IG_0606 Additional Maths

Logarithmic and Exponential Funtions Exercise

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Logarithmic and exponential functions

Q1. Variables x and y are such that when $\lg y$ is plotted against x^2, a straight line graph passing through the points $(1, 0.73)$ and $(4, 0.10)$ is obtained.

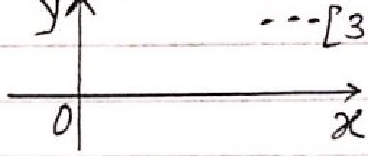
(a) Given that $y = Ab^{x^2}$, find the value of each of the constants A and b. --[4]

SP-20/02/Q7

(b) Find the value of y when $x = 1.5$ ---[2]

(c) Find the positive value of x when $y = 2$ --[2]

Q2 (a)(i) Sketch the graph of $y = e^x - 5$ on the axes, showing the exact coordinates of any points where the graph cuts the coordinates axes. ---[3]



(ii) Find the range of values of k for which the equation $e^x - 5 = k$ has no solution. --[1]

(b) Simplify $\log_a \sqrt{2} + \log_a 8 + \log_a\left(\frac{1}{2}\right)$, giving your answer in the form:

$p \log_a 2$, where p is a constant. ---[2]

(c) Solve the equation $\log_3 x - \log_9 4x = 1$ ---[4]

SP-20/02/Q10

Q3 (a) Solve the following equations:

(i) $5e^{3x+4} = 14$ ---[2]

(ii) $\lg(2y-7) + \lg y = 2\lg 3$ ---[4]

(b) Write: $\dfrac{\log_2 p - \log_2 q}{(\log_2 2)(\log_2 2)}$ as a single logarithm to base 2. ---[2]

M-18/22/Q8

Q4 It is given that $y = Ae^{bx}$, where A and b are constants. When $\ln y$ is plotted against x a straight line graph is obtained, which passes through the points $(1.0, 0.7)$ and $(2.5, 3.7)$.

(i) Find the value of A and b. --- [6]

(ii) Find the value of y when $x = 2$ --- [2]

M-17/12/Q11

Q5 The value, V dollars, of a car aged t years is given by;

$$V = 12000\, e^{-0.2t}$$

(i) Write down the value of the car when it was new. ---[1]

(ii) Find the time it takes for the value to decrease to $2/3$ of the value when it was new. [M-17/22/Q2] ---[2]

Q6 It is given that $y = \log_a(ax) + 2\log_a(4x-3) - 1$, where 'a' is a positive integer.

(i) Explain why x must greater than 0.75 ---[1]

(ii) Show that y can be written as $\log_a(16x^3 - 24x^2 + 9x)$ ---[3]

(iii) Find the value of x for which $y = \log_a(9x)$ ---[2]

[M-17/22/Q6(b)]

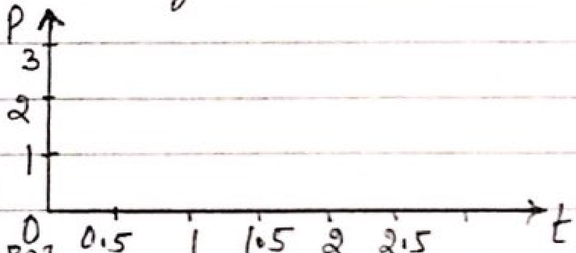
Q7 The table shows values of the variables t and P.

t	1	1.5	2	2.5
P	4.39	8.33	15.8	30.0

(i) Draw the graph of $\ln P$ against t on the grid below: ---[2]

(ii) Use the graph to estimate the value of P when t = 2.2. ---[2]

(iii) Find the gradient of the graph and state the coordinates of the point where the graph meets the vertical axis. ---[2]



(iv) Using your answers to part (iii), show that $P = ab^t$, where a and b are the constants to be found. ---[3]

(v) Given that your equation in part (iv) is valid for values of t upto 10, find the smallest value of t, correct to 1 decimal place, for which P is at least 1000. [S-17/21/Q10] ---[2]

Q8 It is given that $y = A(10^{bx})$, where A and b are constants. The straight graph obtained when $\lg y$ is plotted against x passes through the points (0.5, 2.2) and (1.0, 3.7).

(i) Find the value of A and of b. ---[5]

Using your values of A and b, find (ii) the value of y when $x = 0.6$ --[2]

(iii) the value of x, when $y = 600$.

[S-17/12/Q7]

Logarithmic and Exponential functions

Q9 (a) Given that $a^7 = b$, where a and b are positive constants, find
(i) $\log_a b$ ---[1]
(ii) $\log_b a$ ---[1]

(b) Solve the equation $\log_{81} y = -\frac{1}{4}$ ---[2]

(c) Solve the equation $\frac{32^{x^2-1}}{4^{x^2}} = 16$ ---[3]

S-17/22/Q7

Q10 Solve the equation $7^{2x+5} = 2 \cdot 5$,
giving your answer correct to 2 decimal places. S-17/23/Q1(a) ---[3]

Q11 When $\lg y$ is plotted against x^2 a straight line is obtained which passes through the points $(4,3)$ and $(12,7)$.
(i) Find the gradient of the line. ---[1]
(ii) Use your answer to part (i) to express $\lg y$ in terms of x. ---[2]
(iii) Hence express y in terms of x, giving your answer in the form:
$y = A(10^{bx^2})$ where A and b are constants. W-17/11/Q4 ---[3]

Q12 Solve the equation $\log_5(10x+5) = 2 + \log_5(x-7)$ W-17/21/Q3 ---[4]

Q13 When $\lg y$ is plotted against x, a straight line is obtained which passes through the points $(0 \cdot 6, 0 \cdot 3)$ and $(1 \cdot 1, 0 \cdot 2)$
(i) Find $\lg y$ in terms of x.
(ii) Find y in terms of x, giving answer in the form $y = A(10^{bx})$, where A and b are constants. W-17/12/Q5 ---[3]

Q14 When $\ln y$ is plotted against x^2 a straight line is obtained which passes through the points $(0 \cdot 2, 2 \cdot 4)$ and $(0 \cdot 8, 0 \cdot 9)$.
(i) Express $\ln y$ in the form $px^2 + q$, where p and q are constants; ---[3]
(ii) Hence express y in terms of z, where $z = e^{x^2}$ W-17/13/Q6 [3]

Q15 Solve $\log_5 \sqrt{x} + \log_{25} x = 3$ ---[3]

M-16/12/Q3

Q16 Do not use a calculator in this question:

(i) Find the value of $-\log_p p^2$ ---[1]

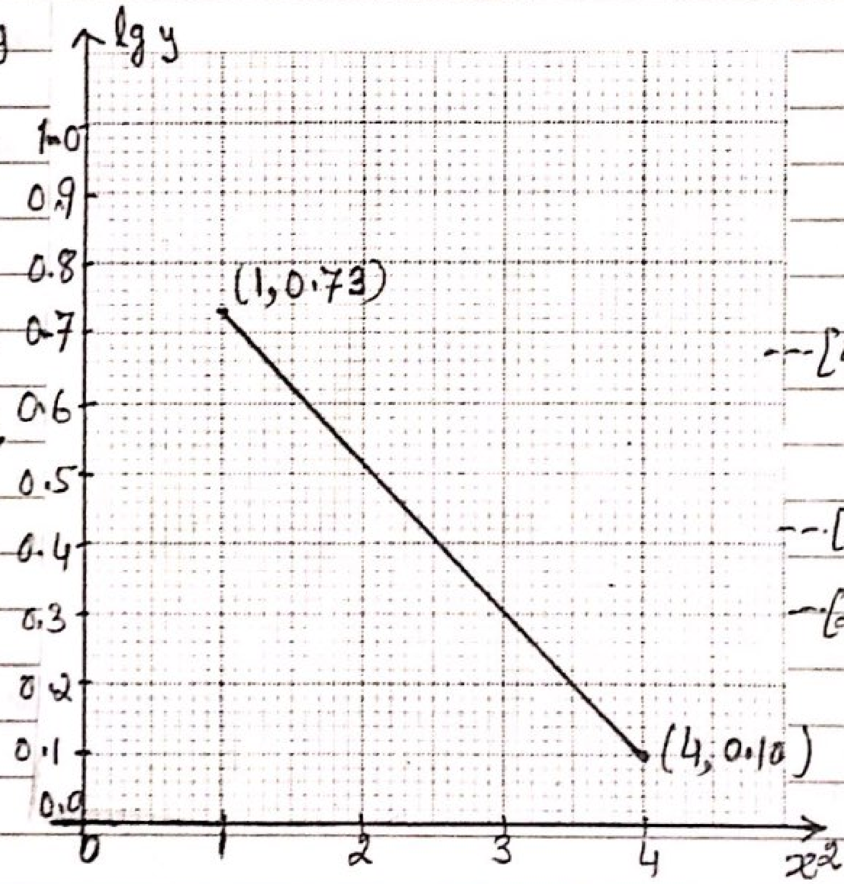
(ii) Find $\lg\left(\frac{1}{10^n}\right)$ ---[1]

(iii) Show that: $\frac{\lg 20 - \lg 4}{\log_5 10} = (\lg y)^2$, where y is a constant to be found. ---[2]

(iv) Solve $\log_2 2x + \log_2 3x = \log_2 600$ ---[2]

S-16/21/Q3

Q17 Variables x and y are such that when $\lg y$ is plotted against x^2, the straight line graph shown, is obtained.



(i) Given that $y = Ab^{x^2}$, find the value of A and b. ---[4]

(ii) Find the value of y when $x = 1.5$ ---[2]

(iii) Find the positive value of x when $y = 2$. ---[2]

S-16/12/Q8

Q18 (a) The graph of the curve $y = p(4^{2x}) - q(4^x)$ passes through the points $(0, 2)$ and $(0.5, 14)$. Find the value of p and of q. ---[3]

(b) The variables x and y are connected by the equation, $y = 10^{2x} - 2(10^x)$. Using substitution $u = 10^x$, or otherwise, Find exact value of x when $y = 24$. ---[3]

(c) Solve $\log_2(x+1) - \log_2 x = 3$ ---[3]

S-16/22/Q10

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Q19 By using substitution $y = \log_3 x$, or otherwise, find the values of x for which. --- [6]

$$3(\log_3 x)^2 + \log_3 x^5 - \log_3 9 = 0$$

W-15/11/Q3

Q20 The variables x and y are such that when $\ln y$ is plotted against x, a straight line graph is obtained. This line passes through the points $x=4$, $\ln y = 0.20$ and $x = 12$, $\ln y = 0.08$.

(i) Given that $y = Ab^x$, find the value of A and of b. --[5]

(ii) Find the value of y when $x = 6$ --[2]

(iii) Find the value of x when $y = 1.1$ W-16/11/Q11 --[2]

Q21 Solve the equation $2\lg x - \lg\left(\frac{x+10}{2}\right) = 1$ ----[5]

W-16/21/Q3

Q22 The number of bacteria, N, present in a culture can be modelled by the equation $N = 7000 + 2000e^{-0.05t}$, where t is measured in days. Find

(i) the number of bacteria when $t = 10$, ---[1]

(ii) the value of t when the number of bacteria reaches 7500, --[3]

(iii) the rate at which the number of bacteria is decreasing after 8 days

W-16/21/Q4 ---[3]

Q23 (i) Given that $\log_9 xy = \frac{5}{2}$ show that $\log_3 x + \log_3 y = 5$ ---[3]

(ii) Hence solve the equations. ---[5]

$$\log_9 xy = \frac{5}{2}$$

$$\log_3 x \times \log_3 y = -6$$

W-16/13/Q5

Q24 Solve the equation $e^{3x} = 6e^x$ --[3]

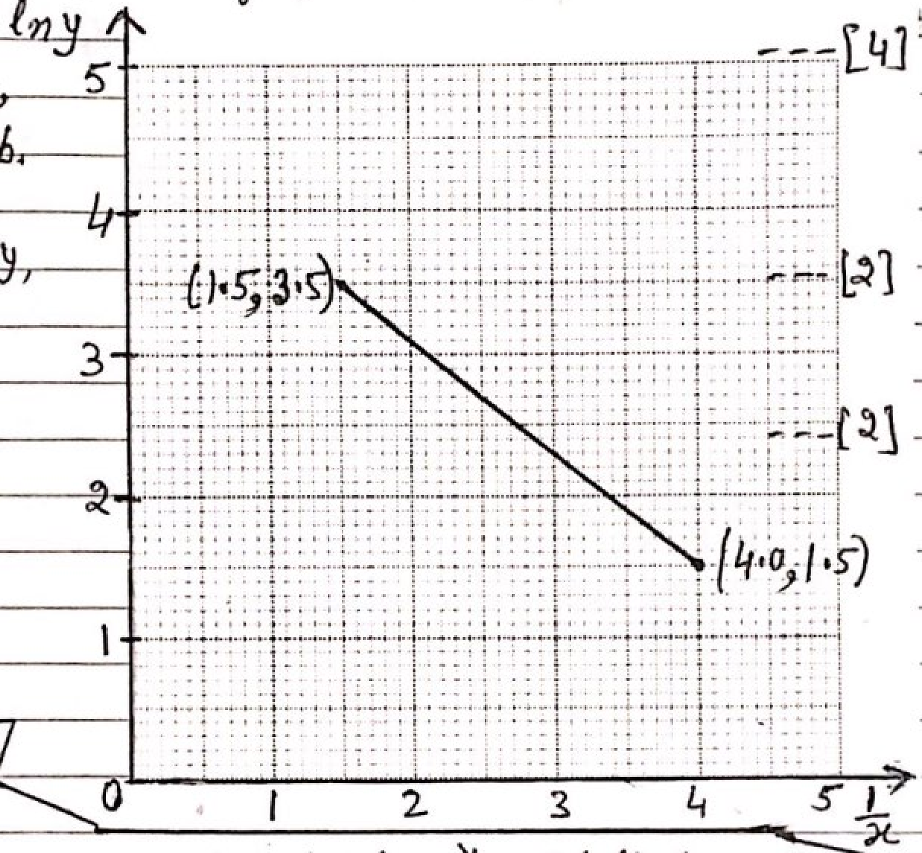
W-16/23/Q2

Q25 The variables x and y are such that when $\ln y$ is plotted against $\frac{1}{x}$ the straight line graph shown is obtained:

(i) Given that $y = Ae^{b/x}$, find the value of A and of b. ---[4]

(ii) Find the value of y, when $x = 0.32$ ---[2]

(iii) Find the value of x, when $y = 20$. ---[2]



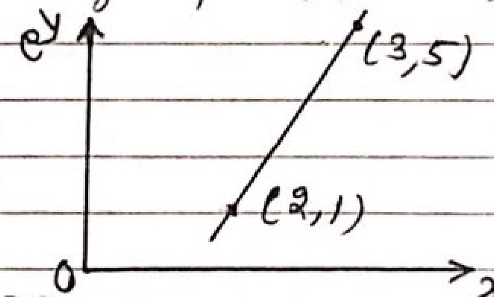
W-16/13/Q7

Q26 Variables x and y are such that when e^y is plotted against x a straight line graph is obtained. The diagram shows this straight line graph which passes through the points $(2,1)$ and $(3,5)$

(i) Express y in terms of x. ---[4]

(ii) State the value of x for which y exists. ---[1]

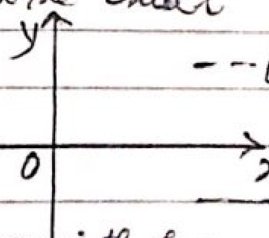
(iii) Find the value of x when $y = \ln 6$ ---[1]



M-15/12/Q5

Q26' (a) (i) Sketch the graph of $y = e^x - 5$ on the axes, show the exact coordinates of any points where the graph meets the coordinate axes. ---[3]

(ii) Find the range of values of k for which the equation $e^x - 5 = k$ has no solution. ---[1]



(b) Simplify $\log_a \sqrt{2} + \log_a 8 + \log_a\left(\frac{1}{2}\right)$, giving your answer in the form $p\log_a 2$, where a is constant. ---- [2]

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Q26(c) Solve the equation $\log_3 x - \log_9 4x = 1$ [M-15/22/Q10] ---[4]

Q27 (a) Write $\log_{27} x$ as a logarithm to base 3. ---[2]

(b) Given that $\log_a y = 3(\log_a 15 - \log_a 3) + 1$, express y in terms of a ---[3]

[S-15/21/Q1]

Q28 (a) Solve $6^{x-2} = \frac{1}{4}$ ---[2]

(b) Solve $\log_a 2y^2 + \log_a 8 + \log_a 16y - \log_a 64y = 2\log_a 4$ ---[4] [S-15/22/Q6]

Q29 Two variables, x and y are such that $y = Ax^b$, where A and b are constants. When $\ln y$ is plotted against $\ln x$, a straight line graph is obtained, which passes through the points (1.4, 5.8) and (2.2, 6.0).

(i) Find the value of A and of b. ---[4]

(ii) Calculate the value of y when $x = 5$ [W-15/11/Q7] ---[2]

Q30 Solve the equation: $\log_2(29x - 15) = 3 + \frac{2}{\log_x 2}$ [W-15/23/Q6] ---[5]

Q31 (a) Solve $2^{x^2-5x} = \frac{1}{64}$ ---[4]

(b) By changing the base of $\log_{2a} 4$, express $(\log_{2a} 4)(1 + \log_a 2)$ as a single logarithm to base a.

[S-14/21/Q11] ---[4]

Q32 (a)(i) State the value of u for which $\lg u = 0$ ---[1]

(ii) Hence solve $\lg|2x+3| = 0$ ---[2]

(b) Express $2\log_3 15 - (\log_a 5)(\log_3 a)$ where $a > 1$, as a single logarithm to base 3. [S-14/22/Q6] ---[4]

Q33 Using the substitution $u = \log_3 x$,

Solve, for x, the equation $\log_3 x - 12\log_x 3 = 4$ ---[5]

[S-14/23/Q2]

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Q34 The number of bacteria B in a culture, t days after the first observation is given by $B = 500 + 400e^{0.2t}$

(i) Find the initial number present. --- [1]

(ii) Find the number present after 10 days. --- [1]

(iii) Find the rate at which the bacteria are increasing after 10 days. --- [2]

(iv) Find the value of t when $B = 10\,000$ [W-14/21/Q5] --- [3]

Q35 Solve the equation $1 + 2\log_5 x = \log_5 (18x - 9)$ [W-14/13/Q7] --- [5]

Q36 The profit \$$P$ made by a company in its n_{th} year is modelled by,

$$P = 1000\,e^{an+b}$$

In the first year the company made \$2000 profit.

(i) Show that $a + b = \ln 2$ --- [1]

In the second year company made \$ 3297 profit.

(ii) Find another linear equation connecting a and b --- [2]

(iii) Solve the two equations from part (i) and part (ii) to find the value of a and b. --- [2]

(iv) Using your values of a and b, find the profit in the 10_{th} year --- [2]

[W-14/23/Q4]

Q37 (i) Given that $\log_4 x = \frac{1}{2}$, find the value of x. --- [1]

(ii) Solve. $2\log_4 y - \log_4 (5y - 12) = \frac{1}{2}$ [S-13/11/Q4] --- [4]

Q38 Given that $\log_p X = 5$ and $\log_p Y = 2$ find.

(i) $\log_p X^2$ --- [1]

(ii) $\log_p \frac{1}{X}$ --- [1]

(iii) $\log_{XY} p$ [W-13/21/Q4] --- [2]

Q39 Solve $2\lg y - \lg (5y + 60) = 1$ [W-13/13/Q2] --- [5]

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Answers

Q1(a) $\lg y = x^2 \lg b + \lg A$

$\lg b = \pm 0.21 \rightarrow b = 0.617$ ✓

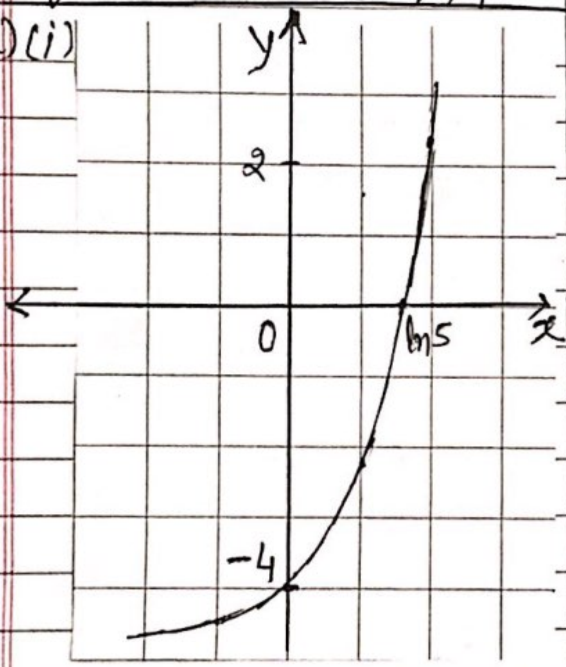
$\lg A = 0.94 \rightarrow A = 8.71$ ✓

(b) $x = 1.5,\ x^2 = 2.25$

$y = 2.93$

(c) $\lg y = 0.301 \rightarrow x = 1.74$

Q2 (a) (i)



(ii) $k \leq -5$

(b) $2\frac{1}{2}\log_a 2$

(c) $\log_9 4x = \dfrac{\log_3 4x}{\log_3 9} = \frac{1}{2}\log_3 4x$

$\therefore \log_3 x - \log_9 4x = 1 \rightarrow \log_3 x - \frac{1}{2}\log_3 4x = 1$

$\rightarrow \log_3 \dfrac{x}{(4x)^{1/2}} = \log_3 3 \rightarrow \dfrac{x^2}{4x} = 9$

$\rightarrow x = 36$ ✓

Q3(a) (i) $3x + 4 = \ln\left(\frac{14}{5}\right)$

$= \ln 14 - \ln 5$

$\rightarrow x = -0.99$ ✓

(ii) $\lg(2y^2 - 7y = \lg 3^2$

$\rightarrow 2y^2 - 7y - 9 = 0 \rightarrow y = 4.5$ ✓

Q3(b) $\log_2 \left(\frac{b}{q}\right)$

Q4 (i) $\ln y = \ln A + bx$

$0.7 = \ln A + b$

$3.7 = \ln A + 2.5b$

$\rightarrow b = 2$ ✓, $\ln A = -1.3$

$\Rightarrow A = 0.273$ ✓

(ii) $\ln y = -1.3 + 2x$

$x = 2,\ \ln y = 2.7 \rightarrow y = 14.9$ ✓

Q5 (i) \$12000

(ii) $\dfrac{8000}{12000} = e^{-0.2t} \rightarrow t = 2.0273$ ✓ years

Q6 (i) In $\log_a(4x-3)$; $4x - 3 > 0$

$\Rightarrow x > 0.75$ ✓

(ii) $y = \log_a ax + \log_a (4x-3)^2 - \log_a a$

$= \log_a \dfrac{\cancel{a}x(4x-3)^2}{\cancel{a}}$

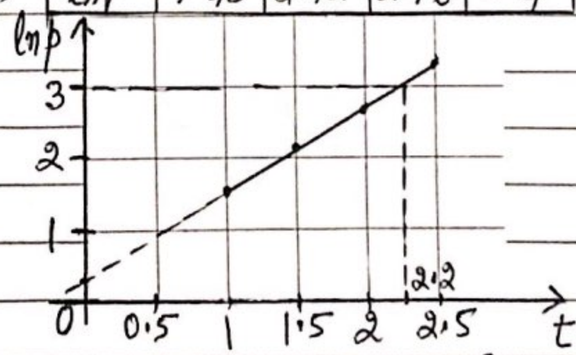
$= \log_a (16x^3 - 24x^2 + 9x)$ ✓

(iii) $\log_a (16x^3 - 24x^2 + 9x) = \log_a 9x$

$\Rightarrow x^2(16x - 24) = 0 \Rightarrow x = \frac{24}{16} = \frac{3}{2}$ ✓

Q7

t	1	1.5	2	2.5
(i) lnP	1.48	2.12	2.76	3.4



(ii) $x = 2.2 \rightarrow \ln P = 3 \rightarrow P = e^3 = 20$ ✓

(iii) $m = \dfrac{3.4 - 1.48}{2.5 - 1} = \dfrac{1.92}{1.5} = 1.28$ ✓

y-int $= 0.2$ ✓

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Answers

$(2 = \log_5 25)$

Q7(iv) $\ln P = 1.28t + 0.2$ $(y = mx + c)$

$\Rightarrow P = e^{0.2} \cdot e^{1.28t}$

$\Rightarrow P = 1.22\, e^{1.28t}$ ✓

(v) $e^{0.2} \times e^{1.28t} \geq 1000$

$\Rightarrow t = 5.3$ ✓

Q8(i) $\lg y = \lg A + bx$

Gradient $b = 3$ ✓

Using substitution, $2.2 = \lg A + 0.5b$, $3.7 = \lg A + b$

$\Rightarrow \lg A = 0.7 \rightarrow A = 5$ ✓

(ii) $\lg y = \lg A + bx$

$x = 0.6 \rightarrow \lg y = 0.7 + 3 \times 0.6 = 2.5$

$\rightarrow y = 10^{2.5} = 316$ ✓

(iii) $\lg 600 = 0.7 + 3x \Rightarrow x = 0.693$ ✓

Q9 (a)(i) 7

(ii) $1/7$

(b) $y = (81)^{-\frac{1}{4}} = 3^{-1} = \frac{1}{3} = 0.333$ ✓

(c) $\dfrac{2^{5(x^2-1)}}{(2^2)^{x^2}} = 16 \Rightarrow 2^{5(x^2-1)-2x^2} = 2^4$

$\Rightarrow 3x^2 - 5 = 4 \rightarrow x = \pm\sqrt{3}$ ✓

Q10 $\log_7 2.5 = 2x + 5 \Rightarrow x = \dfrac{\log_7 2.5 - 5}{2}$

$\rightarrow x = \frac{1}{2}\left(\dfrac{\log 2.5}{\log 7} - 5\right) = -2.26$ ✓

Q11(i) $\dfrac{7-3}{12-4} = m = \frac{1}{2}$ ✓

(ii) $\lg y = mx^2 + c$

or $\lg y = \frac{1}{2}x^2 + 1$ ✓

(iii) $y = 10^{(\frac{1}{2}x^2 + 1)}$

$y = 10 \cdot 10^{x^2/2}$ ✓

Q12 $\log_5(10x+5) = \log_5 25 + \log_5(x-7)$

$\Rightarrow \log_5(10x+5) = \log_5 25(x-7)$

$\Rightarrow 10x + 5 = 25(x-7) \Rightarrow x = 12$ ✓

Q13 (i) Gradient $= -0.2$

$\lg y = -0.2x + 0.42$ ✓

(ii) $y = 10^{(0.42 - 0.2x)}$

or $y = 10^{0.42} \cdot 10^{-0.2x}$

or $y = 2.63\; 10^{-0.2x}$ ✓

Q14 (i) Gradient $= -2.5$

$\ln y = -\frac{5}{2}x^2 + c$

$\ln y = -\frac{5}{2}x^2 + 2.9$ ✓

(ii) $y = e^{(-5/2 x^2 + 2.9)}$

$= e^{2.9} \cdot e^{-5/2 x^2}$

$y = 18.2\, z^{-5/2}$ ✓

Q15 $\log_5 \sqrt{x} + \log_{25} x = 3$

$\frac{1}{2}\log_5 x + \dfrac{\log_5 x}{\log_5 25} = 3 \rightarrow \log_5 x = 3$

$\rightarrow x = 5^3 = 125$ ✓

Q16 (i) -2 (ii) $-n$

(iii) $\dfrac{\lg 20 - \lg 4}{1/\lg 5} = (\lg\ )^2$ ✓

(iv) $\log_2 6x^2 = \log_2 600 \Rightarrow x = 10$ ✓

Q17(i) $\lg y = x^2 \lg b + \lg A$

$\lg b = -0.0.21 \Rightarrow b = 0.617$ or 0.62 ✓

$\lg A = 0.94 \rightarrow A = 8.71$ ✓

(ii) $x = 1.5 \rightarrow x^2 = 2.25 \rightarrow y = 2.93$ ✓

(iii) $\lg y = 0.301 \rightarrow x = 1.74$ ✓

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Answers

Q18 (a) $2 = p - q$ and $14 = 4p - 2q$

$\Rightarrow p = 5,\ q = 3$ ✓

(b) $10^{2x} - 2.10^{x} - 24 = 0$

or $u^2 - 2u - 24 = 0 \rightarrow u = 6$

$\therefore 10^{x} = 6 \rightarrow x = \lg 6$ ✓

(c) $\frac{x+1}{x} = 2^3 \Rightarrow x = \frac{1}{7}$ or 0.143 ✓

Q19 $3y^2 + 5y - 2 = 0 \Rightarrow y = \frac{1}{3},\ y = -2$

$\therefore x = 3^{1/3}$, $x = 3^{-2}$

$x = 1.44$ or $x = 1/9$

Q20 (i) $\ln y = \ln A + x \ln b$

Gradient $\ln b = -\frac{0.12}{8} = -0.015$

$b = 0.985$ ✓

Intercept $\ln A = 0.26 \Rightarrow A = 1.30$ ✓

(ii) when $x = 6$, $\ln y = 0.17$

$\Rightarrow y = 1.19$ ✓

(iii) when $y = 1.1$; $\ln y = 0.095$

$\rightarrow x = 11$

Q21 $2\lg x - \lg\left(\frac{x+10}{2}\right) = 1$

$\Rightarrow \lg x^2 - \lg\left(\frac{x+10}{2}\right) = \lg 10$

$\Rightarrow \lg\left(\frac{2x^2}{x+10}\right) = \lg 10$

$\Rightarrow 2x^2 - 10x - 100 = 0$

$\rightarrow 2(x+5)(x-10) = 0 \Rightarrow x = 10$ ✓

Q22 (i) $t \rightarrow 10 \Rightarrow N = 7000 + 2000\,e^{-0.5}$

or $N = 8213$ ✓

(ii) $N = 7500 = 7000 + 2000\,e^{-0.05t}$

$\Rightarrow e^{-0.05t} = \frac{500}{2000} = 0.25$

$\Rightarrow -0.05t = \ln 0.25 \rightarrow t = \frac{\ln 0.25}{-0.05}$

days $= 27.7$ ✓

(iii) $\frac{dN}{dt} = -100\,e^{-0.05t}$

$\therefore t = 8 \rightarrow \frac{dN}{dt} = -67$

Q23 (i) $\log_9 xy = \log_9 x + \log_9 y$

$= \frac{\log_3 x}{\log_3 9} + \frac{\log_3 y}{\log_3 9}$

or $\frac{\log_3 x}{2} + \frac{\log_3 y}{2} = \frac{5}{2} \Rightarrow \log_3 x + \log_3 y = 5.$

(ii) $\log_3 x\,(5 - \log_3 x) = -6$

$\rightarrow (\log_3 x)^2 - 5\log_3 x - 6 = 0$

$\Rightarrow \log_3 x = 6$ or $\log_3 x = -1$

$x = 729$, $y = \frac{1}{3}$; $x = 1/3$, $y = 729$

Q24 $\ln e^{3x} = \ln 6e^{x}$

$3x = \ln 6 + \ln e^{x}$

$3x = \ln 6 + x$

$\Rightarrow x = \frac{1}{2}\ln 6 = 0.896$ ✓

Q25 (i) $\ln y = \ln A + \frac{b}{x}$

Gradient $b = -0.8$ ✓

Intercept $\ln A = 4.7 \rightarrow A = 110$ ✓

(ii) $x = 0.32 \rightarrow \frac{1}{x} = 3.125$, $\ln y = 2.2$

$\rightarrow y = 9$ ✓

(iii) $y = 20 \rightarrow \ln y = 3$, $\frac{1}{x} = 2.125$

$\rightarrow x = 0.47$ ✓

Q26 (i) gradient $= 4$

using $(2,1)$ or $(3,5)$, $c = -7$

$\rightarrow e^{y} = 4x - 7 \rightarrow y = \ln(4x-7)$ ✓

(ii) $x > \frac{7}{4}$

(iii) $\ln 6 = \ln(4x-7) \rightarrow x = \frac{13}{4}$ ✓

Q26' same as Q2

Logarithmic and Exponential Functions

Answers continued →

Q27 (a) $\dfrac{\log_3 x}{\log_3 27} = \dfrac{\log_3 x}{3}$ ✓

(b) $\left(\log_a 15 - \log_a 3 = \log_a 5\right)$; $1 = \log_a a$

$\therefore \log_a y = \log_a 5^3 \cdot a = \log_a 125a$

$\Rightarrow \underline{y = 125a}$ ✓

Q28 (a) $(x-2)\log 6 = \log\left(\frac{1}{4}\right)$

$\Rightarrow (x-2) = \dfrac{\log \frac{1}{4}}{\log 6} = \log_6 \frac{1}{4}$

$\therefore x = \underline{1.23}$ ✓

(b) $\log 2 + 2\log y + 3\lg 2 + 4\lg 2 + \log y$
$- 6\log 2 - \log y = 4\log 2$

$\Rightarrow \underline{y = 2}$ ✓

Q29 Gradient $= \dfrac{0.2}{0.8} = 0.25 = b$ ✓

(i) and $6 = 0.25(2.2) + c$

$\Rightarrow c = 5.45 = \ln A$

$\Rightarrow A = e^{5.45} = 233$ ✓

(ii) $y = 233 \times 5^{0.25}$ (for $x = 5$)

or $\ln y = 0.25 \ln 5 + \ln 233$

$\longrightarrow \underline{y = 348}$ ✓

Q30 $\log_x 2 = \dfrac{\log_2 2}{\log_2 x}$

$\therefore \dfrac{2}{\log_x 2} = 2\log_2 x = \log_2 x^2$
and $3 = \log_2 8$

$\therefore \log_2(29x-15) = 3 + 2/\log_x 2$

$\Rightarrow \log_2(29x-15) = \log_2 8 + \log_2 x^2$

(continued →)

Q30 $\rightarrow 8x^2 - 29x + 15 = 0$

$(8x-5)(x-3) = 0 \rightarrow \underline{x = 5/8,\ x = 3}$ ✓

Q31 (a) $2^{x^2-5x} = 2^{-6}$

$\Rightarrow x^2 - 5x + 6 = 0 \rightarrow x = 2,\ x = 3$ ✓

(b) $\log_{2a} 4 = \dfrac{\log_a 4}{\log_a 2a} = \dfrac{\log_a 4}{\log_a 2 + \log_a a}$

$\therefore \log_{2a} 4 = \dfrac{\log_a 4}{\log_a 2 + 1}$

$\therefore \log_{2a} 4 \cdot (1 + \log_a 2) = \dfrac{\log_a 4}{(1+\log_a 2)} \times (1 + \log_a 2)$

$= \underline{\log_a 4}$ ✓

Q32 (a) (i) 1

(ii) $x = -1$ or -2

(b) $2\log_3 15 - (\log_a 5)(\log_3 a)$

$= \log_3 15^2 - \dfrac{\log_3 5}{\log_3 a} \times \log_3 a = \log_3\left(\dfrac{15^2}{5}\right)$

$= \underline{\log_3 45}$ ✓

Q33 $\log_3 x - 12\log_x 3 = 4$

$\rightarrow \log_3 x - 12 \times \dfrac{1}{\log_3 x} = 4$

$\rightarrow u^2 - 4u - 12 = 0 \quad [u = \log_3 x]$

$u = 6, -2$

$\log_3 x = 6$ or $\log_3 x = -2$

$x = 729,\ x = \frac{1}{9}$ ✓

Answers

Q34 (i) $B = 900$

(ii) $B = 500 + 400e^2 = 3455$

(iii) $\frac{dB}{dt} = 80e^{0.2t}$

$t = 10 \rightarrow \frac{dB}{dt} = 80e^2 = 591$ (days)

(iv) $10000 = 500 + 400e^{0.2t}$

$\rightarrow e^{0.2t} = 23.75$

$0.2t = \ln 23.75 \Rightarrow t = 15.8$ ✓ (days)

Q35 $\log_5 5 + \log_5 x^2 = \log_5 (18x - 9)$

$5x^2 = 18x - 9 \rightarrow (5x-3)(x-3) = 0$

$x = 3/5, 3$

Q36 (i) $2000 = 1000e^{a+b} \rightarrow a+b = \ln 2$ ✓

(ii) $3297 = 1000e^{2a+b} \Rightarrow 2a+b = \ln 3.297$

(iii) $a = 0.5$ and $b = 0.193$

(iv) $n = 10 \rightarrow P = 1000e^{5.193}$

$= \$180000.$ ✓

Q37 (i) 2

(ii) $\log_4 y^2 - \log_4 (5y-12) = \log_4 2$

$\log_4 \frac{y^2}{5y-12} = \log_4 2$

$\rightarrow y^2 - 10y + 24 = 0$

$y = 4, 6$ ✓

Q38 (i) 10 (ii) -5

(iii) $\log_p XY = \frac{1}{\log_{p} XY}$

$= \frac{1}{\log_p X + \log_p Y}$

$= \frac{1}{5+2} = \frac{1}{7}$ ✓

Q39 $\lg \frac{y^2}{5y+60} = \lg 10$

$\rightarrow y^2 - 50y - 600 = 0$

$\rightarrow y = -10^{\times}, 60$

$\therefore y = 60$ ✓

⟵ X ——— X ⟶