$

(iii) $2.352\times10 + 0.790\times24$

$= 42.5$ cm.

(iv) $\left(\frac{1}{2}\times24^2\times0.79\right) - \left(\frac{1}{2}\times24^2\sin0.79\right)$

$+ \frac{1}{2}\times10^2(2.352 - \sin 2.352)$

$= 22.94 + 82.1 = 105$ ✓

Q8 (i) Length of arc $l = r\theta$

$\angle BAC = \theta = l/r = \frac{6.2}{5} = 1.24$ ✓

(ii) $\frac{BD}{5} = \sin\left(\frac{1.24}{2}\right) \Rightarrow BD = 2.91$

Arc $BFC = \pi\times BD = 9.13$

$\therefore$ Perimeter $= 9.13 + 6.2 = \underline{15.3}$ ✓

(iii) Area $= \left(\frac{1}{2}\pi\times2.91^2\right) - \left(\frac{1}{2}\times5^2\times1.24\right)$

$- \frac{1}{2}\times5^2\sin1.24$

$9.58 \leqslant$ Area $\leqslant 9.62$

Q9 (i) angle $AOB = \frac{arc}{r} = \frac{14.8}{10} = 1.48$ ✓

(ii) $\frac{1}{2}\times10^2\times\theta = 21.8 \Rightarrow \theta = 0.436$

(iii) $\angle BOC = \frac{(2\pi - 1.48 - 0.436)}{2} = 2.18$

$BC = 20\sin\left(\frac{1}{2}BOC\right) = 17.7$ ✓

Perimeter $= 14.8 + 2\times17.7 + 4.36$

$= 54.7$ ✓

(iv) $\frac{1}{2}\times10^2\times1.48 + 21.8 + 2\times\frac{1}{2}\times10^2\sin2.14$

$= 178$ ✓

Q10 (i) $7\times\frac{1}{2}r^2\theta = \frac{1}{2}r^2(2\pi - \theta)$

$\Rightarrow \theta = \frac{\pi}{4}$ ✓

(ii) $r + r + \frac{\pi}{4}r = 20$

$\Rightarrow r = 7.18$ ✓

continued →

Q10 (iii) Perimeter $= \frac{\pi}{4}r + 2r\tan\frac{\pi}{8}$

$= 11.6$ ✓

(iv) Area $= r\times AC - \frac{1}{2}r^2\cdot\frac{\pi}{4}$

$= 21.356 - 20.246$

$1.008 \leqslant$ Area $\leqslant 1.11$ ✓

Q11 $RT = \frac{1}{\tan\theta}$; $RS = \frac{1}{\sin\theta}$

$x = 1 - \frac{1}{2\tan\theta} - \frac{1}{2\sin\theta}$

or $x = 1 - \frac{\cot\theta}{2} - \frac{\operatorname{cosec}\theta}{2}$ ✓

(ii) $A = x + \frac{1}{2}\frac{\cot\theta}{2}$

or $= 1 - \frac{\operatorname{cosec}\theta}{2}$

(iii) $\operatorname{cosec}\theta = \frac{2\sqrt{3}}{3} \Rightarrow \theta = \frac{\pi}{3}$ ✓

Q12 (i) $AB = 2r\sin\theta$ ✓

(ii) $2r\sin\theta + 2r\theta = 20$ (Given)

$r = \frac{10}{\theta + \sin\theta}$ ✓

(iii) $\frac{dr}{d\theta} = \frac{-10(1+\cos\theta)}{(\theta+\sin\theta)^2}$

when $\theta = \frac{\pi}{6}$, $\frac{dr}{d\theta} = -17.8$ ✓

(iv) $\frac{dr}{dt} = 15$;

$\frac{d\theta}{dt} = \frac{dr}{dt} \div \frac{dr}{d\theta}$

$\therefore \frac{d\theta}{dt} = 15 \div (-17.8) = -0.842$ ✓

Q13 (i) $\pi/3$

(ii) Area of triangle $ABC = \frac{1}{2}\times10^2\times\sin\frac{\pi}{3}$

Area of one sector $= \frac{1}{2}\times5^2\times\frac{\pi}{3}$

$\therefore$ Rep $= 25\sqrt{3} - 25\frac{\pi}{3} = 4.03$ ✓

Answers

Q14

(i) $\theta = \frac{arc}{r} = \frac{7}{5} > 1$

(ii) $\theta = 1.4$ rad.

(iii) $\frac{1}{2} \times 5^2 \times 1.4 = 17.5$ ✓

(iv) area of triangle $= \frac{1}{2} \times 5^2 \times \sin 1.4$
$= 12.32$

∴ area of shaded $= 17.5 - 12.32$
$= 5.18$ ✓

Q15 (i) length of arc $= 47 - 24 = 23$

$\Rightarrow \theta = \frac{23}{12} = 1.917$ rad
$= 1.92$,

(ii) $CD = 2 \times 12 \sin \theta/2 = 19.6$ (or 19.7)

(iii) Area of Sector $= 138$

area of $\Delta AOB = 67$ (or 68)

area of segment $= 70$ (or 71)

$AD \times AB$ + seg. Area $= 425$

$\Rightarrow$ $AD = 18.1$ (or 18.0)

Q16 (i) $\cos TOA = \frac{6}{10} \Rightarrow TOA = 0.927$

(ii) area of major sector.
$= \frac{1}{2} \times 6^2 \times (2\pi - 2 \times 0.927)$

Area of kite $= 2 \times \frac{1}{2} \times 6\sqrt{10^2 - 6^2}$

∴ Complete area $= 128$ ✓

(iii) Perimeter $= 6(2\pi - 2 \times 0.927)$
$+ 2 \times \sqrt{10^2 - 6^2}$
$= 42.6$ ✓

Q17 (i) $\theta = \frac{1}{r} = \frac{15\pi}{20} = \frac{3}{4}\pi$ ✓

(ii) Area of Sector + ar of triangle
$= \frac{1}{2} \times 20^2 \times \frac{3}{4}\pi + \frac{1}{2} \times 20^2 \times \sin \frac{\pi}{4}$

$= 613$

Q18 (i) $PT = 8 \cdot \tan\left(\frac{3\pi}{8}\right) = 19.3$ ✓

(ii) Kite OPTQ − area sector
$= 2 \times \frac{1}{2} \times 19.3 \times 8 - \frac{1}{2} \times 8^2 \times \frac{3\pi}{4}$
$= 79.1$ ✓

(iii) Perimeter :
$= 8 \times \frac{3\pi}{4} + 2 \times 8 \tan \frac{3\pi}{8} = 57.5$ ✓

Q19 Area $= \frac{1}{2} \times 12^2 \times 1.7$

(i) $+ \frac{1}{2} \times 12^2 \times \sin(2\pi - 1.7 - 2.4)$
$= 181$ ✓

(ii) $BC = 2 \times 12 \times \sin\left(\frac{2\pi - 4.1}{2}\right)$
$= 21.296$

Perimeter $= 12 \times 1.7 + 12 + 12 + 21.296$
$= 65.7$ ✓

Q20 (i) All sides are equal to radii of equal circles.

(ii) Angle $CBE = \frac{2\pi}{3}$ ✓

(iii) $DE = 10\sqrt{3}$

Arc $CE = 10 \times \frac{2\pi}{3}$

Perimeter $= 20 + 10\sqrt{3} + \frac{20\pi}{3}$
$= 58.3$ ✓

(iv) Area of Sector $= \frac{1}{2} \times 10^2 \times \frac{2\pi}{3} = \frac{100\pi}{3}$

Area of triangle $= \frac{1}{2} \times 10^2 \times \sin \frac{2\pi}{3} = 25\sqrt{3}$

∴ Area $= \frac{100\pi}{3} + 25\sqrt{3} = 148$ ✓

Q21 (i) $\cos ABC = \frac{6^2 + 6^2 - 10^2}{2 \times 6 \times 6}$

$\Rightarrow$ angle $ABC = 1.9702$ ✓

(ii) $XY = 2$,

Arc length $= 6\left(\frac{\pi - 1.9702}{2}\right)$

Perimeter $= 2 + 2 \times 6\left(\frac{\pi - 1.9702}{2}\right)$
$= 9.03$ ✓

(continued →)

Continued

Q21(iii) $\left[\frac{1}{2}\times6^2\left(\frac{\pi-1.970}{2}\right)-\frac{1}{2}\times5\times\sqrt{11}\right]\times2$
$= 4.50$ (or 4.51)

Q22(i) $2r+1\theta=30 \Rightarrow \theta=\frac{30-2r}{r}$
Area $=\frac{1}{2}r^2\theta=\frac{1}{2}r^2\left(\frac{30-2r}{r}\right)$
$\therefore A=15r-r^2$ ✓
(ii) $15-2r=0 \Rightarrow r=7.5$
$\therefore A=56.25$

Q23(i) $\sin\theta/2=6/8 \Rightarrow \frac{\theta}{2}=0.8481$
$\Rightarrow \theta=1.696$ ✓
(ii) Arc length $=(2\pi-1.696)\times8=36.7$
$\therefore$ Perimeter $=12+36.7=48.7$ ✓
(iii) Area $=\frac{1}{2}\times8^2\times(2\pi-1.696)$
$+\frac{1}{2}\times8^2\times\sin1.696$
$=178.5$ ✓

Q24(i) $\frac{1}{2}r^2\cdot(1.6)=500$
$\Rightarrow r=25$ ✓
(ii) $25+25+25\times1.6=90$ ✓

Q25 Area of sector $=\frac{1}{2}x^2\times0.8=0.4x^2$
Area of triangle $=\frac{1}{2}\times5\cos0.8\times5\sin0.8$
$=6.247$
$\Rightarrow \frac{1}{5}\times0.4x^2=6.247$
$\Rightarrow x=8.837$ ✓
(ii) $SQ=8.84-5=3.84$
$PR=8.84-5\cos0.8=5.35$
$PQ=8.84\times0.8=7.07$
$SR=5\times\sin0.8=3.59$
$\therefore$ Perimeter $=19.9$ ✓
(iii) Area PQSR $=4\times$ Area of triangle
$=4\times6.247$
$=25\,cm^2$

Answers

Q26(i) $\frac{OC}{\sin0.9}=\frac{12}{\sin(\pi-1.8)}$
$\Rightarrow OC=9.652$ ✓
(ii) Perimeter $=0.9\times12+9.652$
$+(12-9.652)$
$=22.8$ ✓
(iii) Area $=\frac{1}{2}\times12^2\times0.9-\frac{1}{2}\times(9.652)^2\times\sin(\pi-1.8)$
$=19.4$ ✓

Q27 $\tan\theta/2=8/6 \Rightarrow \frac{\theta}{2}=0.927 \Rightarrow \theta=1.855$ ✓
(ii) $r=10$,
$P=10\times1.855+10+10+16=54.6$ ✓
(iii) $A=256-2\times\frac{1}{2}\times8\times6-\frac{1}{2}\times10^2\times1.855$
$=115$ ✓

Q28(i) Arc $AB=2\cdot\frac{\pi}{3}$
Chord $AB=2r\cdot\sin\pi/6=r$
$\therefore P=r+\pi r/3=r\left(\frac{3+\pi}{3}\right)$ ✓
(ii) $r=12.7$
Area $=\frac{1}{2}r^2\times\left(\frac{\pi}{3}-\sin\frac{\pi}{3}\right)=14.6$ ✓

Q29(i) $AB=2r\sin\theta/2=2\times12\times\sin0.7$
$=15.46$
and Arc $=r\theta=12\times(2\pi-1.4)=58.6$
$\therefore$ Perimeter $=74.1$ ✓
(ii) Area $=\frac{1}{2}\times12^2\times(2\pi-1.4)$
$+\frac{1}{2}\times12^2\times\sin1.4$
$=422$ (or 423) ✓

Q30(i) $\frac{1}{2}\times4^2\cdot\sin\theta=7.5$
$\Rightarrow \sin\theta=\frac{15}{16} \Rightarrow \theta=1.215$ ✓
(ii) $CD=2\times4\times\sin\theta/2=4.567$ ✓
Arc length $=6\times1.215=7.29$
Perimeter $=2+2+7.29+4.567=15.9$ ✓
(iii) Area $=\frac{1}{2}\times6^2\times1.215-7.5$
$=14.4$ ✓