IG_0606 Additional Maths

Functions Exercise

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Functions

Q1 Functions g and h are such that:

$$g(x) = 2 + 4\ln x \quad \text{for } x > 0$$
$$h(x) = x^2 + 4 \quad \text{for } x > 0$$

(a) Find g^{-1}, stating its domain and its range. ---[4]

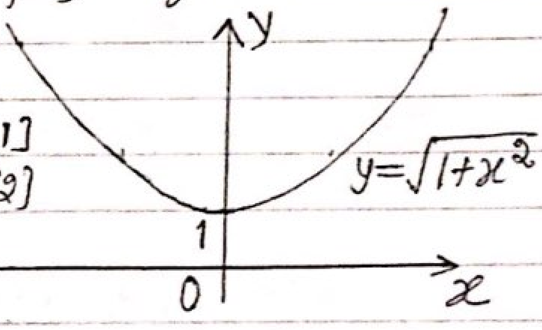
(b) Solve, $gh(x) = 10$ ---[3]

(c) Solve, $g'(x) = h'(x)$ SP-20/02/Q3 ---[3]

Q2 (a) The function f is defined by $f(x) = \sqrt{1+x^2}$, for all values of x. The graph of $y = f(x)$ is given below.

(i) Explain, with reference to the graph, why f does not have an inverse. ---[1]

(ii) Find $f^2(x)$ ---[2]

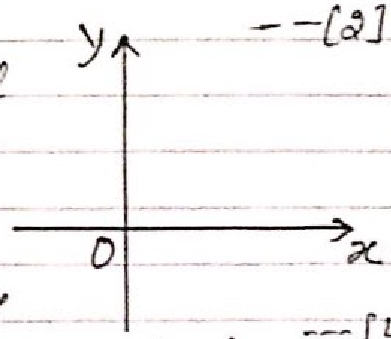


(b) The function g is defined, for $x > k$, by $g(x) = \sqrt{1+x^2}$ and g has an inverse.

(i) Write down a possible value for k. ---[1]

(ii) Find $g^{-1}(x)$ ---[2]

(c) The function h is defined, for all real values of x, by $h(x) = 4e^{x} + 2$. Sketch the graph of $y = h(x)$. Hence, on the same axes, sketch the graph of $y = h^{-1}(x)$. Give the coordinates of any point where your graph meet the coordinate axes. ---[4]



M-18/22/Q10

Q3 The functions f and g are defined by:

$$f(x) = \frac{x^2 - 2}{x} \quad \text{for } x \geq 2$$

$$g(x) = \frac{x^2 - 1}{2} \quad \text{for } x \geq 0$$

(i) State the range of g. ---[1]

(ii) Explain why $fg(1)$ does not exist. ---[2]

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IG-Maths 0606

Functions

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Q3 (iii) Show that $gf(x) = ax^2 + b + \frac{c}{x^2}$, where a, b and c are constants to be found. ---[3]

(iv) State the domain of gf ---[1]

(v) Show that: $f^{-1}(x) = \frac{x + \sqrt{x^2+8}}{2}$ ---[4]

M-17/22/Q11

Q4 (a) It is given that $f(x) = 3e^{-4x} + 5$ for $x \in R$

(i) State the range of f. ---[1]

(ii) Find f^{-1} and state its domain. ---[4]

(b) It is given that $g(x) = x^2 + 5$ and $h(x) = \ln x$ for $x > 0$.
Solve $hg(x) = 2$ ---[3]

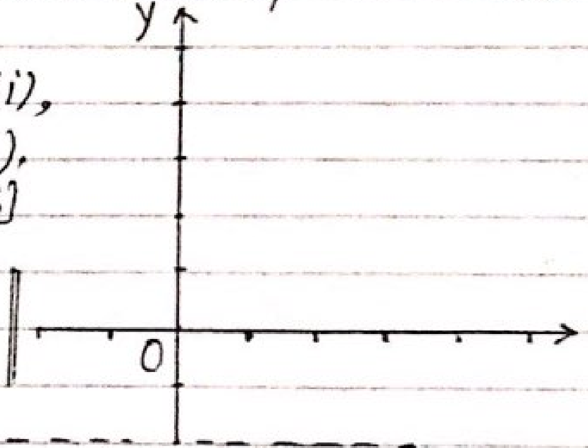
S-17/11/Q4

Q5 A function f is defined, for $x \leq \frac{3}{2}$, by $f(x) = 2x^2 - 6x + 5$.

(i) Express $f(x)$ in the form $a(x-b)^2 + c$ where a, b and c are constants. --[3]

(ii) On the same axis, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$, showing the geometrical relationship between them: --[3]

(iii) Using your answer from part (i), find an expression for $f^{-1}(x)$, stating its domain. ---[3]



S-17/22/Q9

Q6 The function g is defined, for $x > -\frac{1}{2}$, by $g(x) = \frac{3}{2x+1}$

(i) Show that $g'(x)$ is always negative. ---[2]

(ii) Write down the range of g. --[1]

The function h is defined, for all real x, by $h(x) = kx + 3$, where k is a constant.

(iii) Find an expression for $hg(x)$. --[1]

(iv) Given that $hg(0) = 5$, Find the value of k. --[2]

(v) State the domain of hg. --[1]

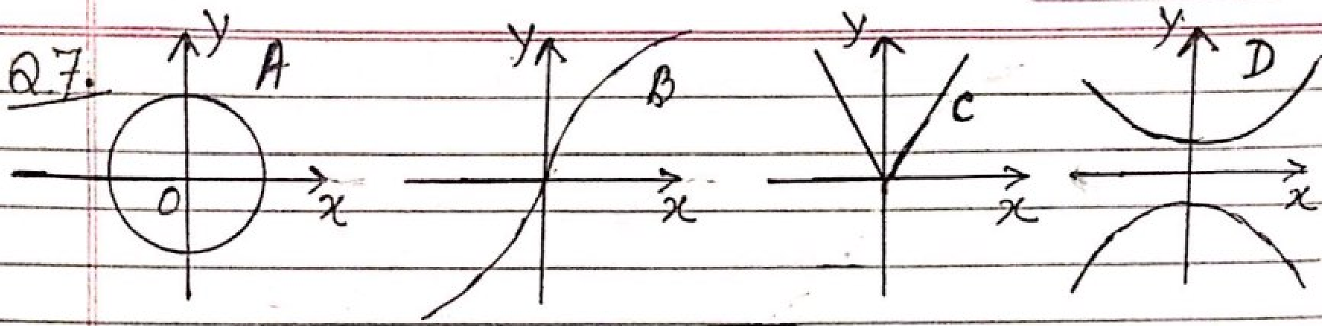
S-17/22/Q12

IG-Maths 0606

Functions

Date P-3
Page No.

Q7.



The four graphs above are labelled A, B, C and D.

(i) Write down the letter of each graph that represents a function, giving the reason for your choice. ---[2]

(ii) Write down the letter of each graph that represents a function which has an inverse, giving a reason for your choice. --[2]

S-17/23/Q2

Q8. The functions f and g are defined, for $x > 1$, by

$$f(x) = 9\sqrt{x-1} \quad \text{and} \quad g(x) = x^2 + 2$$

(i) Find an expression for $f^{-1}(x)$, stating its domain. ---[3]

(ii) Find the exact value of $fg(7)$ ---[2]

(iii) Solve $gf(x) = 5x^2 + 83x - 95$ --- S-17/23/Q9 --[4]

Q9. (a) Functions f and g are such that, for $x \in R$,

$$f(x) = x^2 + 3 \quad \text{and} \quad g(x) = 4x - 1$$

(i) State the range of f. ---[1]

(ii) Solve $fg(x) = 4$ --[3]

(b) A function h is such that $h(x) = \frac{2x+1}{x-4}$ for $x \in R$, $x \neq 4$.

(i) Find $h^{-1}(x)$ and state its range. --[4]

(ii) Find $h^2(x)$, giving your answer in its simplest form. --[3]

W-17/11/Q6

Q10. Functions f and g are defined, for $x > 0$, by

$$f(x) = \ln x \quad ; \quad g(x) = 2x^2 + 3$$

(i) Write down the range of f. ---[1]

(ii) Write down the range of g. --[1]

(iii) Find the exact value of $f^{-1}g(4)$ --[2]

(iv) Find $g^{-1}(x)$ and state its domain. --[3]

W-17/12/Q6

Functions

Q-11. The functions f and g are defined for real value of x by,

$$f(x) = (x+2)^2 + 1,$$

$$g(x) = \frac{x-2}{2x-1}, \quad x \neq \frac{1}{2}$$

(i) Find $f^2(-3)$ ---[2]

(ii) Show that $g^{-1}(x) = g(x)$ ---[3]

(iii) Solve: $gf(x) = \frac{8}{19}$ [W-17/23/Q6] ---[4]

Q12 A function f is such that $f(x) = 6 + e^{4x}$ for $x \in R$

(i) Write down the range of f. ---[1]

(ii) Find $f^{-1}(x)$ and state its domain and range. ---[4]

(iii) Find $f'(x)$. --[1]

(iv) hence find the exact solution of $f(x) = f'(x)$ --[2]

[M-16/12/Q6]

Q13 The function f is defined by $f(x) = 2 - \sqrt{x+5}$ for $-5 \leq x \leq 0$

(i) Write down the range of f. --[2]

(ii) Find $f^{-1}(x)$ and state its domain and range --[4]

The function g is defined by $g(x) = \frac{4}{x}$ for $-5 \leq x \leq -1$

(iii) Solve $fg(x) = 0$ [S-16/11/Q6] --[3]

Q14 (a) A function f is defined by, $f(x) = x - x^2$, for all real x, find the greatest value of $f(x)$ and the value of x for which this occures. ---[3]

(b) The domain of $g(x) = x - x^2$ is such that $g^{-1}(x)$ exists. Explain why $x \geq 1$ is a suitable domain for $g(x)$. ---[1]

(c) The functions h and k are defined by,

$$h : x \to \lg(x+2) \text{ for } x > -2$$

$$k : x \to 5 + \sqrt{x-1} \text{ for } 1 < x < 101$$

(i) Find $hk(10)$ ---[2]

(ii) Find $k^{-1}(x)$, stating its domain and range. --[5]

[S-16/22/Q11]

IG-Maths 0606

Functions

Date P-5

Page No.

Q15 The functions f and g are defined for $x>1$ by :

$f(x) = 2+\ln x$

$g(x) = 2e^{x}+3$

(i) Find $fg(x)$ ---[1]

(ii) Find $ff(x)$ ---[1]

(iii) Find $g^{-1}(x)$ ---[2]

(iv) Solve the equation $f(x)=4$ --[1]

(v) Solve the equation $gf(x)=20$ W-16/23/Q10 -- [4]

Q16.(a) A function f is such that $f(\theta)=\sin 2\theta$ for $0\leqslant\theta\leqslant\frac{\pi}{2}$

(i) Write down the range of f. --[1]

(ii) Write down a suitabe restricted domain for f such that f^{-1} exists. --[1]

(b) Functions g and h are such that

$g(x)=2+4\ln x$ for $x>0$

$h(x)=x^{2}+4$ for $x>0$

(i) Find g^{-1}, stating its domain and range. ---[4]

(ii) Solve $gh(x)=10$ ---[3]

(iii) Solve $g'(x)=h'(x)$ M-15/12/Q8 ---[3]

Q17 It is given that $f(x)=3e^{2x}$ for $x\geqslant 0$

$g(x)=(x+2)^{2}+5$ for $x\geqslant 0$

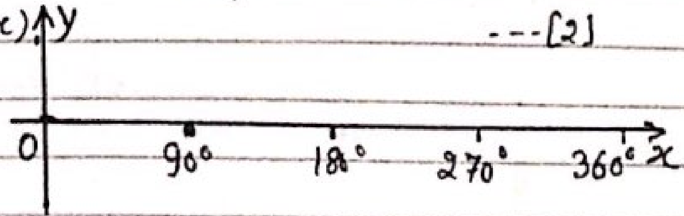
(i) Write down the range of f and of g ---[2]

(ii) Find g^{-1}, stating its domain ---[3]

(iii) Find the exact solution of $gf(x)=41$. ---[4]

(iv) Evaluate $f'(\ln 4)$ S-15/11/Q8 ---[2]

Q18(a) The function f is defined by $f: x\to|\sin x|$ for $0^{\circ}\leqslant x\leqslant 360^{\circ}$, on the axes, sketch the graph of $y=f(x)$. ---[2]



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Functions

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Q18 (b) The functions g and hg are defined, for $x \geq 1$, by

$$g(x) = \ln(4x-3),$$
$$hg(x) = x$$

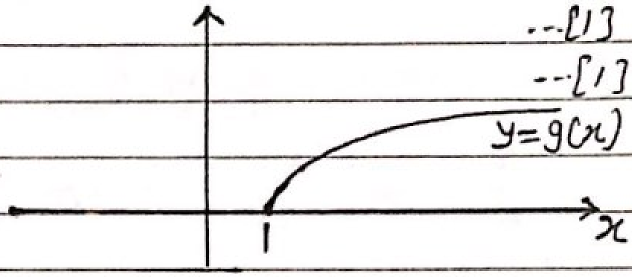
(i) Show that $h(x) = \dfrac{e^x+3}{4}$ ---[2]

(ii) The diagram shows the graph of $y=g(x)$. Given that g and h are inverse functions, sketch, on the same diagram, the graph of $y=h(x)$. Give the coordinates of any point where your graph meets the the coordinate axes. ---[2]

(iii) State the domain of h. ---[1]

(iv) State the range of h. ---[1]

$\boxed{S-15/22/Q10}$



Q19 (a) A function f is such that $f(x) = x^2+6x+4$ for $x \geq 0$

(i) Show that x^2+6x+4 can be written in the form, $(x+a)^2+b$, where a and b are integers. --[2]

(ii) Write down the range of f. ---[1]

(iii) Find $f^{-1}(x)$ and state its domain. ---[3]

(b) Functions g and h are such that, for $x \in R$,

$$g(x) = e^x \quad \text{and} \quad h(x) = 5x+2$$

Solve $h^2g(x) = 37$ $\boxed{W-15/11/Q11}$ ---[4]

Q20 Given that $f(x) = 3x^2+12x+2$

(i) find the value of a, b and c such that $f(x) = a(x+b)^2+c$ ---[3]

(ii) State the minimum value of $f(x)$ and the value of x at which it occurs. ---[2]

(iii) Solve $f(\frac{1}{y}) = 0$, giving each answer for y correct to 2 decimal places. $\boxed{W-15/23/Q9}$ ---[3]

Q21 The functions f and g are defined by.

$$f(x) = \frac{2x}{(x+1)} \text{ for } x>0$$
$$g(x) = \sqrt{x+1} \text{ for } x>-1$$

(i) Find $fg(8)$ ---[2]

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IG-Maths 0606

Functions

P-7

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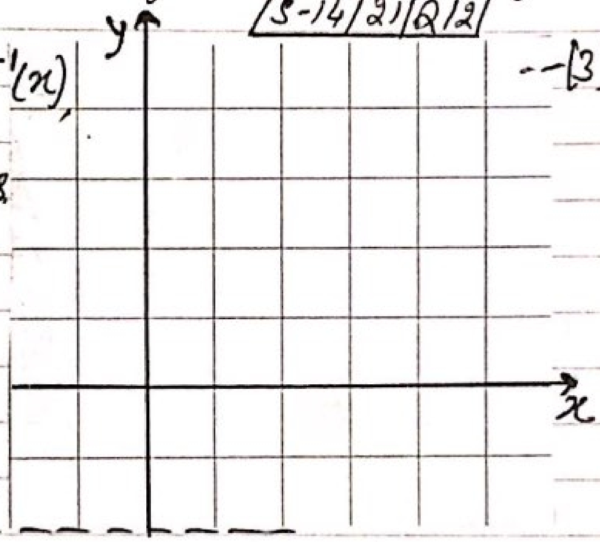
Q21 (ii) Find an expression for $f^2(x)$, giving your answer in the form $\frac{ax}{bx+c}$, where a, b and c are integers to be found. --- [3]

(iii) Find an expression for $g^{-1}(x)$, stating its domain and range. --- [4]

(iv) On the same axes, sketch the graph of $y = g(x)$ and $y = g^{-1}(x)$, indicating the geometrical relationship between the graphs. --- [3]

S-14/21/Q12



S-14/21/Q12

Q22 The functions f and g are defined, for real values of x greater than 2, by,

$$f(x) = 2^x - 1$$
$$g(x) = x(x+1)$$

(i) State the range of f. --- [1]

(ii) Find an expression for $f^{-1}(x)$, stating its domain and range. --- [4]

(iii) Find an expression for $gf(x)$ and explain why the equation $gf(x) = 0$ has no solution. --- [4]

S-14/22/Q11

Q23 The function f is such that $f(x) = 2 + \sqrt{x-3}$ for $4 \leq x \leq 28$

(i) Find the range of f. --- [2]

(ii) Find $f^2(12)$ --- [2]

(iii) Find an expression for $f^{-1}(x)$ --- [2]

The function g is defined by $g(x) = \frac{120}{x}$ for $x \geq 0$

(iv) Find the value of x for which $gf(x) = 20$ --- [3]

S-14/23/Q12

Functions

Q24 The functions f and g are defined for real values of x by.

$$f(x) = \sqrt{x-1} - 3 \quad \text{for } x > 1$$
$$g(x) = \frac{x-2}{2x-3} \quad \text{for } x > 2$$

(i) Find $gf(37)$ --- [2]

(ii) Find an expression for $f^{-1}(x)$ --- [2]

(iii) Find an expression for $g^{-1}(x)$ W-14/21/Q4 --- [2]

Q25 The functions f and g are defined for real values of x by

$$f(x) = \frac{2}{x} + 1 \quad \text{for } x > 1 \text{ and } g(x) = x^2 + 2$$

Find an expression for

(i) $f^{-1}(x)$ --- [2]

(ii) $gf(x)$ W-14/23/Q7 --- [2]

(iii) $fg(x)$ --- [2]

(iv) Show that $ff(x) = \dfrac{3x+2}{x+2}$ and solve $ff(x) = x$ --- [4]

Q26 A one-one function f is defined by $f(x) = (x-1)^2 - 5$ for $x \geq k$

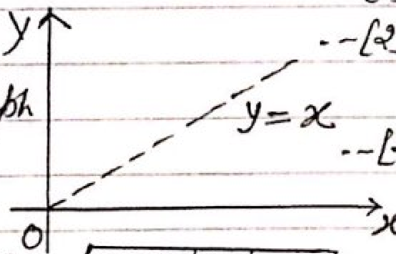
(i) State the least value that k can take. --- [1]

for this least value of k

(ii) Write down the range of f. --- [1]

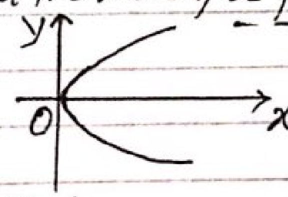
(iii) find $f^{-1}(x)$ --- [2]

(iv) sketch and label, on the axes, the graph of $y = f(x)$ and of $y = f^{-1}(x)$, --- [2]



(v) Find the value of x for which $f(x) = f^{-1}(x)$. S-13/21/Q11 --- [2]

Q27(a) A



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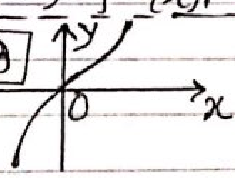

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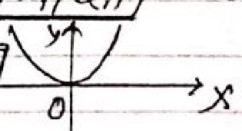

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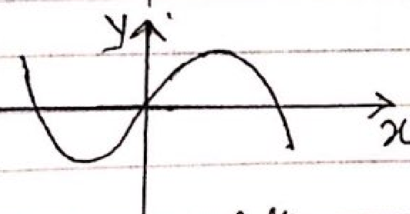

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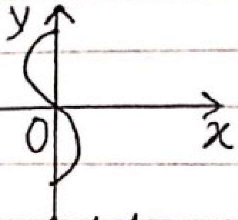

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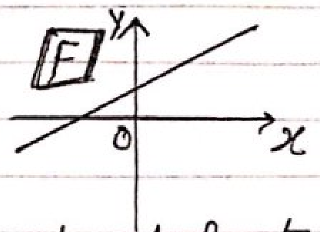

(i) Write down the letter of each graph which does **not** represent a function --- [2]

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Functions

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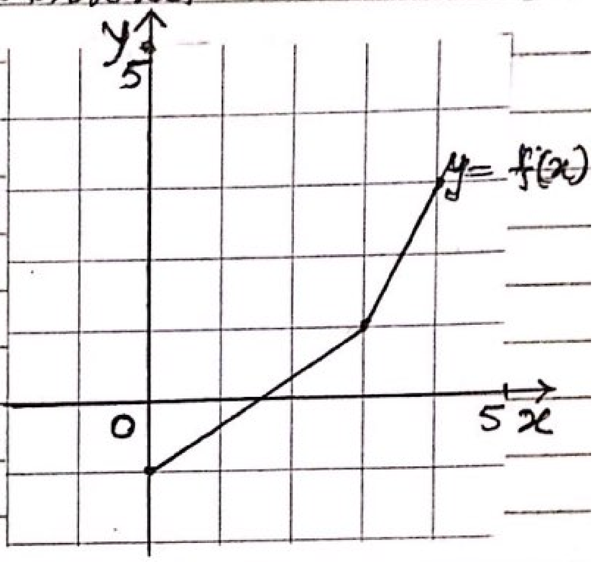
Q27. (ii) Write down the letter of each graph which represent a function that does not have an inverse. --[2]

(b) The diagram shows the graph of a function $y = f(x)$. On the same axes sketch the graph of $y = f^{-1}(x)$. --- [2]



S-13/22/Q3

Q28 (a) A function f is such that $f(x) = 3x^2 - 1$ for $-10 \leq x \leq 8$

(i) Find the range of f --- [3]

(ii) Write down a suitable domain for f for which f^{-1} exists. --[1]

(b) Functions g and h are defined by,

$g(x) = 4e^x - 2$ for $x \in R$

$h(x) = \ln 5x$ for $x > 0$

(i) Find $g^{-1}(x)$ --- [2]

(ii) Solve $gh(x) = 18$ --- [3]

W-13/11/Q12

Q29 For $x \in R$, the functions f and g are defined by.

$f(x) = 2x^3$ and $g(x) = 4x - 5x^2$

(i) Express $f^2\left(\frac{1}{2}\right)$ as a power of 2. --- [2]

(ii) Find the value of x for which f and g are increasing at the same rate with respect to x. --- [4]

W-13/13/Q5

IG-Maths 0606

Functions

Answers

Q1 $g(x)=2+4\ln x$ for $x>0$

$h(x)=x^2+4$ for $x>0$

(a) $y=2+4\ln x \rightarrow \ln x=\frac{y-2}{4}$

$\rightarrow x=e^{\frac{y-2}{4}}$

$\therefore g^{-1}(x)=e^{\frac{x-2}{4}}$, Domain $x\in R$

Range $y>0$

(b) $g(x^2+4)=10$

$\rightarrow 2+4\ln(x^2+4)=10$

$\rightarrow \ln(x^2+4)=2$

$\rightarrow x^2=e^2-4=7.39-4$

$=3.39$

$\therefore x=1.84$ or $\sqrt{e^2-4}$ ✓

(c) $\frac{4}{x}=2x \rightarrow x^2=2 \rightarrow x=\sqrt{2}$

Q2(a) The function is not one to one.

(i) as the graph has a turning point. ✓

(ii) $f^2(x)=\sqrt{1+(\sqrt{1+x^2})^2}$

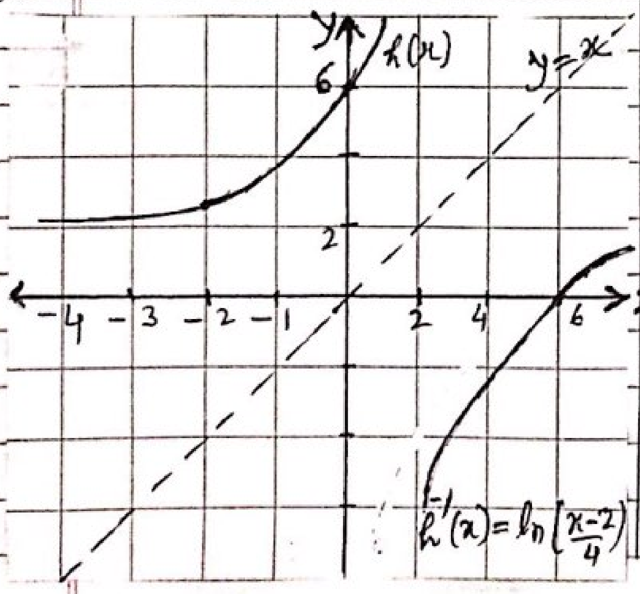
$=\sqrt{2+x^2}$ ✓

10(b)(i) $x\geq 0$

(ii) $y=\sqrt{1+x^2} \Rightarrow x=\sqrt{y^2-1}$

$\rightarrow g^{-1}(x)=\sqrt{x^2-1}$ ✓

(c) $h(x)=4e^x+2$



Q3 (i) $g\geq -\frac{1}{2}$

(ii) $g(1)=0$ does not lie in the domain of $g(x)$, $(x\geq 2)$.

(iii) $gf(x)=\frac{\left(\frac{x^2-2}{x}\right)^2-1}{2}$

$=\frac{1}{2}x^2-\frac{5}{2}+\frac{2}{x^2}$ ✓

(iv) $x\geq 2$

(v) $x^2-yx-2=0$

$x=\frac{y\pm\sqrt{y^2+8}}{2}$

$f^{-1}(x)=\frac{x+\sqrt{x^2+8}}{2}$ ✓

Explain why neg. sq. root should be discarded.

Q4 (a) (i) $f(x)>5$

(ii) $\frac{y-5}{3}=e^{-4x} \rightarrow \frac{x-3}{5}=e^{-4y}$

$\Rightarrow -4x=\ln\left(\frac{y-5}{3}\right)$

$f^{-1}(x)=-\frac{1}{4}\ln\left(\frac{x-5}{3}\right)$

$\rightarrow f^{-1}(x)=\frac{1}{4}\left(\frac{3}{x-5}\right)$

$=\frac{1}{4}[\ln 3-\ln(x-5)]$ ✓

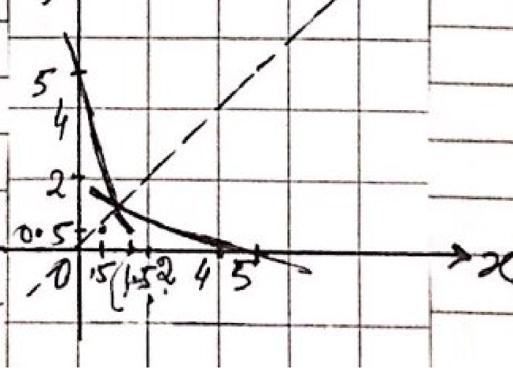
Domain $x>5$ ✓

(b) $\ln(x^2+5)=2$

$x=1.55$ or $\sqrt{e^2-5}$ ✓

Q5 (i) $2(x-1.5)^2+0.5$

(ii) $f(x)=2(x-1.5)^2+0.5$: $x\leq 3/2$



IG-Maths 0606

Functions

Answers

Q5 (iii) $\frac{x-0.5}{2} = (y-1.5)^2$

$\therefore f^{-1}(x) = 1.5 - \sqrt{\frac{x-0.5}{2}}$

Domain is $x \geq \frac{1}{2}$.

Q6 $g(x) = \frac{3}{2x+1}, \quad x > -\frac{1}{2}$

(i) $g'(x) = -6(2x+1)^{-2} = \frac{-2}{(2x+1)^2}$

is always negative as $\frac{2}{(2x+1)^2}$ is +.

(ii) $g > 0$ ✓

(iii) $hg(x) = \frac{3k}{2x+1} + 3$ ✓

(iv) $\frac{3k}{2(0)+1} + 3 = 5 \Rightarrow k = 2/3$ ✓

(v) $x > -\frac{1}{2}$ ✓

Q7 (i) B and C since for each value of x there is a unique value of y.

(ii) B only as it is one-one fnʸ.

Q8 (i) $\frac{y}{9} = \sqrt{x-1} \rightarrow$ Interchange x & y

$x/9 = \sqrt{y-1}$

$f^{-1}(x) = \left(\frac{x}{9}\right)^2 + 1 \ : \ x > 0$ ✓

(ii) $f(51) = 9\sqrt{50}$ ✓

(iii) $gf(x) = (9\sqrt{x-1})^2 + 2$

or $81(x-1) + 2 = 5x^2 + 83x - 95$

$\Rightarrow 5x^2 + 2x - 16 = 0$

$x = 1.6$ ✓

Q9(a) (i) $f \geqslant 3$

(ii) $(4x-1)^2 + 3 = 4$

$\Rightarrow x = 0, \ x = \frac{1}{2}$ ✓

Q9(b)(i) $xy - 4y = 2x + 1$

$x = \frac{4y+1}{y-2}$

$h^{-1}(x) = \frac{4x+1}{x-2}$

Range. $h^{-1}(x) \neq 4$

(ii) $h^2(x) = h\left(\frac{2x+1}{x-4}\right)$

$= \frac{2\left(\frac{2x+1}{x-4}\right) + 1}{\left(\frac{2x+1}{x-4}\right) - 4} = \frac{5x-2}{17-2x}$ ✓

Q10 (i) $y \in R$ ✓

(ii) $y > 3$ ✓

(iii) $f^{-1}(x) = e^x$; $g(4) = 35$

$f^{-1}(g(4)) = e^{35}$ ✓

(iv) $\frac{y-3}{2} = x^2 \rightarrow \frac{x-}{2} = y^2$

$g^{-1}(x) = \sqrt{\frac{x-3}{2}}$ ✓

Domain $x > 3$

Q11(i) $f^2 = f(f(x)) = \left([(x+2)^2+1]+2\right)^2 + 1$

$f^2(-3) = 17$ ✓

(ii) $x = \frac{y-2}{2y-1} \rightarrow 2xy - x = y - 2$

$\rightarrow y = \frac{x-2}{2x-1} = g(x)$ ✓

(iii) $gf(x) = \frac{[(x+2)^2+1]-2}{2[(x+2)^2+1]-1} = \frac{8}{19}$

$\Rightarrow 3(x+2)^2 = 27 \Rightarrow x^2 + 4x - 5 = 0$

$\Rightarrow x = 1, x = -5$ ✓

Q12 (i) $f(x) > 6$

(ii) $f^{-1}(x) = \frac{1}{4}\ln(x-6)$

Domain: $x > 6$

Range: $f^{-1}(x) \in R$

(Continued→)

Functions

Answers

Q12(iii) $f'(x) = 4e^{4x}$

(iv) $6 + e^{4x} = 4e^{4x}$

$\Rightarrow \quad x = \frac{1}{4}\ln 2$ ✓

Q13 (i) $2-\sqrt{5} < f(x) \leq 2$

(ii) $f^{-1}(x) = (2-x)^2 - 5$

Domain $2-\sqrt{5} < x \leq 2$

Range $-5 \leq f^{-1}(x) < 0$

(iii) $f(g(x)) = f\left(\frac{4}{x}\right)$

$= 2 - \sqrt{\frac{4}{x} + 5} = 0$

$\Rightarrow \quad x = -4$ ✓

Q14(a) when $x = \frac{1}{2}$; greatest value $= \frac{1}{4}$ ✓

(b) when $x \geq 1$, $f'(x)$ is always decreasing.

c(i) $h(k(10)) = h(8) = \lg 10 = 1$ ✓

(ii) $(y-5)^2 = (x-1)$

$\therefore k^{-1}(x) = (x-5)^2 + 1$

Domain: $5 < x < 15$

Range: $1 < k^{-1}(x) < 10$

Q15(i) $fg(x) = \ln(2e^x + 3) + 2$ ✓

(ii) $ff(x) = \ln(\ln x + 2) + 2$ ✓

(iii) $x = 2e^y + 3$

$e^y = \frac{x-3}{2}$

$g^{-1}(x) = \ln\left(\frac{x-3}{2}\right)$ ✓

(iv) e^2

(v) $gf(x) = 2e^{(\ln x + 2)} + 3 = 20$

$\Rightarrow 2xe^2 = 17 \Rightarrow x = \frac{17}{2e^2}$ or 1.15 ✓

Q16 (i) Range $0 \leq y \leq 1$

(ii) any suitable domain to give a one-one function.

(b)(i) $y = 2 + 4\ln x$

$\Rightarrow \ln x = \frac{y-2}{4} \rightarrow g^{-1}(x) = e^{\frac{x-2}{4}}$

Domain $x \in \mathbb{R}$ ✓

Range $y > 0$ ✓

(ii) $g(x^2+4) = 10$

$\rightarrow \quad 2 + 4\ln(x^2+4) = 10$

$\Rightarrow \quad x = 1.84$ ✓

(iii) $\frac{4}{x} = 2x \rightarrow x^2 = 2 \rightarrow x = \sqrt{2}$ ✓

Q17 (i) Range of f: $y \geq 3$

Range of g: $y \geq 9$

(ii) $x = -2 + \sqrt{y-5}$

$g^{-1}(x) = -2 + \sqrt{x-5}$

Domain of g^{-1}: $x \geq 9$

(iii) $g(3e^{2x}) = 41$

$\Rightarrow (3e^{2x} + 2)^2 + 5 = 41$

$\Rightarrow 9e^{4x} + 12e^{2x} - 32 = 0$

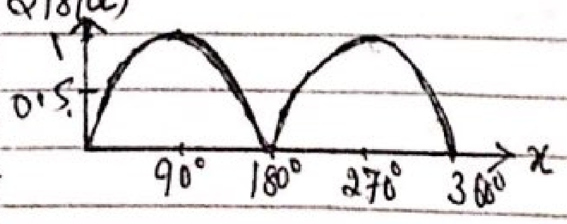
$(3e^{2x} - 4)(3e^{2x} + 8) = 0$

$\Rightarrow x = \frac{1}{2}\ln \frac{4}{3}$ ✓

(iv) $f'(x) = 6e^{2x}$

$\rightarrow f'(\ln 4) = 6e^{2\ln 4} = 96$ ✓

Q18(a)



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IG-Maths 0606

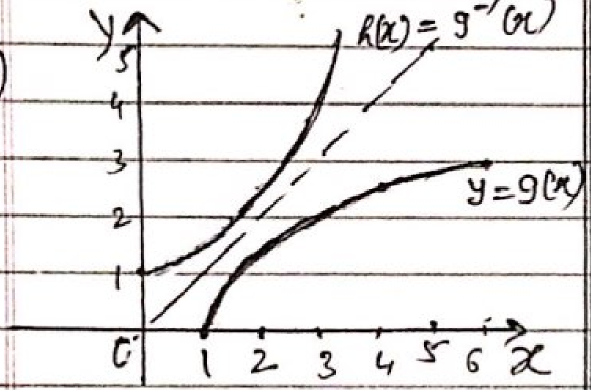
Functions.

Answers

Q18(b)(i) $h(g(x)) = e^{\frac{\ln(4x-3)+3}{4}}$

$= x$

(ii)



(iii) $x \geq 0$

(iv) $y \geq 1$

Q19 (a) (i) $(x+3)^2 - 5$

(ii) $y \geq 4$ or $f \geq 4$

(iii) $y = \sqrt{x+5} - 3$

Domain $x \geq 4$

(b) $h^2 g(x) = h^2(e^x)$

$= h(5e^x + 2)$

$= 25e^x + 12 = 37$

$\Rightarrow e^x = 1 \Rightarrow x = 0$ ✓

Q20(i) $f(x) = 3(x+2)^2 - 10$

$a = 3,\ b = 2,\ c = -10$

(ii) Min $f(x) = -10$ at $x = -2$

(iii) $f(\frac{1}{y}) = 0 \rightarrow \frac{1}{y} = \pm\sqrt{\frac{10}{3}} - 2$

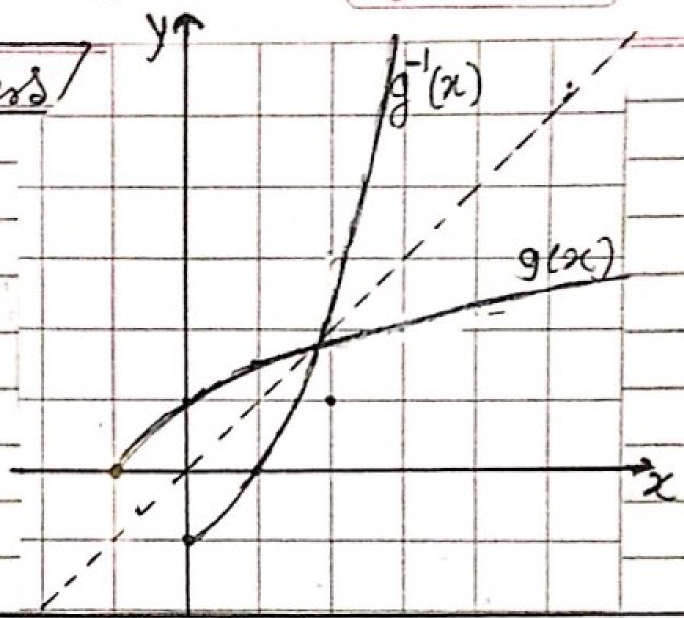
$y = -5.74,\ -0.26$

Q21(i) $f(g(8)) = f(3) = \frac{6}{4}$ ✓

(ii) $\dfrac{2\left(\frac{2x}{2x+1}\right)}{\frac{2x}{2x+1} + 1} = \dfrac{4x}{3x+1}$ ✓

(iii) $g^{-1}(x) = x^2 - 1$

Domain $x > 0$; Range $g^{-1}(x) > -1$



Q22 (i) $f(x) > 3$ $(\because x > 2)$

(ii) $f^{-1}(x) = \log_2(x+1)$

Domain $x > 3$, Range $f^{-1}(x) > 2$

(iii) $g(2^x - 1) = 2^x(2^x - 1)$ ✓

Now $2^x(2^x - 1) = 0$

$\Rightarrow 2^x = 0$ Not possible

and $2^x = 1 \Rightarrow x = 0$ is not in the domain, $\therefore$ No Solution.

Q23 (i) $3 \leq f \leq 7$

(ii) $f(f(12)) = f(5) = (2 + \sqrt{2})$ ✓

(iii) $f^{-1}(x) = (x-2)^2 + 3$

(iv) $gf(x) = \dfrac{120}{\sqrt{x-3} + 2} = 20$

$\rightarrow x = 19$ ✓

Q24 (i) $gf(x) = \dfrac{\sqrt{x-1} - 3 - 2}{2(\sqrt{x-1} - 3) - 3}$

$gf(37) = \frac{1}{3}$ ✓

(ii) $y = \sqrt{x-1} - 3 \rightarrow (y+3)^2 = (x-1)$

$f^{-1}(x) = (x+3)^2 + 1$ ✓

(iii) $g^{-1}(x) = \dfrac{3x-2}{2x-1}$

Answers

Q25 (i) $x = \frac{2}{y} + 1 \rightarrow y = \frac{2}{x-1}$

$f^{-1}(x) = \frac{2}{(x-1)}$ ✓

(ii) $gf(x) = \left(\frac{2}{x}+1\right)^2 + 2$

(iii) $fg(x) = \frac{2}{x^2+2} + 1$

(iv) $ff(x) = \frac{2}{\frac{2}{x}+1} + 1 = \frac{3x+2}{x+2}$

$\rightarrow \frac{3x+2}{x+2} = x \rightarrow x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$ ✓ only

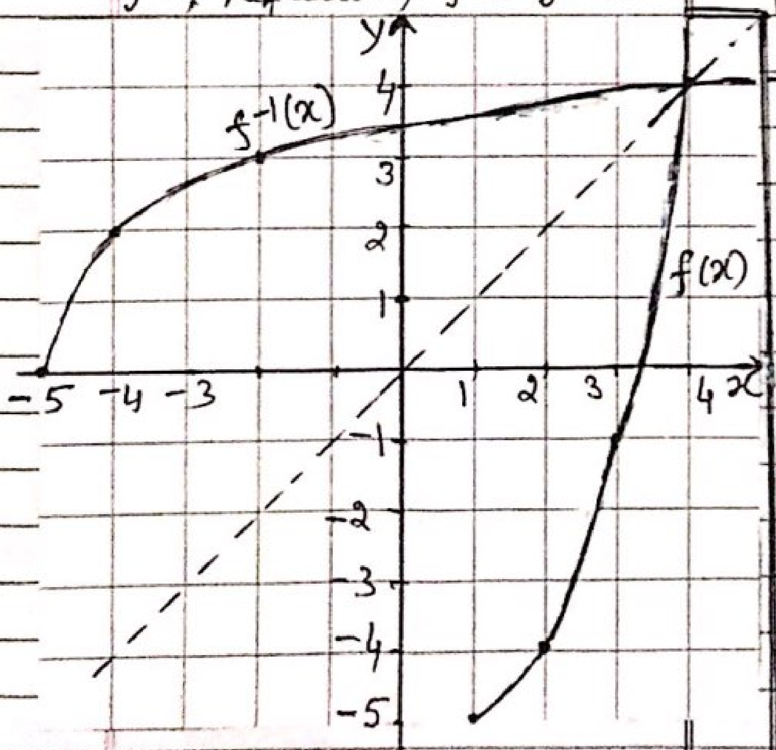
Q26 (i) 1 ✓

(ii) $f \geq -5$ ✓

(iii) $f^{-1}(x) = 1 + \sqrt{x+5}$ ✓

(iv) f: Positive quad. curve correct range and domain

f^{-1}; Reflection of f in $y = x$



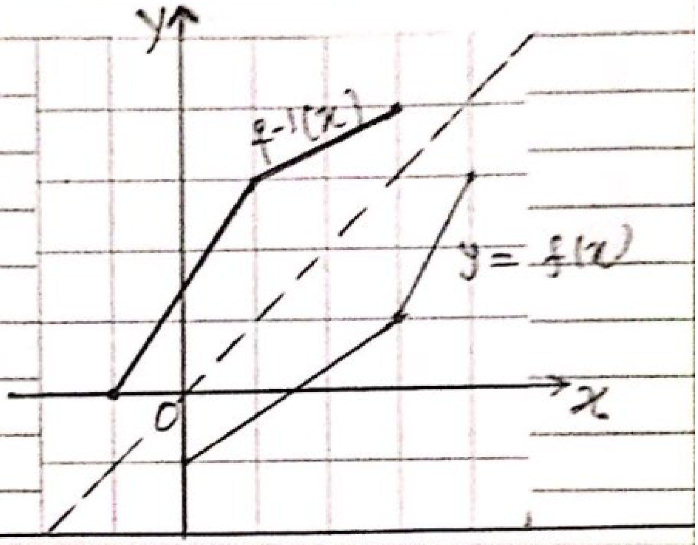
(v) $f(x) = f^{-1}(x) \rightarrow (x-1)^2 - 5 = 1 + \sqrt{x+5}$

$\Rightarrow x = 4$ ✓ (see graph).

Q27(a)(i) A and E.

(ii) C and D

(b)



Q28(a)(i) Range $-1 \leq y \leq 299$

(ii) $x \geq 0$

(b)(i) $g^{-1}(x) = \ln\left(\frac{x+2}{4}\right)$

(ii) $gh(x) = g(\ln 5x) = 4e^{\ln 5x} - 2$

$\therefore 20x - 2 = 18 \rightarrow x = 1$ ✓

Q29 (i) $f^2(x) = f(2x^3) = 2(2x^3)^3$

$\Rightarrow f^2(\frac{1}{2}) = 2^{-5}$ ✓

(ii) $f'(x) = g'(x)$

$6x^2 = 4 - 10x$

$(3x-1)(x+2) = 0$

$x = \frac{1}{3}, -2$ ✓

$\longleftarrow X \longrightarrow X \longrightarrow$