IG- Maths

0606

Addtional Maths

Series Exercise

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IG-Maths 0606

Series

Q1 Do not use a calculator in this question.
In the expansion of $(1+2x)^n$, the coefficient of x^4 is ten times the coefficient of x^2. Find the value of positive integer n. [SP-20/01/Q9] ---[6]

Q2 (a) An arithmetic progression has a first term of 5 and a common difference of -3. Find the number of terms such that the sum of n terms is first less than -200. ---[4]

(b) A geometric progression is such that its 3rd term is $\frac{81}{64}$, 5th term is equal to $\frac{729}{1024}$. [SP-20/01/Q10]

(i) Find the first term of this progression and the positive common ratio of this progression. ---[5]

(ii) Hence find the sum to infinity of this progression. --[1]

Q3 The first 3 terms in the expansion of $(2+ax)^n$ are equal to $1024 - 1280x + bx^2$, where a, b and n are constants.

(i) Find the value of n, a and b. --[5]

(ii) Hence find the term independent of x in the expansion of, $(2+ax)^n\left(x-\frac{1}{x}\right)^2$ [M-18/12/Q5] --[3]

Q4 The first three terms in the expansion of $\left(a+\frac{x}{4}\right)^5$ are $32 + bx + cx^2$. Find the value of each of the constants a, b and c. ---[5] [M-17/12/Q3]

Q5 (i) Given that a is a constant, expand $(2+ax)^4$, in ascending powers of x, simplify each term of your expansion. ---[2]
Given also that the coefficient of x^2 is equal to the coefficient of x^3.

(ii) Show that $a=3$ --[1]

(iii) Use your expansion to show that the value of 1.97^4 is 15.1 to 1 decimal place. [S-17/21/Q5] ---[2]

Q6 The first three terms in the expansion of, $\left(3-\frac{x}{6}\right)^n$ are $81 + ax + bx^2$. Find the value of each of the constants n, a and b. [S-17/12/Q4] ---[5]

Q7 The first three terms of the binomial expansion of $(2-ax)^n$ are $64-16bx+100bx^2$. Find the value of each of the integers n, a and b. $\boxed{S\text{-}17/23/Q6}$ ---[7]

Q8 (i) Expand $(1+x)^4$, simplify all coefficients. ---[1]

(ii) Expand $(6-x)^4$, simplifying all coefficients ---[2]

(iii) Hence express $(6-x)^4-(1+x)^4=175$ in the form, $ax^3+bx^2+cx+d=0$, where a, b, c and d are integers ---[2]

(iv) Show that $x=2$ is a solution of the equation in part (iii) and show that this equation has no other real roots. $\boxed{W\text{-}17/21/Q9}$ ---[5]

Q9 (i) Find, in ascending powers of x, the first three terms in the expansion of $\left(2-\frac{x^2}{4}\right)^5$. ---[3]

(ii) Hence find the term independent of x in the expansion of $\left(2-\frac{x^2}{4}\right)^5\left(\frac{1}{x}-\frac{3}{x^2}\right)^2$ ---[3] $\boxed{W\text{-}17/12/Q3}$

Q10 (i) Find, in ascending powers of x, the first 3 terms in the expansion of $\left(2-\frac{x^2}{4}\right)^6$. Give each term in its simplest form. ---[3]

(ii) Hence find the coefficient of x^2 in the expansion of:

$$\left(2-\frac{x^2}{4}\right)^6\left(\frac{1}{x}+x\right)^2$$ ---[4] $\boxed{W\text{-}17/13/Q7}$

Q11 (i) Find, in ascending powers of x, the first three terms of the expansion of $(3+kx)^7$, where k is a constant. Give each term in its simplest form. ---[3]

(ii) Given that, in the expansion of $(3+kx)^7$, the coefficient of x^2 is twice the coefficient of x, find the value of k. $\boxed{M\text{-}16/22/Q5}$ [2]

Q12 (a)(i) Use the Binomial theorem to expand $(a+b)^4$, giving each term in its simplest form. -- [2]

(ii) Hence find the term independent of x, in $\left(2x+\frac{1}{5x}\right)^4$ ---[2]

(b) The coefficient of x^3 in the expansion of $\left(1+\frac{x}{2}\right)^n$ equals $\frac{5n}{12}$. Find the value of the positive integer n. $\boxed{S\text{-}16/21/Q8}$ --[3]

Series.

Q13 (i) The first three terms in the expansion of $\left(2-\frac{1}{4x}\right)^5$ are $a+\frac{b}{x}+\frac{c}{x^2}$, Find the value of each of the integers a, b and c. --- [3]

(ii) Hence find the term independent of x in the expansion of,

$$\left(2-\frac{1}{4x}\right)^5 \cdot (3+4x)$$ --- [2]

S-16/12/Q2

Q.14 (i) Find the first three terms in the expansion of $\left(2x^2-\frac{1}{3x}\right)^5$, in descending powers of x. --- [3]

(ii) Hence find the coefficient of x^7 in the expansion of,

$$\left(3+\frac{1}{x^3}\right)\left(2x^2-\frac{1}{3x}\right)^5$$ -- [2]

W-16/11/Q4

Q15 (i) Find, in ascending powers of x, the first 3 terms in the expansion of $\left(2-\frac{x}{4}\right)^6$ --- [3]

(ii) Hence find the term independent of x in the expansion of

$$\left(4+\frac{2}{x}+\frac{3}{x^2}\right)\left(2-\frac{x}{4}\right)^6$$ --- [3]

W-16/13/Q4

Q16 (i) Write down, in ascending powers of x, the first three terms in the expansion of $(3+2x)^6$. Give each term in its simplest form. --- [3]

(ii) Hence find the coefficient of x^2 in the expansion of

$$(2-x)(3+2x)^6$$ [2]

M-15/12/Q4

Q17 (i) Find the first 4 terms in the expansion of $(2+x^2)^6$ in ascending powers of x. --- [3]

(ii) Find the term independent of x in the expansion of,

$$(2+x^2)^6 \cdot \left(1-\frac{3}{x^2}\right)^2$$ --- [3]

S-15/11/Q3

Q18 In the expansion of $(1+2x)^n$, the coefficient of x^4 is ten times the coefficient of x^2. Find the value of the positive integer n. --- [6]

S-15/22/Q7

Series

Q19 (i) Find, in the simplest form, the first three terms of the expansion of $(2-3x)^6$, in ascending powers of x. ---[3]

(ii) Find the coefficient of x^2 in the expansion of $(1+2x)(2-3x)^6$ ---[2]

W-15/21/Q2

Q20 (a) Given that the first 4 terms in the expansion of $(2+kx)^8$ are $256+256x+px^2+qx^3$, find the value of k, p and q. ---[3]

(b) Find the term that is independent x in the expansion of $\left(x-\frac{2}{x^2}\right)^9$ ---[3]

W-15/13/Q8

Q21 (a) Find the coefficient of x^5 in the expansion of $(3-2x)^8$ ---[2]

(b) (i) Write down the first three terms in the expansion of $(1+2x)^6$ in ascending powers of x. ---[2]

(ii) In the expansion of $(1+ax)(1+2x)^6$, the coefficient of x^2 is 1.5 times the coefficient x. Find value of constant a. ---[4]

S-14/21/Q6

Q22 (i) Find and simplify the first three terms of the expansion, in ascending powers of x, of $(1-4x)^5$ ---[2]

(ii) The first three terms in the expansion of $(1-4x)^5(1+ax+bx^2)$ are $1-23x+222x^2$. Find the value of each of the constants a and b. S-14/22/Q5 ---[4]

Q23 (i) The first three terms in the expansion of $(2-5x)^6$, in ascending power of x are $p+qx+rx^2$. Find the value of each of the integers p, q and r. ---[3]

(ii) In the expansion of $(2-5x)^6(a+bx)^3$, the constant terms is equal to 512 and the coefficient x is zero. Find the value of each of the constants a and b. S-14/13/Q5 ---[4]

Q24 (i) Given that the coefficient of x^2 in the expansion of $(2+px)^6$ is 60, find the value of positive constant p. ---[3]

(ii) Using your value of p, find the coefficient of x^2 in the expansion of $(3-x)(2+px)^6$. W-14/11/Q6 ---[3]

Series

Q25 (a) Given that the first three terms in the expansion of $(5-qx)^p$ are $625-1500x+rx^2$, find the value of each of the integers p, q and r. ---[5]

(b) Find the value of the term that is independent of x in the expansion of $\left(2x+\frac{1}{4x^3}\right)^{12}$. [W-14/13/Q9] ---[3]

Q26 (i) Find the first four terms in the expansion of $(2+x)^6$ in ascending powers of x. ---[3]

(ii) Hence find the coefficient of x^3 in the expansion of $(1+3x)(1-x)(2+x)^6$ [S-13/21/Q7] ---[4]

Q27 (i) Given that n is a positive integer, find the first 3 terms in the expansion of $(1+\frac{1}{2}x)^n$ in ascending powers of x. --[2]

(ii) Given that the coeffient of x^2 in the expansion of $(1-x)(1+\frac{1}{2}x)^n$ is $\frac{25}{4}$, find the value of n. [S-13/12/Q9] --[5]

Q28 (a)(i) Find the coefficient of x^3 in the expansion of $(1-2x)^6$ --[2]

(ii) Find the coefficient of x^3 in the expansion of $(1+\frac{x}{2})(1-2x)^6$ --[3]

(b) Expand $\left(2\sqrt{x}+\frac{1}{\sqrt{x}}\right)^4$ in a series of powers of x with integer coefficients. [W-13/21/Q6] --[3]

Q29 The coefficient of x^2 in the expansion of $(2+px)^6$ is 60.

(i) Find the value of the positive constant p. ---[3]

(ii) Using your value of p, find the coefficient of x^2 in the expansion of $(3-x)(2+px)^6$ [W-13/13/Q1] ---[3]

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Answers

Q1 $\frac{n(n-1)(n-2)(n-3)}{4\times3\times2\times1}\times 2^4 = 10\times\frac{n(n-1)}{2\times1}\cdot 2^2$

$\Rightarrow n^2-5n-24=0$

$(n+3)(n-8)=0$

$\therefore n=8$ ✓

Q2(a) $\frac{n}{2}[2\times5+(n-1)(-3)] < -200$

$\Rightarrow 3n^2-13n-400>0$

$n=13.9$

$\therefore n=14$ ✓ is the first term needed.

(b)(i) $ar^2=\frac{81}{64}$ and $ar^4=\frac{729}{1024}$

$\therefore r^2=\frac{9}{16}\Rightarrow r=\frac{3}{4}$ ✓

and $a=9/4$ ✓

(ii) $S_\infty=\frac{a}{1-r}=9$ ✓

Q3 (i) $2^n=1024\Rightarrow n=10$ ✓

$10\times2^9\times a=-1280\Rightarrow a=-\frac{1}{4}$ ✓

${}^{10}C_2\, 2^8\times(-\frac{1}{4})^2=b\Rightarrow b=720$ ✓

(ii) $(1024-1280x+720x^2)(\frac{1}{x^2}-2+x^2)$

Term Independent of $x=720-2048$
$=-1328$ ✓

Q4 $a^5+5a^4\cdot\frac{x}{4}+10a^3\cdot(\frac{x}{4})^2 = 32+bx+cx^2$

$\Rightarrow a^5=32\Rightarrow a=2$ ✓

$b=5\times\frac{1}{4}(2)^4=20$ ✓

$c=10\times\frac{1}{16}\times2^3=5$ ✓

Q5 (i) $16+32ax+24a^2x^2+8a^3x^3+a^4x^4$

(ii) $24a^2=8a^3\Rightarrow a=3$ ✓

(iii) $x=-0.01$ or $ax=-0.03$

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Q5(iii) $16+32(3)(-0.01)+24\times9\times(-0.01)^2$

$=15.06$ ✓

Q6 $3^n-n\cdot3^{n-1}(\frac{x}{6})+n(n-1)3^{n-2}\cdot(\frac{x}{6})^2$

$\therefore 3^n=81\Rightarrow n=4$ ✓

$4\times3^3\times(-\frac{1}{6})=a\Rightarrow a=-18$ ✓

$4\times3\times3^2\times\frac{1}{36}=b\Rightarrow b=\frac{3}{2}$ ✓

Q7 $64=2^n\Rightarrow n=6$

$n\times2^{n-1}\times(-a)=-16b$

$\Rightarrow 6\times2^5(-a)=-16b$ — (i)

and $\frac{6(6-1)}{2}\times2^4\times(-a)^2=100b$ — (ii)

Solving (i) and (ii)

$a=5$ and $b=60$ ✓

Q8 (i) $1+4x+6x^2+4x^3+x^4$

(ii) $1296-864x+216x^2-24x^3+x^4$

(iii) $1295-868x+210x^2-28x^3=175$

$\Rightarrow 28x^3-210x^2+868x-1120=0$

(iv) $28(2^3)-210\times2^2+868\times2-1120=0$

$\therefore (x-2)$ is factor

$(x-2)(28x^2-154x+560)=0$

for quad factor $b^2-4ac<0$

Q9 (i) $32-20x^2+5x^4$

(ii) $(32-20x^2+5x^4)(\frac{1}{x^2}---+\frac{9}{x^4})$

$\therefore$ Term Independent of x
$=-20+45$
$=25$ ✓

Q10 (i) $64-48x^2+15x^4$

(ii) $(64-48x^2+15x^4)(\frac{1}{x^2}+2+x^2)$

$\therefore$ Coeff of $x^2=64+15-96$
$=-17$ ✓

IG-Maths 0606

Series

Date P-7

Answers

Q11 (i) $2187 + 5103kx + 5103k^2x^2$

(ii) $2(5103k) = 5103k^2$

$\Rightarrow k = 2$ ✓

Q12 (i) $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

(ii) $6(2x)^2 \cdot \left(\frac{1}{5x}\right)^2 = \frac{24}{25}$ ✓

(b) $\frac{1}{8}\left(\frac{n(n-1)(n-2)}{6}\right) = \frac{5n}{12}$

$\Rightarrow n^2 - 3n - 18 = 0$

$(n-6)(n+3) = 0$

$\therefore n = 6$ ✓

Q13 (i) $32 - \frac{20}{x} + \frac{5}{x^2}$

(ii) $\left(32 - \frac{20}{x} + \frac{5}{x^2}\right)(3 + 4x)$

The term independent of x

$= 32 \times 3 - 20 \times 4 = 16$ ✓

Q14 (i) $32x^{10} - \frac{80}{3}x^7 + \frac{80}{9}x^4$

(ii) $\left(3 + \frac{1}{x^3}\right)\left(32x^{10} - \frac{80}{3}x^7 + \frac{80}{9}x^4\right)$

$\therefore$ Coeff $x^7 = -80 + 32$

$= 48$ ✓

Q15 (i) $64 - 48x + 15x^2$

(ii) $\left(4 + \frac{2}{x} + \frac{3}{x^2}\right)(64 - 48x + 15x^2 + \dots)$

$\therefore$ Coeff of $x^2 = 4 \times 64 + 2 \times (-48) + 3 \times 15$

$= 205$ ✓

Q16 (i) $729 + 2916x + 4860x^2$

(ii) $(729 + 2916x + 4860x^2)(2 - x)$

$\therefore$ The Coefficient of x^2

$= 2 \times 4860 - 2916$

$= 6804$ ✓

Q17 (i) $64 + 192x^2 + 240x^4 + 160x^6$

(ii) $(64 + 192x^2 + 240x^4 + 160x^6)\left(1 - \frac{6}{x^2} + \frac{9}{x^4}\right)$

The term independent of x,

$= 64 \times 1 - 192 \times 6 + 240 \times 9$

$= 1072$ ✓

Q18

$\frac{n(n-1)(n-2)(n-3)}{4 \times 3 \times 2 \times 1} \cdot 2^4 = 10\,\frac{n(n-1)}{2 \times 1} \times 2^2$

$\Rightarrow n^2 - 5n - 24 = 0$

$(n+3)(n-8) = 0$

$\therefore n = 8$ ✓

Q19 (i) $64 - 576x + 2160x^2 \dots$

(ii) $(1 + 2x)(64 - 576x + 2160x^2 - \dots)$

$\therefore$ Coeff of $x^2 = 2160 - 2 \times 576$

$= 1008$ ✓

Q20

$(2 + kx)^8 = 256 + 1024kx + 1792k^2x^2 + 1792k^3x^3$

$\therefore k = \frac{1}{4}$, $p = 112$, $q = 28$

(b) ${}^9C_3\, x^6 \left(-\frac{2}{x^2}\right)^3$

$= 84x^6 \times \left(-\frac{8}{x^6}\right) = -672$

Q21 (a) ${}^8C_3 \times 3^3 \times (-2)^5$

$= -48384$ ✓

(b)(i) $1 + 12x + 60x^2$

(ii) $(1 + ax)(1 + 12x + 60x^2 + \dots)$

Coeff of $x^2 = 60 + 12a$

Coeff of $x = 12 + a$

Given $(60 + 12a) = 1.5(12 + a)$

$\Rightarrow a = -4$ ✓

IG-Maths 0606

Series

Date P-8

Answers

Q22 (i) $1-20x+160x^2$

(ii) $(1-20x+160x^2)(1+ax+bx^2) = 1-23x+222x^2$

$\Rightarrow$ Coeff of x, $a-20=-23$

$\Rightarrow a=-3$ ✓

Coeff of x^2, $b-20a+160=222$

$b-20\times(-3)+160=222$

$\Rightarrow b=2$ ✓

Q23 (i) $64-960x+6000x^2$

(ii) $(64-960x+6000x^2)(a^3+3a^2bx+\ldots)$

Constant term $64a^3=512$

$\Rightarrow a=2$ ✓

Coeff of x; $-960a^3+3a^2b\times64=0$

or $-960\times2^3+3\times2^2\times b\times64=0$

$\Rightarrow b=10$

Q24 (i) $64+192px+240p^2x^2\ldots$

Coeff of x^2 $240p^2=60$

$p=\frac{1}{2}$ ✓

(ii) $(3-x)(64+192px+240p^2x^2)$

Coeff of $x^2 = 3\times240p^2-192p$

$=720\times\left(\frac{1}{2}\right)^2-192\times\frac{1}{2}$

$=84$ ✓

Q25 (a) $5^p+p\cdot5^{p-1}(-qx)+\frac{p(p-1)}{2!}5^{p-2}\cdot(-qx)^2$

given $=625-1500x+rx^2$

$\Rightarrow 5^p=625 \Rightarrow p=4$

and $4\times5^{4-1}\cdot(-q)=-1500$ [$\because p=4$]

$\Rightarrow q=3$

and $\frac{4\times3}{2}\times5^2\times q^2=r$

$150\times3^2=r$

$r=1350$ ✓

Q25 (b) The term independent of x; ${}^{12}C_3(2x)^9\left(\frac{1}{4x^3}\right)^3$

$=1760$ ✓

Q26 (i) $64+192x+240x^2+160x^3$

(ii) $(1+3x)(1-x)(2+x)^6$

$=(1+2x-3x^2)(64+192x+240x^2+160x^3)$

$\therefore$ Coeff of x^3

$=1\times160+2\times240-3\times192$

$=64$ ✓

Q27 (i) $1+n\cdot\frac{x}{2}+\frac{n(n-1)}{2}\cdot\left(\frac{x}{2}\right)^2$

(ii) $(1-x)\left(1+\frac{nx}{2}+\frac{n(n-1)}{2}\times\frac{x^2}{4}+\ldots\right)$

Coeff of x^2: $\frac{n(n-1)}{8}-\frac{n}{2}=\frac{25}{4}$ (given)

$\Rightarrow n^2-5n-50=0 \Rightarrow n=10$ ✓

Q28 (a)(i) ${}^6C_3(-2x)^3=-160x^3$

$\therefore$ Coeff $x^3=-160$ ✓

(ii) $(1+x/4)\left(1+{}^6C_1(-2x)+{}^6C_2(-2x)^2-160x^3\right)$

$\therefore$ Coeff $x^3=\frac{1}{4}\times60-160=-130$ ✓

(b) $16x^2+32x+24+\frac{8}{x}+\frac{1}{x^2}$ ✓

Q29 (i) ${}^6C_2\,2^4\cdot(px)^2$

$\therefore 240p^2=60$ (given)

$p=\frac{1}{2}$ ✓

(ii) $(3-x)(2+\frac{1}{2}x)^6$

$=(3-x)\left(2^6+{}^6C_1\times2^5\times\frac{1}{2}x+{}^6C_2\,2^4\left(\frac{1}{2}x\right)^2\right)$

$\therefore$ Coeff $x^2=3\times60-96$

$=84$ ✓

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