IG-0606
Additional Maths

Diff. and Integration
Exercise. 1

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IG-Maths 0606

Differentiation and Integration

Q1. Variable x and y are related by the equation $y = x\sqrt{x}$.

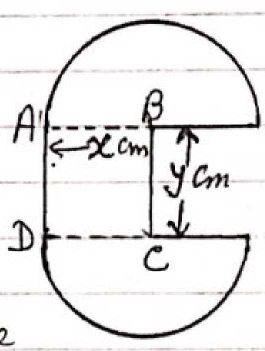
(a) Find $\frac{dy}{dx}$ ----[2] SP-20/01/Q2

(b) Hence find the approximate change in x when y increases from 8 by the small amount 0.015 ----[3]

Q2 Find the equation of the normal to the curve $y = \frac{2x-1}{\sqrt{x^2+5}}$, at the point where $x = 2$, give your answer in the form $ax + by = c$, where a, b and c are integers. --[8]

SP-20/01/Q6

Q3 The diagram shows a badge, made of thin sheet metal, consisting of two semi-circular pieces, centres B and C, each of radius x cm. They are attached to each other by a rectangular piece of thin sheet metal, ABCD, such that AB and CD are the radii of the semicircular pieces and AD = BC = y cm.



(a) Given that the area of the badge is $20\,cm^2$, show that the perimeter, P cm, of the badge is given by,

$$P = 2x + \frac{40}{x}$$ ---[4]

(b) Given that x can vary, find the minimum value of P, justifying that this value is minimum. SP-20/01/Q7 --[5]

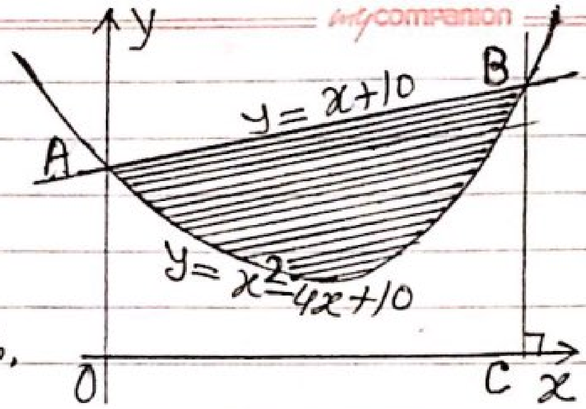
Q4(a) Giving your answer in its simplest form, find the exact value of (i) $\int_{0.2}^{1} e^{5x-1}\,dx$, --[4]

(ii) $\int_{1}^{2} \left(x + \frac{1}{x^2}\right)^2 dx$ --[2]

SP-20/01/Q8

(b) $\int \sin\frac{x}{6}\,dx$ --[2]

Q5. The graph of $y=x^2-4x+10$ cuts the y-axis at point A. The graph of $y=x^2-4x+10$ and $y=x+10$ intersect one another at the points A and B. The line BC is perp. to the x-axis. Calculate the area of the shaded region enclosed by the curve and line AB. [SP-20/01/Q11] --[8]



Q6 A particle P travels in a straight line such that, t s after passing through a fixed point O, its velocity v ms^{-1} is given by $v=\left(e^{\frac{t^2}{8}}-4\right)^3$

(a) Find the speed of P at O. --[1]

(b) Find the value of t for which P is instantaneously at rest. --[2]

(c) Find the acceleration of P when t=1. [SP-20/02/Q6] --[4]

Q7 A curve has equation, $y=4+5\sin 3x$,

(i) Find $\frac{dy}{dx}$ ---[2]

(ii) Hence find the equation of the tangent to the curve, $y=4+5\sin 3x$ at the point where $x=\frac{\pi}{3}$ ---[3] [M-18/12/Q2]

Q8 It is given that $y=\frac{\ln(4x^2-1)}{(x+2)}$

(i) Find the value of x for which y is not defined. --[2]

(ii) Find $\frac{dy}{dx}$. [M-18/12/Q4] --[3]

(iii) Hence find the approximate increase in y, when x increases from 2 to 2+h, where h is small. --[2]

Q9. A particle P, moving in a straight line, passes through a fixed point O at time t=0 s. At time t s after leaving O, the displacement of the particle is x m and its velocity is v ms^{-1}, where

$v=12e^{2t}-48t,\ t\geqslant 0$

(i) Find x in terms of t. (continued→) ---[4]

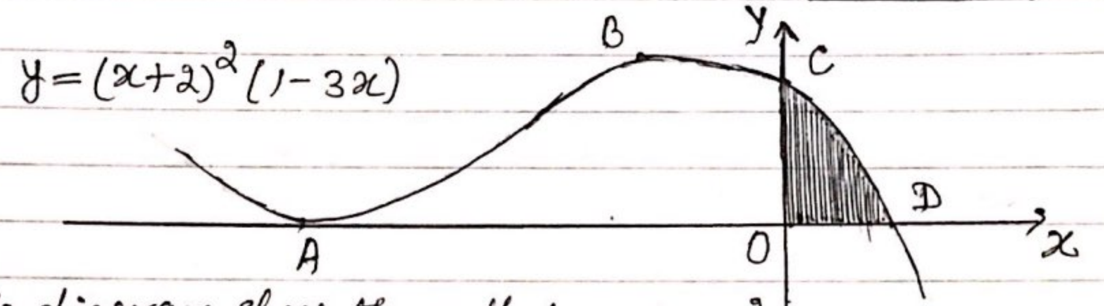
(Continued→)

Q9 (ii) Find the value of t when acceleration of P is zero. ---[3]

(iii) Find the velocity of P when the acceleration is zero. ---[2]

M-18 | 12 | Q 8

Q10



The diagram shows the graph of $y=(x+2)^2(1-3x)$. The curve has a minimum at the point A, a maximum at the point B, and intersects the y-axis and the x-axis at the points C and D resp.

(i) Find the x-coordinate of A and of B. --[5]

(ii) Write down the coordinates of C and of D. ---[2]

(iii) Show all your working, find the area of the shaded region. --[5]

M-18 | 12 | Q 10

Q11 (i) Differentiate $1+\tan\left(\frac{x}{3}\right)$ with respect to x. ---[2]

(ii) Hence find $\int \sec^2\left(\frac{x}{3}\right)dx$ ---[2]

M-18 | 22 | Q 6

Q12 The volume, V, and surface area, S, of a sphere of radius r are given by $V=\frac{4}{3}\pi r^3$ and $S=4\pi r^2$ respectively. The volume of a sphere increases at a rate of 200 cm^3 per second. At the instant when the radius of the sphere is 10 cm, find

(i) the rate of increase of the radius of the sphere, ---[4]

(ii) the rate of increase of the surface area of the sphere --[3]

M-18 | 22 | Q 12

Q13 A curve has equation $y=4+5\sin 3x$

(i) Find $\frac{dy}{dx}$ ---[2]

(ii) Hence find the equation of tangent to the curve; $y=4+5\sin 3x$ at the point where $x=\frac{\pi}{3}$ ---[3]

M-18 | 12 | Q 2

Q14 A curve is such that $\frac{d^2y}{dx^2} = 4\sin 2x$. A curve has a gradient of 5 at the point where $x = \frac{\pi}{2}$. ---[4]

(i) Find an expression for the gradient of the curve at the point (x, y).

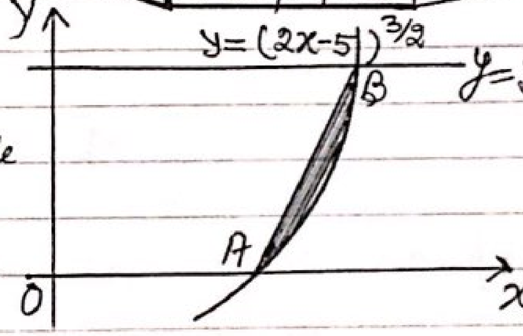
The curve passes through the point $P\left(\frac{\pi}{12}, -\frac{1}{2}\right)$

(ii) Find the equation of the curve. ---[4]

(iii) Find the equation of the normal to the curve at the point P, giving your answer in the form $y = mx + c$, where m and c are constant correct to 3 decimal places. ---[3]

M-17/12/Q8

Q15 The curve shows part of the curve $y = (2x-5)^{3/2}$ and the line $y = 3\sqrt{3}$. The curve meets the x-axis at point A and the line $y = 3\sqrt{3}$ at the point B. Find the area of the shaded region



enclosed by the line AB and the curve, giving your answer in the form $\frac{p\sqrt{3}}{20}$, where p is an integer. You must show all your working. M-17/12/Q10 ---[8]

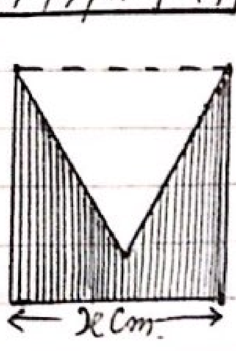
Q16 (a) Find $\int e^{2x+1}\,dx$ ---[2]

(b)(i) Given that $y = \frac{x}{\ln x}$; find $\frac{dy}{dx}$ ---[3]

(ii) Hence $\int \left(\frac{1}{\ln x} - \frac{1}{(\ln x)^2} + \frac{1}{x^2}\right) dx$ ---[3]

M-17/22/Q9

Q17 The diagram shows a shape made by cutting an equilateral triangle out of a rectangle of width x cm.



The perimeter of the shape is 20 cm.

(i) Show that the area, A cm², of the shape is: $A = 10x - \left(\frac{6+\sqrt{3}}{4}\right)x^2$ ---[3]

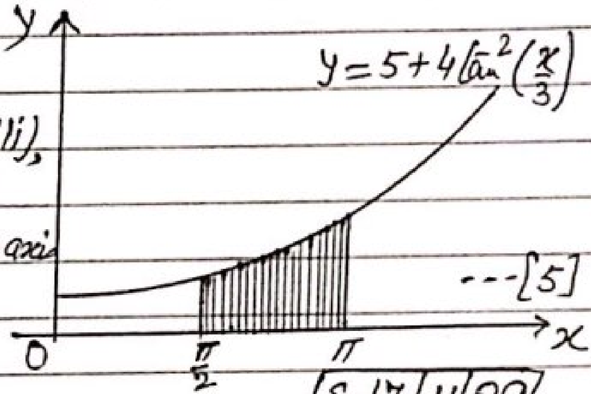
(ii) Given that x can vary, find the value of x, which produces the maximum area and calculate this maximum area. Give your answers to 2 significant figures. M-17/22/Q12 ---[4]

Q18 Show that the curve $y=(3x^2+8)^{5/3}$ has only one stationary point. Find the coordinate of this stationary point and determine its nature. --[8]
S-17/11/Q7

Q19 (i) Show that $5+4\tan^2\left(\frac{x}{3}\right)=4\sec^2\left(\frac{x}{3}\right)+1$ --[1]

(ii) Given that $\frac{d}{dx}\left(\tan\left(\frac{x}{3}\right)\right)=\frac{1}{3}\sec^2\left(\frac{x}{3}\right)$; find $\int \sec^2\left(\frac{x}{3}\right)dx$ --[1]

(iii) The diagram shows part of the curve $y=5+4\tan^2\left(\frac{x}{3}\right)$. Using the results from part (i) and (ii), find the exact area of the shaded region enclosed by the curve, the x-axis and the lines $x=\frac{\pi}{2}$ and $x=\pi$. ---[5]



S-17/11/Q9

Q20 (a) Given that $y=\dfrac{e^{3x}}{4x^2+1}$, find $\dfrac{dy}{dx}$. ---- [3]

(b) Variables x, y and t are such that:

$$y=4\cos\left(x+\frac{\pi}{3}\right)+2\sqrt{3}\sin\left(x+\frac{\pi}{3}\right) \text{ and } \frac{dy}{dt}=10$$

(i) Find the value of $\frac{dy}{dx}$ when $x=\frac{\pi}{2}$ ---[3]

(ii) Find the value of $\frac{dx}{dt}$ when $x=\frac{\pi}{2}$ ---[2]
S-17/11/Q10

Q21 Find the equation of the curve which passes through the point $(2,17)$ and for which $\frac{dy}{dx}=4x^3+1$. S-17/21/Q1 --[4]

Q22 The variables x and y are such that $y=\ln(x^2+1)$

(i) Find an expression for $\frac{dy}{dx}$ --[2]

(ii) Hence, find the approximate change in y when x increases from 3 to $3+h$, where h is small. --[2]
S-17/21/Q3

Diff and Integration

Q23 A particle moves in a straight line so that, t seconds after passing through a fixed point O, its displacement, s m, from O is given by:

$$s = 1 + 3t - \cos 5t$$

(i) Find the distance between the particle's first two positions of instantaneous rest. --[7]

(ii) Find the acceleration when $t = \pi$. [S-17/21/Q12] --[2]

Q24 It is given that $y = \dfrac{(5x^2+4)^{1/2}}{(x+1)}$, show all your working, find the exact value of $\dfrac{dy}{dx}$ when $x = 3$ [S-17/12/Q2] --[5]

Q25 A particle 'P' moves in a straight line, such that its displacement, x m, from a fixed point O, t s after passing O, is given by,

$$x = 4\cos(3t) - 4$$

(i) Find the velocity of P at time t. ---[1]

(ii) Hence write down the maximum speed at P. ---[1]

(iii) Find the smallest value of t for which the accelation of P is zero. --[3]

(iv) For value of t found in part (iii), find the distance of P from O. --[1]

[S-17/12/Q5]

Q26 The curve $y = f(x)$ passes through the point $\left(\frac{1}{2}, \frac{7}{2}\right)$ and is such that $f'(x) = e^{2x-1}$

(i) Find the equation of the curve. --[4]

(ii) Find the value of x for which $f''(x) = 4$, giving your answer in the form $a + b\ln\sqrt{2}$, where a and b are constants. --[4]

[S-17/12/Q11]

Q27 The point P lies on the curve $y = 3x^2 - 7x + 11$. The normal to the curve at P has equation $5y + x = k$. Find the coordinates of P and the value of k. [S-17/22/Q4] ---[6]

Q28 (i) Show that $\frac{d}{dx}[0.4x^5(0.2 - \ln 5x)] = kx^4 \ln 5x$, where k is an integer to be found. --[2]

(ii) Express $\ln 125x^3$ in terms of $\ln 5x$. --[1]

(iii) Hence find $\int (x^4 \ln 125x^3)\,dx$ ---[2] [S-17/22/Q5]

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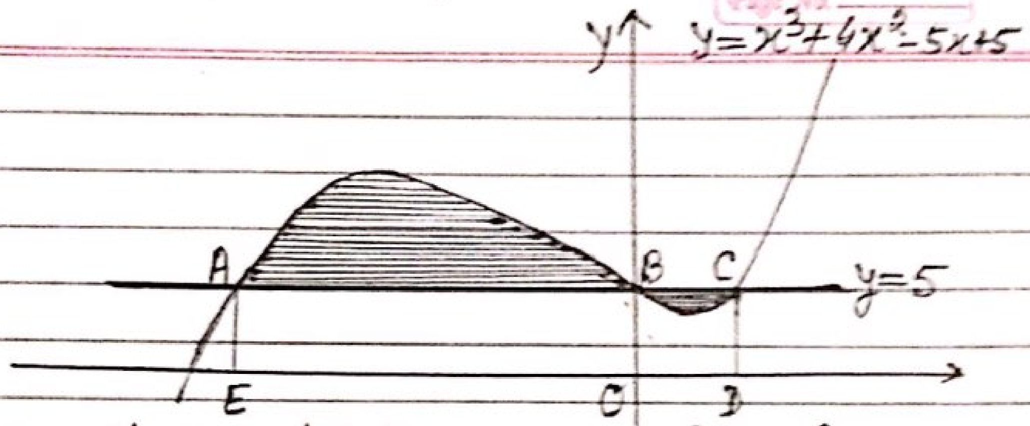
Diff. and Integration

Q29



The diagram shows part of the curve $y=x^3+4x^2-5x+5$ and the line $y=5$. The curve and the line intersect at the points A, B and C. The points D and E are on the x-axis and the lines AE and CD are parallel to the y-axis.

(i) Find $\int(x^3+4x^2-5x+5)\,dx$ ---[2]

(ii) Find the area of each of the rectangles OEAB and OBCD --[4]

(iii) Hence calculate the total area of the shaded region enclosed between the line and the curve. You must show all your working. --[4]

[S-17/22/Q11]

Q30 The normal to the curve $y=\sqrt{4x+9}$, at the point where $x=4$, meets the x-axis and y-axis at the points A and B. Find the coordinates of the mid point of line AB. [S-17/13/Q5] --[7]

Q31 It is given that $\int_{-k}^{k}(15e^{5x}-5e^{-5x})\,dx=6$

(i) Show that $e^{5k}-e^{-5k}=3$ ---[5]

(ii) Hence using substitution $y=e^{5k}$, or otherwise find the value of k. [S-17/13/Q9] --[3]

Q32 It is given that $y=(10x+2)\ln(5x+1)$.

(i) Find $\frac{dy}{dx}$ ---[4]

(ii) Hence show that: $\int\ln(5x+1)\,dx=\frac{(ax+b)}{5}\ln(5x+1)-x+c$, where a and b are integers and c is a constant of integration. --[3]

(iii) Hence find $\int_0^{1/5}\ln(5x+1)\,dx$, giving your answer in the form, $\frac{d+\ln f}{5}$, where d and f are integers. --[2]

[S-17/13/Q10]

Q33 A curve has equation $y = 6x - x\sqrt{x}$

(i) Find the coordinates of the stationary point of the curve. ---[4]

(ii) Determine the nature of this stationary point. --[2]

(iii) Find the approximate change in y when x increases from 4 to 4+h, where h is small. [S-17/13/Q11] --[3]

Q34 A particle moves in a straight line, such that the velocity, v ms^{-1}, t s after passing a fixed point O, is given by,

$v = 2 + 6t + 3\sin 2t$.

(i) Find the acceleration of the particle at time t. ---[2]

(ii) Hence find the smallest value of t for which the acceleration of the particle is zero. ---[2]

(iii) Find the displacement, x m from O, of the particle at time t. --[5]

[S-17/13/Q12]

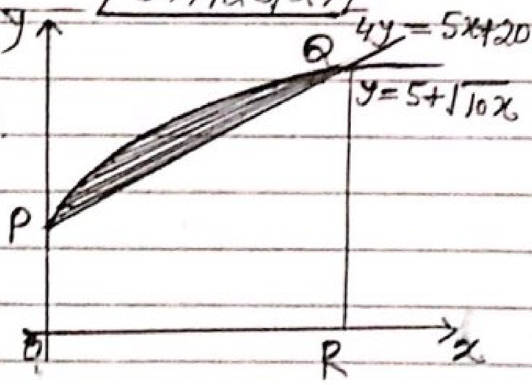
Q35 Differentiate with respect to x.

(i) $(1+4x)^{10}\cos x$, ---[4]

(ii) $\dfrac{e^{4x-5}}{\tan x}$ ---[4]

[S-17/23/Q7]

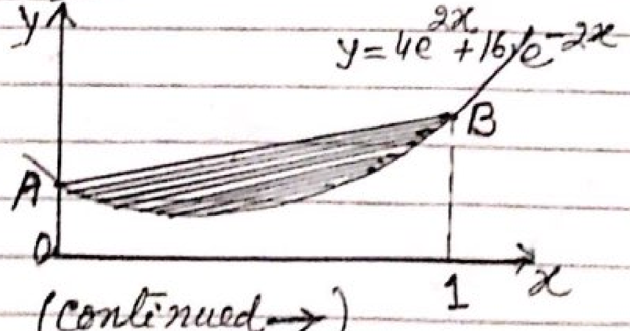
Q36 The diagram shows part of the curve $y = 5 + \sqrt{10x}$ and the line $4y = 5x + 20$. The line and curve intersect at the points P(0,5) and Q. The line QR is parallel to the y-axis.



(i) Find the coordinates of Q. ---[4]

(ii) Find the area of the shaded region. You must show all your working. [S-17/23/Q11] --[6]

Q37 The diagram shows part of the graph of $y = 4e^{2x} + 16e^{-2x}$ meeting the y-axis at the point A and the line $x = 1$ at the point B.



(i) Find the coordinates of A. ---[1]

(continued→)

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Q37(ii) Find the coordinates of B. ---[1]

(iii) Find $\int (4e^{2x} + 16e^{-2x})\,dx$ ---[2]

(iv) hence find the area of the shaded region enclosed by the curve and the line AB. You must show all your working. ---[4]

W-17/11/Q5

Q38 (i) Write $\ln\left(\frac{2x+1}{2x-1}\right)$ as the difference of two logarithms. ---[1]

A curve has equation $y = \left(\frac{2x+1}{2x-1}\right) + 4x$ for $x > \frac{1}{2}$

(ii) Using your answer to part (i) show that $\frac{dy}{dx} = \frac{ax^2+b}{4x^2-1}$, where a and b are integers. ---[4]

(iii) Hence find the x-coordinate of the stationary point on the curve. ---[2]

(iv) determine the nature of this stationary point. ---[2]

W-17/11/Q7

Q39 (i) Find $\frac{d}{dx}\left(\frac{5}{3x+2}\right)$ ---[2]

(ii) use your answer to part (i) to find $\int \frac{30}{(3x+2)^2}\,dx$ ---[2]

(iii) Hence evaluate $\int_1^2 \frac{30}{(3x+2)^2}\,dx$ ---[2]

W-17/21/Q5

Q40 Find y in terms of x, given that $\frac{d^2y}{dx^2} = 6x + \frac{2}{x^3}$ and that when $x = 1$, $y = 3$ and $\frac{dy}{dx} = 1$ ---[6]

W-17/21/Q7

Q41 (i) differentiate $(\cos x)^{-1}$ with respect to x. ---[2]

(ii) Hence find $\frac{dy}{dx}$; given that $y = \tan x + 4(\cos x)^{-1}$ ---[2]

(iii) using your answer to part (ii) find the value of x, in the range $0 \le x \le 2\pi$ such that $\frac{dy}{dx} = 4$ ---[6]

W-17/21/Q12

Q42 Given that $y = \frac{\ln(3x^2+2)}{x^2+1}$, find the value of $\frac{dy}{dx}$ when $x = 2$, giving your answer as $a + b\ln 14$, where a and b are fractions in their simplest form. ---[6]

W-17/12/Q4

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Diff and Integration

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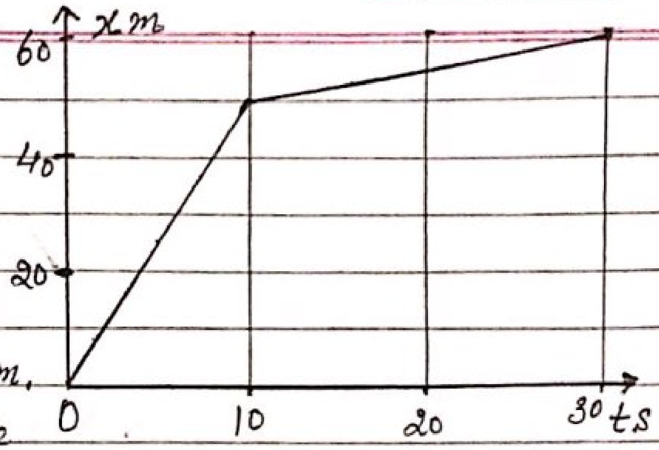
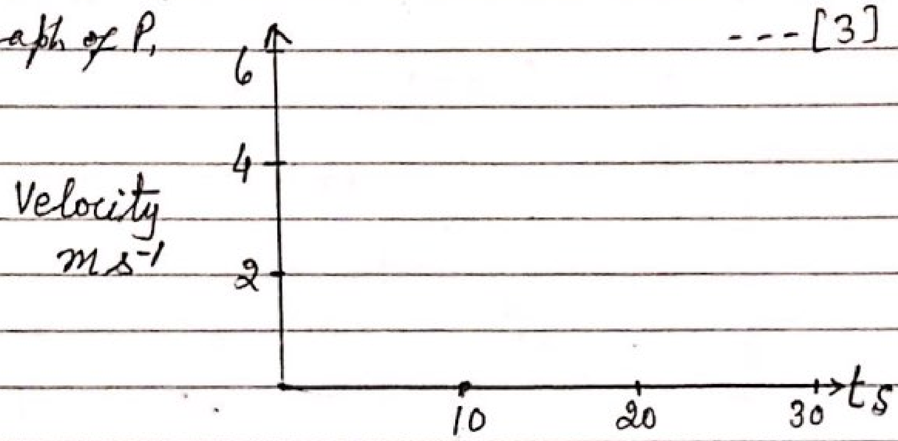
Q43 (a) The diagram shows the displacement-time graph of a particle P which moves in a straight line such that, t s after leaving a fixed point O, its displacement from O is x m.



On the axes below, draw the velocity-time graph of P. ---[3]



(b) A particle Q moves in a straight line such that its velocity, t s after passing through a fixed point O, is given by,

$$V = 3e^{-5t} + \frac{3t}{2}, \quad \text{for } t \geq 0$$

(i) Find the velocity of Q when $t = 0$ ---[1]

(ii) Find the value of t, when the acceleration of Q is zero. ---[3]

(iii) Find the distance of Q from O when $t = 0.5$ $\boxed{W\text{-}17/12/Q9}$ ---[4]

Q44 The volume of a closed cylinder of base radius x cm and height h cm is 500 cm^3.

(i) Express h in terms of x. ---[1]

(ii) Show that the total surface area of the cylinder is given by,

$$A = 2\pi x^2 + \frac{1000}{x} \text{ cm}^2$$ ---[2]

(iii) Given that: x can vary, find the stationary value of A and show that this value is a minimum. ---[5]

$\boxed{W\text{-}17/22/Q6}$

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Diff and Integration

Q45 The gradient of the normal to a curve at the point with coordinates (x, y) is given by $\frac{\sqrt{x}}{1-3x}$.

(i) Find the equation of the curve, given that the curve passes through the point $(1, -10)$ ---[5]

(ii) Find, in the form $y = mx + c$, the equation of the tangent to the curve at the point where $x = 4$. [W-17/22/Q7] ---[4]

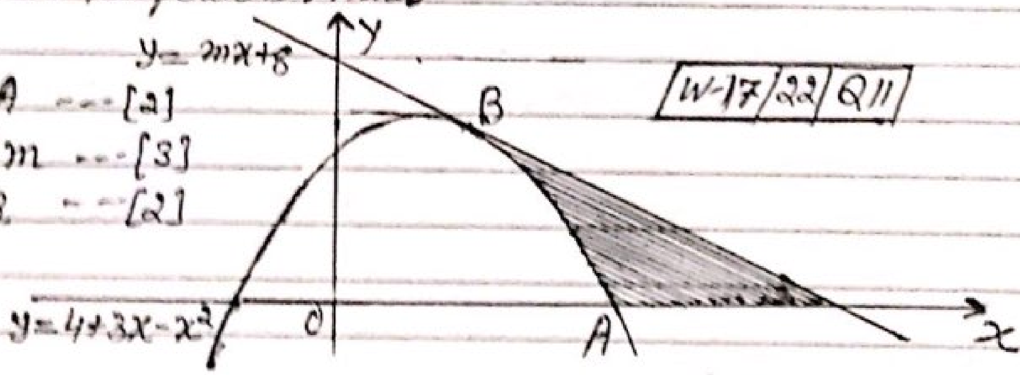
Q46 (i) Find $\frac{d}{dx}(x \cdot \ln x)$ ---[2]

[W-17/22/Q9]

(ii) Hence find $\int \ln x \, dx$ ---[2]

(iii) Hence, given that $k > 0$, show that $\int_k^{2k} \ln x \, dx = k[\ln 4k - 1)$ ---[4]

Q47 The diagram shows the curve $y = 4 + 3x - x^2$ intersecting the positive x-axis at the point A. The line $y = mx + 8$ is a tangent to the curve at the point B. Find



[W-17/22/Q11]

(i) Coordinates of A ---[2]

(ii) the value of m ---[3]

(iii) Coordinates of B. ---[2]

(iv) the area of the shaded region, showing all your working. ---[5]

Q48 (i) Find $\int (7x - 10)^{-3/5} dx$ [W-17/13/Q5] ---[2]

(ii) Given that $\int_6^a (7x-10)^{-3/5} dx = \frac{35}{14}$, find the exact value of a. ---[3]

Q49 It is given that $y = (x-4)(3x-1)^{5/3}$

(i) Show that $\frac{dy}{dx} = (3x-1)^{2/3}(Ax+B)$, where A and B are integers to be found. ---[5]

(ii) Hence find, in terms of h, where h is small, the approximate change in y when x increases from 3 to $3+h$. [W-17/13/Q8] ---[3]

Q50 A particle moves in a straight line passes through a fixed point O. Its velocity, $v\,ms^{-1}$, ts after passing through O, is given by $v = 3\cos 2t - 1$ for $t \geq 0$

(i) Find the value of t when the particle is first at rest. --- [2]

(ii) Find the displacement from O of the particle when $t = \frac{\pi}{4}$ --- [3]

(iii) Find the acceleration of the particle when it is at rest. --- [3]

[W-17/23/Q7]

Q51 (i) Show that $\frac{d}{dx}\left(\frac{\ln x}{x^3}\right) = \frac{1-3\ln x}{x^4}$ --- [3]

(ii) Find the exact coordinates of the stationary point on the curve $y = \frac{\ln x}{x^3}$ --- [3]

(iii) Use the result from part (i) to find, $\int\left(\frac{\ln x}{x^4}\right) dx$. [W-17/23/Q9] --- [4]

Q52 (i) Find $\frac{d}{dx}\left(x(2x-1)^{3/2}\right)$ --- [3]

(ii) Hence show that, $\int x(2x-1)^{1/2} dx = \frac{(2x-1)^{3/2}}{15}(px+q)+c$,

where c is a constant of integration, and p and q are integers to be found. --- [6]

(iii) Hence find $\int_{0.5}^{1} x(2x-1)^{1/2} dx$ [M-16/12/Q10] --- [2]

Q53. Two variables x and y are such that $y = \frac{5}{\sqrt{x-9}}$ for $x > 9$

(i) Find an expression for $\frac{dy}{dx}$ --- [2]

(ii) Hence, find the approximate change in y as x increases from 13 to 13+h, where h is small. [M-16/22/Q1] --- [2]

Q54. Find the equation of curve which passes through the point (1,7) and for which $\frac{dy}{dx} = \frac{9x^4-3}{x^2}$ [M-16/22/Q3] --- [4]

Q55 A curve has equation; $y = \frac{x}{x^2+1}$,

(i) Find the coordinates of the stationary points of the curve. ---- [5]

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Diff and Integrations

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Q55(ii) Show that $\frac{d^2y}{dx^2} = \frac{px^3+qx}{(x^2+1)^3}$, where p and q are integers to be found, and determine the nature of the stationary points of the curve. [M-16/22/Q11] ---[5]

Q56 A particle P is projected from the origin O so that it moves in a straight line. At time t seconds after projection, the velocity of the particle, $v\ ms^{-1}$, is given by

$$v = 9t^2 - 63t + 90$$

(i) Show that P first comes to instantaneous rest when $t = 2$ ---[2]

(ii) Find the acceleration of P when $t = 3.5$ ---[2]

(iii) Find an expression for the displacement of P from O at time t seconds. ---[3]

(iv) Find the distance travelled by P

(a) in first 2 seconds. --[2]

(b) in first 3 seconds. [M-16/22/Q12] --[2]

Q57 Find the equation of the normal to the curve $y = \ln(2x^2 - 7)$ at the point where the curve crosses the positive x-axis. Give your answer in the form $ax + by + c = 0$, where a, b and c are are integers. [S-16/11/Q3] ---[5]

Q58 Do not use a calculator in this question.

(i) Show that $\frac{d}{dx}\left(\frac{e^{4x}}{4} - xe^{4x}\right) = pxe^{4x}$, where p is an integer to be found. --[4]

(ii) Hence find the exact value of $\int_0^{\ln 2} xe^{4x}\,dx$, giving your answer in the form, ---[4]

$a\ln 2 + \frac{b}{c}$, where a, b, and c are integers to be found. [S-16/11/Q5]

Q59 (i) Find $\int (3x - x^{3/2})\,dx$ ---[2]

The diagram shows part of the curve $y = 3x - x^{3/2}$ and the lines $y = 3x$ and $2y = 27 - 3x$. The curve and the line $y = 3x$ meet the x-axis at O and the curve and the line $2y = 27 - 3x$ meet the x-axis at A.

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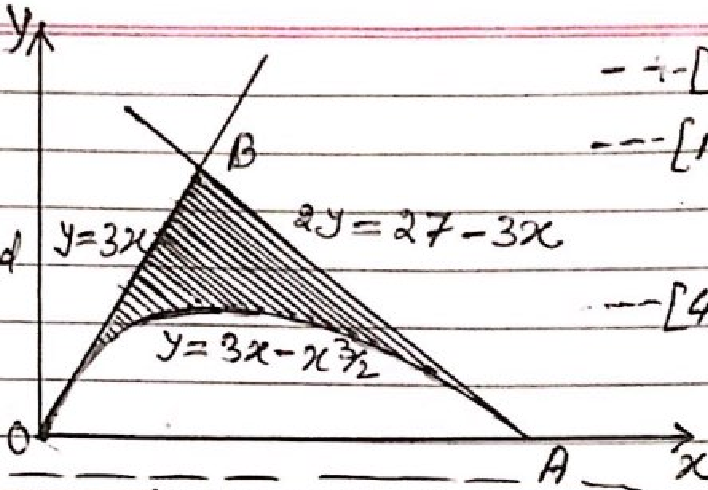
Q59 (ii) Find the coordinates of A. ---[1]

(iii) Verify that the coordinates of B are (3, 9) ---[1]

(iv) Find the area of the shaded region. ---[4]



[S-16/21/Q11]

Q60 A curve has equation: $y = \frac{2x-5}{x-1} - 12x$

(i) Find $\frac{dy}{dx}$, ---[3]

(ii) Find $\frac{d^2y}{dx^2}$ ---[2]

(iii) Find the coordinates of the stationary points of the curve and determine their nature. [S-16/21/Q12] ---[5]

Q61 Show that $\frac{d}{dx}\left(e^{3x}\cdot\sqrt{4x+1}\right)$ can be written in the form $\frac{e^{3x}(px+q)}{\sqrt{4x+1}}$, where p and q are integers to be found. ---[5]

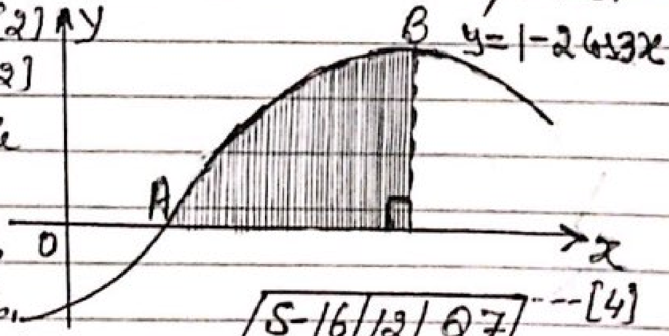
[S-16/12/Q6]

Q62 The diagram shows part of the graph of $y = 1 - 2\cos 3x$, which crosses x-axis at the point A and has a maximum at point B.

(i) Find the coordinate of A. ---[2]

(ii) Find the coordinates of B. ---[2]

(iii) Showing all your working, find the area of the shaded region bounded by the curve, the x-axis and the perp. from B to the x-axis. ---[4]



[S-16/12/Q7]

Q63 A curve passes through the point $(2, -\frac{4}{3})$ and is such that $\frac{dy}{dx} = (3x+10)^{-\frac{1}{2}}$

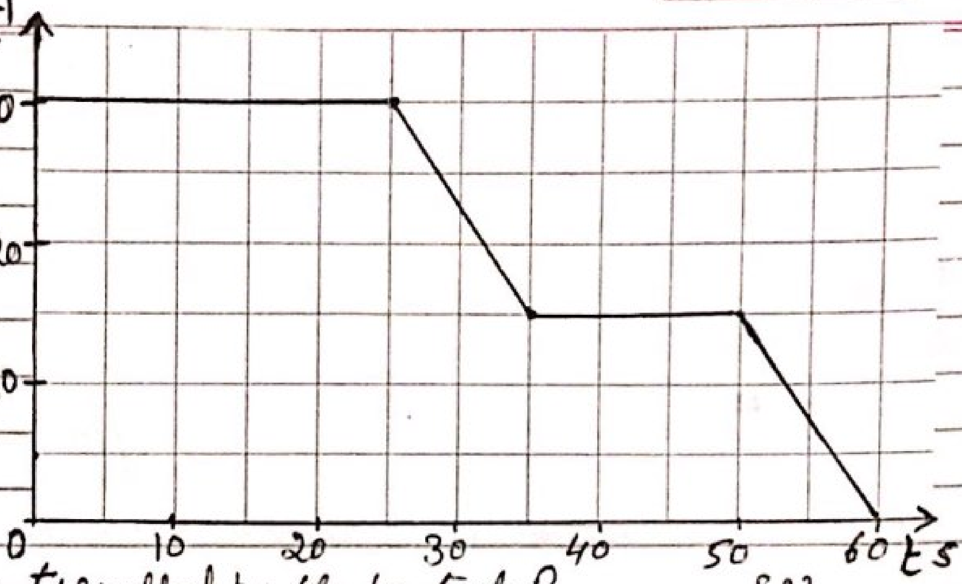
(i) Find the equation of the curve. ---[4]

The normal to the curve, at the point where $x = 5$, meets the line $y = -5/3$ at point P.

(ii) Find the x-coordinate of P. [S-16/12/Q9] ---[6]

Q64

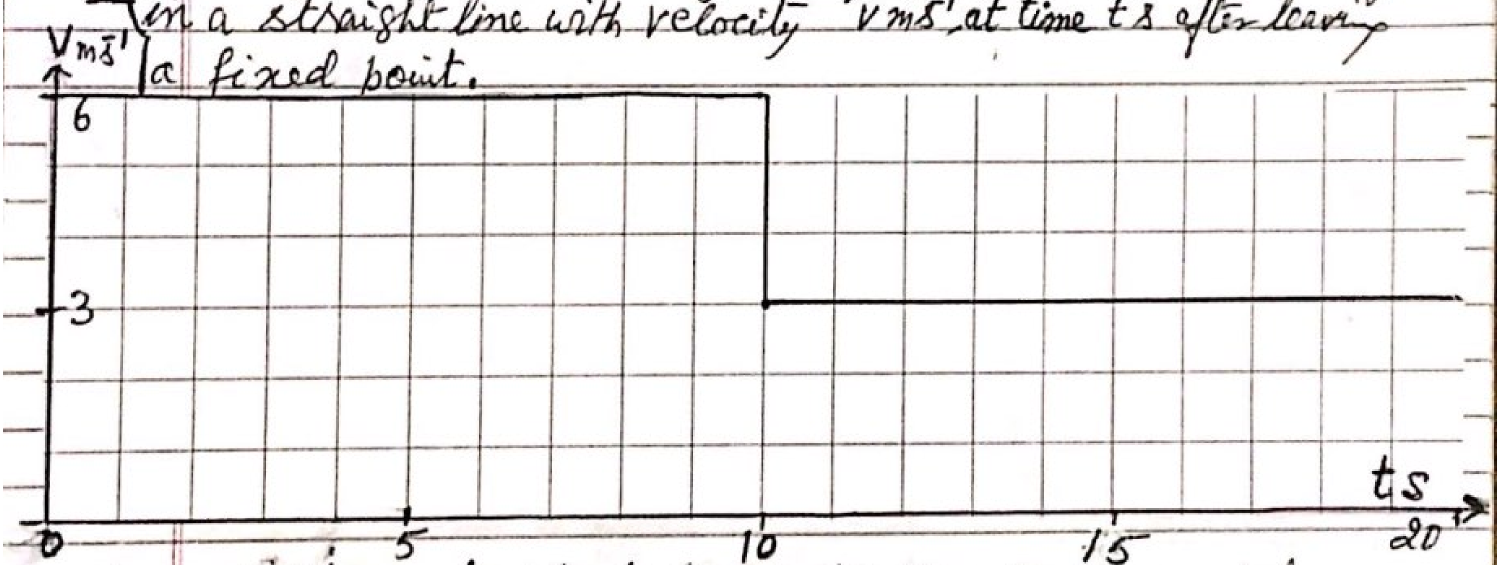
(a) The diagram shows the velocity-time graph of a particle 'P' moving in a straight line with Velocity ms^{-1} at time t s after leaving a fixed point.



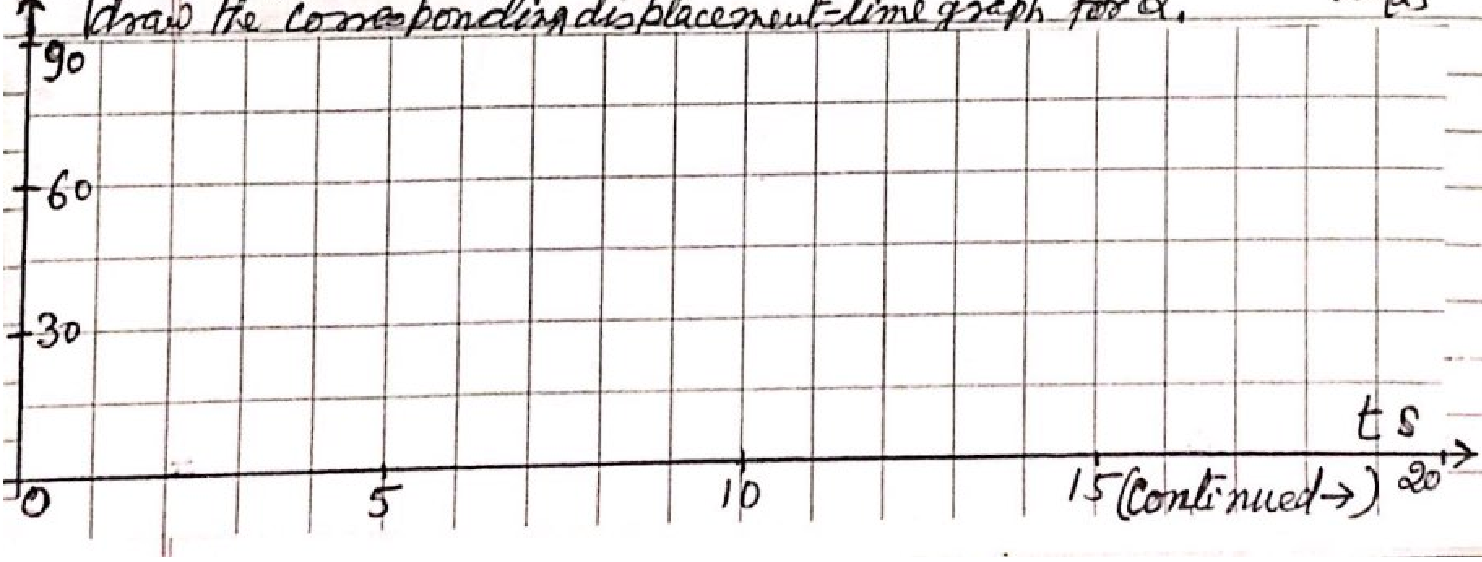
(i) Find the distance travelled by the particle P. ---[2]

(ii) Write down the deceleration of the particle when $t=30$. --[1]

(b) The diagram shows a velocity-time graph of a particle 'Q' moving in a straight line with velocity v ms^{-1} at time t s after leaving a fixed point.



The displacement of Q at time t s is s m. On the axes below, draw the corresponding displacement-time graph for Q. ---[2]



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Q64(c): The velocity $v\,ms^{-1}$, of a particle 'R' moving in a straight line, t s after passing through a fixed point O, is given by $v = 4e^{2t} + 6$

(i) Explain why the particle is never at rest. ---[1]

(ii) Find the exact value of t for which the acceleration of 'R' is $12\,ms^{-2}$. ---[2]

(iii) Show all your working, find the distance travelled by 'R' in the interval between $t = 0.4$ and $t = 0.5$ ---[4]

S-16/12/Q11

Q65 Variables x and y are related by the equation $y = \frac{5x-1}{3-x}$

(i) Find $\frac{dy}{dx}$, Simplify your answer. [2]

(ii) Hence find the approximate change in x when y increases from 9 by the small amount 0.07. ---[3]

S-16/22/Q2

Q66 (a) Find $\int \frac{x^3+x^2+1}{x^2}\,dx$ ---[3]

(b) (i) Find $\int \sin(5x+\pi)\,dx$ ---[2]

(ii) Hence evaluate $\int_{-\frac{\pi}{5}}^{0} \sin(5x+\pi)\,dx$ ---[2]

S-16/22/Q9

Q67 Find the equation of the normal to the curve $y = \ln(2x^2-7)$ at the point where the curve crosses the positive x-axis. Give your answer in the form $ax+by+c=0$, where a, b and c are integers. S-16/13/Q3 ---[5]

Do not use a calculator in this question:

Q68 (i) Show that, $\frac{d}{dx}\left(\frac{e^{4x}}{4} - xe^{4x}\right) = pxe^{4x}$, where p is an integer to be found. ---[4]

(ii) Hence find the exact value of, $\int_0^{\ln 2} xe^{4x}\,dx$, giving your answer in the form, $a\ln 2 + \frac{b}{c}$, where a, b and c are integers to be found. ---[4]

S-16/13/Q5

Q69 (i) Find the equation of the normal to the curve, $y = \frac{1}{2}\ln(3x+2)$ at the point 'P' where $x = -\frac{1}{3}$, --- [4]

The normal to the curve at the point P intersects the y-axis at the point 'Q'. The curve $y = \frac{1}{2}\ln(3x+2)$ intersects the y-axis at the point R.

(ii) Find the area of the triangle PQR. [W-16/11/Q5] -- [3]

Q70 The diagram shows part of the graph of $y = 2\cos(x - \frac{\pi}{6})$. The graph intersects the y-axis at the point A, has a maximum point at B and intersects the x-axis at the point C.

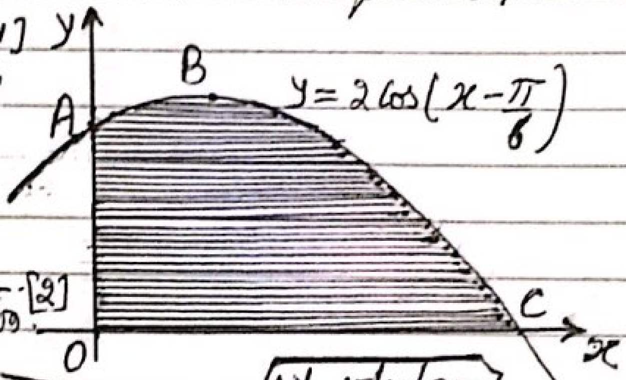
(i) Find the coordinates of A. --- [1]

(ii) Find the coordinates of B --- [2]

(iii) Find the coordinates of C. --- [2]

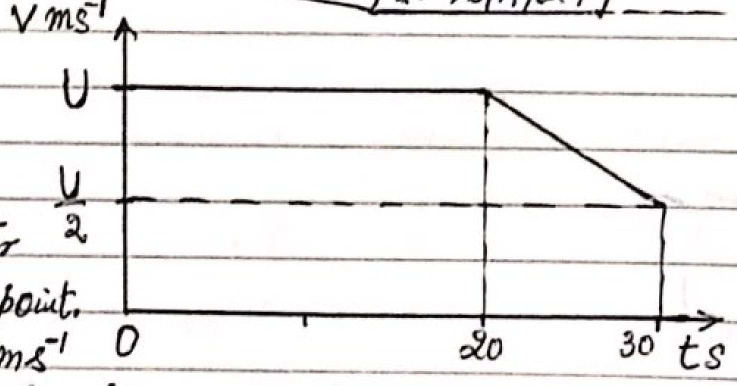
(iv) Find, $\int 2\cos\left(x - \frac{\pi}{6}\right)dx$ --- [1]

(v) Hence find the area of shaded region. --- [2]



[W-16/11/Q7]

Q71 (a) The diagram shows part of the velocity-time graph for a particle, moving at $v\,ms^{-1}$ in a straight line, t s after passing through a fixed point.



The particle travels at $U\,ms^{-1}$ for 20 s and then decelerates uniformly for 10 s, to a velocity of $\frac{U}{2}\,ms^{-1}$. In this 30 s interval, the particle travels 165 m.

(i) Find the value of U. --- [3]

(ii) Find the acceleration of the particle between $t = 20$ and $t = 30$. -- [2]

(b) A particle P travels in a straight line such that, t s after passing through a fixed point O, its velocity $v\,ms^{-1}$, is given by,

$$v = \left(e^{\frac{t^2}{8}} - 4\right)^3$$

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Diff. and Integration

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Q71 (b) (i) Find the speed of 'P' at O. ---[1]

(ii) Find the value of t for which 'P' is instantaneously at rest. --[2]

(iii) Find the acceleration at P when $t=1$ $\boxed{W\text{-}16/11/Q10}$ --[4]

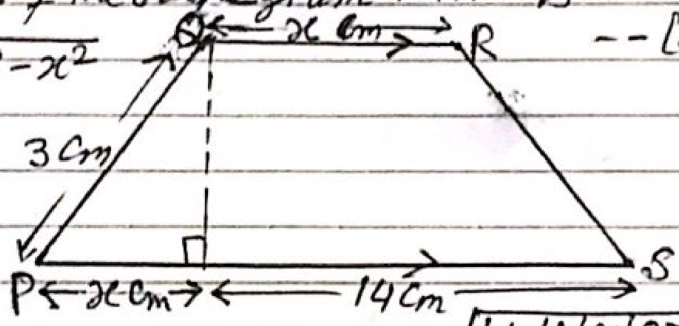
Q72 The curve with equation $y = x^3 + 2x^2 - 7x + 2$ passes through the point $A(-2, 16)$, Find

(i) the equation of the tangent to the curve at the point A. ---[3]

(ii) the coordinates of the point where the tangent meets the curve again. $\boxed{W\text{-}16/21/Q5}$ --[5]

Q73 (i) Show that the area, $A\,cm^2$, of the trapezium PQRS is given by $A = (7+x)\sqrt{9-x^2}$ --[2]

(ii) Given that x can vary, find the stationary value of A. ---[7]



$\boxed{W\text{-}16/21/Q7}$

Q74 The function $f(x)$ is given by $f(x) = \dfrac{3x^3-1}{x^3+1}$ for $0 \leqslant x \leqslant 3$

(i) Show that $f'(x) = \dfrac{kx^2}{(x^3+1)^2}$, where k is constant to be found. ---[3]

(ii) Find $\int \dfrac{x^2}{(x^3+1)^2}\,dx$ and hence evaluate $\int_1^2 \dfrac{x^2}{(x^3+1)^2}\,dx$ ---[4]

(iii) Find $f^{-1}(x)$, stating its domain. $\boxed{W\text{-}16/21/Q8}$ --[4]

Q75 (i) Find $\dfrac{d}{dx}\left(\ln(3x^2-11)\right)$ ---[2]

(ii) Hence show that $\int \dfrac{x}{3x^2-11}\,dx = p\ln(3x^2-11) + c$, ---[1]

where p is a constant to be found and c is a constant of integration.

(iii) Given that $\int_2^a \dfrac{x}{3x^2-11} = \ln 2$, where $a > 2$. --[4]

find the value of a.

$\boxed{W\text{-}16/13/Q6}$

Q76 A curve $y = f(x)$ is such that $f'(x) = 6x - 8e^{2x}$

(i) Given that the curve passes through the point $P(0, -3)$, find the equation of the curve. ---[5]

Normal to the curve $y = f(x)$ at P meet the line $y = 2 - 3x$ at the point Q.

(ii) Find the area of the triangle OPQ, where O is origin. ---[5]

W-16/13/Q10

Q77 A particle moving in a straight line has a velocity of $v\,ms^{-1}$ such that, t s after leaving a fixed point, $v = 4t^2 - 8t + 3$.

(i) Find the acceleration of the particle when $t = 3$. ---[2]

(ii) Find the values of t for which the particle is momentarily at rest. ---[2]

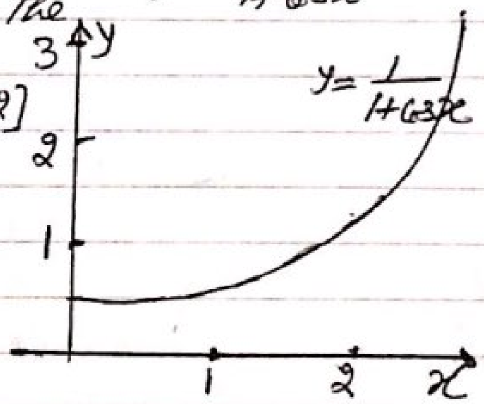
(iii) Find the total distance the particle has travelled when $t = 1.5$ ---[5]

W-16/13/Q11

Q78 (i) Show that $\frac{d}{dx}\left(\frac{\sin x}{1+\cos x}\right) = \frac{1}{1+\cos x}$ ---[4]

W-16/23/Q3

(ii) The diagram shows part of the graph of $y = \frac{1}{1+\cos x}$. Use the result of part (i) to find the area enclosed by the graph and the lines $x = 0$, $x = 2$ and $y = 0$ ---[2]



W-16/23/Q3

Q79 A curve has equation $y = 7 + \tan x$ find,

(i) the equation of the tangent to the curve at the point where $x = \frac{\pi}{4}$. ---[4]

(ii) the value of x between 0 and π radian for which $\frac{dy}{dx} = y$ ---[4]

W-16/23/Q6

Answers

Q1 (a) $\frac{dy}{dx} = \frac{3}{2}\sqrt{x}$

$y = 8 \Rightarrow x = 4$

(b) $\lim_{x\to 4} \frac{\delta y}{\delta x} = \left(\frac{dy}{dx}\right)_{x=4} = 3$

$\frac{0.015}{\delta x} = 3 \Rightarrow \delta x = 0.005$ ✓

Q2 $\frac{dy}{dx} = \boxed{\frac{2\sqrt{x^2+5} - \frac{1}{2}\times 2x(x^2+5)^{-\frac{1}{2}}(2x-1)}{x^2+5}}$

or $\frac{dy}{dx} = \boxed{2(x^2+5)^{-1/2} - \frac{1}{2}\times 2x(x^2+5)^{-3/2}(2x-1)}$

$x = 2, y = 1$

$\left(\frac{dy}{dx}\right)_{x=2} = \frac{4}{9}$

∴ Gradient of normal $= -\frac{9}{4}$

∴ Eqn of normal. $y - 1 = -\frac{9}{4}(x-2)$

or $9x + 4y = 22$ ✓

Q3 Area $= \pi x^2 + xy = 20$

$y = \frac{20 - \pi x^2}{x}$ --- (1)

$P = 2\pi x + 2x + 2y$

$= 2\pi x + 2x + 2\left(\frac{20-\pi x^2}{x}\right)$ by (1)

$\Rightarrow P = 2x + \frac{40}{x}$ ✓

$\frac{dP}{dx} = 2 - \frac{40}{x^2}$; for Max/Min $\frac{dP}{dx} = 0$

$\Rightarrow x = 2\sqrt{5}$ ✓

$\frac{d^2P}{dx^2} = \frac{80}{x^3} \Rightarrow \left(\frac{dP}{dx}\right)_{x=2\sqrt{5}} = \frac{80}{(2\sqrt{5})^3} > 0$

∴ Min at $x = 2\sqrt{5}$

∴ Min P Value $= 2\times 2\sqrt{5} + \frac{40}{2\sqrt{5}}$

$= 8\sqrt{5}$

$= 17.9$ ✓

Q4 (i) $\frac{1}{5}\left[e^{5x-1}\right]_{0.2}^{1}$

$= \frac{1}{5}\left[e^4 - e^0\right] = \frac{1}{5}(e^4 - 1)$ ✓

(ii) $\int_1^2 \left(x^2 + 2\times\frac{1}{x} + \frac{1}{x^4}\right)dx$

$= \left[\frac{x^3}{3} + 2\ln x - \frac{x^{-3}}{3}\right]_1^2$

$= \left(\frac{8}{3} + 2\ln 2 - \frac{1}{24}\right) - \left(\frac{1}{3} + 2\ln 1 - \frac{1}{3}\right)$

$= 2\ln 2 + \frac{21}{8}$ ✓

(b) $\frac{-\cos x/6}{\frac{1}{6}} = -6\cos\left(\frac{x}{6}\right)$ ✓

Q5 Curve $y = x^2 - 4x + 10$ — (1)

line $y = x + 10$ — (2)

Solving (1) & (2) x-coord of B = 5,

y-coord of A = 10

y-coord of B = 15

Area of shaded region:

= Area of Tra. OABC − Area under the curve

$= \frac{1}{2}(10+15)\times 5 - \int_0^5 (x^2 - 4x + 10)\,dx$

$= \frac{125}{2} - \left[\frac{x^3}{3} - 2x^2 + 10x\right]_0^5$

$= \frac{125}{2} - \frac{125}{3} = \frac{125}{6}$ ✓

Q6 (a) 27 ✓

(b) $t^2 = 8\ln 4 \Rightarrow t = 3.33$ ✓

(c) Acc. $\frac{dv}{dt} = 3\times\frac{2t}{8}\times e^{t^2/8}\left(e^{t^2/8} - 4\right)^2$

Acc. at $t = 1$, Acc $= 6.98$ ✓

Q7 (i) $\frac{dy}{dx} = 15\cos 3x$

at $x = \frac{\pi}{3}$, $y = 4$, $\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{3}} = -15$

(ii) Equation of tangent

$y - 4 = -15\left(x - \frac{\pi}{3}\right)$

$y = -15x + 5\pi + 4$ or $y = -15x + 19.7$ ✓

Diff. and Integration

Answers

Q8 (i) -2 ; $-\frac{1}{2} \le x \le \frac{1}{2}$

(ii) $\frac{dy}{dx} = \frac{(x+2)\left(\frac{8x}{4x^2-1}\right) - \ln(4x^2-1)}{(x+2)^2}$ ✓

(iii) when $x = 2$, $\frac{dy}{dx} = \frac{4}{15} - \frac{\ln 15}{16}$

$= 0.0974$

$\therefore \delta y = 0.0974h$ ✓

Q9 (i) $x = 6e^{2t} - 24t^2 + c$

$t = 0, x = 0 \Rightarrow c = -6$

$\therefore x = 6e^{2t} - 24t^2 - 6$ ✓

(ii) $\frac{d^2x}{dt^2} = 24e^{2t} - 48$

$acc = 0$ $t = ?$ $\Rightarrow e^{2t} = 2$

$\Rightarrow t = \frac{1}{2}\ln 2 = \ln\sqrt{2}$

or $t = 0.347$ ✓

(iii) Vel at $t = \frac{1}{2}\ln 2$

$= 24 - 24\ln 2 = 7.36$ ✓

Q10 (i) $y = -3x^3 - 11x^2 - 8x + 4$

$\frac{dy}{dx} = -9x^2 - 22x - 8$

$\frac{dy}{dx} = 0 \Rightarrow x = -2$ ✓A

and $x = -\frac{4}{9}$ ✓B

(ii) $C(0,4)$, $D(\frac{1}{3}, 0)$

(iii) $\text{Area} = \int_0^{1/3} (-3x^3 - 11x^2 - 8x + 4)\,dx$

$= \left[-\frac{3}{4}x^4 - \frac{11}{3}x^3 - 4x^2 + 4x\right]_0^{1/3}$

$= \frac{241}{324} = 0.744$ ✓

Q11 (i) $\frac{1}{3}\sec^2 x/3$ ✓

(ii) $3\tan x/3 + c$. ✓

Q12 (i) $\frac{dV}{dr} = 4\pi r^2$

$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$

$\therefore = \frac{1}{4\pi \times 10^2} \times 200 = \frac{1}{2\pi}$

$= 0.159$ ✓

(ii) $\frac{ds}{dr} = 8\pi r$

$\frac{ds}{dt} = \frac{ds}{dr} \times \frac{dr}{dt} = 8\pi \times 10 \times 0.159$

$= 40$ ✓

Q13 (i) $\frac{dy}{dx} = 15\cos 3x$

(ii) $\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{3}} = -15$

$x = \frac{\pi}{3}$, $y = 4$

Equation of tangent:

$y - 4 = -15(x - \frac{\pi}{3})$

or $y = -15x + 19.7$ ✓

Q14 (i) $\frac{dy}{dx} = -2\cos 2x + c$

$5 = -2\cos\pi + c \Rightarrow c = 3$

$\therefore \frac{dy}{dx} = 3 - 2\cos 2x$ ✓

(ii) $y = 3x - \sin 2x + c$

$-\frac{1}{2} = \frac{\pi}{4} - \frac{1}{2} + c \Rightarrow c = -\frac{\pi}{4}$

$\therefore y = 3x - \sin 2x - \frac{\pi}{4}$ ✓

(iii) $\frac{dy}{dx} = 3 - \sqrt{3}$, when $x = \frac{\pi}{12}$

$\therefore$ Normal equation: $y = -\frac{1}{2}$

$y + \frac{1}{2} = \frac{1}{\sqrt{3} - 3}(x - \frac{\pi}{12})$

$y = -0.789 - 0.294$

Q15 $(2x-5)^{3/2} = 3\sqrt{3} \Rightarrow x = 4$ at B.

$x = 2.5$ at A.

$\text{Area} = \frac{1}{2} \times \frac{3}{2} \times 3\sqrt{3} - \int_{2.5}^{4} (2x-5)^{3/2}\,dx$

$= \frac{9\sqrt{3}}{4} - \left[\frac{(2x-5)^{2.5}}{5}\right]_{2.5}^{4}$

$= \frac{9\sqrt{3}}{4} - \frac{1}{5} \times 3^{2.5} = \frac{9\sqrt{3}}{20}$ ✓

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Answers

Q16 (a) $\frac{1}{2}e^{2x+1}+c$

(b)(i) $\frac{dy}{dx} = \frac{\ln x \times 1 - x \times \frac{1}{x}}{(\ln x)^2}$

$= \frac{\ln x - 1}{(\ln x)^2}$ ✓

(ii) $\int \frac{\ln x - 1}{(\ln x)^2} + \int \frac{1}{x^2}\,dx$

$= \frac{x}{\ln x} - \frac{1}{x} + c$

Q17 (i) length of Rect $= \frac{20-3x}{2}$

$A = x\left(\frac{20-3x}{2}\right) - \frac{1}{2}\times x \times x \times \sin 60^\circ$

$= 10x - \left(\frac{6+\sqrt{3}}{4}\right)x^2$

(ii) $\frac{dA}{dx} = 10 - 2\left(\frac{6+\sqrt{3}}{4}\right)x = 0$

$\Rightarrow x = 2.6$

$A = 13$ ✓

Q18 $y = (3x^2+8)^{5/3} \Rightarrow \frac{dy}{dx} = 6x(3x^2+8)^{2/3}$

$\frac{dy}{dx} = 0 \Rightarrow x = 0$

Stationary Point at $(0, 32)$

minimum.

Q19 (i) $5 + 4\tan^2(x/3) = 5 + 4\left[\sec^2\left(\frac{x}{3}\right) - 1\right]$

$= 4\sec^2 x/3 + 1$ ✓

(ii) $3\tan x/3 + c$ ✓

(iii) Area $= \int_{\frac{\pi}{2}}^{\pi} 4\sec^2 x/3 + 1\ dx$

$= \left[12\tan x/3 + x\right]_{\pi/2}^{\pi}$

$= \left(8\sqrt{3} + \frac{\pi}{2}\right)$ ✓

Q20 (a) $\frac{dy}{dx} = \frac{3e^{3x}}{4x^2+1} - \frac{8xe^{3x}}{(4x^2+1)^2}$

b(i) $\frac{dy}{dx} = -4\sin\left(x+\frac{\pi}{3}\right) + 2\sqrt{3}\cos\left(x+\frac{\pi}{3}\right)$

when $x = \frac{\pi}{2}$, $\frac{dy}{dx} = -5$

(ii) $\frac{dy}{dx} \times \frac{dx}{dt} = \frac{dy}{dt} \Rightarrow -5 \times \frac{dx}{dt} = 10$

$\Rightarrow \frac{dx}{dt} = -2$ ✓

Point $(2, 17)$

Q21 $y = x^4 + x + c$

$17 = 2^4 + 2 + c \Rightarrow c = -1$

$\therefore$ equation of curve

$y = x^4 + x - 1$ ✓

Q22 (i) $\frac{dy}{dx} = \frac{2x}{x^2+1}$; $\left(\frac{dy}{dx}\right)_{x=3} = \frac{6}{10}$

(ii) $\delta y = \left(\frac{dy}{dx}\right)_{x=3} \times h = \frac{6}{10}h$ ✓

Q23 (i) $V = \frac{ds}{dt} = 3 + 5\sin 5t$

$3 + 5\sin 5t = 0 \Rightarrow t = 0.76;\ 1.13$

Values of $s = (4.07, 3.58)$

and $(0.48, 0.49)$. ✓

(ii) Acc. $\frac{d^2s}{dt^2} = 25\cos 5t$

acc $= \left(\frac{d^2s}{dt^2}\right)_{t=\pi} = -25$ ✓

Q24

$\frac{dy}{dx} = \frac{(x+1)\times\frac{1}{2}\times 10x(5x^2+4)^{-1/2} - (5x^2+4)^{1/2}}{(x+1)^2}$

$\therefore \frac{dy}{dx} = \frac{11}{112}$ ✓ when $x = 3$

Q25 (i) $V = \frac{dx}{dt} = -12\sin 3t$ ✓

(ii) 12

(iii) acc $= -36\cos 3t = 0$

$\Rightarrow 3t = \frac{\pi}{2} \Rightarrow t = \frac{\pi}{6}$ or 0.524

(iv) 4

Q26 (i) $f(x) = \frac{1}{2}e^{2x-1} + c$

Passes $\left(\frac{1}{2}, \frac{7}{2}\right) \Rightarrow \frac{7}{2} = \frac{1}{2} + c \Rightarrow c = 3$

$\therefore f(x) = \frac{1}{2}e^{2x-1} + 3$ ✓

(ii) $f''(x) = 2e^{2x-1} = 4$

$\Rightarrow 2x - 1 = \ln 2$

$x = 1 + \ln\sqrt{2}$ ✓

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Diff. and Integration

Answers

Q27 $\frac{dy}{dx} = 6x - 7$

$m_{normal} = -\frac{1}{5}$

$\therefore m_{tangent} = 5$

$\Rightarrow 6x - 7 = 5$

$\Rightarrow x = 2, \quad y = 9$

$K = 47$

Q28 (i) $\frac{dy}{dx} = -2x^4 \ln 5x$

(ii) $\ln 125x^3 = 3 \ln 5x$ ✓

(iii) $-\frac{3}{2}\int -2x^4 \ln 5x \, dx$

$= -\frac{3}{2}\left[0.4x^5(0.2 - \ln 5x)\right] + c$

Q29 (i) $\frac{x^4}{4} + \frac{4x^3}{3} - \frac{5x^2}{2} + 5x + c$ ✓

(ii) $x^3 + 4x^2 - 5x + 5 = 5$

$x(x^2 + 4x - 5) = 0$

$x = -5, \ 1$

$OEAB = 25, \quad OBCD = 5$ ✓

(iii) Area $= \left[\frac{x^4}{4} + \frac{4x^3}{3} - \frac{5x^2}{2} + 5x\right]_{-5}^{0}$

$\left[\frac{x^4}{4} + \frac{4x^3}{3} - \frac{5x^2}{2} + 5x\right]_{0}^{1}$

$= \left(\frac{1175}{12} - OEAB\right) + \left(OBCD - \frac{49}{12}\right)$

$= \frac{886}{12}$ or 73.83 ✓

Q30 when $x = 4 \Rightarrow y = 5$

$\frac{dy}{dx} = \frac{1}{2} \times 4(4x+9)^{-1/2}$

$\left(\frac{dy}{dx}\right)_{x=4} = 2/5$

$\therefore$ grad of normal $= -5/2$

Eqn of Normal $y - 5 = -5/2(x-4)$

or $y = -\frac{5}{2}x + 15$ ✓

$A(6,0), B(0,15)$, MidPoint $\left(3, \frac{15}{2}\right)$ ✓

Q31 $\left[3e^{5x} + e^{-5x}\right]_{-k}^{k} = 6$

(i) $\Rightarrow e^{5k} - e^{-5k} = 3$ ✓

(ii) $y^2 - 3y - 1 = 0, \quad y = e^{5k}$

$y = e^{5k} = 3.303$

$\Rightarrow k = 0.239$

Q32 (i) $\frac{dy}{dx} = (10x+2) \times \frac{5}{(5x+1)} + 10 \ln(5x+1)$ ✓

or $\frac{dy}{dx} = 10 + 10 \ln(5x+1)$ --- (i)

(ii) $\therefore \int (10 + 10\ln(5x+1))\, dx = y + c$

$= (10x+2)\ln(5x+1) + c$

$\Rightarrow 10\int \ln(5x+1)\, dx = (10x+2)\ln(5x+1) - 10x + c$

$\Rightarrow \int \ln(5x+1)\, dx = \frac{(5x+1)}{5}\ln(5x+1) - x + c$

(iii) $\left[(x + 0.2)\ln(5x-1) - x\right]_0^{1/5}$

$= -\frac{1}{5} + \frac{2}{5}\ln 2$ ✓

Q33 (i) $\frac{dy}{dx} = 6 - \frac{3}{2}x^{1/2}$

For stationary point $\frac{dy}{dx} = 0$

$\longrightarrow x = 16, \quad y = 32$ ✓

(ii) $\frac{d^2y}{dx^2} = -\frac{3}{4}x^{-\frac{1}{2}}$

$\left(\frac{d^2y}{dx^2}\right)_{x=16} = -\frac{3}{4\sqrt{16}} < 0, \therefore$ Max. ✓

(iii) $x = 4 \Rightarrow \frac{dy}{dx} = 3$

$\delta y = \frac{dy}{dx} \times h = 3h$ ✓

Q34 (i) acc $\frac{dv}{dt} = 6 + 6\cos 2t$ ✓

(ii) $6 + 6\cos 2t = 0 \Rightarrow \cos 2t = -1 \Rightarrow t = \frac{\pi}{2}$ ✓

(iii) $x = -\frac{3}{2}\cos 2t + 3t^2 + 2t + c$

when $t = 0, x = 0 \Rightarrow c = 3/2$

$\therefore x = \frac{3}{2} - \frac{3}{2}\cos 2t + 3t^2 + 2t$ ✓

Diff. and Integration

Answers

Q35 (i) $4\times10(1+4x)^9\cdot\cos x+(1+4x)^{10}(-\sin x)$

$=-\sin x(1+4x)^{10}+40\cos x(1+4x)^9$

(ii) $\dfrac{\tan x(4e^{4x-5})-e^{4x-5}\cdot\sec^2x}{\tan^2 x}$ ✓

Q36 $5+\sqrt{10x}=\dfrac{5x+20}{4}$

$\Rightarrow \sqrt{x}=\dfrac{4\sqrt{10}}{5}\Rightarrow x=6.4,\ y=13$ ✓

(ii) Area of Tra $=\frac{1}{2}(5+13)\times6.4=57.6$

and $\int_0^{6.4}(5+\sqrt{10x})\,dx=\left[5x+\dfrac{2(10x)^{3/2}}{3\times10}\right]_0^{6.4}$

$A=\left[5\times6.4+\dfrac{2(10\times6.4)^{3/2}}{30}\right]-57.6$

$=\dfrac{128}{15}=8.53$ ✓

Q37 (i) $(0,20)$

(ii) 31.7

(iii) $2e^{2x}-8e^{-2x}+c$

(iv) Area of Tra $=\frac{1}{2}(20+31.7)\times1$

$=25.86$

and $\left[2e^{2x}-8e^{-2x}\right]_0^1=19.7$

$\therefore$ Required Area $=25.86-19.7$

$=6.16$ ✓

Q38 (i) $\ln(2x+1)-\ln(2x-1)$ ✓

(ii) $\dfrac{dy}{dx}=\dfrac{2}{2x+1}-\dfrac{2}{2x-1}+4$

$=\dfrac{16x^2-8}{4x^2-1}$ ✓

(iii) for stationary point $\dfrac{dy}{dx}=0$

$\Rightarrow 16x^2-8=0\Rightarrow x=\dfrac{1}{\sqrt2}$ only

(iv) $\dfrac{d^2y}{dx^2}=\dfrac{32x(4x^2-1)-8x(16x^2-8)}{(4x^2-1)^2}$

$\left(\dfrac{d^2y}{dx^2}\right)_{x=\frac{1}{\sqrt2}}>0$ $\therefore$ Minimum ✓

Q39 (i) $-5(3x+2)^{-2}\times3$

(ii) $\int\dfrac{30}{(3x+2)^2}dx=\dfrac{-10}{(3x+2)}$

(iii) $\left[\dfrac{-10}{(3x+2)}\right]_1^2=-\dfrac{10}{8}+\dfrac{10}{5}=\dfrac{3}{4}$ ✓

Q40 $\dfrac{dy}{dx}=3x^2-\dfrac{1}{x^2}+c$

$x=1,\dfrac{dy}{dx}=1\Rightarrow c=-1$

$y=x^3+\dfrac{1}{x}-x+D$

$x=1,\ y=3\Rightarrow D=2$

$\therefore\ y=x^3+\dfrac{1}{x}-x+2$

Q41 (i) $\dfrac{-1}{(\cos x)^2}\times-\sin x=\dfrac{\sin x}{\cos^2x}$ ✓

(ii) $\dfrac{dy}{dx}=\sec^2x+\dfrac{4\sin x}{\cos^2x}$

(iii) $\dfrac{1}{\cos^2x}+\dfrac{4}{\cos x}\times\dfrac{\sin x}{\cos x}=4$

$\Rightarrow 1+4\sin x=4\cos^2x$

$\Rightarrow 4\sin^2x+4\sin x-3=0$

$(2\sin x-1)(2\sin x+3)=0$

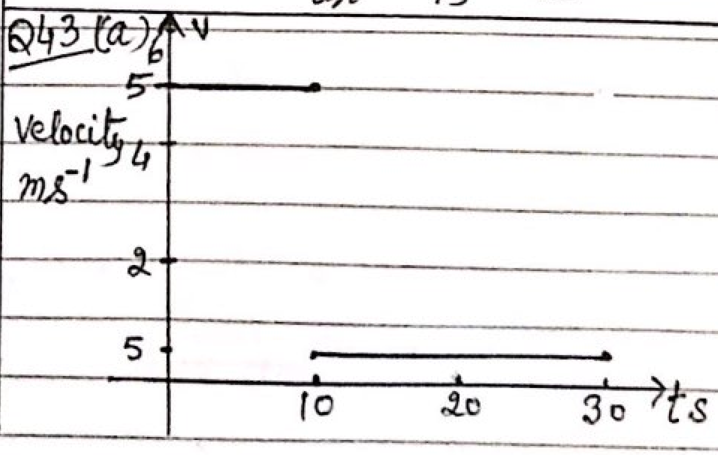
$\sin x=\frac{1}{2}$ or $-\frac{3}{2}$ ✗

$x=\dfrac{\pi}{6};\ \dfrac{5\pi}{6}$ ✓

Q42 $\dfrac{dy}{dx}=\dfrac{(x^2+1)\left(\dfrac{6x}{3x^2+2}\right)-2x\cdot\ln(3x^2+2)}{(x^2+1)^2}$

when $x=2$, $\dfrac{dy}{dx}=\dfrac{6}{15}-\dfrac{4}{25}\ln14$ ✓

Q43 (a)



Answers

Q43(b)(i) 3 ✓

(ii) $\frac{dv}{dt} = -15e^{-5t} + \frac{3}{2}$

$\frac{dv}{dt} = 0 \Rightarrow e^{-5t} = 0.1$

$\Rightarrow t = 0.461$ ✓

(iii) $s = -\frac{3}{5}e^{-5t} + \frac{3}{4}t^2 + C$

when $t = 0$, $s = 0 \Rightarrow C = 3/5$

when $t = 0.5 \Rightarrow s = 0.738$ ✓

Q44(i) $\pi x^2 h = 500 \Rightarrow h = \frac{500}{\pi x^2}$ ✓

(ii) $A = 2\pi x^2 + 2\pi x h$

$= 2\pi x^2 + 2\pi x \times \frac{500}{\pi x^2}$

$= 2\pi x^2 + \frac{1000}{x}$ ✓

(iii) $\frac{dA}{dx} = 4\pi x - \frac{1000}{x^2}$

$\frac{dA}{dx} = 0 \Rightarrow x = \sqrt[3]{\frac{1000}{4\pi}} \Rightarrow x = 4.3$

$A = 2\pi(4.3)^2 + \frac{1000}{4.3} = 349\ cm^2$ ✓

$\frac{d^2A}{dx^2} = 4\pi + \frac{2000}{x^3} > 0,\ x > 0$

$\therefore$ Min ✓

Q45(i) Gradient $\frac{dy}{dx} = \frac{3x-1}{\sqrt{x}} = 3x^{1/2} - x^{-1/2}$

$y = 2x^{3/2} - 2x^{1/2} + C$

$-10 = 2 - 2 + C \Rightarrow C = -10$

$\therefore y = 2x^{3/2} - 2x^{1/2} - 10$ ✓

(ii) $x = 4 \Rightarrow y = 2$

and $\left(\frac{dy}{dx}\right)_{x=4} = 6 - \frac{1}{2} = 5.5$

Equation of tangent at (4,2)

$\frac{y-2}{x-4} = 5.5$

$\Rightarrow y = 5.5x - 20$ ✓

Q46(i) $\frac{d}{dx}(x\ln x) = x \times \frac{1}{x} + \ln x$

$= 1 + \ln x$

Q46(ii) $\int(1 + \ln x)dx = x\ln x$

$\therefore \int \ln x\, dx = x\ln x - x + C$

(iii) $\int_k^{2k} \ln x\, dx = (2k\ln 2k - 2k) - (k\ln k - k)$

$= k(2\ln 2k - \ln k - 1)$

$= k(\ln 4k - 1)$ ✓

Q47(i) A(4,0) ✓

(ii) $4 + 3x - x^2 = mx + 8$

$x^2 + (m-3)x + 4 = 0$

for tangent $b^2 - 4ac = 0$

$\Rightarrow (m-3)^2 = 16 \Rightarrow m = -1$ ✓

(iii) $x^2 + (m-3)x + 4 = 0$

$\Rightarrow x^2 - 4x + 4 = 0$ for $m = -1$

$(x-2)^2 = 0 \Rightarrow x = 2,\ y = 6$

B(2,6) ✓

(iv)

Area under the curve $= \int_2^4 (4 + 3x - x^2)dx$

$= \left[4x + \frac{3}{2}x^2 - \frac{x^3}{3}\right]_2^4 = 7\frac{1}{3}$

Line intersects x-axis at (8,0)

area of triangle $= \frac{1}{2} \times (8-2) \times 6 = 18$

$\therefore$ shaded area $= 18 - 7\frac{1}{3} = 10\frac{2}{3}$ ✓

Q48(i) $\frac{5}{14}(7x-10)^{2/5}$

(ii) $\frac{5}{14}\left[(7x-10)^{2/5}\right]_6^a = \frac{25}{14}$

$\Rightarrow (7a-10)^{2/5} = 9$

$\Rightarrow a = \frac{253}{7} = 36\frac{1}{7}$ ✓

Q49 (i) $\frac{dy}{dx} = (3x-1)^{2/3} \cdot (8x-21)$

(ii) $\left(\frac{dy}{dx}\right)_{x=3} = 12$

$\therefore \delta y = 12h$ ✓

Answers

Q50 (i) $V=0 \Rightarrow \cos 2t = \frac{1}{3}$

$\Rightarrow t = 0.615$ ✓

(ii) $s = \frac{3}{2}\sin 2t - t + c$

$t=0, s=0, c=0$

$\therefore s = \frac{3}{2}\sin 2t - t$

when $t = \frac{\pi}{4} \Rightarrow s = 1.5 - \frac{\pi}{4} = 0.715$ ✓

(iii) $acc = -6\sin 2t$

$t = 0.615 \Rightarrow a = -5.66$ ✓

Q51 (i) $\frac{d}{dx}\left(\frac{\ln x}{x^3}\right) = \frac{x^3 \times \frac{1}{x} - 3x^2 \ln x}{x^6}$

$= \frac{1 - 3\ln x}{x^4}$ ✓

(ii) $\frac{dy}{dx} = 0 \Rightarrow 1 - 3\ln x = 0 \Rightarrow x = e^{1/3}$

$y = \frac{1}{3e}$ ✓

(iii) $\frac{\ln x}{x^3} = \int \frac{1 - 3\ln x}{x^4}\, dx$

$= \int \frac{1}{x^4}\, dx - 3\int \frac{\ln x}{x^4}\, dx$ ✓

$\Rightarrow \int \frac{\ln x}{x^4}\, dx = -\frac{1}{9x^3} - \frac{\ln x}{3x^3} + c$

Q52 (i) $3x(2x-1)^{3/2} + (2x-1)^{3/2}$ ✓

(ii) $3\int x(2x-1)^{3/2} dx = x(2x-1)^{3/2} - \int (2x-1)^{3/2} dx$

$= x(2x-1)^{3/2} - \frac{1}{2} \times \frac{2}{5}(2x-1)^{5/2}$

$\therefore \int x(2x-1)^{3/2} dx = \frac{1}{3}(2x-1)^{3/2}\left[x - \frac{1}{5}(2x-1)\right]$

$= \frac{(2x-1)^{3/2}(3x+1)}{15}$ ✓

(iii) $\frac{4}{15}$ ✓

Q53 (i) $\frac{dy}{dx} = -\frac{5}{2}(x-9)^{-3/2}$ ✓

(ii) $\left(\frac{dy}{dx}\right)_{x=13} = -0.3125$

$\therefore \delta y = -0.3125h$ ✓

Q54 $y = \int (9x^2 - 3x^{-2})\, dx$

$y = \frac{9x^3}{3} + \frac{3}{x} + c$

$x=1, y=7 \Rightarrow c = 1$

$\therefore y = 3x^3 + 3x^{-1} + 1$

Q55 (i) $\frac{dy}{dx} = \frac{1-x^2}{(x^2+1)^2}$

for stationary point $\frac{dy}{dx} = 0$

$\Rightarrow 1 - x^2 = 0 \Rightarrow x = 1, -1$

$y = 0.5, -0.5$

$(1, 0.5)$ and $(-1, -0.5)$ ✓

(ii) $\frac{d^2y}{dx^2} = \frac{2x^3 - 6x}{(x^2+1)^3}$

$\left(\frac{d^2y}{dx^2}\right)_{x=1} = \frac{2-6}{2^3} < 0 \therefore$ Max.

and $\left(\frac{d^2y}{dx^2}\right)_{x=-1} = \frac{-2+6}{2^3} > 0$, Min

Q56 $v = 9t^2 - 63t + 9$ ✓

(i) $v = (9t-18)(t-5) = 0$ at rest

$t = 2$ is the smaller value ✓

(ii) acc. $\frac{d^2v}{dt^2} = 18t - 63$

$\left(\frac{d^2v}{dt^2}\right)_{t=3.5} = 18 \times 3.5 - 63 = 0$ ✓

(iii) $s = \int (9t^2 - 63t + 90)\, dt$

$= \frac{9t^3}{3} - \frac{63t^2}{2} + 90t$

(iv) (a) at $t=2$, $s = \frac{9 \times 2^3}{3} - \frac{63 \times 2^2}{2} + 90 \times 2$

$= 78\,m$ ✓

(b) at $t=3$, $s = \frac{9 \times 3^3}{3} - \frac{63 \times 3^2}{2} + 90 \times 3$

$= 67.5$

$\therefore$ distance in 3rd sec $= 78 - 67.5 = 10.5$

$\therefore$ Distance travelled in 3 sec.

$78 + 10.5 = 88.5\,m$ ✓

Answers

Q57 On x-axis $y = \ln(2x^2-7) = 0$

$\Rightarrow 2x^2 - 7 = 1$

$\Rightarrow x = 2$

$x = 2,\ \frac{dy}{dx} = 8$

Grad. of normal $= -\frac{1}{8}$

Equation of normal $y = -\frac{1}{8}(x-2)$

or $x + 8y - 2 = 0$ ✓

Q58 (i) $e^{4x} - (x \times 4e^{4x} + e^{4x})$

$= -4xe^{4x}$ ✓

(ii) $\int_0^{\ln 2} x e^{4x}\,dx = -\frac{1}{4}\left[\frac{e^{4x}}{4} - xe^{4x}\right]_0^{\ln 2}$

$= 4\ln 2 - \frac{15}{16}$ ✓

Q59 (i) $\frac{3x^2}{2} - \frac{2x^{5/2}}{5} + c$

(ii) $(9, 0)$

(iii) Substitute $(3, 9)$ into both the lines.

(iv) Area of $AOB = \frac{1}{2} \times 9 \times 9 = 81/2$

Area under curve $\left[\frac{3 \times 9^2}{2} - \frac{2 \times 9^{5/2}}{5}\right] = 24.3$

$\therefore$ Req. Area $= \frac{81}{2} - 24.3 = 16.2$ ✓

Q60 (i) $\frac{dy}{dx} = \frac{2(x-1) - (2x-5)}{(x-1)^2} - 12$

$\frac{dy}{dx} = \frac{3}{(x-1)^2} - 12$ ✓

(ii) $\frac{d^2y}{dx^2} = -6(x-1)^{-3}$ ✓

(iii) $\frac{3}{(x-1)^2} - 12 = 0 \Rightarrow x = 0.5, 1.5$

$y = 2, -22$

at $x = 0.5$, $\frac{-6}{(-0.5)^3} > 0$, Min. ✓

at $x = 1.5$, $\frac{-6}{(0.5)^3} < 0$, Max. ✓

Q61. $\frac{e^{3x}(12x+5)}{(4x+1)^{1/2}}$ ✓

Q62 (i) $\cos 3x = \frac{1}{2} \rightarrow x = \frac{\pi}{9}$ or 0.35

$A(0.35, 0)$

(ii) $B(\frac{\pi}{3}, 3)$ or $(1.05, 3)$

(iii) Area $= \int_{\frac{\pi}{9}}^{\pi/3} (1 - 2\cos 3x)\,dx$

$= \left[x - \frac{2}{3}\sin 3x\right]_{\frac{\pi}{9}}^{\pi/3}$

$= \frac{2\pi}{9} + \frac{\sqrt{3}}{3} = 1.28$ ✓

Q63 (i) $y = \frac{2}{3}(3x+10)^{1/2} + c$

Passes through $(2, -\frac{4}{3}) \Rightarrow c = -4$

$\therefore y = \frac{2}{3}(3x+10)^{1/2} - 4$ ✓

(ii) at $x = 5$, $y = -2/3$

$\left(\frac{dy}{dx}\right)_{x=5} = \frac{1}{5} \Rightarrow$ grad. of normal $= -5$

Eqⁿ of normal $y + 2/3 = -5(x-5)$

when $y = -5/3$

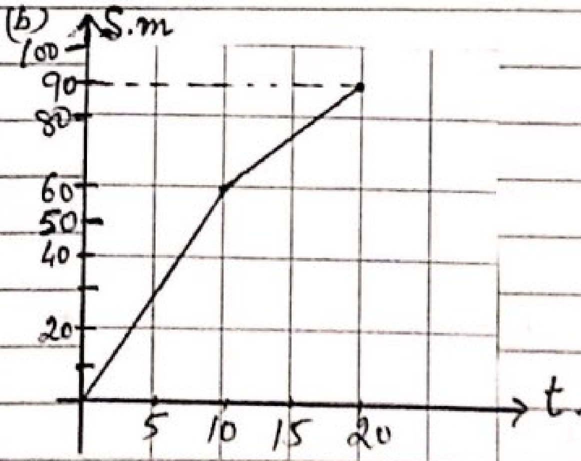
$\Rightarrow x = 5.2$ ✓

Q64 (a)(i) distance = Area under the graph

$= 1275$ ✓

(ii) deceleration $= 1.5$ ✓

(b)



(c)(i) e^{2t} is always positive.

(Continued →)

Diff. and Integration

Answers

Q64 (c)(ii) acc $a = 8e^{2t} = 12$
$\Rightarrow e^{2t} = 3/2$
$t = \frac{1}{2}\ln\frac{3}{2}$ ✓

(iii) $s = \left[2\frac{e^{2t}}{2} + 6t\right]_{0.4}^{0.5}$
$= 8.436 - 6.851 = 1.59$ ✓

Q65 (i) $\frac{dy}{dx} = \frac{14}{(3-x)^2}$ ✓

(ii) $y = 9$, $x = 2$
$\left(\frac{dy}{dx}\right)_{x=2} = 14 = dt$ $\frac{\delta y}{\delta x} = \frac{0.07}{\delta x}$
$\Rightarrow \delta x = \frac{0.07}{14} = 0.005$ ✓

Q66 (a) $\frac{x^2}{2} + x - \frac{1}{x} + c$

b(i) $-\frac{\cos(5x+\pi)}{5} + c$

(ii) $-\frac{\cos(5\times 0+\pi)}{5} - \frac{-\cos(5(-\frac{\pi}{5})+\pi)}{5}$
$= 0.4$ ✓

Q67 On x-axis, $2x^2 - 7 = 1$
$\Rightarrow x = 2$
$\frac{dy}{dx} = \frac{4x}{2x^2-7}$
when $x = 2$, $\frac{dy}{dx} = 8$
Gradient of normal $= -\frac{1}{8}$
Equ^n of normal at $(2,0)$
$y = -\frac{1}{8}(x-2)$
or $x + 8y - 2 = 0$ ✓

Q68 (i) $\frac{d}{dx}\left(\frac{e^{4x}}{4} - xe^{4x}\right)$
$= e^{4x} - (x \times 4e^{4x} + e^{4x})$
$= -4xe^{4x}$ ✓

(ii) $\int_0^{\ln 2} xe^{4x}\,dx = -\frac{1}{4}\left[\frac{e^{4x}}{4} - xe^{4x}\right]_0^{\ln 2}$
$= 4\ln 2 - \frac{15}{16}$ ✓

Q69 (i) $\frac{dy}{dx} = \frac{3}{2(3x+2)}$
when $x = -\frac{1}{3}$, $y = 0$, $\frac{dy}{dx} = 3/2$
Equ^n of normal. $y = -\frac{2}{3}(x + 1/3)$ ✓

(ii) $Q(0, -2/9)$, $R(0, \frac{1}{2}\ln 2)$
Area PQR $= \frac{1}{2}\left(\frac{1}{2}\ln 2 + \frac{2}{9}\right) \times \frac{1}{3}$
$= 0.0948$ ✓

Q70 (i) $(0, \sqrt{3})$ A
(ii) $(\frac{\pi}{6}, 2)$ B
(iii) $C(\frac{2\pi}{3}, 0)$
(iv) $2\sin(x - \frac{\pi}{6})$
(V) Area $= \left[2\sin(x - \frac{\pi}{6})\right]_0^{\frac{2\pi}{3}}$
$= 3$ ✓

Q71 (a)(i) $20U + \frac{1}{2}\left(U + \frac{U}{2}\right) \times 10 = 165$
$\Rightarrow U = 6$ ✓

(ii) Gradient of line: -0.3 ✓

b(i) 27 ✓
(ii) $t^2 = 8\ln 4 \Rightarrow t = 3.33$ ✓
(iii) acc $= 3 \times \frac{2t}{8} \cdot e^{t^2/8}(e^{t^2/8} - 4)^2$
when $t = 1$, $a = 6.98$ ✓

Q72 (i) $\frac{dy}{dx} = 3x^2 + 4x - 7$
$\left(\frac{dy}{dx}\right)_{x=-2} = -3$
Equation of tangent $\frac{y-16}{x+2} = -3 \Rightarrow y = -3x + 10$ ✓

(ii) $x^3 + 2x^2 - 7x + 2 = -3x + 10$
$x^3 + 2x^2 - 4x - 8 = 0$
$(x+2)(x+2)(x-2) = 0$
$x = 2$, $y = 4$ ✓

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Diff. and Integration

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Answers

Q73 (i) $h = \sqrt{9 - x^2}$

Area $A = \frac{\sqrt{9-x^2}}{2}(14 + x + x)$

$= \sqrt{9-x^2}\,(7+x)$ ✓

(ii) $\frac{dA}{dx} = \sqrt{9-x^2} + \frac{(7+x)(9-x^2)^{-\frac{1}{2}}}{2} \times -2x$

$\frac{dA}{dx} = 0 \Rightarrow 9 - x^2 = 7x + x^2$

$\Rightarrow 2x^2 + 7x - 9 = 0$

$\Rightarrow x = 1$

$A = 16\sqrt{2}$ ✓

Q74 (i) $f'(x) = \frac{(x^3+1) \times 9x^2 - (3x^3-1) \times 3x^2}{(x^3+1)^2}$

$= \frac{12x^2}{(x^3+1)^2}$ ✓

(ii) $\int_1^2 \frac{x^2}{(x^3+1)^2}\,dx = \frac{1}{12}\left[\frac{3x^3-1}{x^3+1}\right]_1^2$

$= \frac{1}{12}\left(\frac{23}{9} - \frac{2}{2}\right)$

$= \frac{7}{54}$ ✓

(iii) $x = \frac{3y^3-1}{y^3+1}$

$\Rightarrow y^3 = \frac{x+1}{3-x}$

$f^{-1}(x) = \sqrt[3]{\frac{x+1}{3-x}}$ ✓

Domain $-1 \le x \le 2\frac{6}{7}$ ✓

Q75 (i) $\frac{6x}{3x^2-1}$ ✓

(ii) $\int \frac{x}{3x^2-1}\,dx = \frac{1}{6}\ln(3x^2-1)$ $\therefore p = \frac{1}{6}$ ✓

(iii) $\frac{1}{6}\ln(3a^2-11) - \frac{1}{6}(\ln 1) = \ln 2$

$\Rightarrow \ln(3a^2-11) = \ln 2^6$

$\Rightarrow 3a^2 - 11 = 64$

$\Rightarrow a = 5$ only. ✓

Q76 (i) $f(x) = 3x^2 - 4e^{2x} + c$

Passes through $(0,-3)$

$-3 = 0 - 4e^0 + c \Rightarrow c = 1$

$\therefore f(x) = 3x^2 - 4e^{2x} + 1$ ✓

(ii) $f'(0) = -8$

Normal at $(0,-3)$, $y + 3 = \frac{1}{8}x$

or $y = \frac{1}{8}x - 3$

and given line $y = 2 - 3x$

solving $x = 8/5$

Area $= \frac{1}{2} \times 3 \times 8/5 = 2.4$ ✓

Q77 (i) $a = 8t - 8$

at $t = 3$, acc $= 16$ ✓

(ii) 0.5 ; 1.5 ✓

(iii) $s = \frac{4}{3}t^3 - 4t^2 + 3t$

when $t = \frac{1}{2}$, $s = 2/3$

when $t = 3/2$, $s = 0$

$\therefore$ Total distance travelled $= \frac{4}{3}$ ✓

Q78 (i) $\frac{d}{dx}\left(\frac{\sin x}{1+\cos x}\right) = \frac{(1+\cos x)\cdot\cos x + \sin x \cdot \sin x}{(1+\cos x)^2}$

$= \frac{1+\cos x}{(1+\cos x)^2} = \frac{1}{(1+\cos x)}$ ✓

(ii) $A = \int_0^2 \frac{1}{1+\cos x}\,dx = \left[\frac{\sin x}{1+\cos x}\right]_0^2 = 1.56$ ✓

Q79 (i) $\frac{dy}{dx} = \sec^2 x$

$x = \frac{\pi}{4} \Rightarrow \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}} = 2$

$\therefore$ Equation of tangent:

$\frac{y-8}{x-\frac{\pi}{4}} = 2 \Rightarrow y = 2x + 6.429$ ✓

(ii) $\sec^2 x = \tan x + 7 \Rightarrow \tan^2 x - \tan x - 6 = 0$

$(\tan x - 3)(\tan x + 2) = 0 \Rightarrow \tan x = 3, -2$

$\therefore x = 1.25$ or 2.03 ✓