

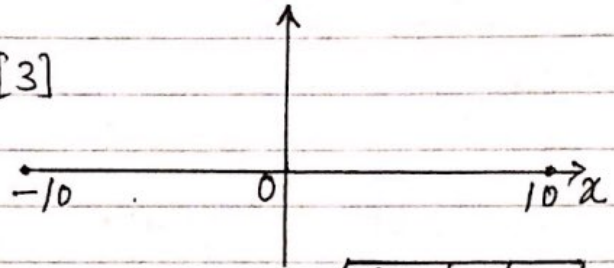
IG-0606
Additional Maths.

Equations-Inequalities
and Graphs.
Exercise

Suresh Goel
(Director)
Alliance World School,
Noida, Delhi. NCR. India.

Q1(a) On the axes below, sketch the graph of $y = |2x+5|$ and the graph of $y = |2-x|$, stating the coordinates of the points where graph meets the coordinate axes, --- [4]

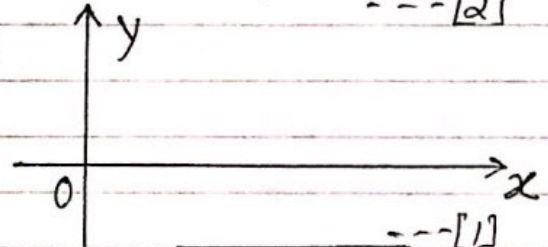
(b) Solve $|2x+5| \leq |2-x|$ --- [3]



SP-20/01/Q5

Q2(a) On the axes, sketch the graph of $y = \frac{1}{5}(x-2)(x-4)(x+5)$, showing the coordinates of the points where the graph meets the coordinate axes, --- [2]

(b) Explain why your sketch in part (a) can be used to solve $(x-2)(x-4)(x+5) \leq 0$



(c) Hence solve $(x-2)(x-4)(x+5) \leq 0$ --- [1]

SP-20/02/Q2

Q3 Solve the equation $|5-3x|=10$ --- [3]

M-17/22/Q1

Q4(i) On the axes below sketch the graphs of $y = |2x-5|$ and $9y = 80x - 16x^2$, --- [5]

(ii) Solve $|2x-5|=4$ --- [3]

(iii) Hence show that the

graphs of $y = |2x-5|$ and

$9y = 80x - 16x^2$ intersect at the points where $y=4$. --- [1]

(iv) Hence find the values of x ,

for which $9|2x-5| \leq 80x - 16x^2$ --- [2]

S-17/11/Q8

Q5 Solve $|5x+3|=|1-3x|$ --- [3]

S-17/22/Q1

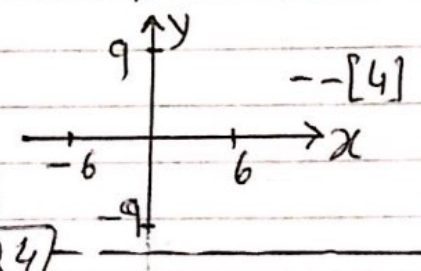
Q6 Solve the inequality: $(x-1)(x-5) > 12$ [W-17/21/Q1] --- [4]

Q7 Solve the equation: $\frac{2x^{1.5} + 6x^{-0.5}}{x^{0.5} + 5x^{-0.5}} = x$ [W-17/22/Q2] --- [5]

Q8 Solve the inequality: $|3x-1| > 3+x$ [W-17/22/Q3] --- [3]

Q9 Solve the equation: $|3x-1| = |5+x|$ [W-17/23/Q2]

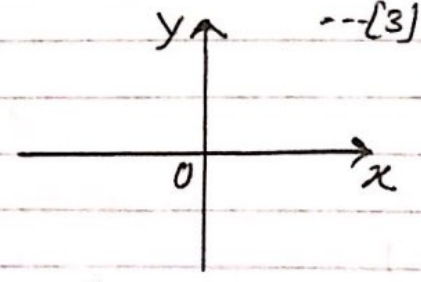
Q10 (i) on the axes, sketch the graphs of $y = 2-x$ and $y = |3+2x|$ [4]
 (ii) Solve $|3+2x| = 2-x$ [3]
 [M-16/12/Q4]



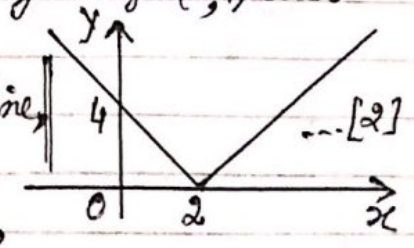
Q11 Find the value of x for which $(x-4)(x+2) > 7$ [S-16/21/Q1] [3]

Q12 Solve the equation $|2x-3| = x$ [W-16/21/Q1] --- [3]

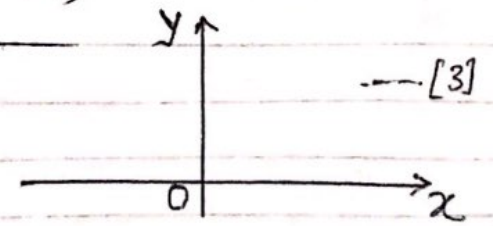
Q13 (i) on the axes below sketch the graph of $y = |4-5x|$, stating the coordinates of the points where the graph meets the coordinate axes. [3]
 (ii) Solve $|4-5x| = 9$ [3]



Q14(a) The diagram shows the graph of $y = |f(x)|$ passing through $(0,4)$ and touching the x -axis at $(2,0)$. Given that the graph of $y = f(x)$ is a straight line, write down the two possible expressions for $f(x)$. [2]



(b) on the axes, sketch the graph of $y = e^{-x} + 3$, starting the coordinates of any point of intersection with the coordinate axes. [3]



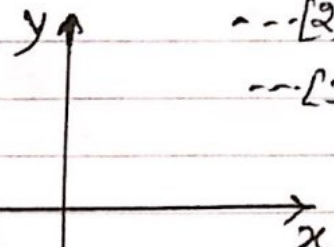
[S-15/21/Q2]

Q15 Find the set of values of x for which $x(x+2) < x$... [3]

S-14/21/Q1

Q16 (i) On the axes below, sketch the graph of $y = |(x-4)(x+2)|$ showing the coordinates of the points where the curve meets the x -axis. ... [2]

(ii) Find the set of values of k , for which $|(x-4)(x+2)| = k$ has four solutions. ... [3]



S-14/21/Q3

Q17 (i) Sketch the graph of $y = |(2x+1)(x-2)|$ for $-2 \leq x \leq 3$, showing the coordinates of the points where the curve meets the x - and y -axes. ... [3]

S-14/12/Q3

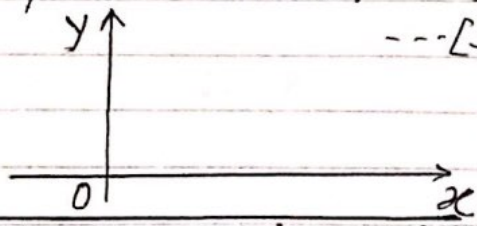
(ii) Find the non-zero values of k for which the equation $|(2x+1)(x-2)| = k$ has two solutions only. ... [2]

Q18 Solve the inequality $9x^2 + 2x - 1 < (x+1)^2$... [3]

W-14/21/Q2

Q19 (i) Sketch the graph of $y = |4x-2|$ on the axes below, showing the coordinates of the points where the graph meets the axes. ... [3]

(ii) Solve the equation $|4x-2| = x$... [3]



S-13/21/Q4

Q20 (a) The graph of $y = k(3^x) + c$ passes through the points $(0, 14)$ and $(-2, 6)$. Find the value of k and of c [3]

(b) The variables x and y are connected by the equation,
 $y = e^x + 25 - 24e^{-x}$

(i) Find the value of y when $x = 4$... [1]

(ii) Find the value of e^x when $y = 20$ and hence find the corresponding value of x [4]

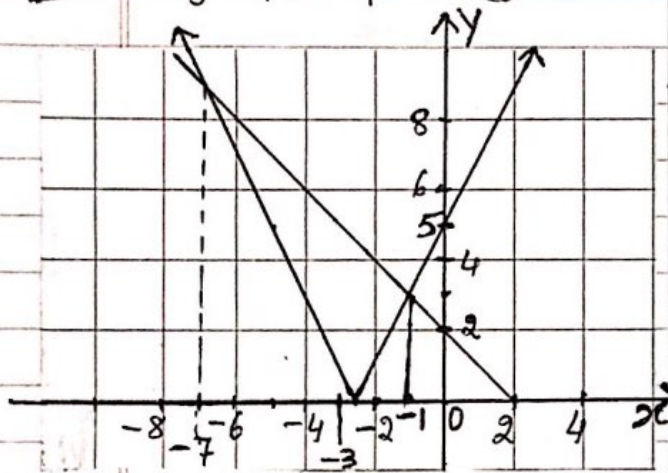
S-13/22/Q9

Q2) Find the set of values of x for which $x^2 < 6 - 5x$ --- [3]

W-13/21/Q1

Answers

Q1(a) $y = |2x+5|$ --- ①
 $y = |2-x|$ --- ②

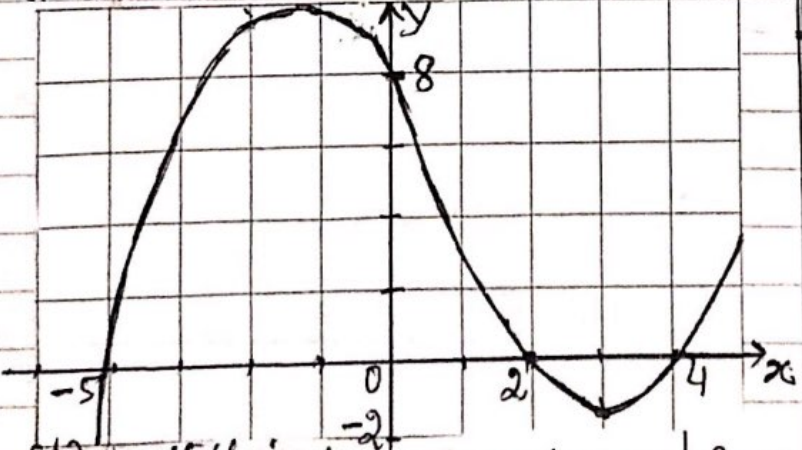


graph ① Intersect $\begin{cases} x\text{-axis } x = -2.5 \\ y\text{-axis } y = 5 \end{cases}$

graph ② Intersect $\begin{cases} x\text{-axis } x = 2 \\ y\text{-axis } y = 2 \end{cases}$

(b) Solve $|2x+5| \leq |2-x|$
or $(2x+5)^2 \leq (2-x)^2$
 $(x+1)(x+7) \leq 0$
critical points $-1, -7$
 $\Rightarrow -7 \leq x \leq -1$ ✓

Q2(a) $y = \frac{1}{5}(x-2)(x-4)(x+3)$

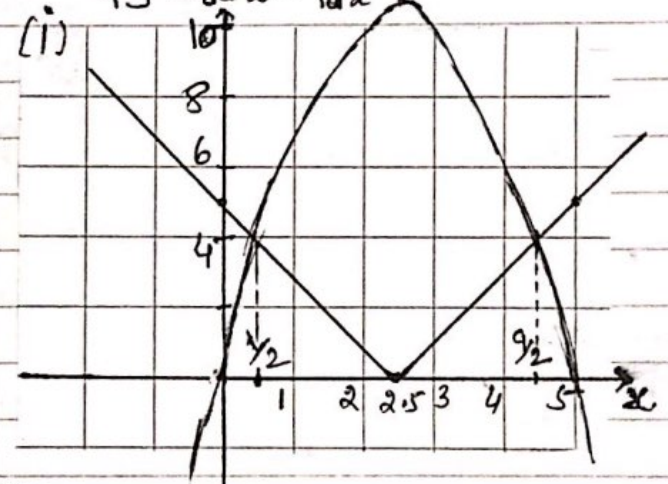


(b) multiplying by 5 does not change the value of x

Q2(c) $x \leq -5$; $2 \leq x \leq 4$

Q3 $|5-3x| = 10$
 $\Rightarrow (5-3x)^2 = 100 \Rightarrow (x-5)(3x+5) = 0$
 $\Rightarrow x = 5, -5/3$ ✓

Q4 $y = |2x-5|$ --- ①
 $9y = 80x - 16x^2$



(ii) solve $|2x-5| = 4$
 $\Rightarrow 2x-5 = \pm 4 \Rightarrow x = \frac{1}{2}$; $\frac{9}{2}$ ✓

(iii) $16(\frac{1}{2})^2 - 80(\frac{1}{2}) = 4$
and $16(\frac{9}{2})^2 - 80(\frac{9}{2}) = 4$ ✓

(iv) using the graph and values in (iii)
 $\frac{1}{2} \leq x \leq \frac{9}{2}$

Q5. $5x+3 = |-3x|$ or
 $5x+3 = 3x-1$
or $x = -2$; $x = -0.25$ ✓

Q6 $x^2 - 6x - 7 > 0$
 $(x-7)(x+1) > 0$
critical points 7 and -1
 $x < -1$ and $x > 7$

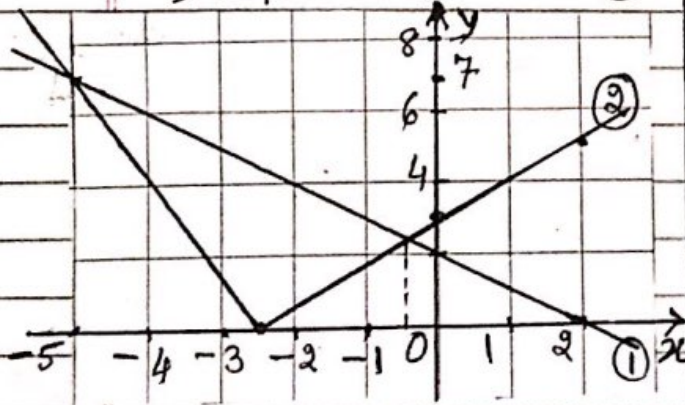
Answers

Q7 $2x^{1.5} + 6x^{-0.5} = x(x^{0.5} + 5x^{-0.5})$
 $\Rightarrow x^{1.5} + 6x^{-0.5} - 5x^{0.5} = 0$
 $\Rightarrow x^2 - 5x + 6 = 0$
 $(x-3)(x-2) = 0 \Rightarrow x = 3, 2 \checkmark$

Q8 $|3x-1| > 3+x$
 or $(3x-1)^2 > (3+x)^2$
 or $2x^2 - 3x - 2 > 0$
 $(x-2)(2x+1) > 0$
 critical points $2, -\frac{1}{2}$
 $\therefore x < -\frac{1}{2}, x > 2$

Q9 $3x-1=5+x$ or $3x-1=-5-x$
 $\therefore x = 3$ or $x = -1 \checkmark$

Q10 (i) $y = 2 - x$ ----- (1)
 $y = |3 + 2x|$ ----- (2)



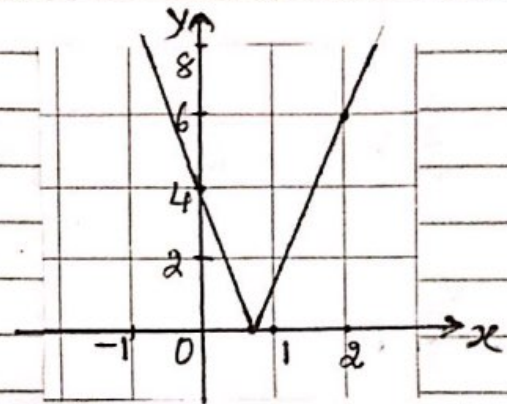
(ii) $2-x = 3+2x$ or $2-x = -3-2x$
 $\Rightarrow x = -\frac{1}{3}$ or $x = -5 \checkmark$

Q11 $x^2 - 2x - 15 > 0$
 $(x-5)(x+3) > 0$
 critical points $-3, 5$

$\therefore x < -3$ and $x > 5$

Q12 $4x-3=x$ or $4x-3=-x$
 $\therefore x = 1$ or $x = 0.6 \checkmark$

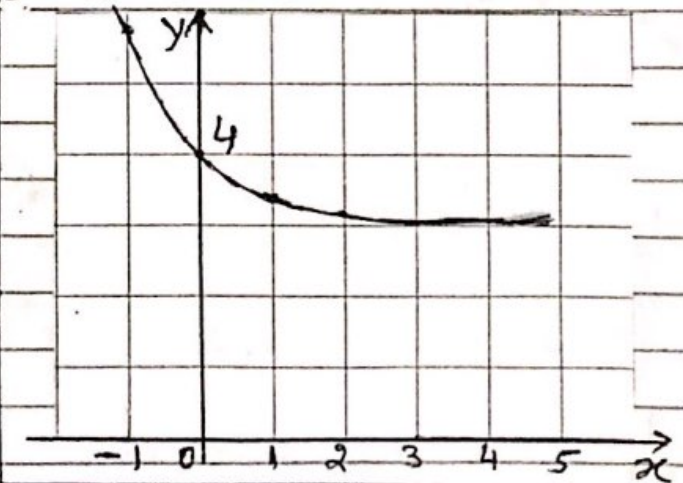
Q13 (i) $y = |4-5x|$



(ii) $|4-5x| = \pm 9$
 $x = -1, x = \frac{13}{5} \checkmark$

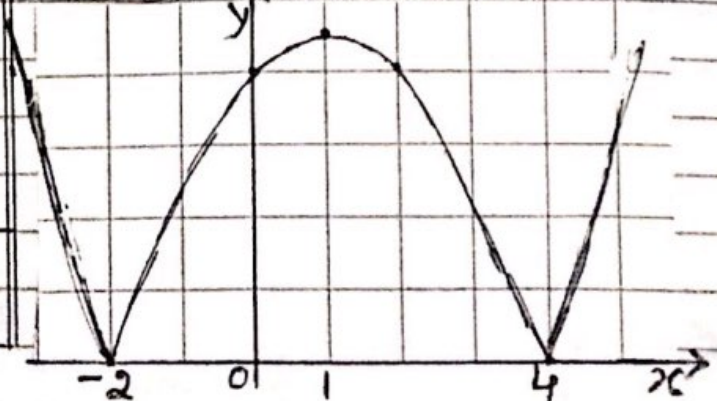
Q14 (a) $f(x) = 2x-4$ or $f(x) = 4-2x \checkmark$

(b) $y = e^{-x} + 3$



Q15 $x^2 + x < 0$ or $x(x+1) < 0$
 critical points $0, -1$
 $\therefore -1 < x < 0$

Q16 (i) $y = |(x-4)(x+2)|$

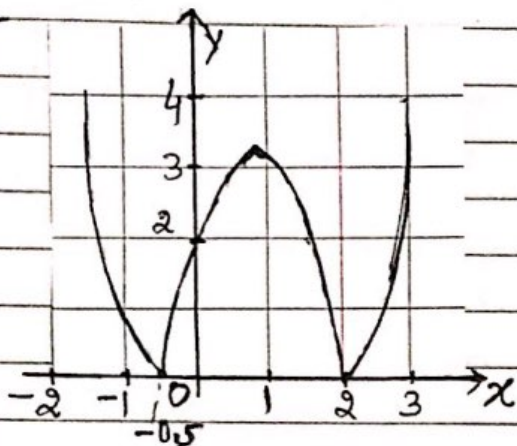


Q16(ii) $0 < k < 9$

Answers

Q17

$$y = |(2x+1)(x-2)|$$



Maximum at $y = \frac{35}{8}$

$$\therefore k > \frac{35}{8} \checkmark$$

Q18 $9x^2 + 2x - 1 < (x+1)^2$

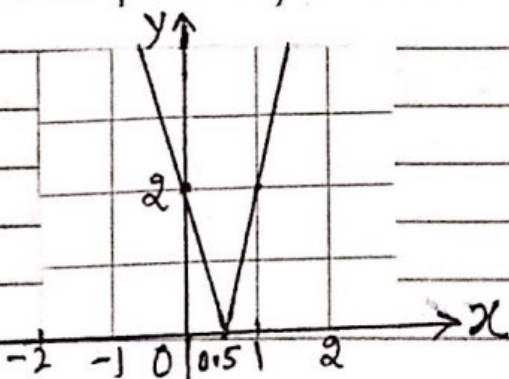
$$\Rightarrow 4x^2 - 1 < 0$$

$$\Rightarrow (2x-1)(2x+1) < 0$$

Critical points $x = \frac{1}{2}, -\frac{1}{2}$

$$\therefore -\frac{1}{2} < x < \frac{1}{2}$$

Q19(i) $y = |4x - 2|$



(ii) $4x - 2 = x$ or $4x - 2 = -x$

$$x = \frac{2}{3} \text{ or } x = \frac{2}{5} \checkmark$$

Q20, $14 = k + c$ and $6 = \frac{k}{9} + c$

$$\Rightarrow c = 5, k = 9$$

(b)(i) 79.2

(ii) $e^{2x} + 5e^x - 24 = 0$

$$(e^x)^2 + 5e^x - 24 = 0$$

$$e^x = 3 \quad \text{or } e^x = -8$$

$$x = \ln 3 \quad \text{or } |0| \checkmark$$

Q21 $(x+6)(x-1) < 0$

critical values -6 and 1

$$\therefore -6 < x < 1$$

