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IG-0606
Additional Maths

Indices and Surds
Exercise

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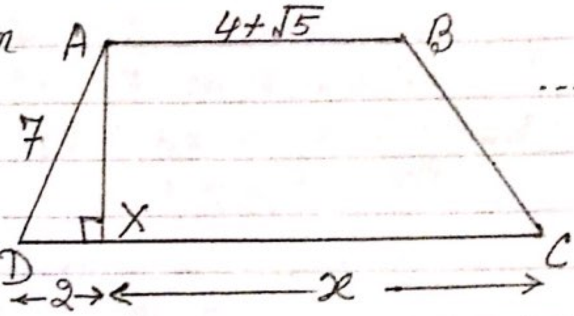
IG-Maths
0606

Indices and surds

Q1 Do not use a calculator in this Qution. SP-20/01/Q4

The diagram shows a trapezium ABCD, in which AD = 7cm and AB = $(4+\sqrt{5})$cm. AX is perpendicular to DC with DX = 2cm and XC = $x$cm. Given that the area of trapezium ABCD is $15(\sqrt{5}+2)\,cm^2$,



---[6]

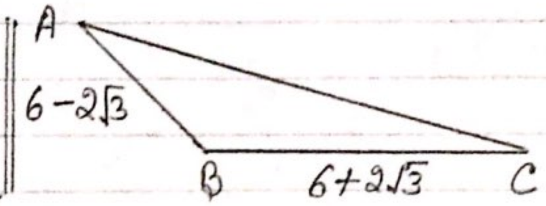
Obtain an expression for x in the form $a+b\sqrt{5}$, where a and b are integers.

Q2 Do not use a calculator in this question.

(a) Simplify $\frac{(3+2\sqrt{5})(6-2\sqrt{5})}{(4-\sqrt{5})}$ giving your answer in the form $a+b\sqrt{5}$, where a and b are integers. ---[3]

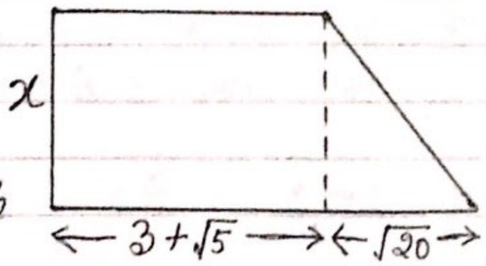
(b) In this part, all lengths are in centimetres.

The diagram shows the triangle ABC, with AB = $6-2\sqrt{3}$, Given that $\cos ABC = -\frac{1}{2}$, find the length of AC in the form $c\sqrt{d}$, where c and d are integers.



M-18/12/Q3 ---[3]

Q3 The diagram show a trapezium made from a rectangle and a right angled triangle. The dimensions, in centimetres, of the rectangle and triangle are shown.



The area, in square centimetres, of the trapezium is $13+5\sqrt{5}$. Without using a calculator, find the value of x in the form $p+q\sqrt{5}$, where p and q are integers. M-17/22/Q5 --[5]

Q4(a)(i) Express $(\sqrt[3]{-8x^9})(6\sqrt{x^{-3}})$ in the form ax^b, where a and b are constants to be found. ---[2]

(ii) Hence solve the equation $(\sqrt[3]{-8x^9})(6\sqrt{x^{-3}}) = -6250$ --[2]

M-17/22/Q6(a)

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Q5 (a) Simplify $\sqrt{x^8y^{10}} \div \sqrt[3]{x^3y^{-6}}$, giving your answer in the form $x^a y^b$, where a and b are integers. ---[2]

(b)(i) Show that $4(t-2)^{1/2}+5(t-2)^{3/2}$ can be written in the form $(t-2)^p(qt+r)$, where p, q, r are constants to be found. ---[3]

(ii) Hence solve the equation; $4(t-2)^{\frac{1}{2}}+5(t-2)^{3/2}=0$ ---[1]

S-17/11/Q3

Q6 Do not use a calculator in this question.

(a) Show that $\sqrt{24}\times\sqrt{27}+\dfrac{9\sqrt{30}}{\sqrt{15}}$ can be written in the form $a\sqrt{2}$, where a is an integer. ---[3]

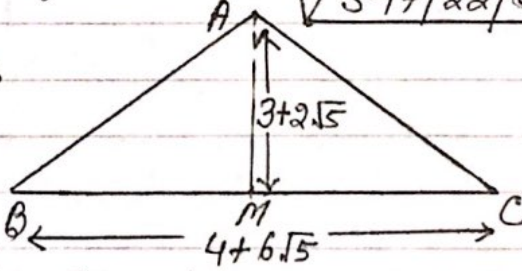
(b) Solve the equation $\sqrt{3}(1+x)=2(x-3)$, giving your answer in the form $b+c\sqrt{3}$, where b and c are integers. ---[3]

S-17/21/Q2

Q7 Without using a calculator, express $\left(\dfrac{1+\sqrt{5}}{3-\sqrt{5}}\right)^{-2}$ in the form $a+b\sqrt{5}$, where a and b are integers. ---[5]

S-17/22/Q2

Q8 In this question, all dimensions are in centimetres.



The diagram shows an isosceles triangle ABC, where AB = AC. The point M is the mid point of BC. Given that $AM = 3+2\sqrt{5}$ and $BC = 4+6\sqrt{5}$, find without using a calculator,

(i) the area of triangle ABC. ---[2]

(ii) tan ABC, giving your answer in the form $\dfrac{a+b\sqrt{5}}{c}$ where a, b and c are positive integers. ---[3]

S-17/13/Q4

Q9 Express $\dfrac{(5\sqrt{q})^3}{(625p^{12}q)^{1/4}}$ in the form $5^a p^b q^c$, where a, b, c are constants. ---[3]

S-17/23/Q1(b)

Q10(a) Given that $T=2\pi l^{\frac{1}{2}} g^{-\frac{1}{2}}$, express l in terms of T, g and π. ---[2]

(b) By using the substitution $y=x^{1/3}$, or otherwise, Solve, $x^{2/3}-4x^{1/3}+3=0$ ---[4]

W-17/11/Q3

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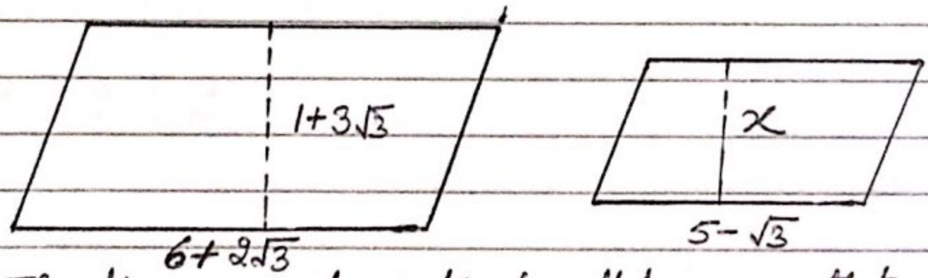
Q11 Given that $z = a+(a+3)\sqrt{3}$ and $z^2 = 79+b\sqrt{3}$, Find the Value of each of the integers a and b. [W-17/21/Q8] --- [6]

Q12 If $z = 2+\sqrt{3}$, find the integers a and b such that $az^2+bz = 1+\sqrt{3}$ ---- [5] [W-17/22/Q1]

Q13 Find the integers p and q such that $\frac{p}{\sqrt{3}-1}+\frac{1}{\sqrt{3}+1} = q+3\sqrt{3}$ [W-17/23/Q3] --- [4]

Q14 Given that $\frac{p^{-2}qr^{-\frac{1}{2}}}{\sqrt{p^{1/3}q^2r^{-3}}} = p^a q^b r^c$ find the a, b and c. --- [3] [M-16/12/Q2]

Q15 Do not use a calculator in this question.



The diagram shows two parallelograms that are similar. The base and height, in centimetres, of each parallelogram is shown. Given that x, the height of the smaller parallelogram, is $\frac{p+q\sqrt{3}}{6}$, find the value of each of the integers p and q. --- [5] [M-16/22/Q6]

Q16 (a) Solve the equation $16^{3x-1} = 8^{x+2}$ --- [3]

(b) Given that $\frac{(a^{1/3}b^{-1/2})^3}{a^{-2/3}b^{1/2}} = a^p b^q$, find the value of each of the constants p and q. --- [2] [S-16/11/Q2]

Q17 Do not use calculator:

(a) Express $\frac{\sqrt{8}}{\sqrt{7}-\sqrt{5}}$ in the form $\sqrt{a}+\sqrt{b}$, where a and b are integers. --- [3]

(b) Given that $28+p\sqrt{3} = (q+2\sqrt{3})^2$, where p and q are integers, find the values of p and of q. --- [3]

[S-16/21/Q5]

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Q18 Do not use a calculator in this question.
Find the positive value of x for which $(4+\sqrt{5})x^2+(2-\sqrt{5})x-1=0$, giving your answer in the form $\frac{a+\sqrt{5}}{b}$, where a and b are integers. [S-16/12/Q4] ---[6]

Q19 (a) Solve the equation $16^{3x-1}=8^{x+2}$ ---[3]

(b) Given that, $\frac{(a^{1/3}\cdot b^{-\frac{1}{2}})}{a^{-2/3}\cdot b^{1/2}}=a^p\cdot b^q$, find the value of each of constants p and q. [S-16/13/Q2] ---[2]

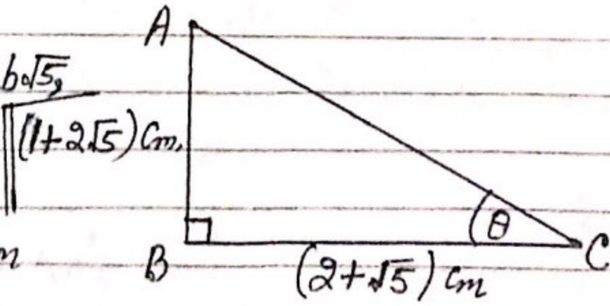
Q20 Given that $\frac{p^{1/3}\cdot q^{-\frac{1}{2}}r^{3/2}}{p^{-2/3}\sqrt{(qr)^5}}=p^a q^b r^c$, find the value of each of the integers a, b and c. [W-16/11/Q2] ---[3]

Q21 Without using a calculator, find the integers a and b such that, $\frac{a}{\sqrt{3}+1}+\frac{b}{\sqrt{3}-1}=\sqrt{3}-3$ ---[5] [W-16/21/Q2]

Q22 Express $\frac{4m\sqrt{m}-\frac{9}{\sqrt{m}}}{2\sqrt{m}+\frac{3}{\sqrt{m}}}$ in the form $Am+B$, where A and B are integers to be found. ---[3] [W-16/13/Q2]

Q23 Without using a calculator, show that:
$\frac{\sqrt{5}+3\sqrt{3}}{\sqrt{5}+\sqrt{3}}=\sqrt{K}-2$ where K is an integer to be found. --[3] [W-16/23/Q1]

Q24 The diagram shows triangle ABC which is right-angled at the point B. The AB $=(1+2\sqrt{5})$ cm. and the side BC $=(2+\sqrt{5})$ cm. Angle BCA $=\theta$



(i) Find $\tan\theta$ in the form $a+b\sqrt{5}$, where a and b are integers to be found. ---[3]

(ii) Hence find $\sec^2\theta$ in the form $c+d\sqrt{5}$, where c and d are integers to be found. ---[3]

[M-15/12/Q10]

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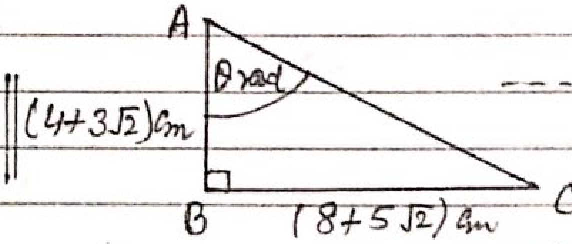
~~Do not use a calculator in this question.~~

Q25 The diagram shows the triangle ABC where angle B is a right angle, $AB = (4+3\sqrt{2})$ cm, $BC = (8+5\sqrt{2})$ cm. and angle $BAC = \theta$ rad. Showing all your working, find

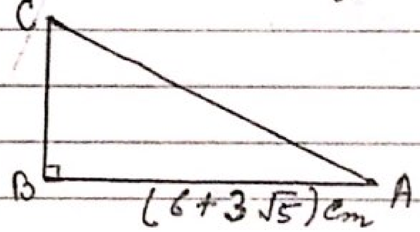


(i) $\tan\theta$ in the form $a+b\sqrt{2}$, where a and b are integers. --- [2]

(ii) $\sec^2\theta$ in the form $c+d\sqrt{2}$, where c and d are integers. --- [3]

S-15/11/Q2

Q26 The diagram shows the right-angled triangle ABC, where $AB = (6+3\sqrt{5})$ cm, and angle $B = 90°$. The area of this triangle is $\left(\frac{36+15\sqrt{5}}{2}\right)$ cm^2.



(i) Find the length of the side BC in the form $(a+b\sqrt{5})$ cm, where a and b are integers. --- [3]

(ii) Find AC^2 in the form $(c+d\sqrt{5})$ cm^2, where c and d are integers. --- [2]

S-15/22/Q3

Q27 (a) Solve the equations to find p and q.

$$8^{q-1} \times 2^{2p+1} = 4^7$$

$$9^{p-4} \times 3^q = 81$$

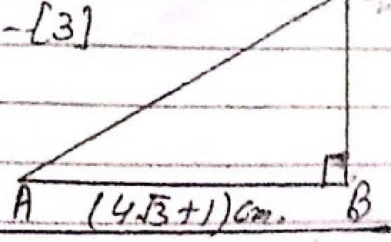
W-15/21/Q5 --- [4]

(b) Solve the equation $\lg(3x-2) + \lg(x+1) = 2 - \lg 2$ --- [5]

Q28 The diagram shows triangle ABC with side $AB = (4\sqrt{3}+1)$ cm. Angle B is a right angle. It is given that the area of this triangle is $\frac{47}{2}$ cm^2.

(i) Find the length of the side BC in the form $(a\sqrt{3}+b)$ cm, where a and b are integers. --- [3]

(ii) Hence find the length of the side AC in the form $p\sqrt{2}$, where p is an integer. W-15/13/Q4 --- [2]



Q29 (a) Given that $2^{2x-1} \times 4^{x+y} = 128$ and $\frac{9^{2y-x}}{27^{y-4}} = 1$, find the value of each of the integers x and y. --- [4]

(b) Solve $2(5)^{2z} + 5^z - 1 = 0$

W-15/13/Q12 --- [4]

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Q30 Express $6(1+\sqrt{3})^{-2}$ in the form $a+b\sqrt{3}$, where a and b are integers to be found. [S-14/21/Q2] ---[4]

Q31 (i) Given that $2^{5x} \times 4^{y} = \frac{1}{8}$, show that $5x+2y=-3$ ---[3]

(ii) Solve the simultaneous equations:

$$2^{5x} \times 4^{y} = \frac{1}{8} \quad \text{and} \quad 7^{x} \times 49^{2y} = 1$$ ---[4]

[S-14/12/Q5]

Q32 Express $\frac{(2+\sqrt{5})^2}{\sqrt{5}-1}$ in the form $a+b\sqrt{5}$, where a and b are constants to found. [S-14/22/Q1] ---[4]

Q33 Given that $2^{4x} \times 4^{y} \times 8^{x-y} = 1$ and $3^{x+y} = \frac{1}{3}$, ---[4]
find the values of x and of y. [S-14/13/Q2]

Q34 Do not use a calculator in this question.

(i) Show that $(2\sqrt{2}+4)^2 - 8(2\sqrt{2}+3) = 0$ ---[2]

(ii) Solve the equation $(2\sqrt{2}+3)x^2 - (2\sqrt{2}+4)x + 2 = 0$, giving your answer in the form $a+b\sqrt{2}$ where a and b are integers. [S-14/23/Q5] ---[3]

Q35 (i) Using the substitution $y=5^x$, show that the equation $5^{2x+1} - 5^{x+1} + 2 = 2(5^x)$ can be written in the form, $ay^2+by+2=0$, where a and b are the constants to be found. --- [2]

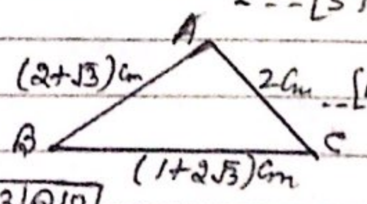
(ii) Hence solve the equation $5^{2x+1} - 5^{x+1} + 2 = 2(5^x)$ [W-14/11/Q4] --- [4]

Q36 Integers a and b are such that $(a+3\sqrt{5})^2 + a - b\sqrt{5} = 51$. Find the possible values of a and the corresponding values of b. [W-14/21/Q9] --- [6]

Q37 (a) Solve the following simultaneous equations.

$$\frac{5^x}{25^{3y-2}} = 1 \quad \text{and} \quad \frac{3^x}{27^{y-1}} = 81$$ ---[5]

(b) The diagram shows a triangle ABC such that $AB = (2+\sqrt{3}\text{ cm})$, $BC = (1+2\sqrt{3})$ cm and $AC = 2$ cm. --[4]

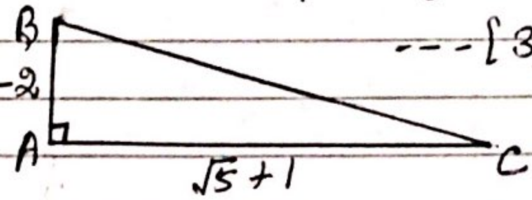


Find the value of $\cos A$ in the form $a+b\sqrt{3}$. [W-14/13/Q10]

Indices and Surds

Q38 Calculator must not be used in this question.
The diagram shows a triangle ABC in which angle $A = 90^\circ$, sides AB and AC are $\sqrt{5}-2$ and $\sqrt{5}+1$ respectively. Find,

(i) $\tan B$ in the form $a+b\sqrt{5}$, where a and b are integers. ---[3]

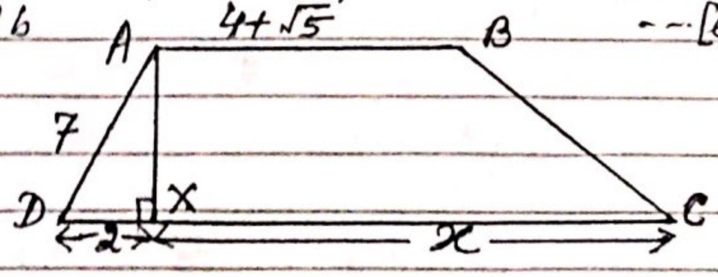


(ii) $\sec^2 B$ in the form $c+d\sqrt{5}$, where c and d are integers. S-13/11/Q7 ---[4]

Q39 (a) Solve the equation $3^{p+1} = 0.7$, giving your answer to 2 decimal places. ---[3]

(b) Express $\dfrac{y \times (4x^3)^2}{\sqrt{8y^3}}$ in the form $2^a \times x^b \times y^c$, ---[3]
where a, b and c are constants.
S-13/22/Q2

Q40 The diagram shows a trapezium ABCD in which AD = 7 cm and AB = $(4+\sqrt{5})$ cm. AX is perpendicular to DC with DX = 2 cm, and XC = x cm. Given that the area of trapezium ABCD is $15(\sqrt{5}+2)\text{cm}^2$, obtain an expression for x in the form $a+b\sqrt{5}$, where a and b are integers. --[6]



S-13/22/Q5

Q41 Express $\dfrac{(4\sqrt{5}-2)^2}{\sqrt{5}-1}$ in the form $p\sqrt{5}+q$, where p and q are integers.
W-13/21/Q2 --[4]

Indices and Surds

Answers

Q1 $AX = \sqrt{45} = 3\sqrt{5}$

$\therefore$ Area of Trap $= \frac{1}{2}[(4+\sqrt{5})+(2+x)] \times \sqrt{45}$

$= 15(\sqrt{5}+2)$

$\Rightarrow \frac{1}{2}(6+\sqrt{5}+x) \times 3\sqrt{5} = 15(\sqrt{5}+2)$

$\therefore x = (4+3\sqrt{5})$ ✓

Q2(a) $\frac{18+12\sqrt{5}-6\sqrt{5}-20}{4-\sqrt{5}}$

$= \frac{6\sqrt{5}-2}{4-\sqrt{5}} \times \frac{4+\sqrt{5}}{4+\sqrt{5}} = \frac{22\sqrt{5}+22}{11}$

$= 2\sqrt{5}+2$ ✓

$AC^2 = AB^2 + BC^2 - 2AB \cdot BC \cdot \cos ABC$

$= (6-2\sqrt{3})^2 + (6+2\sqrt{3})^2$

$-2(6-2\sqrt{3})(6+2\sqrt{3})\left(-\frac{1}{2}\right)$

$\Rightarrow AC = 2\sqrt{30}$ ✓

Q3. $(3+\sqrt{5})x + \frac{1}{2}x \times 2\sqrt{5}$ $(\because \sqrt{20} = 2\sqrt{5})$

$= 13+5\sqrt{5}$ given

$\Rightarrow (3+2\sqrt{5})x = 13+5\sqrt{5}$

$x = \frac{13+5\sqrt{5}}{2+2\sqrt{5}} \times \frac{3-2\sqrt{5}}{3-2\sqrt{5}}$

or $x = 1+\sqrt{5}$ ✓

Q4(a)(i) $-2x^3 \times x^{-1/2} = ax^b$

or $-2x^{5/2} = ax^b$

$\Rightarrow a = -2;\ b = 5/2$ ✓

(ii) $x = \left(\frac{-6250}{-2}\right)^{2/5} = 25$ ✓

Q5(a) x^3y^7

(b)(i) $(t-2)^{1/2}[4+5(t-2)]$

$= (t-2)^{1/2}(5t-6)$ ✓

(ii) $t = 2$ and $6/5$ ✓

Q6(a) $2\sqrt{6} \times 3\sqrt{3} + 9\sqrt{2}$

$= 6\sqrt{18} + 9\sqrt{2}$

$= 18\sqrt{2} + 9\sqrt{2} = 27\sqrt{2}$ ✓

(b) $x = \frac{6+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$= 15+8\sqrt{3}$ ✓

Q7 Consider $\frac{3-\sqrt{5}}{1+\sqrt{5}} \times \frac{1-\sqrt{5}}{1-\sqrt{5}}$

$= (-2+\sqrt{5})$

$\therefore$ Req $= (-2+\sqrt{5})^2$

$= 9-4\sqrt{5}$ ✓

Q8(i) Area $= \frac{1}{2}(3+2\sqrt{5})(4+6\sqrt{5})$

$= 36+13\sqrt{5}$ ✓

(ii) $\tan ABC = \frac{3+2\sqrt{5}}{2+3\sqrt{5}} \times \frac{2-3\sqrt{5}}{2-3\sqrt{5}}$

$= \frac{24+5\sqrt{5}}{41}$ ✓

Q9 $5^2 p^{-3} q^{5/4}$

Q10(a) $Tg^{1/2} = 2\pi l^{1/2}$

$\therefore l = \frac{T^2g}{4\pi^2} = \left(\frac{Tg^{1/2}}{2\pi}\right)^2$

(b) $y^2 - 4y + 3 =$ (where $y = x^{1/3}$)

$\Rightarrow y = 1, 3$

$\Rightarrow x^{1/3} = 1$ or 3

$x = 1$ or 27 ✓

Q11 $z^2 = a^2 + 3(a+3)^2 + 2a(a+3)\sqrt{3}$

$= 79 + b\sqrt{3}$

$\Rightarrow a^2 + 3(a+3)^2 = 79$ and $2a(a+3) = b$

$4a^2 + 18a + 27 = 79$

$4a^2 + 18a - 52 = 0$

$(a-2)(4a+26) = 0 \Rightarrow a = b,\ b = 20$ ✓

Indices and Surds

Answers

Q12 $x^2 = 7+4\sqrt{3}$

$a(7+4\sqrt{3})+b(2+\sqrt{3}) = 1+\sqrt{3}$

$\Rightarrow \begin{cases} 7a+2b=1 \\ 4a+b=1 \end{cases}$

Solving, $a=1$ and $b=-3$ ✓

Q13 $\dfrac{p(\sqrt{3}+1)+(\sqrt{3}-1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = q+3\sqrt{3}$

$\Rightarrow (p+1)\sqrt{3}+(p-1) = q+6\sqrt{3}$

$\Rightarrow \begin{cases} p+1=6 \\ p-1=2q \end{cases}$

$\Rightarrow p=5$ and $q=2$ ✓

Q14 $a=-\frac{13}{6},\ b=0,\ c=1$

Q15 $\dfrac{x}{1+3\sqrt{3}} = \dfrac{5-\sqrt{3}}{6+2\sqrt{3}}$

$x = \dfrac{-4+14\sqrt{3}}{6+2\sqrt{3}} \times \dfrac{6-2\sqrt{3}}{6-2\sqrt{3}} = \dfrac{p+q\sqrt{3}}{6}$

$\Rightarrow p=-27,\ q=23$

Q16 (a) $2^{4(3x-1)} = 2^{3(x+2)}$

$\Rightarrow 4(3x-1) = 3(x+2)$

$\Rightarrow x = \frac{10}{9}$ ✓

(b) $p=\frac{5}{3}$ and $q=-2$ ✓

Q17 (a) $\dfrac{\sqrt{8}}{\sqrt{7}-\sqrt{5}} \times \dfrac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}}$

$= \sqrt{14}+\sqrt{10}$ ✓

(b) $28+p\sqrt{3} = q^2+4q\sqrt{3}+12$

$\Rightarrow q^2+12=28 \Rightarrow q=\pm 4$ ✓

and $p=4q \Rightarrow q=\pm 16$ ✓

Q18 $x = \dfrac{-(2-\sqrt{5}) \pm \sqrt{(2-\sqrt{5})^2-4(4+\sqrt{5})(-1)}}{2(4+\sqrt{5})}$

$= \dfrac{(3+\sqrt{5})}{2(4+\sqrt{5})} \times \dfrac{(4-\sqrt{5})}{(4-\sqrt{5})}$

$= \dfrac{7+\sqrt{5}}{22}$ ✓

Q19 (a) $x = 10/9$

(b) $p = 5/3$ and $q=-2$

Q20 $a=1,\ b=-3$ and $c=-1$

Q21 $a(\sqrt{3}-1)+b(\sqrt{3}+1) = 2(\sqrt{3}-3)$

$\Rightarrow \begin{cases} a+b=2 \\ -a+b=-6 \end{cases} \Rightarrow \begin{matrix} b=-2 \\ a=4 \end{matrix}$ ✓

Q22 $\dfrac{4m^2-9}{2m+3}$

$= 2m-3$ ✓

Q23 $\dfrac{\sqrt{5}+3\sqrt{3}}{\sqrt{5}+\sqrt{3}} \times \dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

$= \sqrt{15}-2$ ✓

Q24 (i) $\tan\theta = \dfrac{1+2\sqrt{5}}{2+\sqrt{5}} \times \dfrac{2-\sqrt{5}}{2-\sqrt{5}}$

$= (8-3\sqrt{5})$

(ii) $\sec^2\theta = 1+\tan^2\theta$

$= 1+(8-3\sqrt{5})^2$

$= 1+64-48\sqrt{5}+45$

$= 110-48\sqrt{5}$ ✓

Q25 (i) $\tan\theta = \dfrac{(8+5\sqrt{2})}{(4+3\sqrt{2})} \times \dfrac{(4-3\sqrt{2})}{(4-3\sqrt{2})}$

$= (-1+2\sqrt{2})$ ✓

(ii) $\sec^2\theta = 1+\tan^2\theta = 1+(-1+2\sqrt{2})^2$

$= 10-4\sqrt{2}$ ✓

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Answers

Q26 (i) $BC = \frac{(36+15\sqrt{5})}{(6+3\sqrt{5})} \times \frac{(6-3\sqrt{5})}{(6-3\sqrt{5})}$

$= 1+2\sqrt{5}$ ✓

(ii) $AC^2 = (6+3\sqrt{5})^2 + (1+2\sqrt{5})^2$

$= 102+40\sqrt{5}$ ✓

Q27 (a) $2^{3(q-1)} \times 2^{2p+1} = 2^{14}$

$3^{2(p-4)} \times 3^{q} = 3^4$

$\Rightarrow \begin{cases} 3q+2p=16 \; ; \\ q+2p=12 \end{cases}$

$\Rightarrow p=5, \; q=2$ ✓

(b) $x=4$

Q28 (i) $BC = \frac{47}{(4\sqrt{3}+1)} \times \frac{4\sqrt{3}-1}{4\sqrt{3}-1}$

$= (4\sqrt{3}-1)$ ✓

(ii) $AC^2 = (4\sqrt{3}+1)^2 + (4\sqrt{3}-1)^2$

$= 98 \quad \Rightarrow AC = 7\sqrt{2}$ ✓

Q29 (a) $2^{2x-1} \times 2^{2(x+y)} = 2^7$

and $\frac{3^{2(2y-x)}}{3^{3(y-4)}} = 1$

$\Rightarrow \begin{cases} 2x-1+2(x+y)=7 \\ 2(2y-x)=3(y-4) \end{cases}$

$\Rightarrow x=4 \; ; \; y=-4$

(b) $(2(5^z)-)(5^z+1)=0$

$\Rightarrow 5^z = \frac{1}{2} = 0.5$ & $5^z = -1$ ✗

$\Rightarrow z = \frac{\log 0.5}{\log 5} = -0.431$ ✓

Q30 $\frac{6}{(1+\sqrt{3})^2}$

$= \frac{6}{4+2\sqrt{3}} \times \frac{4-2\sqrt{3})}{4-2\sqrt{3}}$

$= (6-3\sqrt{3})$ ✓

Q31 (i) $2^{5x} \times 2^{2y} = 2^{-3}$

$\Rightarrow 5x+2y=-3$ ✓

(ii) $7^x \times 49^{2y} = 1$

$\Rightarrow x+4y=0$

and $5x+2y=-3$ from Part (i)

$\Rightarrow x = -2/3 \; ; \; y = \frac{1}{6}$

Q32 $\frac{(2+\sqrt{5})^2}{(\sqrt{5}-1)} \times \frac{(\sqrt{5}+1)}{(\sqrt{5}+1)}$

$= \frac{(9+4\sqrt{5})(\sqrt{5}+1)}{4}$

$= \frac{29}{4} + \frac{13}{4}\sqrt{5}$ ✓

Q33 $2^{4x} \times 2^{2y} \times 2^{3(x-3)} = 1$

$\Rightarrow 7x-y=0$ —(i)

and $3^{x+y} = \frac{1}{3} \Rightarrow x+y=-1$ —(ii)

from (i) and (ii) $x = -\frac{1}{8} \; ; \; y = -\frac{7}{8}$ ✓

Q34 (i) $(2\sqrt{2}+4)^2 = 8+16\sqrt{2}+16$

(ii) $x = \frac{(2\sqrt{2}+4) \pm \sqrt{(2\sqrt{2}+4)^2 - 4\times2(2\sqrt{2}+3)}}{2(2\sqrt{2}+3)}$

$= \frac{(2\sqrt{2}+4) \pm 0}{2(2\sqrt{2}+3)}$ from (i)

$= 2-\sqrt{2}$ ✓

Q35 (i) $5y^2-7y+2=0$

(ii) $(5y-2)(y-1)=0$

$\Rightarrow y = 2/5$ or $y=1$

or $y=0.4$ or $y=1$

$y=5^x$

$x = \log_5 y$

$\Rightarrow x = \frac{\ln 0.4}{\ln 5} = -0.569$ ✓

and $y=1 \Rightarrow x=0$ ✓

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Indices and Surds

Answers

Q36 $a^2+a+45+6\sqrt{5}a = 51 - b\sqrt{5}$

$\Rightarrow$ $a^2+a-6=0$ & $b=6a$

$(a+3)(a-2)=0$

$\begin{cases} a=-3, 2 \\ b=-18, 12 \end{cases}$

Q37(a) $\dfrac{5^x}{5^{2(3y-2)}}=1$ and $\dfrac{3^x}{3^{3(y-1)}}=3^4$

$\Rightarrow x=6y-4$ and $x=3y+1$

$\Rightarrow x=6$ and $y=5/3$ ✓

(b) $\cos A = \dfrac{(2+\sqrt{3})^2+2^2-(1+2\sqrt{3})^2}{2\times 2\times(2+\sqrt{3})}$

or $\cos A = -1+\dfrac{\sqrt{3}}{2}$ ✓

Q38(i) $\tan B = \dfrac{\sqrt{5}+1}{\sqrt{5}-2} \times \dfrac{\sqrt{5}+2}{\sqrt{5}+2}$

$= 7+3\sqrt{5}$ ✓

(ii) $\sec^2 B = 1+\tan^2 B$

$= 1+(7+3\sqrt{5})^2$

$= 95+42\sqrt{5}$ ✓

Q39 (a) $(p+1)\ln 3 = \ln 0.7$

$\Rightarrow p = \dfrac{\ln 0.7}{\ln 3} - 1 = -1.32$ ✓

(b) $2^{5/2} \times x^6 \times y^{-1/2}$

$a=5/2,\ b=6,\ c=-\frac{1}{2}$

In $\triangle AXD$

Q40 $AX=\sqrt{7^2-2^2}=\sqrt{45}=3\sqrt{5}$

Area $= \frac{1}{2}(4+\sqrt{5}+2+x)\cdot\sqrt{45} = 15(\sqrt{5}+2)$

$\Rightarrow x = 4+3\sqrt{5}$

Q41 $\dfrac{(4\sqrt{5}-2)^2}{\sqrt{5}-1}$

$= \dfrac{(84-16\sqrt{5})}{(\sqrt{5}-1)} \times \dfrac{(\sqrt{5}+1)}{(\sqrt{5}+1)}$

$= 17\sqrt{5}+1$ ✓

$\longleftarrow$ X —— X $\longrightarrow$