IG_0606
Additional Maths

Diff. and Integration
Exercise_2

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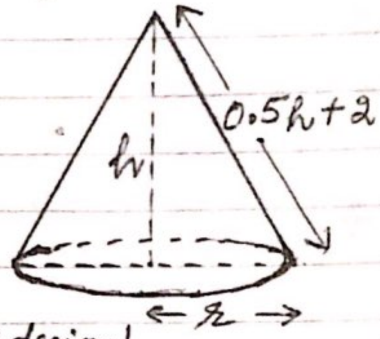
Q 80 In this question all lengths are in metres.
A conical tent is to be made with height h, base radius r, and slant height $0.5h+2$, as shown in the diagram.



(i) Show that $r^2 = 2h + 4 - 0.75h^2$ ---[2]

The volume of the tent, V, is given by $\frac{1}{3}\pi r^2 h$

(ii) Given that h can vary find, correct to 2 decimal places, the value of h which gives a stationary value of V. ---[5]

(iii) Determine the nature of this stationary value. [W-16/23/Q7] --[2]

Q 81. (i) Given that $y = \frac{\tan 2x}{x}$, find $\frac{dy}{dx}$ ---[3]

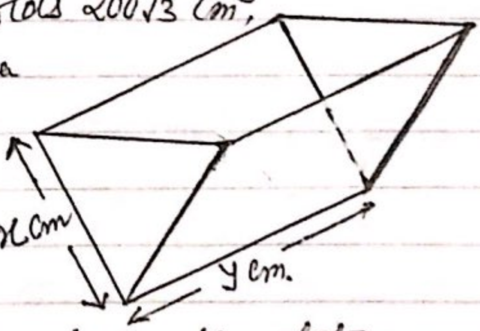
(ii) Hence find the equation of the normal to the curve $y = \frac{\tan 2x}{x}$ at the point where $x = \frac{\pi}{8}$ ---[3]

[M-15/12/Q6]

Q 82 The diagram shows an empty container in the form of an open triangular prism. The triangular faces are equilateral with a side of x cm and the length of each rectangular face is y cm. The container is made from thin sheet metal. When full, the container holds $200\sqrt{3}$ cm^3.

(i) Show that A cm^2, the total area of the thin sheet metal used, is given by,

$A = \frac{\sqrt{3}}{2}x^2 + \frac{1600}{x}$ ---[5]



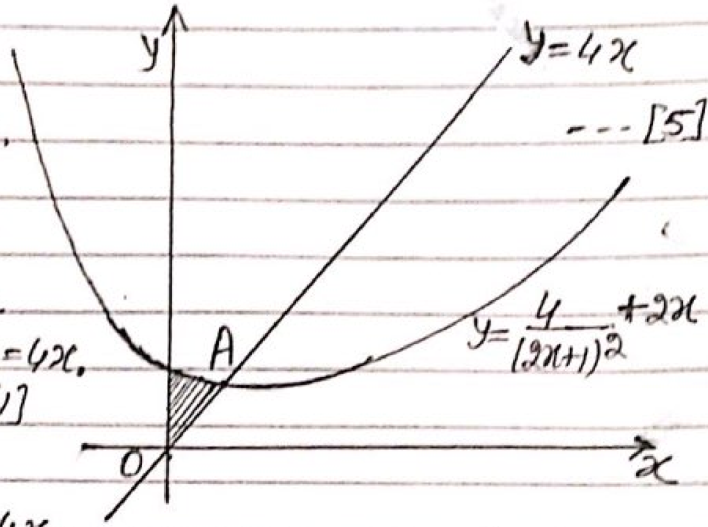
(ii) Given that x and y can vary, find the stationary value of A and determine its nature. ---[6]

[M-15/12/Q9]

Q 83 (i) Differentiate $\sin x \cdot \cos x$ with respect to x, giving your answer in terms of $\sin x$. ---[3]

(ii) Hence find $\int \sin^2 x \, dx$ [M-15/22/Q4] ---[3]

Q84 The diagram shows part of the curve $y = \frac{4}{(2x+1)^2} + 2x$ and the line $y = 4x$.



(i) Find the coordinates of A, the stationary point of the curve. --- [5]

(ii) Verify that A is also the point of intersection of the curve $y = \frac{4}{(2x+1)^2} + 2x$ and the line $y = 4x$. --- [1]

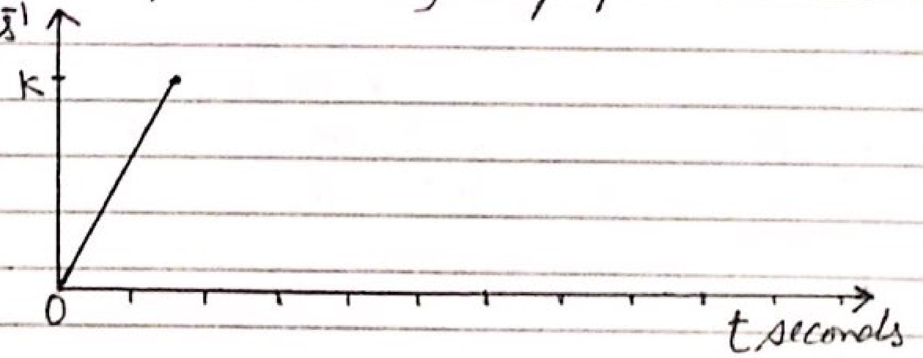
(iii) Find the area of the shaded region enclosed by the line $y = 4x$, the curve and the y-axis. --- [6]

M-15/22/Q9

Q85(a) A particle moves in a straight line. Starting from rest, P moves with constant acceleration for 30 seconds after which it moves with constant velocity, $k\,ms^{-1}$ for 90 seconds. P then moves with constant deceleration until it comes to rest, the magnitude of the deceleration is twice the magnitude of the initial acceleration.

(i) Use the information to complete the velocity-time graph. --- [2]

(ii) Given that the particle travels 450 metres while it is accelerating, Find the value of k and the acceleration of the particle. --- [4]



(b) A body Q moves in a straight line such that, t seconds after passing a fixed point, its acceleration, $a\,ms^{-2}$, is given by $a = 3t^2 + 6$. When $t = 0$, the velocity of the body is $5\,ms^{-1}$. Find the velocity when $t = 3$. --- [5]

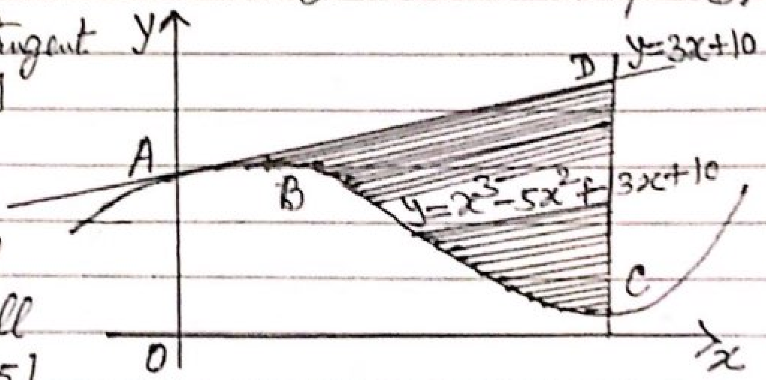
M-15/22/Q11

Diff and Integration

Q86 The point A, where $x=0$, lies on the curve $y=\frac{\ln(4x^2+3)}{x-1}$.
The normal to the curve at 'A' meets the x-axis at the point B.
(i) Find the equation of this normal. ---[7]
(ii) Find the area of the triangle AOB, where O is origin. ---[2]

[S-15/11/Q7]

Q87 The diagram shows part of the line $y=3x+10$ and the curve $y=x^3-5x^2+3x+10$. The line and the curve both pass through the point A on the Y-axis. The curve has a maximum at the point B and a minimum at the point C. The line through C, parallel to Y-axis, intersects the line $y=3x+10$ at the point D.
(i) Show that the line AD is a tangent to the curve at A. ---[2]
(ii) Find the x-coordinates of B and of C. ---[3]
(iii) Find the area of the shaded region ABCD, showing all your working. ---[5]



[S-15/11/Q9]

Q88 A particle 'P' is projected from the origin O so that it moves in a straight line. A time t seconds after projection, the velocity of the particle, $v\,ms^{-1}$, is given by $v=2t^2-14t+12$
(i) Find the time at which 'P' first comes to instantaneous rest. --[2]
(ii) Find an expression for the displacement of P from O at time t seconds. --[3]
(iii) Find the acceleration of P when $t=3$. [S-15/21/Q6] --[2]

Q8.9 (a) (i) Find $\int e^{4x+3}\,dx$ --[2]
(ii) Hence evaluate $\int_{2.5}^{3} e^{4x+3}\,dx$ --[2]
(b) (i) Find $\int \cos\left(\frac{x}{3}\right)dx$ --[2]
(ii) Hence evaluate $\int_0^{\frac{\pi}{6}} \cos\left(\frac{x}{3}\right)dx$ --[2]
(c) Find. $\int (x^{-1}+x)^2\,dx$ --[4]

[S-15/21/Q8]

Q90 A particle moves in a straight line such that its displacement, x m, from a fixed point O after t s, is given by

$$x = 10\ln(t^2+4) - 4t$$

(i) Find the initial displacement of the particle from O. ---[1]

(ii) Find the value of t when the particle is instantaneously at rest. --[4]

(iii) Find the value of t when the acceleration of the particle is zero. --[5]

[S-15/12/Q6]

Q91 (i) Find $\int (10e^{2x} + e^{-2x})\,dx$ ---[2]

(ii) Hence find $\int_{-k}^{k} (10e^{2x} + e^{-2x})\,dx$ in terms of the constant k. --[2]

(iii) Given that $\int_{-k}^{k} (10e^{2x} + e^{-2x})\,dx = -60$

Show that $11e^{2k} - 11e^{-2k} + 120 = 0$ --[2]

(iv) Using a substitution of $y = e^{2k}$ or otherwise, find the value of k in the form $a\ln b$, where a and b are constant. --[3]

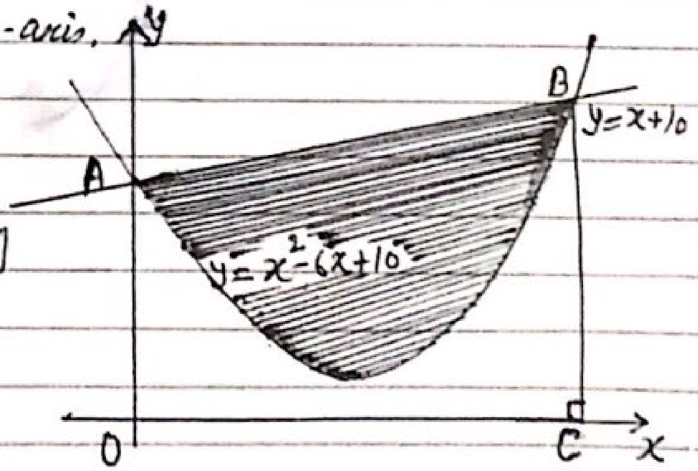
[S-15/12/Q8]

Q92 A curve has equation $y = 4x + 3\cos 2x$. The normal to the curve at the point where $x = \frac{\pi}{4}$ meets the x-axis and y-axis at the points A and B respectively. Find the exact area of the triangle AOB, where O is the origin. --[8]

[S-15/12/Q9]

Q93 The graph of $y = x^2 - 6x + 10$ cuts the y-axis at A. The graph of $y = x^2 - 6x + 10$ and $y = x + 10$ cut one another at A and B. The line BC is perpendicular to the x-axis.

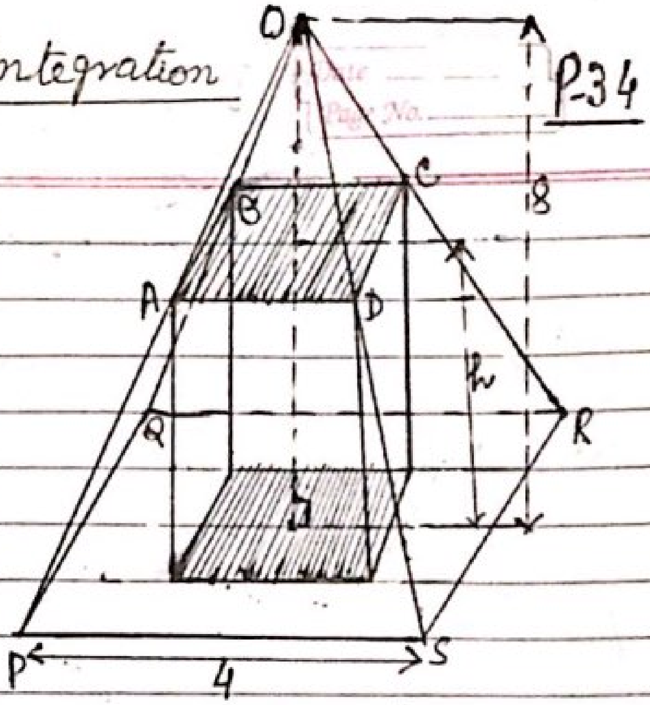
Calculate the area of the shaded region enclosed by the curve and the line AB, showing all your working. ----[8]



[S-15/22/8]

Diff. and Integration

Q94 The diagram shows a cuboid of h units inside a right pyramid OPQRS of height 8 units and with square base of 4 units. The base of the cuboid sits on the square base PQRS of the pyramid. The points A, B, C and D are corners of the cuboid and lie on the edges OP, OQ, OR and OS, respectively, of the the pyramid OPQRS. The pyramids OPQRS and OABCD are similar.



(i) Find an expression for AD in terms of h and hence show that the volume of the cuboid is given by $V = \frac{h^3}{4} - 4h^2 + 16h$ unit^3. ---[4]

(ii) Given that h can vary, find the value of h for which V is a maximum. [S-15/22/Q11] ---[4]

Q95 A curve, showing the relationship between two variables x and y, passing through the point $P(-1,3)$. The curve has a gradient of 2 at P. Given that $\frac{d^2y}{dx^2} = -5$, find the equation of the curve. ---[4]

[W-15/11/Q2]

Q96 Variables x and y are such that $y = (x-3)\ln(2x^2+1)$

(i) Find the value of $\frac{dy}{dx}$ when $x = 2$ ---[4]

(ii) Hence find the approximate change in y when x changes from 2 to 2.03. ---[2]

[W-15/11/Q5]

Q97 Find the equation of tangent to the curve $y = \frac{2x-1}{\sqrt{x^2+5}}$ at the point where $x = 2$ [W-15/11/Q8] --- [7]

Q98 Find the equation of the normal to the curve $y = 5\tan x - 3$ at the point where $x = \frac{\pi}{4}$. --[5]

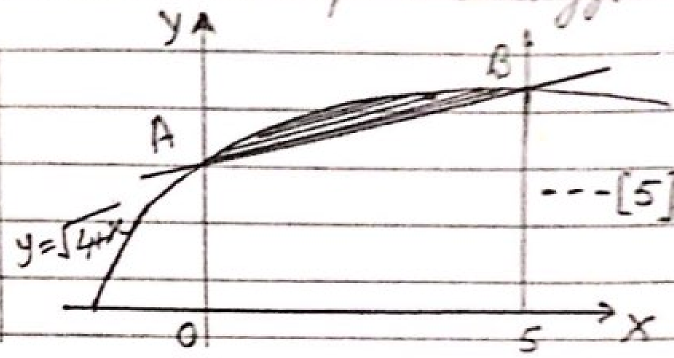
[W-15/13/Q5]

Diff and Integration

You are not allowed to use a calculator in this question.

Q99 (i) Find $\int \sqrt{4+x}\,dx$ ---[2]

(ii) The diagram shows the graph of $y=\sqrt{4+x}$, which meets the y-axis at the point A and the line $x=5$ at the point B. Using your answer to part (i), find the area of the region enclosed by the curve and the straight line AB. ---[5]



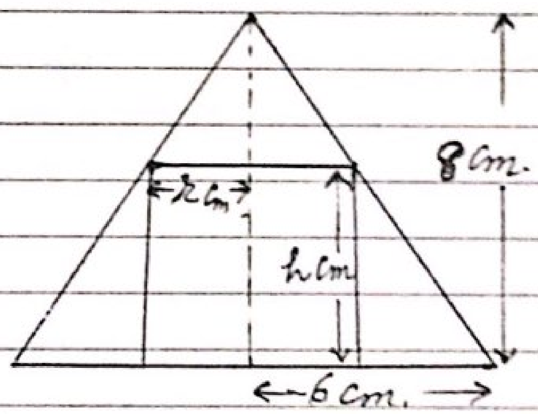
[W-15/11/Q9]

Q100 A cone, of height 8 cm and base radius 6 cm, is placed over a cylinder of radius r cm and height h cm and is in contact with the cylinder along the cylinder's upper rim. The arrangement is symmetrical and the diagram shows a vertical cross-section through the vertex of the cone.

(i) Use similar triangles to express h in terms of r. ---[2]

(ii) Hence show that the volume, $V\,cm^3$, of the cylinder is given by,
$$V = 8\pi r^2 - \frac{4}{3}\pi r^3$$ --[1]

(iii) Given that r can vary, find the value of r which gives a stationary value of V. Find this stationary value of V in term of π and determine its nature.



[W-15/21/Q7] --[6]

Q101 A particle is moving in a straight line such that its velocity, $v\,ms^{-1}$, t seconds after passing a fixed point O is $v = e^{2t} - 6e^{-2t} - 1$.

(i) Find an expression for the displacement, s m, from O of the particle after t seconds. ---[3]

(ii) Using substitution $u = e^{2t}$, or otherwise, find the time when the particle is at rest. --[3]

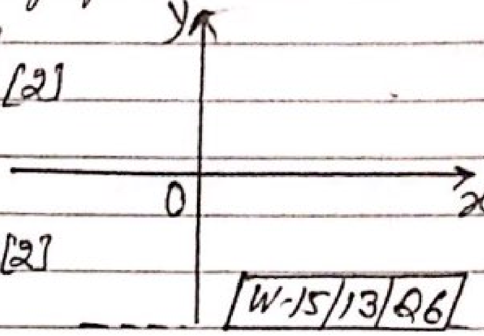
(iii) Find the acceleration at this time. ---[2]

[W-15/21/Q10]

Q102 (i) On the axes below, sketch the graph of $y = |x^2 - 4x - 12|$ showing the coordinates of the points where the graph meets the axes. ---[3]

(ii) Find the coordinates of the stationary point on the curve $y = |x^2 - 4x - 12|$ ---[2]

(iii) Find the values of k such that the equation $|x^2 - 4x - 12| = k$ has only 2 solutions. -- ---[2]



W-15/13/Q6

Q103 A curve, showing the relationship between two variables x and y, is such that $\frac{d^2y}{dx^2} = 6\cos 3x$. Given that the curve has a gradient of $4\sqrt{3}$ at the point $\left(\frac{\pi}{9}, -\frac{1}{3}\right)$, find the equation of the curve. --[6]

W-15/13/Q7

Q104 Find the equation of the tangent to the curve, ---[5]
$y = x^3 + 3x^2 - 5x - 7$ at the point where $x = 2$.

W-15/23/Q1

Q105 (a) Given that $y = \frac{x^3}{2 - x^2}$, find $\frac{dy}{dx}$ --[3]

(b) Given that $y = x\sqrt{4x+6}$, show that,

$$\frac{dy}{dx} = \frac{k(x+1)}{\sqrt{4x+6}}$$ and state the value of k. ---[3]

W-15/23/Q3

Q106 The velocity, $v\,ms^{-1}$, of a particle travelling in a straight line, t seconds after passing through a fixed point O, is given by $v = \frac{10}{(2+t)^2}$.

(i) Find the acceleration of the particle when $t = 3$. ---[3]

(ii) Explain why the particle never comes to rest. --[1]

(iii) Find an expression for the displacement of the particle from O after time t s. ---[3]

(iv) Find the distance travelled by the particle between $t = 3$ and $t = 8$. --[2]

W-15/23/Q7

Q107 (i) Given that $\frac{d}{dx}\left(e^{2-x^2}\right) = kxe^{2-x^2}$, state the value of k. --- [1]

(ii) Using your result from part (i) find $\int 3xe^{2-x^2}\,dx$ -- [2]

(iii) Hence find the area enclosed by the curve $y = 3xe^{2-x^2}$, the x-axis and the lines $x = 1$ and $x = \sqrt{2}$ -- [2]

(iv) Find the coordinates of the stationary point on the curve, $y = 3xe^{2-x^2}$. -- [4]

W-15/23/Q10

Q108 (i) Given that $y = e^{x^2}$, find $\frac{dy}{dx}$ --- [2]

(ii) Use your answer to part (i) to find $\int xe^{x^2}\,dx$ -- [2]

(iii) Hence evaluate $\int_0^2 xe^{x^2}\,dx$ --- [2]

S-14/11/Q5

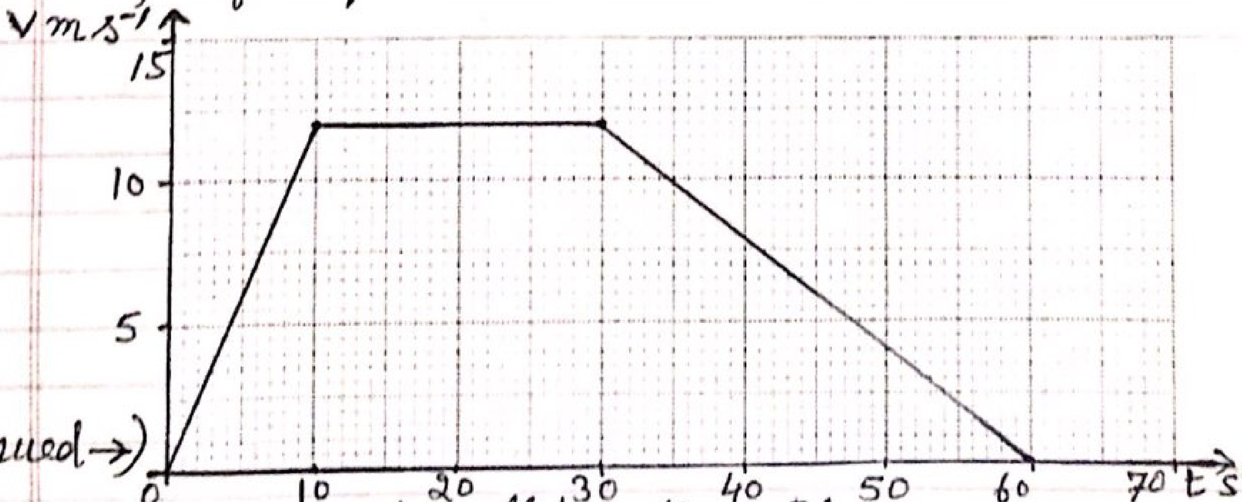
Q109. A curve is such that $\frac{dy}{dx} = 4x + \frac{1}{(x+1)^2}$ for $x > 0$. The curve passes through the point $\left(\frac{1}{2}, \frac{5}{6}\right)$.

(i) Find the equation of the curve. --- [4]

(ii) Find the equation of the normal to the curve at the point where $x = 1$ --- [4]

S-14/11/Q7

Q110 (a) The diagram shows the velocity-time graph of a particle P moving in a straight line with velocity $v\,ms^{-1}$ at time t s after leaving a fixed point.

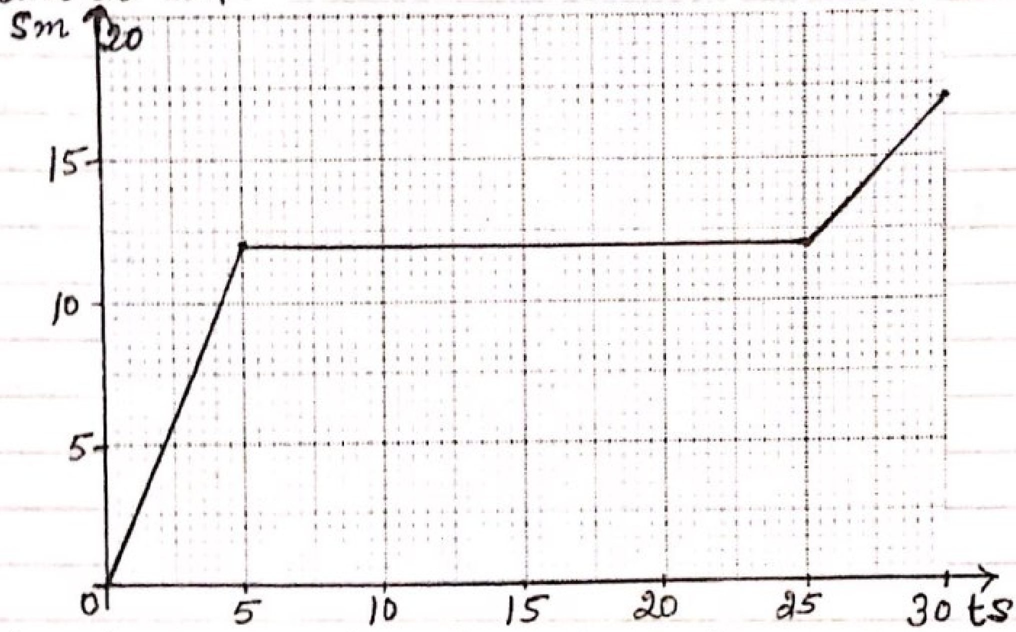


(Continued →)

Find the distance travelled by the particle P. --- [2]

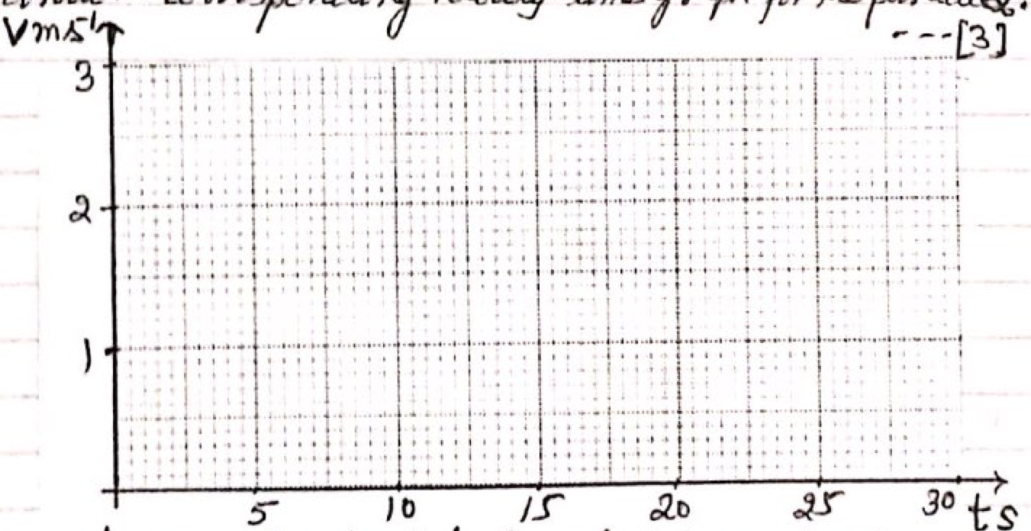
Continued.

Q110(b) The diagram shows the displacement-time graph of a particle Q moving in a straight line with displacement s m from a fixed point at t s.



On the axis below, draw the corresponding velocity-time graph for the particle Q.

--- [3]



(c) The displacement s m of a particle R, which is moving in a straight line, from a fixed point at time t s is given by,

$s = 4t - 16\ln(t+1) + 13.$

(i) Find the value of t for which the particle R is instantaneously at rest. --- [3]

(ii) Find the value of t for which the acceleration of the particle R is $0.25\,ms^{-2}$. --- [2]

S-14/11/Q9

Q11.1 Given that a curve has equation $y = \frac{1}{x} + 2\sqrt{x}$, where $x > 0$, find. (i) $\frac{dy}{dx}$ --[2]

(ii) $\frac{d^2y}{dx^2}$ ---[2]

Hence or otherwise, find

(iii) the coordinates and nature of the stationary point of the curve. [S-14/21/Q7] ---[4]

Q11.2 Find $\frac{dy}{dx}$ when

(i) $y = \cos 2x \sin\left(\frac{x}{3}\right)$ ---[4]

(ii) $y = \frac{\tan x}{1+\ln x}$ -- [4]

[S-14/21/Q10]

Q11.3 The region enclosed by the curve $y = 2\sin 3x$, the x-axis and the line $y = a$, where $0 < a < 1$ radian, lies entirely above x-axis. Given that the area of the region is $\frac{1}{3}$ square unit, find the value of a. ...[6]

[S-14/12/Q4]

Q11.4 A solid circular cylinder has a base radius of r cm and a volume of $4000\ cm^3$.

(i) Show that the total surface area, $A\ cm^2$, of the cylinder is given by. $A = \frac{8000}{r} + 2\pi r^2$ ---[3]

(ii) Given that r can vary, find the minimum total surface area of the cylinder, justify that this area is a minimum. --[6]

[S-14/12/Q9]

Q11.5 Differentiate with respect to x

(i) $x^4 . e^{3x}$ ---[2]

(ii) $\ln(2 + \cos x)$ -- [2]

(iii) $\frac{\sin x}{1+\sqrt{x}}$ [S-14/22/Q7] ---[3]

Q11.6 A curve is such that $\frac{dy}{dx} = (2x+1)^{\frac{1}{2}}$, the curve passes through the point $(4, 10)$.

(i) Find the equation of the curve. --[4]

(ii) Find $\int y\,dx$ and hence evaluate $\int_0^{1.5} y\,dx$. [S-14/22/Q9] [5] myCOMPANION

Q117 A curve has equation $y = x^3 - 9x^2 + 24x$

(i) Find the set of values of x for which $\frac{dy}{dx} \geq 0$ ---[4]

The normal to the curve at the point on the curve where $x = 3$ cuts the y-axis at the point P.

(ii) Find the equation of the normal and the coordinates of P. --[5]

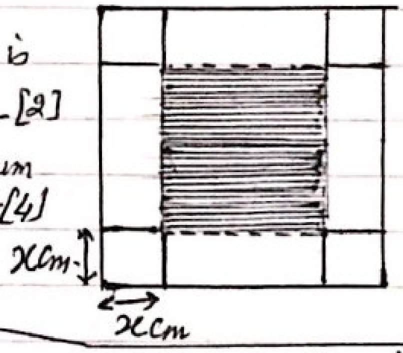
[S-14/22/Q/2]

Q118 The diagram shows a thin square sheet of metal measuring 24 cm by 24 cm. A square of side x cm is cut off from from each corner. The remainder is then folded to form an open box, x cm deep, whose square base is shown shaded in the diagram.

(i) Show that the volume, $V\,cm^3$, of the box is given by $V = 4x^3 - 96x^2 + 576x$ ---[2]

(ii) Given that x can vary, fin the maximum volume of the box. --[4]

[S-14/13/Q4]



Q119. Find the equation of normal to the curve $y = x(x^2 - 12)^{1/3}$ at the point on the curve where $x = 2$ [S-14/13/Q6] ---[6]

Q120 A particle moves in a straight line such that, t s, after passing through a fixed point O, its velocity $v\,ms^{-1}$, is given by $v = 5 - 4e^{-2t}$

(i) Find the velocity of the particle at O. ---[1]

(ii) Find the value of t when the acceleration of the particle is $6\,ms^{-2}$. --[3]

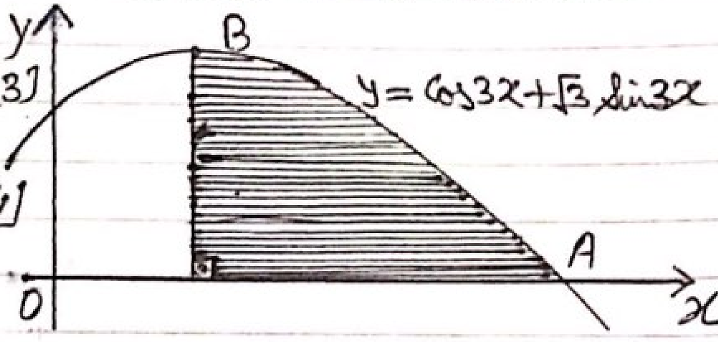
(iii) Find the distance of the particle from O when $t = 1.5$ ---[5]

(iv) Explain why the particle does not return at O. --[1]

[S-14/13/Q8]

Diff and Integration

Q121 The diagram shows the graph of $y = \cos 3x + \sqrt{3} \sin 3x$, which crosses the x-axis at A and has a maximum point at B.



(i) Find the x-coordinate of A. ---[3]

(ii) Find $\frac{dy}{dx}$ and hence find the x-coord. of B. ---[4]

(iii) Showing all your working, find the area of the shaded region bounded by the curve, the x-axis and the line through B parallel to the y-axis. $\boxed{\text{S-14/13/Q11}}$ ---[5]

Q122 A curve is such that $\frac{dy}{dx} = 6x^2 - 8x + 3$

(i) Show that the curve has no stationary point. ---[2]

Given that the curve passes through the point $P(2,10)$.

(ii) Find the equation of the tangent to the curve at the point P. ---[2]

(iii) Find the equation of the curve. $\boxed{\text{S-14/23/Q8}}$ ---[4]

Q123 (i) Given that $y = \frac{2x}{\sqrt{x^2+21}}$, show that $\frac{dy}{dx} = \frac{k}{\sqrt{(x^2+21)^3}}$, where k is a constant to be found.

(ii) Hence find $\int \frac{6}{\sqrt{(x^2+21)^3}}\,dx$ and evaluate $\int_2^{10} \frac{6}{\sqrt{(x^2+21)^3}}\,dx$ ---[3] $\boxed{\text{S-14/23/Q10}}$

Q124 Find the coordinates of the stationary point on the curve $y = x^2 + \frac{16}{x}$ ---[4]
$\boxed{\text{W-14/11/Q1}}$

Q125 A curve is such that $\frac{dy}{dx} = \frac{2}{\sqrt{x+3}}$ for $x > -3$. The curve passes through the point $(6,10)$.

(i) Find the equation of the curve. ---[4]

(ii) Find the x-coordinate of the point on the curve where $y = 6$. ---[1]
$\boxed{\text{W-14/11/Q3}}$

Q126 (i) Find the equation of the tangent to the curve $y = x^3 - \ln x$, at the point on the curve where $x = 1$. $\boxed{\text{W-14/11/Q5}}$ ---[4]

(ii) Show that this tangent bisects the line joining the points $(-2,16)$ and $(12,2)$ ---[2]

Q127 A particle moving in a straight line passes through a fixed point O. The displacement, x metres, of the particle, t second after it passes through O, is given by $x = t + 2\sin t$

(i) Find an expression for the velocity, $v\,ms^{-1}$, at time t. --- [2]

When the particle is first at instantaneous rest, find

(ii) the value of t. --- [2]

(iii) its displacement and acceleration. W-14/21/Q7 --- [3]

Q128 (i) Given that $y = \frac{x^2}{2+x^2}$, show that $\frac{dy}{dx} = \frac{kx}{(2+x^2)^2}$, where k is a constant to be found. --- [3]

(ii) Hence find $\int \frac{x\,dx}{(2+x^2)^2}$ --- [2]

W-14/21/Q8

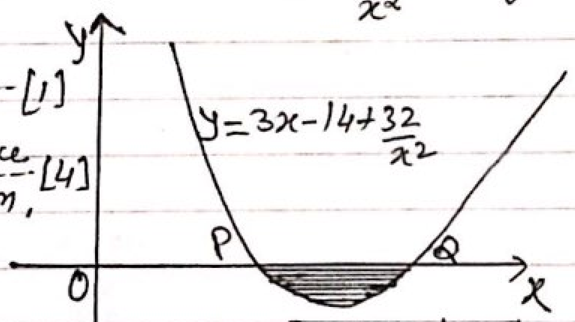
Q129 (i) Show that $x-2$ is a factor of $3x^3 - 14x^2 + 32$ --- [1]

(ii) Hence factorise completely: $3x^3 - 14x^2 + 32$ --- [4]

The diagram below shows part of the curve $y = 3x - 14 + \frac{32}{x^2}$ cutting the x-axis at the points P and Q.

(iii) State the x-coordinates of P and Q. --- [1]

(iv) Find $\int \left(3x - 14 + \frac{32}{x^2}\right) dx$ and hence determine the area of the shaded region. --- [4]



W-14/21/Q12

Q130 Find the coordinates of the stationary point on the curve $y = x^2 + \frac{16}{x}$

W-14/12/Q1 --- [4]

Q131 A curve is such that $\frac{dy}{dx} = \frac{2}{\sqrt{x+3}}$ for $x > -3$. The curve passes through the point (6,10).

(i) Find the equation of the curve. --- [4]

(ii) Find the x-coordinate of the point on the curve where $y=6$ --- [1]

W-14/12/Q3

Q132 (i) Given that $f(x) = x \cdot \ln x^3$, show that $f'(x) = 3(1+\ln x)$ --- [3]

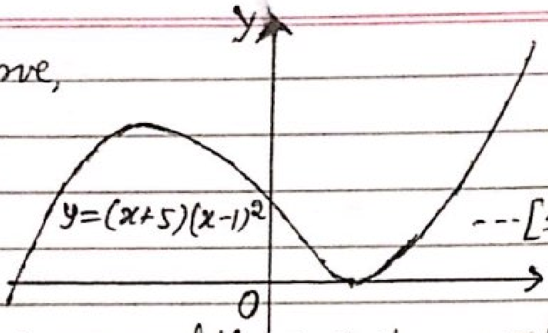
(ii) Hence find. $\int (1+\ln x)\,dx$ --- [2]

(iii) Hence find. $\int_1^2 \ln x\,dx$ in the form $p + \ln q$, where p and q are integers. --- [3]

W-14/13/Q8

Diff. and Integration

Q133. The diagram shows part of the curve, $y=(x+5)(x-1)^2$



(i) Find the x-coordinates of the stationary points of the curve. ---[5]

(ii) Find $\int (x+5)(x-1)^2\,dx$ ---[3]

(iii) Hence find the area enclosed by the curve and the x-axis. ---[2]

(iv) Find the set of positive values of k for which the equation $(x+5)(x-1)^2=k$ has only one real solution. ---[2]

W-14/13/Q11

Q134. A particle moving in a straight line passes through a fixed point O. The displacement, x metres, of the particle, t seconds after it passes through O, is given by $x=5t-3\cos 2t+3$

(i) Find expression for the velocity and acceleration of the particle after t seconds. ---[3]

(ii) Find the maximum velocity of the particle and the value of t at which this occurs. ---[3]

(iii) Find the value of t when the velocity of the particle is first equal to $2\,ms^{-1}$ and its acceleration at this time. W-14/23/Q8 ---[3]

Q135 (i) Determine the coordinates and nature of each of the two turning points on the curve. $y=4x+\frac{1}{x-2}$ --[6]

(ii) Find the equation of the normal to the curve at the point $(3,13)$ and find the x-coordinate of the point where this normal cuts the curve again. W-14/23/Q9 --[6]

Q136 (i) Find $\int\left(1-\frac{6}{x^2}\right)dx$ ---[2]

(ii) Hence find the value of the positive constant k for which $\int_k^{3k}\left(1-\frac{6}{x^2}\right)dx=2$ --[4]

S-13/11/Q5

Q137. The point A, whose x-coordinate is 2, lies on the curve with equation $y=x^3-4x^2+x+1$

(i) Find the equation of the tangent to the curve at A. ---[4]

(Continued→)

(→ Continued)

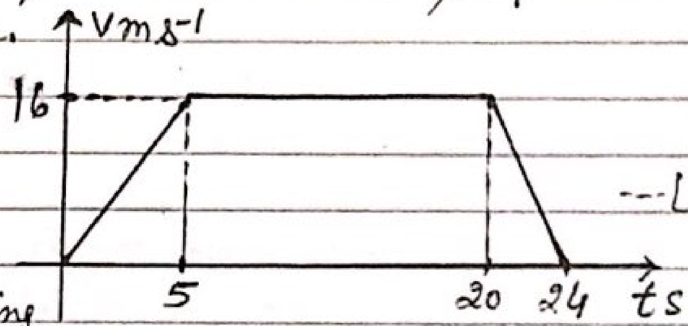
Q137. This tangent meets the curve again at the point B.

(ii) Find the coordinates of B. ---[4]

(iii) Find the equation of the perpendicular bisector of the line AB.

$\boxed{S-13/11/Q10}$ ---[4]

Q138. The velocity-time graph represents the motion of a particle moving in a straight line.



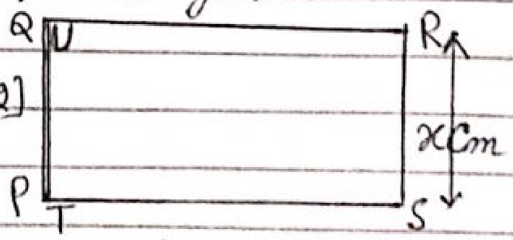
(i) Find the acceleration during the first 5 seconds. ---[1]

(ii) Find the length of time for which the particle is travelling with constant velocity. ---[1]

(iii) Find the total distance travelled by the particle. $\boxed{S-13/21/Q2}$ ---[3]

Q139. Variables x and y are related by the equation $y = 10 - 4\sin^2 x$, where $0 \leq x \leq \frac{\pi}{2}$. Given that x is increasing at a rate of 0.2 radians per second, find the corresponding rate of change of y when $y = 8$. $\boxed{S-13/21/Q3}$ ---[6]

Q140. A piece of wire of length 96 cm is formed into the rectangular shape PQRSTU shown in the diagram. It is given that $PQ = TU = SR = x$ cm. It may be assumed that PQ and TU coincide and that TS and QR have the same length.



(i) Show that the area, $A\,cm^2$, enclosed by the wire is given by $A = \frac{96x - 3x^2}{2}$ ---[2]

(ii) Given that x can vary, find the stationary value of A and determine the nature this stationary values. ---[4]

$\boxed{S-13/21/Q5}$

Q141. Find the equation of the normal to the curve $y = \frac{x^2+8}{x-2}$ at the point on the curve where $x = 4$ $\boxed{S-13/21/Q6}$ ---[6]

Diff. and Integration

Q142 The normal to the curve $y+2 = 3\tan x$, at the point on the curve whose $x = \frac{3\pi}{4}$, cuts the y-axis at the point P. Find the coordinates of P. [S-13/12/Q6] ---[6]

Q143 (a) (i) Find $\int \sqrt{2x-5}\,dx$ --[2]

(ii) Hence evaluate $\int_3^{15} \sqrt{2x-5}\,dx$ ---[2]

(b) (i) Find $\frac{d}{dx}(x^3 \cdot \ln x)$ ---[2]

(ii) Hence find $\int x^2 \ln x\,dx$ [S-13/12/Q10] ---[3]

Q144 A particle moves in a straight line such that, t s after leaving a point O, its velocity $v\,ms^{-1}$ is given by $v = 36t - 3t^2$ for $t \geq 0$

(i) Find the value of t when the velocity of P stops increasing. ---[2]

(ii) Find the value of t, when P comes to instantaneous rest. --[2]

(iii) Find the value of distance P from O when P is at instantaneous rest. [3]

(iv) Find the speed of P, when P is again at O. [S-13/12/Q12] ---[4]

Q145 Differentiate, with respect to x,

(i) $(3-5x)^{12}$ ---[2]

(ii) $x^2 \cdot \sin x$ ---[2]

(iii) $\frac{\tan x}{1+e^{2x}}$ ---[4]

[S-13/22/Q7]

Q146 A curve has equation $y = 3x + \frac{1}{(x-4)^3}$

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. ---[4]

(ii) Show that the coordinates of the stationary points of the curve are (5,16) and (3,8) ---[2]

(iii) Determine the nature of each of these stationary points, ---[2]

(iv) Find $\int \left(3x + \frac{1}{(x-4)^3}\right) dx$ ---[2]

(v) Hence find the area of the region enclosed by the curve, the line $x = 5$, the x-axis and the line $x = 6$. ---[2]

[S-13/22/Q11]

Diff. and Integration

Q147 (i) Find $\int (9+\sin 3x)\,dx$ ---[3]

(ii) Hence show that $\int_{\frac{\pi}{9}}^{\pi} (9+\sin 3x)\,dx = a\pi + b$, where a and b are constants to be found. $\boxed{W-13/11/Q5}$ ---[3]

Q148 (a) Differentiate $4x^3 \ln(2x+1)$ with respect to x. ---[3]

(b) (i) Given that $y = \dfrac{2x}{\sqrt{x+2}}$, show that $\dfrac{dy}{dx} = \dfrac{x+4}{(\sqrt{x+2})^3}$ ---[4]

(ii) Hence find $\int \dfrac{5x+20}{(\sqrt{x+2})^3}\,dx$ ---[2]

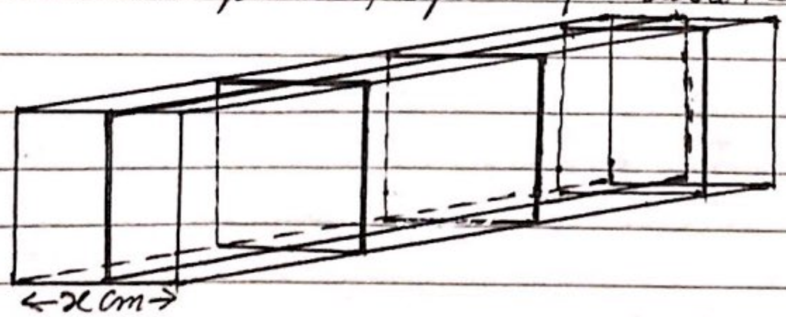
(iii) Hence evaluate. $\int_2^7 \dfrac{5x+20}{(\sqrt{x+2})^3}\,dx$ ---[2]

$\boxed{W-13/11/Q9}$

Q149 (i) Given that $y = \left(\frac{1}{4}x - 5\right)^8$, find $\dfrac{dy}{dx}$ --[2]

(ii) Hence find the approximate change in y as x increases from 12 to $12+p$, where p is small. $\boxed{W-13/21/Q3}$ --- [2]

Q150 The diagram shows a box in the shape of a cuboid with a square cross-section of side x cm. The volume of the box is $3500\ cm^3$. Four pieces of tape are fasted round the box as shown. The pieces of tape are parallel to the edges of the box.



(i) Given that the total length of the four pieces of tape is L cm, show that $L = 14x + \dfrac{7000}{x^2}$ ---[3]

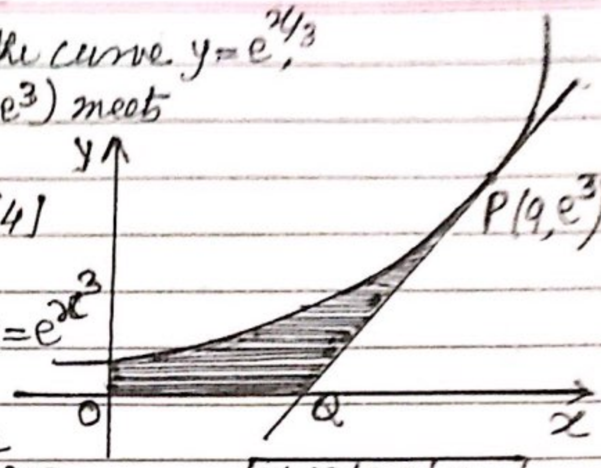
(ii) Given that x can vary, find the stationary value of L and determine the nature of this stationary. ---[5]

$\boxed{W-13/21/Q7}$

Q151) The diagram shows part of the curve $y = e^{x/3}$.
The tangent to the curve at $P(9, e^3)$ meets the x-axis at Q.

(i) Find the coordinates of Q ---[4]

(ii) Find the area of the shaded region bounded by the curve, the coordinate axes and the tangent to the curve at P. ---[6]



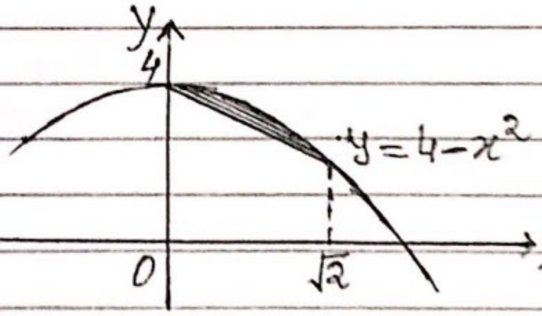
[W-13/21/Q11]

Q152) A curve has equation $\frac{e^{2x}}{(x+3)^2}$

(i) Show that $\frac{dy}{dx} = \frac{Ae^{2x}(x+2)}{(x+3)^3}$, where A is constant to be found, ---[4]

(ii) Find the exact coordinates of the point on the curve where $\frac{dy}{dx} = 0$ [W-13/13/Q4] ---[2]

Q153 The diagram shows part of the curve $y = 4 - x^2$. Show that the area of the shaded region can be written in the form $\frac{\sqrt{2}}{p}$, where p is an integer to be found.



[W-13/13/Q6] ---[6]

Q154 (i) Given that $\int_0^k \left(2e^{2x} - \frac{5}{2}e^{-2x}\right)dx = \frac{3}{4}$, where k is a constant, show that $4e^{4k} - 12e^{2k} + 5 = 0$ --[5]

(ii) Using a substitution of $y = e^{2k}$, or otherwise, find the possible values of k. [W-13/13/Q11] --[4]

Q155 Find the coordinates of the stationary point on the curve.
$y = x^3 - 6x^2 - 36x + 16$. [W-13/23/Q1] --[5]

Q156. A particle travels in a straight line so that, t s after passing through a fixed point O, its velocity, $v\ ms^{-1}$, is given by
$v = 3 + 6\sin 2t$

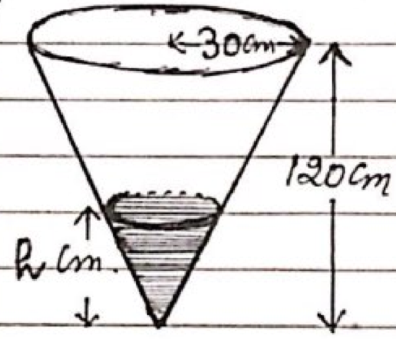
(i) Find the velocity of the particle when $t = \frac{\pi}{4}$ ---[1]

(ii) Find the acceleration of the particle when $t = 2$ ---[3]

The particle first comes to instantaneous rest at the point P.

(iii) Find the distance OP. $\boxed{W\text{-}13/23/Q9}$ --[5]

Q157. The diagram shows a container in the shape of a cone of height 120 cm and radius 30 cm. Water is poured into the container at a rate of $20\pi\ cm^3 s^{-1}$.



(i) At the instant when the depth of water in the cone is h cm, the volume of water in the cone is $V\ cm^3$.
Show that $V = \frac{\pi h^3}{48}$ ----[3]

(ii) Find the rate at which h is increasing when $h = 50$ ---[3]

(iii) Find the rate at which the circular area of water's surface is increasing when $h = 50$. ---[4]

$\boxed{W\text{-}13/23/Q10}$

Volume of a cone $= \frac{1}{3}\pi R^2 H$
R = Radius of cone
H = Height.

Answers

Q80 (i) $r^2+h^2=(0.5h+2)^2$

$r^2=0.25h^2+2h+4-h^2$

$r^2=2h+4-0.75h^2$ ✓

(ii) $V=\frac{1}{3}\pi r^2 h=\frac{\pi}{3}(2h^2+4h-0.75h^3)$

$\frac{dV}{dh}=\frac{\pi}{3}(4h+4-2.25h^2)$

$\frac{dV}{dh}=0 \Rightarrow 2.25h^2-4h-4=0$

$h=2.49$ Only

(iii) $\frac{d^2V}{dh^2}=\frac{\pi}{3}(4-4.5h)$

$=-7.545<0$ Max.

Q81 (i) $\frac{dy}{dx}=\frac{x(2\sec^2 2x)-\tan 2x}{x^2}$

(ii) when $x=\frac{\pi}{8}$, $y=\frac{8}{\pi}$

$\frac{dy}{dx}=\left(\frac{32}{\pi}-\frac{64}{\pi^2}\right)$

∴ Equation of the normal:

$y-\frac{8}{\pi}=-\frac{\pi^2}{32(\pi-2)}\left(x-\frac{\pi}{8}\right)$

or $y=-0.27x+2.65$ ✓

Q82 Area of Triangular face $=\frac{\sqrt{3}}{4}x^2$

(i) Volume of prism $=\frac{\sqrt{3}}{4}x^2\times y$

$\frac{\sqrt{3}}{4}x^2y=200\sqrt{3}$

∴ $x^2y=800$

∴ $A=2\times\frac{\sqrt{3}x^2}{4}+2xy$

$=\frac{\sqrt{3}}{2}x^2+\frac{1600}{x}$ ✓

(ii) $\frac{dA}{dx}=\sqrt{3}x-\frac{1600}{x^2}=0$

$\Rightarrow x^3=\frac{1600}{\sqrt{3}} \Rightarrow x=9.74$

∴ $A=246$ ✓

$\frac{d^2A}{dx^2}=\sqrt{3}+\frac{3200}{x^3}>0$ for $x=9.74$

∴ Min.

Q83 (i) $\sin x(-\sin x)+\cos x(\cos x)$

$=\cos^2x-\sin^2x$

$=1-2\sin^2x$ ✓

(ii) $\int(1-2\sin^2x)\,dx=\sin x\cos x+c$

$\Rightarrow -2\int\sin^2x\,dx=\sin x\cos x-\int 1\,dx$

$\Rightarrow \int\sin^2x\,dx=+\frac{x}{2}-\frac{1}{2}\sin x\cos x+c$ ✓

Q84 (i) $\frac{dy}{dx}=-8(2x+1)^{-3}\times 2+2$

$\frac{dy}{dx}=0 \Rightarrow x=\frac{1}{2},\ y=2$

(ii) $y=4\times\frac{1}{2}=2$ ✓

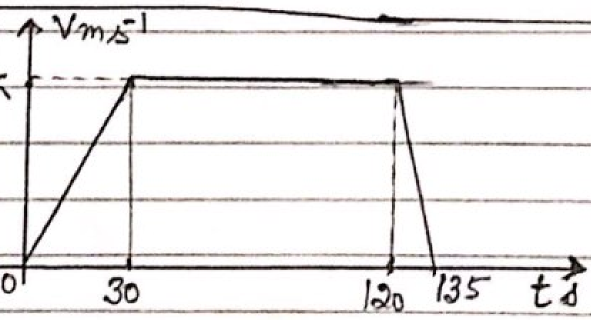
(iii) $\int\left(\frac{4}{(2x+1)^2}+2x\right)dx=\frac{4(2x+1)^{-1}}{-2}+\frac{2x^2}{2}$

$\left[\frac{4(2x+1)^{-1}}{-2}+\frac{2x^2}{2}\right]_0^{0.5}=\frac{5}{4}$

∴ Shaded area $=\frac{5}{4}-\frac{1}{2}=\frac{3}{4}$ ✓

Q85

(i)



(ii) $450=\frac{1}{2}\times 30\times k \Rightarrow k=30$

and acc $a=1\ ms^{-2}$

(b) $V=\int a\,dt=\int(3t^2+6)\,dt$

$V=t^3+6t+5$

when $t=3$, $V=50\ ms^{-1}$.

Q86 (i) $\frac{dy}{dx}=\frac{(x-1)\cdot\frac{8x}{(4x^2+3)}-\ln(4x^2+3)}{(x-1)^2}$

When $x=0$, $y=-\ln 3$

$\frac{dy}{dx}=-\ln 3 \longrightarrow$ grad of normal $=\frac{1}{\ln 3}$

∴ Equation of normal, $y+\ln 3=\frac{1}{\ln 3}x$

$\Rightarrow y=0.91x-1.1$ ✓

Diff. and Integration

Answers

Q86(ii) when $x=0$, $y=-\ln 3$

when $y=0$, $x=(\ln 3)^2$

Area $= \frac{1}{2}(\ln 3)^3 = 0.66$ ✓

Q87 (i) $\frac{dy}{dx} = 3x^2 - 10x + 3$

when $x=0$, $\frac{dy}{dx} = 3$,

gradient of the line is also 3, so line is a tangent. ✓

(ii) $\frac{dy}{dx} = 0 \Rightarrow (3x-1)(x-3)=0$

$x = \frac{1}{3}$, $x=3$ ✓

(iii) Area $= \frac{1}{2}(10+19)\times 3 - \int_0^3 x^3 - 5x^2 + 3x + 10\,dx$

$= \frac{87}{2} - \left[\frac{x^4}{4} - \frac{5x^3}{3} + \frac{3}{2}x^2 + 10x\right]_0^3$

$= \frac{87}{2} - \left(\frac{81}{4} - 45 + \frac{27}{2} + 30\right)$

$= 24.7$ ✓

Q88 (i) $2t^2 - 14t + 12 = 0$

$(t-1)(t-6) = 0$

$\therefore t = 1$ ✓ (first time)

(ii) $s = \int (2t^2 - 14t + 12)\,dt$

$= \frac{2t^3}{3} - 7t^2 + 12t$ ✓

(iii) $a = \frac{dv}{dt} = 4t - 14$

$a = \left(\frac{dv}{dt}\right)_{t=3} = (12-14) = -2$ ✓

Q89(a)(i) $\frac{1}{4}e^{4x+3} + c$

(ii) $\frac{1}{4}\left[e^{4\times 3+3} - e^{4\times 2.5+3}\right)$

$= 707000$ ✓

(b)(i) $3\sin x/3 + c$

(ii) $\left(3\sin\frac{\pi}{6}\times\frac{1}{3} - 3\sin 0\right) = 0.521$ ✓

(c) $\int (x^{-1}+x)^2\,dx = \int (x^{-2} + 2 + x^2)\,dx = \frac{x^{-1}}{-1} + 2x + \frac{x^3}{3} + c$

Q94 $\frac{AD}{4} = \frac{8-h}{8} \Rightarrow AD = \frac{(8-h)}{2}$

$V_{cuboid} = h\left(\frac{8-h}{2}\right)^2$

$= \frac{h^3}{4} - 4h^2 + 16h$ ✓

(ii) $\frac{dV}{dh} = \frac{3}{4}h^2 - 8h + 16 = 0$

$\Rightarrow 3h^2 - 32h + 64 = 0$

$(h-8)(3h-8) = 0$

$\Rightarrow h = \frac{8}{3}$ only $(h = 8^{\times})$

Q95 $\frac{dy}{dx} = -5x + c$

at $x=-1$, $\frac{dy}{dx} = 2 \Rightarrow c = -3$

$\therefore \frac{dy}{dx} = -5x - 3$

$\Rightarrow y = -\frac{5x^2}{2} - 3x + d$

$x=-1, y=3 \Rightarrow d = \frac{5}{2}$

$\therefore y = 5/2 - \frac{5x^2}{2} - 3x$ ✓

Q90 (i) $10\ln 4$ or 13.9

(ii) $\frac{dx}{dt} = \frac{20t}{t^2+4} - 4 = 0$

$\Rightarrow t^2 - 5t + 4 = 0$

$t=1, t=4$

(iii) $v = \frac{20t}{t^2+4} - 4$

$a = \frac{dv}{dt} = \frac{20(t^2+4) - 20t\times 2t}{(t^2+4)^2}$

$= \frac{20(4-t^2)}{(t^2+4)^2} = 0 \Rightarrow t = 2$ ✓

Q91 (i) $5e^{2x} - \frac{1}{2}e^{-2x}$ ✓

(ii) $(5e^{2k} - \frac{1}{2}e^{-2k}) - (5e^{-2k} - \frac{1}{2}e^{2k})$ ✓

(iii) $= -60$

or $\frac{11}{2}e^{2k} - \frac{11}{2}e^{-2k} = -60$

$11e^{2k} - e^{-2k} + 120 = 0$ ✓

(continued →)

Answers

Q91(iv) $11y^2+120y-11=0$

$(11y-1)(y+11)=0$

$y=\frac{1}{11}$ or $y=-11$

$e^{2k}=\frac{1}{11}$ or -11

$\Rightarrow k=\frac{1}{2}\ln\frac{1}{11}$ or $-\frac{1}{2}\ln 11$

or $k=\ln\frac{1}{\sqrt{11}}$, $-\frac{1}{2}\ln 11$

Q92 $\frac{dy}{dx}=4-6\sin 2x$

when $x=\frac{\pi}{4}, y=\pi, \frac{dy}{dx}=-2$

$\therefore$ gradient of Normal $=\frac{1}{2}$

Normal Equation: $y-\pi=\frac{1}{2}\left(x-\frac{\pi}{4}\right)$

when $x=0$, $y=\frac{7\pi}{8}$

$y=0$, $x=-\frac{7\pi}{4}$

$\therefore$ Area $=\frac{1}{2}\times\frac{7\pi}{4}\times\frac{7\pi}{8}=\frac{49\pi^2}{64}$

Q93 $x_B=x_C=7$

$\int_0^7(x^2-6x+10)dx=\left[\frac{x^3}{3}-\frac{6x^2}{2}+10x\right]_0^7$

Area under the curve $=\frac{112}{3}$

Area under line $=\frac{1}{2}(10+17)\times 7=\frac{189}{2}$

$\therefore$ Shaded area $=\frac{189}{2}-\frac{112}{3}=\frac{343}{6}=57.2$ ✓

Q94 and Q95 — on Page – 50

Q96(i) $\frac{dy}{dx}=(x-3)\cdot\frac{4x}{(2x^2+1)}+\ln(2x^2+1)$

when $x=2$, $\frac{dy}{dx}=-\frac{8}{9}+\ln 9$

$=1.31$ ✓

(ii) $\delta y=1.31\times 0.03$

$=0.0393$ ✓

Q97 $\frac{dy}{dx}=\frac{2(x^2+5)^{\frac{1}{2}}-\frac{1}{2}\cdot 2x\cdot(x^2+5)^{-\frac{1}{2}}\cdot(2x-1)}{x^2+5}$

$\left(\frac{dy}{dx}\right)_{x=2}=4/9$, $x=2, y=1$

$\therefore$ Eqn of tangent: $y-1=\frac{4}{9}(x-2)$

or $9y=4x+1$ ✓

Q98 when $x=\frac{\pi}{4}$, $y=2$

$\frac{dy}{dx}=5\sec^2 x\Rightarrow\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}}=10$

$\therefore$ Equation of Normal:

$y-2=-\frac{1}{10}\left(x-\frac{\pi}{4}\right)$

or $10y+x-20-\frac{\pi}{4}=0$

or $10y+x-20.8=0$ ✓

Q99(i) $\frac{2}{3}(4+x)^{3/2}+c$

(ii) Area of trapezium $=\frac{1}{2}\times 5\times 5=12.5$ ✓

Area $=\int_0^5\sqrt{4+x}\,dx-12.5$

$=\left[\frac{2}{3}(4+x)^{3/2}\right]_0^5-12.5$

$=\frac{2}{3}\times 27-\frac{16}{3}-12.5$

$=\frac{1}{6}$ or 0.17 ✓

Q100(i) $\frac{h}{8}=\frac{6-r}{6}\Rightarrow h=\frac{4}{3}(6-r)$ ✓

(ii) $V=\pi r^2 h=\pi r^2\times\frac{4}{3}(6-r)$

$=8\pi r^2-\frac{4}{3}\pi r^3$

(iii) $\frac{dV}{dr}=16\pi r-4\pi r^2$

$\frac{dV}{dr}=0\Rightarrow r=4$

$\therefore V=\frac{128\pi}{3}$ ✓

$\frac{d^2V}{dr^2}=16\pi-8\pi r<0$ when $r=4$

$\therefore$ Max. Volume

Answers

Q101 (i) $s = \frac{1}{2}e^{2t} + 3e^{-2t} - t + c$

$t = 0,\ s = 0 \Rightarrow c = -3.5$

$s = \frac{1}{2}e^{2t} + 3e^{-2t} - t - 3.5$ ✓

(ii) $v = 0 \Rightarrow u^2 - u - 6 = 0$

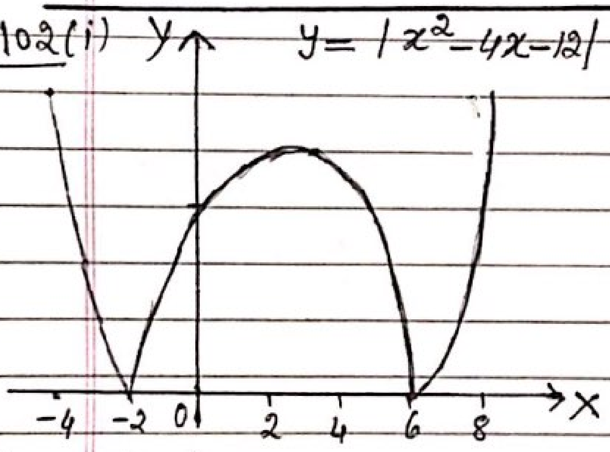
$(u-3)(u+2) = 0$

$\Rightarrow u = 3 \Rightarrow t = \frac{1}{2}\ln 3$ or 0.549

(iii) $a = 2e^{2t} + 12e^{-2t}$

when $t = \frac{1}{2}\ln 3 \Rightarrow a = 6 + 4 = 10$ ✓

Q102 (i)



(ii) $(2, 16)$

(iii) $k = 0$ or $k > 16$

Q103 $\frac{dy}{dx} = 2\sin 3x + c$

$4\sqrt{3} = 2\cdot\frac{\sqrt{3}}{2} + c \Rightarrow c = 3\sqrt{3}$

$\therefore \frac{dy}{dx} = 2\sin 3x + 3\sqrt{3}$

$y = -\frac{2}{3}\cos 3x + 3\sqrt{3}x + d$

$-\frac{1}{3} = -\frac{2}{3}\cos\frac{\pi}{3} + 3\sqrt{3}\cdot\frac{\pi}{9} + d \Rightarrow d = -\frac{\sqrt{3}}{3}\pi$

$y = -\frac{2}{3}\cos 3x + 3\sqrt{3}x - \frac{\sqrt{3}}{3}\pi$ ✓

Q104 $\frac{dy}{dx} = 3x^2 + 6x - 5$

$\left(\frac{dy}{dx}\right)_{x=2} = 19;\quad x = 2,\ y = 3$

$\therefore$ Equation of tangent, $\frac{y-3}{x-2} = 19$

or

$y = 19x - 35$ ✓

Q105 (a) $\frac{dy}{dx} = \frac{(2-x^2)\cdot 3x^2 - x^3(-2x)}{(2-x^2)^2}$

$= \frac{(6x^2 - x^4)}{(2-x^2)^2}$ ✓

(b) $\frac{dy}{dx} = x(4x+6)^{-\frac{1}{2}} \times 4 + (4x+6)^{\frac{1}{2}}$

$= \frac{6(x+1)}{\sqrt{4x+6}}$; $k = 6$ ✓

Q106. $a = -\frac{20}{(t+2)^3}$

$t = 3$; $a = -0.16\ m/s^2$ ✓

(ii) $\frac{10}{(t+2)^2}$ is never zero. ✓

(iii) $s = \frac{-10}{t+2} + 5$

(iv) $s = \left[\frac{-10}{t+2}\right]_3^8 = -1 + 2 = 1$ ✓

Q107 (i) $\frac{d}{dx}\left(e^{2-x^2}\right) = -2xe^{2-x^2}$ $\therefore k = -2$ ✓

(ii) $-\frac{3e^{2-x^2}}{2} + c$ ✓

(iii) $\left[-\frac{3e^{2-x^2}}{2}\right]_1^{\sqrt{2}} = -\frac{3}{2} + \frac{3}{2}e$

(iv) $y = xe^{2-x^2}$

$\frac{dy}{dx} = 3x\left(-2xe^{2-x^2}\right) + 3e^{2-x^2}$

$\frac{dy}{dx} = 0 \Rightarrow x = \pm\frac{1}{\sqrt{2}} = \pm 0.707$

$y = \pm\frac{3}{\sqrt{2}}e^{1.5} = \pm 9.51$

Q108 (i) $\frac{dy}{dx} = 2xe^{x^2}$ ✓

(ii) $\frac{1}{2}e^{x^2}$ ✓

(iii) $\frac{1}{2}e^4 - \frac{1}{2} = 26.8$ ✓

Scanned with CamScanner

Answers

Q109 (i) $y = 2x^2 - \frac{1}{x+1} + c$

when $x = \frac{1}{2}$, $y = \frac{5}{6}$; $\frac{5}{6} = \frac{1}{2} - \frac{2}{3} + c$

$\Rightarrow c = 1$

$\therefore y = 2x^2 - \frac{1}{(x+1)} + 1$ ✓

(ii) $x = 1$, $y = 5/2$

$\left(\frac{dy}{dx}\right)_{x=1} = \frac{17}{4}$; Grad. of Normal $= -\frac{4}{17}$

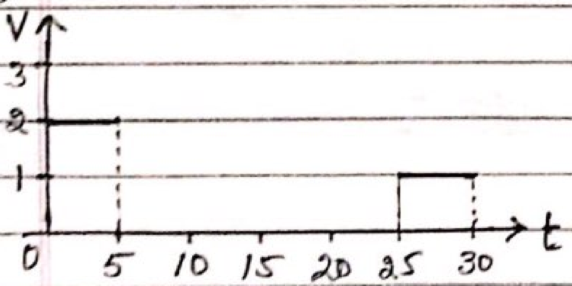
Equation of Normal, $y - \frac{5}{2} = -\frac{4}{17}(x-1)$

or $8x + 34y - 93 = 0$ ✓

Q110 (a) Distance = Area under graph

$= \frac{1}{2}(60+20) \times 12 = 480$ ✓

(b)



(C)(i) $V = 4 - \frac{16}{t+1}$, $V = 0$, $t = 3$ ✓

(ii) $a = \frac{16}{(t+1)^2}$; $0.25(t+1)^2 = 16$

$\Rightarrow t = 7$ ✓

Q111 (i) $\frac{dy}{dx} = -\frac{1}{x^2} + \frac{1}{x^{1/2}}$ ✓

(ii) $\frac{d^2y}{dx^2} = \frac{2}{x^3} - \frac{1}{2x^{3/2}}$ ✓

(iii) $\frac{dy}{dx} = 0 \Rightarrow x = 1$, $y = 3$

$\left(\frac{d^2y}{dx^2}\right)_{x=1} = 2 - \frac{1}{2} > 0$

$\therefore$ Min at $(1, 3)$ ✓

Q112 (i) $\frac{1}{3}\cos\frac{x}{3}\cos 2x - 2\sin 2x \sin\frac{x}{3}$ ✓

(ii) $\frac{\sec^2 x(1+\ln x) - \frac{1}{x}\tan x}{(1+\ln x)^2}$ ✓

Q113. $\int_0^a 2\sin 3x\, dx = \frac{1}{3}$

$\Rightarrow \left[-\frac{2}{3}\cos 3x\right]_0^a = \frac{1}{3}$

$-\frac{2}{3}\cos 3a - \left(-\frac{2}{3}\right) = \frac{1}{3}$

$\cos 3a = 0.5$

$\Rightarrow 3a = \frac{\pi}{3} \Rightarrow a = \frac{\pi}{9}$ ✓

Q114 (i) $\pi r^2 h = 4000$

$\Rightarrow h = 4000/\pi r^2$

$\therefore A = 2\pi r h + 2\pi r^2$

$= 2\pi r \times \frac{4000}{\pi r^2} + 2\pi r^2$

$A = \frac{8000}{r} + 2\pi r^2$ ✓

(ii) $\frac{dA}{dr} = -\frac{8000}{r^2} + 4\pi r = 0$

$\Rightarrow r^3 = \frac{8000}{4\pi}$

$A = 1395$

$\frac{d^2A}{dr^2} = \frac{16000}{r^3} + 4\pi > 0$ $\therefore$ Min ✓

Q115 (i) $x^4(3e^{3x}) + 4x^3e^{3x}$ ✓

(ii) $\frac{1}{2+\cos x}(-\sin x)$

(iii) $\frac{(1+\sqrt{x})\cos x - \frac{1}{2}x^{-1/2} \times \sin x}{(1+\sqrt{x})^2}$ ✓

Q116 (i) $y = \frac{(2x+1)^{3/2}}{2 \times 3/2} + c$ Passes through $(4, 10)$

$10 = \frac{2}{6}\left[2 \times 4 + 1\right]^{3/2} + c \Rightarrow c = 1$

$\therefore y = \frac{(2x+1)^{3/2}}{3} + 1$ ✓

(Continued →)

IG-0606
Maths

Diff. and Integration

Answers

Q116(ii) $\int \left(\frac{1}{3}(2x+1)^{3/2}+1\right) dx$

$= \frac{1}{15}(2x+1)^{5/2} + x + \text{constant}$

$\therefore \left[\frac{1}{15}(2x+1)^{5/2} + x\right]_0^{1.5} = \frac{107}{30}$ ✓

Q117 (i) $\frac{dy}{dx} = 3x^2 - 18x + 24$

now $3x^2 - 18x + 24 \geq 0$

$x^2 - 6x + 8 \geq 0$

$x \leq 2, \quad x \geq 4$

(ii) $\left(\frac{dy}{dx}\right)_{x=3} = -3$

$\therefore$ grad of Normal $= \frac{1}{3}$

$x = 3, y = 18$

$\therefore$ Eqn of Normal $y - 18 = \frac{1}{3}(x-3)$

$y = x/3 + 17$ ✓

Normal cuts y-axis $x = 0, y = 17$

$(0, 17)$ ✓

Q118 (i) $V = x(24-2x)^2$

or $V = 4x^3 - 96x^2 + 576x$ ✓

(ii) $\frac{dV}{dx} = 12x^2 - 192x + 576 = 0$

$\Rightarrow 12(x-4)(x-12)$

$x = 4, \quad x = 12^{\times}$

$\therefore$ Max $V = 1024$ ✓

Q119 $x = 2, y = -4$

$\frac{dy}{dx} = x\left(\frac{2x}{3}\right)(x^2-12)^{-2/3} + (x^2-12)^{1/3}$

$\left(\frac{dy}{dx}\right)_{x=2} = -4/3$

Noral $y + 4 = \frac{3}{4}(x-2)$

or $4y = 3x - 22$.

Q120(i) 1

(ii) $a = 8e^{-2t} = 6$

$\Rightarrow -2t = \ln 3/4$

or $t = 0.144$ ✓

Q120(iii) $s = 5t + 2e^{-2t} + c$

$t = 0, s = 0 \Rightarrow c = -2$

when $t = 1.5$, $s = 5.60$ ✓

(iv) Velocity is always +ve, so no change in the direction. ✓

Q121. at A, $\cos 3x + \sqrt{3}\sin 3x = 0$

$\Rightarrow \sin 3x = -\frac{1}{\sqrt{3}} \Rightarrow 3x = \frac{5\pi}{6}$

$\Rightarrow x = \frac{5\pi}{18} = 0.873$ ✓

(ii) $\frac{dy}{dx} = 3\sqrt{3}\cos 3x - 3\sin 3x = 0$

$\Rightarrow \tan 3x = \sqrt{3} \Rightarrow 3x = \frac{\pi}{3}$

$x = \frac{\pi}{9}$ (or 0.349) ✓

(iii) Area $= \left[-\frac{\sqrt{3}}{3}\cos 3x + \frac{1}{3}\sin 3x\right]_{\pi/9}^{5\pi/18}$

$= 2/3$ or 0.667 ✓

Q122 (i) $b^2 - 4ac = -8 < 0$

$\therefore \frac{dy}{dx} = 0$ has no solution.

$\therefore$ no stationary point. ✓

(ii) $x = 2, \frac{dy}{dx} = 11$

$\therefore$ Eqn of tangent $y - 10 = 11(x-2)$

$\Rightarrow y = 11x - 12$ ✓

(iii) $y = 2x^3 - 4x^2 + 3x + c$

Passes through $(2, 10) \Rightarrow c = 4$

$\therefore y = 2x^3 - 4x^2 + 3 + 4$

Q123 (i) $\frac{dy}{dx} = \dfrac{2\sqrt{x^2+21} - 2x \times \frac{x}{\sqrt{x^2+21}}}{(x^2+21)}$

$= \frac{42}{(x^2+21)^{3/2}} \quad \therefore k = 42$ ✓

(ii) $\frac{1}{7} \times \frac{2x}{\sqrt{x^2+21}}$; $\left[\frac{2x}{7\sqrt{x^2+21}}\right]_2^{10}$

$= \frac{2}{7}\left(\frac{10}{11} - \frac{2}{5}\right) = \frac{8}{55} = 0.145$ ✓

Answers

Q124. $\frac{dy}{dx} = 2x - \frac{16}{x^2} = 0$

$x = 2,\ y = 12$

Q125 (i) $y = 4(x+3)^{1/2} + c$

$x = 6,\ y = 10 \Rightarrow c = -2$

$\therefore\ y = 4(x+3)^{1/2} - 2$ ✓

(ii) $6 = 4(x+3)^{1/2} - 2$

$\Rightarrow x = 1$ ✓

Q126 (i) $\frac{dy}{dx} = 3x^2 - \frac{1}{x}$

when $x = 1, y = 1$; $\frac{dy}{dx} = 2$

tangent $y - 1 = 2(x-1)$

or $y = 2x - 1$ ✓

(ii) Mid point $(5,9)$

$9 = 2\times 5 - 1$ True ✓

Q127 (i) $v = 2\cos t + 1$

(ii) $2\cos t + 1 = 0 \Rightarrow t = \frac{2\pi}{3}$ or 2.09 ✓

(iii) $t = \frac{2\pi}{3} \Rightarrow x = 2\sin\frac{2\pi}{3} + \frac{2\pi}{3} = 3.83$ ✓

and $a = -2\sin t$

$t = \frac{2\pi}{3} \rightarrow a = -\sqrt{3} = -1.73\ m s^{-2}$ ✓

Q128 (i) $\frac{dy}{dx} = \frac{(2+x^2)\cdot 2x - x^2 \times 2x}{(2+x^2)^2}$

$= \frac{4x}{(2+x^2)^2}$; $k = 4$ ✓

(ii) $\int \frac{x}{(2+x^2)^2}dx = \frac{1}{4} \times \frac{x^2}{2+x^2} + c$ ✓

Q129 (i) $f(2) = 3(2)^3 - 14(2)^2 + 32 = 0$

(ii) $f(x) = (x-2)(3x^2 - 8x - 16)$

$= (x-2)(x-4)(3x+4)$

(iii) $x = 2, 4$

(iv) $\int\left(3x - 14 + \frac{32}{x^2}\right)dx = 1.5x^2 - 14x - \frac{32}{x}$

Area $= \left[1.5x^2 - 14x - \frac{32}{x}\right]_2^4 = |-2| = 2$ ✓

Q130 $\frac{dy}{dx} = 2x - \frac{16}{x^2} = 0$

$\Rightarrow x = 2,\ y = 12$

Q131 Same as Q125

Q132 (i) $f'(x) = x \times \frac{3x^2}{x^3} + \ln x^3$

$= 3 + 3\ln x = 3(1 + \ln x)$ ✓

(ii) $\int 3(1+\ln x)dx = x\ln x^3 = 3x\ln x$

$\therefore \int (1+\ln x)dx = x\ln x$ ✓

(iii) $x\ln x - \int 1\,dx = \left[x\ln x - x\right]_1^2$

$= 2\ln 2 - 2 + 1$

$= -1 + \ln 4$ ✓

Q133 (i) $\frac{dy}{dx} = (x+5)\cdot 2(x-1) + (x-1)^2$

$= (x-1)(3x+9) = 0$

$\Rightarrow x = 1,\ x = -3$ ✓

(ii) $\int (x^3 + 3x^2 - 9x + 5)\,dx$

$= \frac{x^4}{4} + x^3 - \frac{9}{2}x^2 + 5x + C$ ✓

(iii) $\left[\frac{x^4}{4} + x^3 - \frac{9}{2}x^2 + 5x\right]_{-5}^{1} = 108$ ✓

(iv) when $x = -3,\ y = 32$

$k > 32$ ✓

Q134 $v = 5 + 6\sin 2t$ ✓

(i) $a = 12\cos 2t$ ✓

(ii) $a = 12\cos 2t = 0 \Rightarrow t = \frac{\pi}{4}$ or 0.785 ✓

$\therefore$ Max $v = 5 + 6\sin\frac{\pi}{2} = 11$ ✓

(iii) $v = 2 \Rightarrow \sin 2t = -\frac{1}{2} \Rightarrow t = \frac{7\pi}{12}$ or 1.83 ✓

$a = 12\cos\frac{7\pi}{6} = -6\sqrt{3}$ or -10.4 ✓

Answers

Q135 (i) $\frac{dy}{dx} = 4 - \frac{1}{(x-2)^2}$ ✓

$\frac{dy}{dx} = 0 \Rightarrow (x-2)^2 = \frac{1}{4}$

$4x^2 - 16x + 15 = 0$

$x = 2.5$ or 1.5

$y = 12, 4$

$\frac{d^2y}{dx^2} = \frac{2}{(x-2)^3}$

$\left(\frac{d^2y}{dx^2}\right)_{x=2.5} > 0 \rightarrow$ Min

$x = 1.5 \rightarrow \frac{d^2y}{dx^2} < 0$ Max.

(ii) $x = 3$, $\frac{dy}{dx} = 3$

grad of normal $= -\frac{1}{3}$

Equn of normal, $\frac{y-13}{x-3} = -\frac{1}{3}$

Intersection of normal and curve.

$14 - x/3 = 4x + \frac{1}{x-2}$

$13x^2 - 68x + 87 = 0$

$x = \frac{29}{13}$ or 2.23 ✓

Q136 (i) $x + \frac{6}{x} + c$ ✓

(ii) $\left(3k + \frac{6}{3k}\right) - \left(k + \frac{6}{k}\right) = 2$

$2k^2 - 2k - 4 = 0$

$\therefore k = 2$ ✓

Q137 (i) $x = 2, y = -5$

$\frac{dy}{dx} = 3x^2 - 8x + 1$, $\left(\frac{dy}{dx}\right)_{x=2} = -3$

Tangent $y + 5 = -3(x+2)$

or $y = 1 - 3x$ ✓

(ii) $1 - 3x = x^3 - 4x^2 + x + 1$

$x(x-2)^2 = 0$

meet at $(0,1)$ B, $x = 2$ (A)

(iii) Grad of $\perp = \frac{1}{3}$, Mid point $(1, -2)$

Perp. bisector $y + 2 = \frac{1}{3}(x-1)$ ✓

Q138 (i) $a = \frac{16}{5} = 3.2$ ✓

(ii) 15 s

(iii) area of Trap $= \frac{1}{2}(24+15) \times 16$

$= 312$ m ✓

Q139 $\frac{dy}{dx} = -8 \sin x \cos x$

$y = 8 \Rightarrow x = \frac{\pi}{4}$; $\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}} = -4$

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$= -4 \times 0.2 = -0.8$ ✓

Q140 (i) $QR = PS = \frac{96 - 3x}{2}$

Area $= \frac{(96-3x)}{2} \times x$ ✓

(ii) $\frac{dA}{dx} = (48 - 3x) = 0$

$\Rightarrow x = 16$

$\frac{d^2A}{dx^2} = -3 < 0$, Max, $A = 384$ ✓

Q141 $\frac{dy}{dx} = \frac{(x-2) \cdot 2x - (x^2+8)}{(x-2)^2}$

$x = 4, y = 12, \frac{dy}{dx} = -2$

Grad of normal $= \frac{1}{2}$

Equn of Normal $y - 12 = \frac{1}{2}(x-4)$

$y = \frac{1}{2}x + 10$ ✓

Q142 $\frac{dy}{dx} = 3 \sec^2 x$

$x = \frac{3\pi}{4}, y = 5$; $\frac{dy}{dx} = 6$

$\therefore$ Normal Grad $= -\frac{1}{6}$

Equation of normal $y + 5 = -\frac{1}{6}\left(x - \frac{3\pi}{4}\right)$

when $x = 0$, $y = \frac{\pi}{8} - 5 = -4.6$ ✓

$P(0, -4.6)$ ✓

Diff. and Integration

Answers

Q143(a)(i) $\frac{1}{3}(2x-5)^{3/2}$ ✓

(ii) $\frac{125}{3}-\frac{1}{3}=\frac{124}{3}$ ✓

(b)(i) $x^3\times\frac{1}{x}+3x^2\cdot\ln x$

(ii) $3\int x^2\ln x\,dx = x^3\ln x-\int x^2dx$

$\therefore \int x^2\ln x\,dx=\frac{1}{3}\left(x^3\ln x-\frac{x^3}{3}\right)+c$

Q144 (i) $\frac{dv}{dt}=36-6t$

$\frac{dv}{dt}=0\Rightarrow t=6$ ✓

(ii) when $v=0\Rightarrow t=12$ ✓

(iii) $s=18t^2-t^3+c$

when $t=12,\ s=864$ ✓

(iv) $s=0,\ t=18\Rightarrow v=-324$

so speed $=324$ ✓

Q145 (i) $-5\times12(3-5x)^{11}$ ✓

(ii) $x^2\cos x+2x\sin x$

(iii) $\frac{(1+e^{2x})\sec^2x-2e^{2x}\cdot\tan x}{(1+e^{2x})^2}$ ✓

Q146(i) $\frac{dy}{dx}=3-3(x-4)^{-4}$

$\frac{d^2y}{dx^2}=12(x-4)^{-5}$

(ii) $\frac{dy}{dx}=0\Rightarrow x=3$ or 5

$x=3, y=8$ & $x=5, y=16$

(iii) $\left(\frac{d^2y}{dx^2}\right)_{x=5}=12>0$ $\therefore$ Min

$\left(\frac{d^2y}{dx^2}\right)_{x=3}=-12<0$ $\therefore$ Max

(iv) $\frac{3x^2}{2}-\frac{(x-4)^{-2}}{2}+c$

(v) $\left[\frac{3x^2}{2}-\frac{(x-4)^{-2}}{2}\right]_5^6=\frac{135}{8}=16.87$ ✓

Q147 (i) $9x-\frac{1}{3}\cos3x+c$ ✓

(ii) $\left[9x-\frac{1}{3}\cos3x\right]_{\pi/9}^{\pi}=8\pi+\frac{1}{2}$ ✓

Q148(a) $12x^2\cdot\ln(2x+1)+4x^3\times\frac{2}{(2x+1)}$ ✓

(b)(i) $\frac{dy}{dx}=\frac{\sqrt{(x+2)}\times2-2x(x+2)^{-\frac{1}{2}}\times\frac{1}{2}}{(x+2)}$

$=\frac{x+4}{(x+2)^{3/2}}$ ✓

(ii) $\frac{10x}{\sqrt{x+2}}+c$

(iii) $\left[\frac{10x}{\sqrt{x+2}}\right]_2^7=\frac{70}{3}-\frac{20}{2}=\frac{40}{3}$ ✓

Q149(i) $\frac{dy}{dx}=2\left(\frac{1}{4}x-5\right)^7$

(ii) $\delta y=\frac{dy}{dx}\times\delta x$; $\left(\frac{dy}{dx}\right)_{x=12}=-256$

$=-256\times p$ $[\because \delta x=p]$

Q150 (i) $l=\frac{3500}{x^2}$

$L=3\times4x+2x+2l$

$L=14x+\frac{7000}{x^2}$ ✓

(ii) $\frac{dL}{dx}=14-\frac{14000}{x^3}=0\Rightarrow x=10$, $L=210$ ✓

$\frac{d^2L}{dx^2}=\frac{42000}{x^4}>0$, Min.

Q151(i) $\frac{dy}{dx}=\frac{1}{3}e^{x/3}$; $\left(\frac{dy}{dx}\right)_{x=9}=\frac{1}{3}e^3$

$\therefore$ Tangent $y-e^3=\frac{1}{3}e^3(x-9)$

At Q, $y=0\Rightarrow x=6$

(ii) Area $=\int_0^9 e^{x/3}dx$ − Area of triangle

$=\left[3e^{x/3}\right]_0^9-\frac{1}{2}\times3\times e^3$

$=3e^3-3-1.5e^3=(1.5e^3-3)$ ✓

or 27.1

Diff. and Integration

Answers

Q152 $\frac{dy}{dx} = \frac{(x+3)^2 \cdot 2e^{2x} - e^{2x} \cdot 2(x+3)}{(x+3)^4}$

$= \frac{2e^{2x}(x+2)}{(x+3)^3}$; $A = 2$ ✓

(ii) $x = -2, y = e^{-4}$

Q153 $\int_0^{\sqrt{2}} 4 - x^2 dx = \left[4x - \frac{x^3}{3}\right]_0^{\sqrt{2}}$

$= \frac{10\sqrt{2}}{3}$ ✓

Area of Trapezium $= \frac{1}{2}(4+2) \times \sqrt{2}$
$= 3\sqrt{2}$

$\therefore$ Shaded Area $= \frac{10\sqrt{2}}{3} - 3\sqrt{2} = \frac{\sqrt{2}}{3}$ ✓

Q154 (i) $\left[e^{2x} + \frac{5}{4}e^{-2x}\right]_0^k = 3/4$

$\left(e^{2k} + \frac{5}{4}e^{-2k}\right) - \left(1 + \frac{5}{4}\right) = 3/4$

$\Rightarrow 4e^{4k} - 12e^{2k} + 5 = 0$ ✓

(ii) $4y^2 - 12y + 5 = 0$

$e^{2k} = 5/2$ or $\frac{1}{2}$

$k = 0.458, -0.347$

Q155 $\frac{dy}{dx} = 3x^2 - 12x - 36 = 0$ ✓

$x = -2$ and $x = 6$

$y = 56$ and $y = -200$

Q156 (i) 9

(ii) $a = 12\cos 2t$

$t = 2 \rightarrow a = -7.84$

(iii) $s = 3t - 3\cos 2t$

$\left[v = 3 + 6\sin 2t = 0 \right.$
$t = \frac{7\pi}{12}$

$t = 0$ to $\frac{7\pi}{12}$

$\therefore s = 3\left[\frac{7\pi}{12} - \cos\frac{7\pi}{6}\right] - (-3)$

$= \frac{7\pi}{4} + \frac{3\sqrt{3}}{2} + 3$

$= 11.1$ ✓

Q157 (i) radius $r = \frac{h}{4}$

$V = \frac{1}{3}\pi\left(\frac{h}{4}\right)^2 \cdot h$ $\left[\frac{1}{3}\pi r^2 h\right]$

$V = \frac{\pi h^3}{48}$ ✓

(ii) $\frac{dV}{dh} = \frac{\pi h^2}{16}$; $\left(\frac{dV}{dh}\right)_{h=50} = \frac{2500\pi}{16}$

$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$

$= 20\pi \times \frac{16}{2500\pi}$

$= 0.128$ ✓

(iii)

$A = \pi r^2 = \pi \cdot \left(\frac{h}{4}\right)^2 = \frac{\pi h^2}{16}$

$\frac{dA}{dh} = \frac{\pi h}{8}$ at $h = 50$, $\frac{dA}{dh} = \frac{\pi \times 50}{8}$

$\frac{dA}{dt} = \frac{dh}{dt} \times \frac{dA}{dh}$

$= 0.128 \times \frac{\pi}{8} \times 50$

$= 0.8\pi$ or 2.51 ✓

$\longleftarrow$ X ——— X $\longrightarrow$