IG-Math
0606
Addional Maths

Factors of Polynomials
Exercise

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Factors of Polynomials

P-1

Q1 Do not use calculator in this question.
The polynomial $p(x) = 2x^3 - 3x^2 + qx + 56$ has a factor $(x-2)$.

(a) Show that $q = -30$ --- [1]

(b) Factorise $p(x)$ completely and hence state all the solution of $p(x) = 0$ --- [4]

SP-20/01/Q1

Q2 The remainder obtained when the polynomial, --- [5]
$p(x) = x^3 + ax^2 - 3x + b$ is divided by $x+3$ is twice the remainder obtained when $p(x)$ is divided by $x-2$. Given also that $p(x)$ is divisible by $(x+1)$. Find the value of a and of b.

M-18/12/Q1

Q3 The polynomial $p(x)$ is $x^4 - 2x^3 - 3x^2 + 8x - 4$

(i) Show that $p(x)$ can be written as $(x-1)(x^3 - x^2 - 4x + 4)$ --- [1]

(ii) Hence write $p(x)$ as a product of its linear factors, showing all your working. --- [4]

M-17/22/Q3

Q4 It is given that $p(x) = x^3 + ax^2 + bx - 48$. When $p(x)$ is divided by $(x-3)$ the remainder is 6. Given that $p'(1) = 0$, find the value of a and b. --- [5]

S-17/11/Q2

Q5 Without using a calculator:
factorise the expression, $10x^3 - 21x^2 + 4$ --- [5]

S-17/22/Q3

Q6 It is given that $p(x) = 2x^3 + ax^2 + 4x + b$, where a and b are constants. It is given also that $(2x+1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $x-1$ there is a remainder of -12.

(i) Find the value of a and of b. --- [5]

(ii) Using your value of a and b, write $p(x)$ in the form $(2x+1)q(x)$, where $q(x)$ is a quadratic expression. --- [2]

(iii) Hence find the exact solutions of the equation $p(x) = 0$ --- [2]

S-17/13/Q8

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Q 7 The polynomial $p(x)$ is $ax^3+bx^2-13x+4$, where a and b are integers. Given that $(2x-1)$ is a factor of $p(x)$ and also a factor of $p'(x)$,

(i) find the value of a and of b. W-17/11/Q2 ---[5]

Using your values of a and b,

(ii) Find the remainder when $p(x)$ is divided by $x+1$ ---[2]

Q8 A polynomial $p(x)$ is ax^3+8x^2+bx+5, where a and b are integers. It is given that $2x-1$ is a factor of $p(x)$ and that a remainder of -25 is obtained when $p(x)$ is divided by $x+2$.

(i) Find the value of a and of b. ---[5]

(ii) Using your values of a and b, find the exact solutions of $p(x)=5$ W-17/12/Q7 ---[2]

Q9 (i) Without using a calculator,

Solve the equation: $6c^3-7c^2+1=0$ ---[5]

It is given that $y=\tan x+6\sin x$

(ii) find $\frac{dy}{dx}$ ---[2]

(iii) if $\frac{dy}{dx}=7$, show that $6\cos^3 x-7\cos^2 x+1=0$ ---[2]

(iv) Hence solve the equation $\frac{dy}{dx}=7$ for $0\leq x\leq\pi$ radians ---[2]

W-17/22/Q10

Q10. The cubic equation $x^3+ax^2+bx-36=0$ has a repeated positive integral root.

(i) If the repeated root is $x=3$ find the other positive root and the value of a and of b. ---[4]

(ii) There is other possible values of a and b for which the cubic equation has a repeated positive integral root. In each case state all three integral roots of the equation. ---[4]

W-17/23/Q11

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Q11 The polynomial $f(x) = ax^3 + 7x^2 - 9x + b$ is divisible by $(2x-1)$. The remainder when $f(x)$ is divided by $(x-2)$ is 5 times the remainder when $f(x)$ is divided by $x+1$.

(i) Show that $a = 6$ and find the value of b. --[4]

(ii) Using the values from (i), show that $f(x) = (2x-1)(cx^2 + dx + e)$, where c, d and e are integers to be found. --[2]

(iii) Hence factorize $f(x)$ completely. [M-16/12/Q7] --[2]

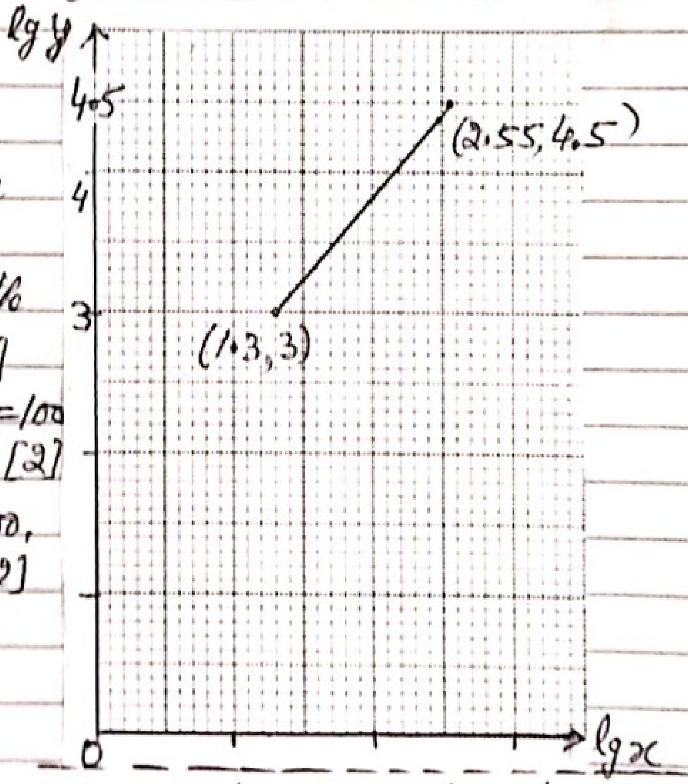
X **Q12** The variables x and y are such that when $\lg y$ is plotted against $\lg x$ the straight line graph shown is obtained.

(i) Given that $y = Ax^b$, find the value of A and b. --[5]

(ii) Find the value of $\lg y$ when $x = 100$ --[2]

(iii) Find the value of x when $y = 8000$. --[2]

[M-16/12/Q8]



Q13 (i) Given that $f(x) = 4x^3 + kx + p$ is exactly divisible by $(x+2)$ and $f'(x)$ is exactly divisible by $2x-1$, find the value of k and p. --[4]

(ii) Using the values of k and p found in part (i), show that, $f(x) = (x+2)(ax^2 + bx + c)$, where a, b and c are integers to be found. --[2]

(iii) Hence show that $f(x) = 0$ has only one solution and state this solution. [S-16/11/Q10] --[2]

Q14 The polynomial $p(x) = 2x^3 - 3x^2 + qx + 56$ has a factor $x-2$.

(i) Show that $q = -30$ [S-16/22/Q4] --[1]

(ii) Factorise $p(x)$ and hence state all solutions of $p(x) = 0$ --[4]

Factors of Polynomials

Q15 Do not use a calculator in this question.
The polynomial $p(x)$ is $ax^3 - 4x^2 + bx + 18$. It is given that $p(x)$ and $p'(x)$ are both divisible by $2x-3$.

(i) Show that $a = 4$ and find the value of b. --[4]

(ii) Using the values of a and b from part (i), factorise $p(x)$. --[2]

(iii) Hence find the values of x for which $p(x) = (x+2)$ --[3]

W-16/11/Q9

Q16 The cubic given by $p(x) = x^3 + ax^2 + bx - 24$ is divisible by $(x-2)$. When $p(x)$ divided by $x-1$ the remainder is -20.

(i) Form a pair of equations in a and b and solve them to find the value of a and of b. ---[4]

(ii) Factorise $p(x)$ completely and hence solve $p(x) = 0$ ---[4]

W-16/23/Q4

Q17 The polynomial $p(x) = ax^3 + bx^2 - 3x - 4$ has a factor of $2x-1$ and leaves a remainder of -10 when divided by $x+2$

(i) Show that $a = 10$ and find the value of b. ---[4]

(ii) Given that $p(x) = (2x-1)(rx^2 + sx + t)$, find the values of each of the integers r, s, and t. --[2]

(iii) Hence find the exact solutions of $p(x) = 0$ --[3]

M-15/12/Q7

Q18 The polynomial $f(x) = ax^3 - 15x^2 + bx - 2$ has a factor of $2x-1$ and a remainder of 5 when divided by $x-1$.

(i) Show that $b = 8$ and find the value of a. ---[4]

(ii) Using the values of a and b from part (i), express $f(x)$ in the form $(2x-1)g(x)$, where $g(x)$ is quadratic factor to be found. --[2]

S-15/11/Q6

(iii) Show that the equation $f(x) = 0$ has only one real root. --[2]

Q19 (i) Show that $x = -2$ is a root of the polynomial equation.

$$15x^3 + 26x^2 - 11x - 6 = 0$$ --[1]

(ii) Find the remainder when $15x^3 + 26x^2 - 11x - 6$ is divided by $x-3$. --[2]

(iii) Find the value of p and of q such that $15x^3 + 26x^2 - 11x - 6$ is a factor of $15x^4 + px^3 - 37x^2 + qx + 6$. --[4]

S-15/22/Q12

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Q20 It is given that $f(x) = 4x^3 - 4x^2 - 15x + 18$

(i) Show that $x+2$ is a factor of $f(x)$ --- [1]

(ii) Hence factorise $f(x)$ completely and solve the equation $f(x) = 0$ --- [4]

W-15/21/Q1

Q21) The roots of the equation $x^3 + ax^2 + bx + c = 0$ are 1, 3 and 3. Show that $c = -9$ and find the value of a and of b. --- [4]

W-15/23/Q5

Q22 (i) Find in terms of p, the remainder when, $x^3 + px^2 + p^2x + 21$ is divided by $x+3$. --- [2]

S-14/13/Q3

(ii) Hence find the set of values of p for which this remainder is negative. --- [3]

Q23 The expression $2x^3 + ax^2 + bx + 12$ has a factor $x-4$ and leaves a remainder of -12 when divided by $x-1$. Find the value of each of the constants a and b. --- [5]

S-14/21/Q4

Q24 (i) Given that $(x+1)$ is a factor of $3x^3 - 14x^2 - 7x + d$, show that $d = 10$ --- [1]

(ii) Show that $3x^3 - 14x^2 - 7x + 10$ can be written in the form $(x+1)(ax^2 + bx + c)$, where a, b and c are constants to be found. --- [2]

(iii) Hence solve the equation $3x^3 - 14x^2 - 7x + 10 = 0$ --- [2]

S-14/22/Q3

Q25 The expression $f(x) = 3x^3 + 8x^2 - 33x + p$ has a factor of $x-2$.

(i) Show that $p = 10$ and express $f(x)$ as a product of a linear factor and a quadratic factor. --- [4]

(ii) Hence solve the equation $f(x) = 0$ --- [2]

W-14/23/Q1

Q26 The function $f(x) = x^3 + x^2 + ax + b$ is divisible by $x-3$ and leaves a remainder of 20 when divided by $x+1$

(i) Show that $b = 6$ and find the value of a. --- [4]

(ii) Using your value of a and taking b as 6, find the non-integer roots of the equation $f(x) = 0$ in the form $p \pm \sqrt{q}$, where p and q are integers. --- [5]

S-13/21/Q12

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Q27 It is given that $f(x) = 6x^3 - 5x^2 + ax + b$ has a factor of $(x+2)$ and leaves a remainder of 27 when divided by $(x-1)$.

(i) Show that $b = 40$ and find the value of a. --- [4]

(ii) Show that $f(x) = (x+2)(px^2 + qx + r)$, where p, q and r are integers to be found. --- [2]

(iii) Hence solve $f(x) = 0$ $\boxed{S-13/12/Q7}$ --- [2]

Q28 The function $f(x) = ax^3 + 4x^2 + bx - 2$, where a and b are constants, is such that $2x-1$ is a factor. Given that the remainder when $f(x)$ is divided by $x-2$ is twice the remainder when $f(x)$ is divided by $x+1$, find the value of a and of b. $\boxed{W-13/11/Q6}$ --- [6]

Q29 The expression $2x^3 + ax^2 + bx + 21$ has a factor $(x+3)$ and leaves a remainder of 65 when divided by $x-2$.

(i) Find the value of a and of b. --- [5]

(ii) Hence find the value of the remainder when the expression is divided by $2x+1$ $\boxed{W-13/23/Q6}$ --- [2]

Factors of Polynomials.

Answers

Q1 (a) $2(2)^3 - 3(2)^2 + 2q + 56 = 0$

$\Rightarrow q = -30$

(b) $(x-2)(2x^2 + x - 28)$

or $p(x) = (x-2)(2x-7)(x+4) = 0$

$x = 2,\ x = -4$ and $x = 3.5$

Q2 Given $p(-3) = 2p(2)$

$\Rightarrow a - b = 22$ --- (i)

also given $p(-1) = 0$

or $a + b = -2$ --- (ii)

Solving (1) & (2) $a = 10,\ b = -12$

Q3 (i) By long Division

$p(x) = (x-1)(x^3 - x^2 - 4x + 4)$

(ii) $p(x) = (x-1)(x-1)(x^2 - 4)$

$= (x-1)^2 \cdot (x+2)(x-2)$

Q4. Given $p(3) = 6$

$\Rightarrow 3a + b = 9$ --- (i)

$p'(x) = 3x^2 + 2ax + b$

$p'(1) = 3 + 2a + b = 0$

or $2a + b = -3$ --- (ii)

Solving (i) and (ii) $a = 12,\ b = -27$

Q5 Check $p(2) = 0$

$\therefore (x-2)$ is factor of $p(x)$

$p(x) = (x-2)(10x^2 - x - 2)$

$= (x-2)(2x-1)(5x+2)$

Q6 Given $p(-\frac{1}{2}) = 0$

$\Rightarrow a + 4b = 9$ --- (i)

also given $p(1) = -12$

$\Rightarrow a + b = -18$ --- (ii)

(i) Solving (1) & (ii) $a = -27,\ b = 9$

(ii) $p(x) = (2x+1)(x^2 - 14x + 9)$

(iii) Solving $p(x) = 0$

$x = -\frac{1}{2}$ and $x = 7 \pm 2\sqrt{10}$

Q7. Given $p(\frac{1}{2}) = 0$

$\Rightarrow a + 2b = 20$ --- (i)

and $p'(x) = 3ax^2 + 2bx - 13$

and $p'(\frac{1}{2}) = 0$

$\Rightarrow 3a + 4b = 52$ --- (ii)

(i) Solving (1) and (ii) $a = 12,\ b = 4$

(ii) $r = P(-1) = 9$

Q8. Given $P(\frac{1}{2}) = 0$

$\Rightarrow a + 4b = -56$ --- (i)

and $P(-2) = -25$

$\Rightarrow 4a + b = 31$ --- (ii)

(i) Solving (i) and (ii) $a = 12;\ b = -17$

(ii) $p(x) = 5$

$\Rightarrow 12x^3 + 8x^2 - 17x = 0$

$\Rightarrow x = 0,\ x = -\frac{1}{3} \pm \frac{\sqrt{55}}{6}$

Q9 Let $P(c) = 6c^3 - 7c^2 + 1$

$P(1) = 0 \Rightarrow (c-1)$ is a factor

$P(c) = (c-1)(6c^2 - c - 1) = 0$

$(c-1)(2c-1)(3c+1) = 0$

$c = 1,\ \frac{1}{2},\ -\frac{1}{3}$

Q10 (i) Let the other positive root is 'p'

$(x-3)^2(x-p) = x^3 + ax^2 + bx - 36$

Comparing the constant term.

$-9p = -36 \Rightarrow p = 4$ the root

$\therefore f(x) = (x-3)^2(x-4)$

$= x^3 - 10x^2 + 33x - 36$

$\therefore a = -10,\ b = 33$

(ii) $x = 6,\ x = 6,\ x = 1$

$x = 2,\ x = 2,\ x = 9$

$x = 1,\ x = 1,\ x = 36$

$\because 36 = 3^2 \times 2^2 \times 1^2$ or $6^2 \times 1^2$

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Answers

Q11

$f(\frac{1}{2}) = 0$

$\Rightarrow a + 8b = 22$ —— (i)

and $f(2) = 5f(-1)$

$\Rightarrow 13a - 4b = 70$ —— (ii)

(i) Solving (i) and (ii) $a = 6, b = 2$ ✓

(ii) $f(x) = (2x-1)(3x^2+5x-2)$ ✓

(iii) $(2x-1)(3x-1)(x+2)$ ✓

Q12 ✗ See in Straight line graph. exercise.

Q13 (i) $f(-2) = 0 \Rightarrow -2k + p = 32$ -- (1)

and $f'(\frac{1}{2}) = 0 \Rightarrow 3 + k = 0$ — (2)

from (1) & (2) $k = -3$; $p = 26$ ✓

(ii) $f(x) = (x+2)(4x^2 - 8x + 13)$

$x + 2 = 0 \Rightarrow x = -2$ is a root

for $4x^2 - 8x + 13 = 0$

$b^2 - 4ac = 64 - 4 \times 4 \times 13 < 0$

No real root.

$\therefore x = -2$ is the only root.

Q14 $p(2) = 0 \Rightarrow q = -30$

Now $p(x) = 2x^3 - 3x^2 - 30x + 56$

$p(x) = (x-2)(2x^2 + x - 28)$

or $(x-2)(2x-7)(x+4)$

$\therefore$ roots of $p(x) = 0$

are $2, -4$ and $x = 3.5$ ✓

Q15 (i) $p(\frac{3}{2}) = 0 \Rightarrow 9a + 4b + 24 = 0$ - (i)

and $p'(3/2) = 0 \Rightarrow 27a + 4b - 48 = 0$ - (ii)

Solving (i) and (ii) $a = 4, b = -15$ ✓

(ii) $(x+2)(2x-3)^2$

(iii) $(x+2)(2x-3)^2 = x + 2$

$\Rightarrow x = -2, x = 1$ and $x = 2$ ✓

Q16 (i) $p(2) = 0 \Rightarrow 4a + 2b = 16$ -- (i)

and $p(1) = -20 \Rightarrow a + b = 3$ — (ii)

Solving (i) and (ii) $a = 5$; $b = -2$ ✓

(ii) $p(x) = (x-2)(x+3)(x+4)$

$p(x) = 0 \Rightarrow x = 2, -3$ and -4 ✓

Q17 (i) $p(\frac{1}{2}) = 0 \Rightarrow a + 2b = 44$ -- (i)

$p(-2) = -10 \Rightarrow 2a - b = 3$ -- (ii)

Solving (i) and (ii) $a = 10, b = 17$ ✓

(ii) $p(x) = (2x-1)(5x^2 + 11x + 4)$

(iii) $x = \frac{1}{2}$ and $x = \frac{-11 \pm \sqrt{41}}{10}$ ✓

Q18 (i) $f(\frac{1}{2}) = 0 \Rightarrow a + 4b = 46$ -- (i)

$f(1) = 5 \Rightarrow a + b = 22$ — (ii)

Solving (i) & (ii) $a = 14, b = 8$ ✓

(ii) $(2x-1)(7x^2 - 4x + 2)$

(iii) $7x^2 - 4x + 2 = 0$ has no real root

as $b^2 - 4ac = 16 - 56 < 0$

Only real root is $\frac{1}{2}$ ✓

Q19 (i) Let $P(x) = 15x^3 + 26x^2 - 11x - 6$

Show that $P(-2) = 0 \therefore -2$ is a root ✓

(ii) $R = 600$ ✓

(iii) Observe that $(x-1)$ is a factor of

$15x^4 + px^3 - 37x^2 + qx + 6$

$= (x-1)(15x^3 + 26x^2 - 11x - 6)$

multiply the R.H.S and comparing the Coeff. on both sides, we get

$p = 11$ and $q = 5$ ✓

Q20 (i) Show $f(-2) = 0$

(ii) $f(x) = (x+2)(4x^2 - 12x + 9)$

$= (x+2)(2x-3)(2x-3)$

$\therefore f(x) = 0 \Rightarrow x = -2, 1.5$ ✓

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Answers

Q21 $(x-3)(x-3)(x-1)=0$

$x^3-7x^2+15x-9=0$

$a=-7,\ b=15,\ c=-9$ ✓

Q22 (i) $f(-3)=9p-3p^2-6$ ✓

(ii) $9p-3p^2-6<0$

or $(p-1)(p-2)>0$

$\Rightarrow p<1$ or $p>2$ ✓

Q23 $f(4)=0 \Rightarrow 16a+4b=0$ -- (i)

and $f(1)=-12 \Rightarrow a+b=-26$ -- (ii)

Solving (1) and (2) $a=-3,\ b=-23$ ✓

Q24 (i) $f(-1)=0 \Rightarrow d=10$ ✓

(ii) $(x+1)(3x^2-17x+10)$ ✓

(iii) $(x+1)(x-5)(3x-2)=0$

$\therefore x=-1,\ 5$ and $2/3$ ✓

Q25 (i) $f(2)=0 \Rightarrow p=10$

$f(x)=(x-2)(3x^2+14x-5)$

(ii) $f(x)=0 \Rightarrow (x-2)(3x-1)(x+5)=0$

$\Rightarrow x=2,\ -5$ and $\frac{1}{3}$ ✓

Q26 (i) $f(3)=0 \Rightarrow 3a+b=-36$ -- (i)

$f(-1)=20 \Rightarrow -a+b=20$ -- (ii)

Solving (i) and (ii)

$a=-14,\ b=6$ ✓

(ii) $f(x)=(x-3)(x^2-4x-2)=0$

Solving $x^2-4x-2=0$

$x=-2\pm\sqrt{6}$ ✓

Q27 (i) $f(-2)=0 \Rightarrow b-2a=68$ -- (1)

$f(1)=27 \Rightarrow a+b=26$ -- (2)

Solving (1) and (2) $a=-14,\ b=40$ ✓

(ii) $f(x)=(x+2)(6x^2-17x+20)$ ✓

(iii) $6x^2-17x+20=0$

$b^2-4ac=289-480<0$

No real root

$\therefore$ Only root is $x=-2$ ✓

Q28 $f(\frac{1}{2})=0 \Rightarrow a+4b=8$ -- (i)

and

$f(2)=2f(-1)$

$\Rightarrow 10a+4b=-10$ -- (ii)

Solving (1) & (2) $a=-2,\ b=\frac{5}{2}$ ✓

Q29 (i) $f(-3)=0 \Rightarrow 9a-3b=33$ -- (1)

and $f(2)=65 \Rightarrow 4a+2b=28$ -- (2)

Solving (1) and (2) $a=5,\ b=4$ ✓

(ii) $f(-\frac{1}{2})=20$ ✓

← X — X →