IG-Maths
0606
Additional Maths

Quadratic Functions
Exercise.

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Quadratic functions

Q1 (a) Express $12x^2-6x+5$ in the form $p(x-q)^2+r$, where p, q and r are constants to be found. ---[3]
(b) Hence find the greatest value of $(12x^2-6x+5)^{-1}$ and state the value of x at which this occur. [SP-20/01/Q3] ---[2]

Q2 Determine the set of values of k for which the equation, $(3-2k)x^2+(2k-3)x+1=0$ has no real roots. ---[5]
[M-18/22/Q2]

Q3 Find the set of values of k for which the line $y=3x+k$ and the curve $y=2x^2-3x+4$ do not intersect. [M-17/22/Q4] --[4]

Q4 The line $y=kx-5$, where k is a positive constant, is a tangent to curve $y=x^2+4x$ at the point A.
(i) Find the exact value of k. ---[3]
(ii) Find the gradient of the normal to the curve at point A; --[2]
giving your answer in the form $a+b\sqrt{5}$, where a and b are constant.
[S-17/11/Q1]

Q5 Show that the roots of $px^2+(p-q)x-q=0$ are real for all real values of p and q. [S-17/22/Q6] ---[4]

Q6 The line $y=kx+3$, where k is a positive constant, is a tangent to the curve $x^2-2x+y^2=8$ at the point P.
(i) Find the value of k. ---[4]
(ii) Find the coordinates of P. ---[3]
(iii) Find the equation of normal to the curve at P. ---[2]
[W-17/21/Q11]

Q7 Find the set of values of k for which the equations ---[4]
$kx^2+3x-4+k=0$ has no real roots. [W-17/13/Q3]

Q8. Find the values of a for which the line $y=ax+9$ intersects the curve $y=-2x^2+3x+1$ at 2 distinct points. ---[4]

[M-16/12/Q1]

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Q9 The line $2y = x + 2$ meets the curve $3x^2 + xy - y^2 = 12$ at the point A and B.
(i) Find the coordinates of the point A and B. ---[5]
(ii) Given that the point C has coordinate (0,6), show that the triangle ABC is right-angled. M-16|22|Q8 ---[2]

Q10 Find the value of k for which the curve $y = 2x^2 - 3x + k$
(i) Passes through the point (4,−7). ---[1]
(ii) Meets the x-axis at one point only S-16|11|Q1 --[2]

Q11 (i) Express $4x^2 + 8x - 5$ in the form $p(x+q)^2 + r$, where p, q and r are constants to be found. ---[3]
(ii) State the coordinates of the vertex of $y = |4x^2 + 8x - 5|$ --[2]
(iii) Sketch the graph of $y = |4x^2 + 8x - 5|$, showing the coordinates of the point where the curve meets the axes. S-16|21|Q6 --[3]

Q12 (i) Given that $x^2 + 2kx + 4k - 3 = 0$ has no real roots, show that k satisfies $k^2 - 4k + 3 < 0$ ---[2]
(ii) Solve the inequality $k^2 - 4k + 3 < 0$ S-16|22|Q1 --[2]

Q13 (i) Given that $3x^2 + p(1-2x) = -3$, show that, for x to be real, $p^2 - 3p - 9 \geq 0$ ---[3]
(ii) Hence find the set of values of p for which x is real, expressing your answer in exact form. W-16|13|Q3 --[3]

Q14 Find the values of k for which the line $y = kx - 3$ does not meet the curve $y = 2x^2 - 3x + k$. M-15|12|Q2 --[5]

Q15 (a) Find the set of values of x for which $4x^2 + 19x - 5 \leq 0$ --[3]
(b)(i) Express $x^2 + 8x - 9$ in the form $(x+a)^2 + b$, where a and b are integers --[2]

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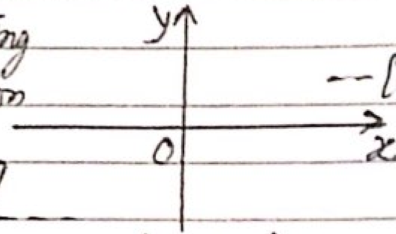
Q15 (b)(ii) Use your answer to part (i) to find the greatest value of $9-8x-x^2$ and the value of x at which this occurs. --[2]

(iii) Sketch the graph of $y=9-8x-x^2$, indicating the coordinates of any points of intersection with the coordinate axes. --[2]



[S-15/21/Q9]

Q16 Given that the graph of $y=(2k+5)x^2+kx+1$ does not meet the x-axis, find the possible values of k. [S-15/12/Q1] --[4]

Q17 Find the range of values of k for which the equation, $kx^2+k=8x-2xk$ has two real distinct roots. --[4]

[W-15/11/Q1]

Q18 Find the value of k for which the line $y=2x+k+2$ cuts the curve $y=2x^2+(k+2)x+8$ in two distinct points. --[6]

[W-15/23/Q2]

Q19 Find the set of values of k for which the line $y=k(4x-3)$ does not intersect the curve $y=4x^2+8x-8$ [S-14/11/Q4] --[5]

Q20 (i) Express $2x^2-x+6$ in the form $p(x-q)^2+r$, where p, q and r are constants to be found. [S-14/21/Q5] --[3]

(ii) Hence state the least value of $2x^2-x+6$ and the value of x at which this occurs. --[2]

Q21 Find the values of k for which the line $y+kx-2=0$ is a tangent to the curve $y=2x^2-9x+4$. [S-14/22/Q2] --[5]

Q22 (i) Express $12x^2-6x+5$ in the form $p(x-q)^2+r$, where p, q and r are constants to be found. --[3]

(ii) Hence find the greatest value of $\dfrac{1}{12x^2-6x+5}$ and state the value of x at which this occurs. --[2]

[S-14/22/Q4]

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Q23 The line $y = x - 5$ meets the curve $x^2 + y^2 + 2x - 35 = 0$ at the points A and B. Find the exact length of AB. --- [6]
$\boxed{S\text{-}14/22/Q8}$

Q24 (i) Show that $y = 3x^2 - 6x + 5$ can be written in the form, $y = a(x-b)^2 + c$, where a, b and c are constants to be found. --- [3]

(ii) Hence, or otherwise, find the coordinates of the stationary point of the curve $y = 3x^2 - 6x + 5$ $\boxed{S\text{-}14/13/Q1}$ --- [1]

Q25 (i) Calculate the coordinates of the points where the line $y = x + 2$ cuts the curve $x^2 + y^2 = 10$ --- [4]

(ii) Find the exact value of m for which the line $y = mx + 5$ is a tangent to the curve $x^2 + y^2 = 10$ $\boxed{W\text{-}14/21/Q6}$ --- [4]

Q26 The line $y = 2x - 8$ cuts the curve $2x^2 + y^2 - 5xy + 32 = 0$ at the points A and B. Find the length of the line AB. $\boxed{S\text{-}13/21/Q8}$ --- [7]

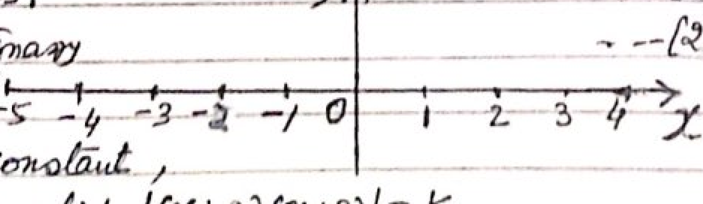
Q27 Find the set of values of k for which the curve $y = 2x^2 + kx + 2k - 6$ lies above the x-axis for all values of x. $\boxed{S\text{-}13/12/Q4}$ --- [4]

Q28 The line $3x + 4y = 15$ cuts the curve $2xy = 9$ at the points A and B. Find the length of line AB. $\boxed{S\text{-}13/12/Q5}$ --- [6]

Q29 Find the set of values of k for which the curve, $y = (k+1)x^2 - 3x + (k+1)$ lies below x-axis. $\boxed{W\text{-}13/11/Q2}$ --- [4]

Q30 (i) On the grid below, sketch the graph of $y = |(x-2)(x+3)|$ for $-5 \leq x \leq 4$, and state the coordinates of the points where the curve meets the coordinate axes. --- [4]

(ii) Find the coordinates of the stationary point on the curve $y = |(x-2)(x+3)|$ --- [2]



(iii) Given that k is a positive constant, state the set of values of k for which $|(x+2)(x+3)| = k$ has only 2 solutions. $\boxed{W\text{-}13/11/Q8}$ --- [1]

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Q31 Find the set of values of k, for which the line $y = 3x - k$ does not meet the curve $y = kx^2 + 11x - 6$ [W-13/23/Q3] --[6]

Answers

Q1 (i) $12\left(x - \frac{1}{4}\right)^2 + \frac{17}{4}$ ✓

(b) $\frac{4}{17}$ is the greatest value at $x = \frac{1}{4}$. ✓

Q2 for No solution $b^2 - 4ac < 0$

$\Rightarrow (2k-3)^2 - 4(3-2k)\times 1 < 0$

$\Rightarrow 4k^2 - 4k - 3 < 0$

or $(2k-3)(2k+1) < 0$

$\Rightarrow -0.5 < k < 1.5$ ✓

Q3 Eliminating y.

$3x + k = 2x^2 - 3x + 4$

or $2x^2 - 6x + 4 - k = 0$

for no solution $b^2 - 4ac < 0$

$(-6)^2 - 4\times 2\times(4-k) < 0$

or $k < -\frac{1}{2}$ ✓

Q4(i) Eliminating y.

$kx - 5 = x^2 + 4x$

or $x^2 + (4-k)x + 5 = 0$

for tangent $b^2 - 4ac = 0$

$\Rightarrow (4-k)^2 - 20 = 0$

$\Rightarrow k = 4 + 2\sqrt{5}$ ✓

(ii) $\frac{-1}{4+2\sqrt{5}} = 1 - \frac{\sqrt{5}}{2}$ ✓

Q5 $b^2 - 4ac = (p-q)^2 - 4p(-q)$

$= (p+q)^2 \geq 0$

∴ has real roots for all value of p and q.

Q6 (i) Eliminating y

$x^2 - 2x + (kx+3)^2 = 8$

or $(1+k^2)x^2 + (6k-2)x + 1 = 0$

for tangent $b^2 - 4ac = 0$

or $(6k-2)^2 - 4(1+k^2) = 0$

$\Rightarrow k = 3/4$ ✓

(ii) $P(-0.8, 2.4)$ $\left[x = -\frac{b}{2a}\right]$

(iii) Eqn of Normal PQ, $\frac{y - 2.4}{x + 0.8} = \frac{-4}{3}$

or $3y = 4 - 4x$ ✓

Q7 for no real roots $b^2 - 4ac < 0$

or $9 - 4k(k-4) < 0$

$\Rightarrow 4k^2 - 16k - 9 > 0$

$(2k-9)(2k+1) > 0$

Critical values $9/2, -\frac{1}{2}$

∴ k: $k < -\frac{1}{2}$ or $k > 9/2$

Q8 Eliminating y

$ax + 9 = -2x^2 + 3x + 1$

or $2x^2 + (a-3)x + 8 = 0$

for 2 distinct points $b^2 - 4ac > 0$

or $(a-3)^2 > 64$

Critical values $-5, 11$

∴ a: $a < -5$ or $a > 11$

Q9 Eliminating y, $13y^2 - 26y = 0$

$\Rightarrow y(y-2) = 0 \Rightarrow \begin{cases} y = 0 \\ x = -2 \end{cases} A$ or $\begin{cases} y = 2 \\ x = 2 \end{cases} B$

$C(0, 6)$

Slope of AB, $m_1 = \frac{1}{2}$ and slope of BC $m_2 = -2$

$m_1 \times m_2 = -1$ ✓

∴ rt triangle.

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Answers

Q10 (i) -27

(ii) meets x-axis $2x^2-3x+k=0$

for one point, $b^2-4ac=0$

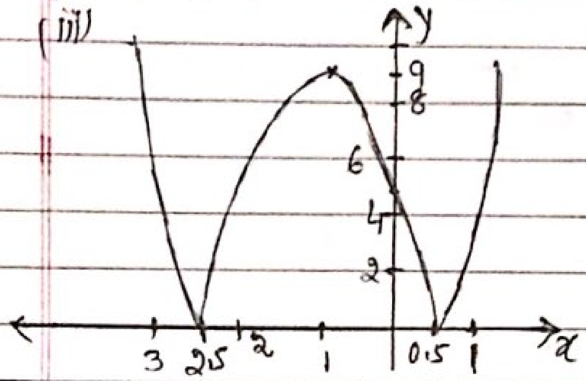
$9-8k=0$

or $k=\frac{9}{8}$

Q11 (i) $4(x+1)^2-9$

(ii) $(-1, 9)$

(iii)



Q12 (i) for no real roots $b^2-4ac<0$

$\Rightarrow (2k)^2-4\times1\times(4k-3)<0$

$\Rightarrow k^2-4k+3<0$

(ii) $(k-1)(k-3)<0$

Critical points 1, 3

$1<k<3$

Q13 (i) for real x, $b^2-4ac\geq0$

$3x^2-2xp+(p+3)=0$

for real roots, $(-2p)^2-4\times3\times(p+3)\geq0$

or $p^2-3p-9\geq0$

(ii) $p^2-3p-9=0$

$\Rightarrow$ Critical values $\frac{3\pm3\sqrt{5}}{2}$

$\therefore\ p\leq\frac{3-3\sqrt{5}}{2}$; $p\geq\frac{3+3\sqrt{5}}{2}$

eliminating y

Q14 $kx-3=2x^2-3x+k$

or $2x^2-(k+3)x+(k+3)=0$

for not meeting the line

$b^2-4ac<0$

$\Rightarrow (k+3)^2-4\times2\times(k+3)<0$

$\Rightarrow (k+3)(k-5)<0$

Critical values $k=-3, 5$

so $-3<k<5$

Q15 (a) $4x^2+19x-5\leq0$

$\Rightarrow (4x-1)(x+5)\leq0$

Critical values $\frac{1}{4}$ and -5

$\therefore\ -5\leq x\leq\frac{1}{4}$

(b)(i) $(x+4)^2-25$

(ii) Greatest value $=25$ at $x=-4$

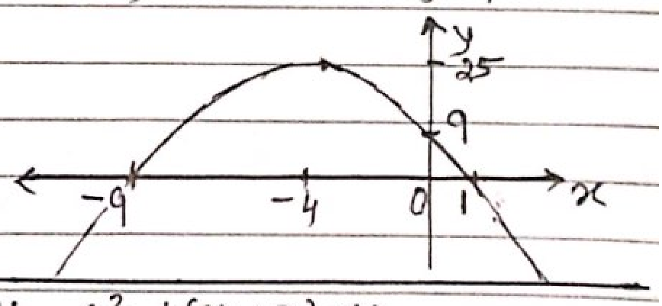
(iii) Graph of $y=9-8x-x^2$

or $y=25-(x+4)^2$

for Int with x-axis $25-(x+4)^2=0$

$\Rightarrow x=1, -9$

Int with y-axis, $x=0\Rightarrow y=9$



Q16 $k^2-4(2k+5)<0$

$k^2-8k-20<0$

$(k-10)(k+2)<0$

critical values 10 and -2

$-2<k<10$

Quadratic Function

Answers

Q17 $kx^2+(2k-8)x+k=0$

for real and distinct roots

$b^2-4ac>0 \Rightarrow (2k-8)^2-4k^2>0$

$\Rightarrow k<2$ ✓

Q18 $2x+k+2=2x^2+(k+2)x+8$

or $2x^2+kx+6-k=0$

for distinct points $b^2-4ac>0$

or $k^2-4\times2\times(6-k)>0$

$k^2+8k-48>0$

$(k+12)(k-4)>0$

$\therefore k<-12$ or $k>4$

Q19 $k(4x-3)=4x^2+8x-8$

$\Rightarrow 4x^2+(8-4k)x+3k-8=0$

for No points of Int, $b^2-4ac<0$

$\Rightarrow (8-4k)^2-4\times4(3k-8)<0$

$\Rightarrow k^2-7k+12<0$

$(k-3)(k-4)<0$

$\therefore 3<k<4$

Q20 (i) $2(x-\frac{1}{4})^2+\frac{47}{8}$

(ii) $\frac{47}{8}$ is min. value when $x=\frac{1}{4}$

Q21 $2-kx=2x^2-9x+4$

or $2x^2+(k-9)+2=0$

for tangent $b^2-4ac=0$

$(k-9)^2-16=0$

$k-9=\pm4$

$k=5$ or 13 ✓

Q22 (i) $12(x-\frac{1}{4})^2+\frac{17}{4}$

(ii) Greatest value is $\frac{4}{17}$ ✓

at $x=\frac{1}{4}$

Eliminating y

Q23 $x^2+(x-5)^2+2x-35=0$

$\Rightarrow 2x^2-8x-10=0$

Solving $x=5,-1$

$\therefore$ Points $(5,0)$ and $(-1,-6)$ ✓

Q24 (i) $y=3(x-1)^2+2$

(ii) $(1,2)$

Q25 (i) $(x+2)^2+x^2=10$

$x^2+2x-3=0$

$(x+3)(x-1)=0 \Rightarrow x=-3,1$

Points $(1,3),(-3,-1)$ ✓

(ii) $m^2x^2+10mx+25+x^2=10$

$(1+m^2)x^2+10mx+15=0$

for tangent $b^2-4ac=0$

$100m^2-60(1+m^2)=0$

$m=\pm\sqrt{\frac{3}{2}}$ ✓

Q26 Eliminating y

$4x^2-8x-96=0$ or $x^2-2x-24=0$

$x=-4$ or 6

Point $A(-4,-16)$, $B(6,4)$

length $AB=\sqrt{10^2+20^2}=\sqrt{500}$

$=10\sqrt{5}$ or 22.4 ✓

Q27 $2x^2+kx+2k-6=0$

has no real roots, $b^2-4ac<0$

$k^2-16k+48<0$

$(k-4)(k-12)<0$

$4<k<12$ ✓

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Answers

Q28 $2x\left(\frac{15-3x}{4}\right)=9$

or $3x^2-15x+18=0$

$3(x-3)(x-2)=0$

$x=3,2$

Points $A(3,3/2)$, $B(2,\frac{9}{4})$

$AB^2=1^2+(0.75)^2$

$\therefore AB=1.25$ ✓

Q29 for not intersecting X-axis

$b^2-4ac<0$

$9-4(k+1)^2<0$

$4k^2+8k-5>0$

$(2k+5)(2k-1)>0$

$\Rightarrow k<-5/2,\ k>\frac{1}{2}\rightarrow(1)$

for curve below X-axis

coeff of x^2, $k+1<0$

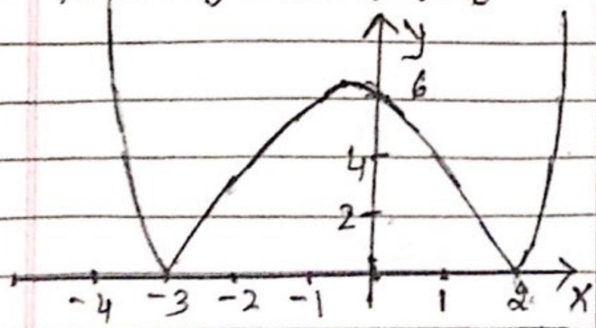
$\Rightarrow k<-1\rightarrow(2)$

from (1) & (2) $k<-5/2$ ✓

Q30 (i) $y=|(x-2)(x+3)|$

Intersects X-axis at $x=-3,2$

Intersects Y-axis at $y=6$



(ii) $\left(-\frac{1}{2},\frac{25}{4}\right)$

(iii) $k>\frac{25}{4}$ or $\frac{25}{4}<k\leq 14$

Q31 Eliminate y

$kx^2+8x+k-6=0$

$b^2-4ac<0$

or $64-4k(k-6)<0$

$-4k^2+24k+64<0$

or $k^2-6k-16>0$

$(k-8)(k+2)>0$

Critical values $8,-2$

$k<-2$ or $k>8$ ✓