IG_0606 Additional Maths

Straight Line Graph Notes

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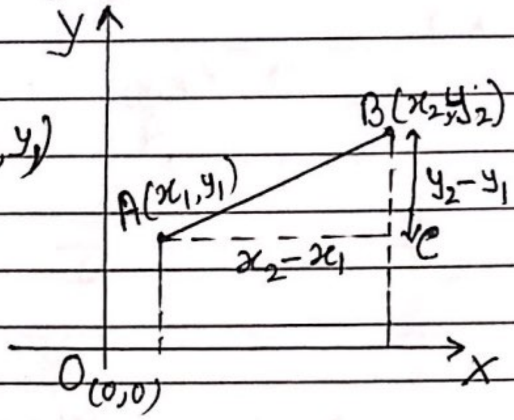
Straight line
(Coordinate Geometry)

§ Distance Formula:

distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$.



$$AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

and

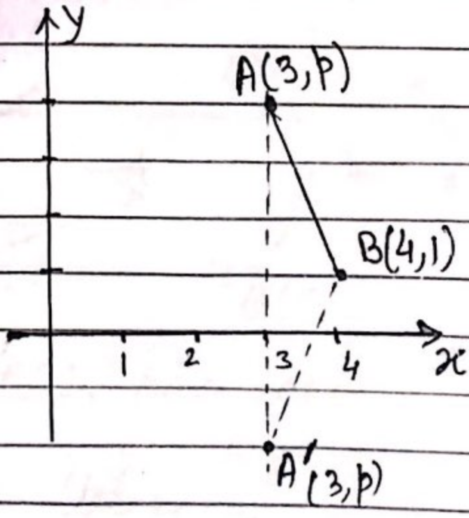
distance $OA = \sqrt{x_1^2 + y_1^2}$

x_1 y_1 x_2 y_2

Example 1. Find the distance between $A(3,4)$ and $B(8,-6)$.

Solution:
$$AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$
$$= \sqrt{(8-3)^2 + (-6-4)^2}$$
$$= \sqrt{5^2 + (-10)^2} = \sqrt{25+100}$$
$$= \sqrt{125}$$
$$= 5\sqrt{5}$$

Example 2. Find the value of p, when the distance between the points $A(3, p)$ and $B(4,1)$ is $\sqrt{10}$.



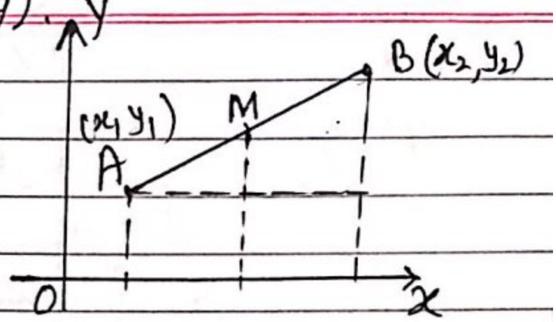
Solution: $AB = \sqrt{(4-3)^2 + (1-p)^2} = \sqrt{10}$ given

square both sides-
$$1^2 + (1-p)^2 = 10$$
or $$(1-p)^2 = 9$$
$$(1-p) = \pm\sqrt{9} = \pm 3$$
$$\Rightarrow -p = 2 \text{ or } -4$$
or $$p = -2 \text{ or } 4$$

A will be.
$(3,4)$ or $(3,-2)$

Straight line (Coordinate Geometry)

§ Mid point of Segment:



Given $A(x_1, y_1)$ and $B(x_2, y_2)$
Mid point of AB,

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Example 3. Find the coordinates of the mid-point of the line joining $P(5,7)$, $Q(-1,1)$

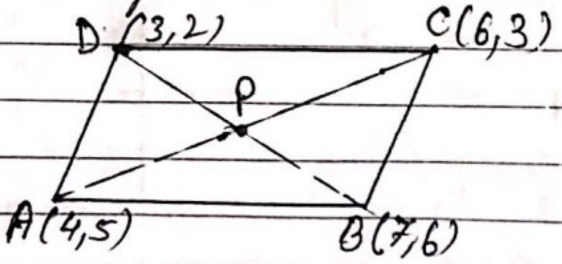
Solution: Mid point of $PQ = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$\therefore$ Mid point $M\left(\frac{5+(-1)}{2}, \frac{7+1}{2}\right)$

or $M(2,4)$ ✓

Example 4. Prove that point $A(4,5)$, $B(7,6)$, $C(6,3)$ and $D(3,2)$ are the vertices of a parallelogram.

Solution:



Midpoint of $AC = \left(\frac{4+6}{2}, \frac{5+3}{2}\right) = (5,4)$

Mid point of $BD = \left(\frac{7+3}{2}, \frac{6+2}{2}\right) = (5,4)$

The diagonal of quadrilateral ABCD, bisect each other at $P(5,4)$.

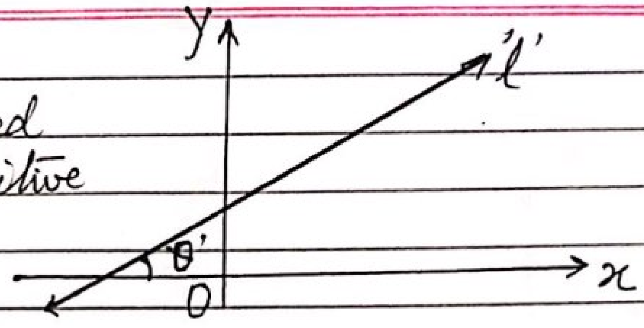
Hence ABCD is a parallelogram.

Straight Line

§ Inclination of a line:

If a line 'l' is inclined at an angle 'θ' with the positive direction of x-axis. Then we say,

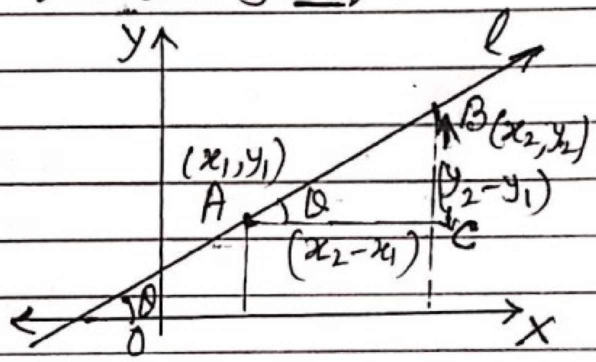
Inclination of line 'l' = θ.✓



§ Gradient (or Slope) of a line 'l' (denoted by 'm'):

Given a line 'l' passing through two points $A(x_1, y_1)$ and $B(x_2, y_2)$,

Gradient $m = \tan\theta$



or $m = \dfrac{\text{rise}}{\text{run}} = \dfrac{BC}{AC}$

or $= \dfrac{(y_2 - y_1)}{(x_2 - x_1)}$, $x_2 \neq x_1$.

Example 5. The coordinates of point P are (−4, −4) and the coordinates of Q are (8, 14), Find the gradient of PQ.

Solution: P(−4, −4), Q(8, 14)

Gradient of line PQ $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{14 - (-4)}{8 - (-4)}$

$= \dfrac{18}{12} = \dfrac{3}{2}$ ✓

Example 6. The point P has coordinates (10, 12) and point Q has (2, k), find the value of k, given that the gradient of line PQ is 2.

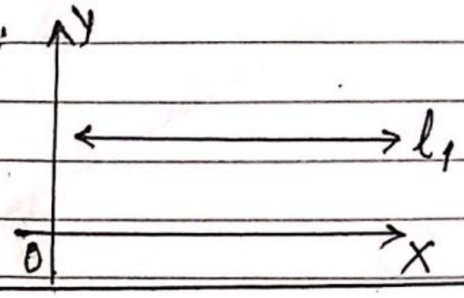
Solution. P(10, 12), Q(2, k)

Gradient of line PQ $= \dfrac{k - 12}{2 - 10} = 2$ (Given)

$\therefore$ $k - 12 = -16$ or $k = -4$ ✓

Straight line
(Coordinate Geometry)

§ **Gradient of a line — Some Special Cases:**



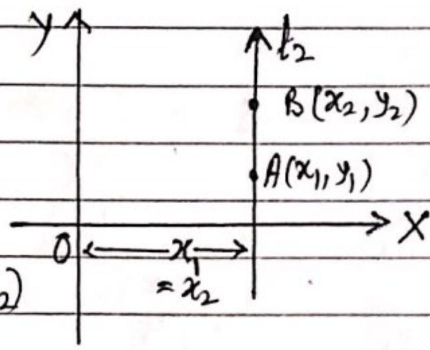
§ (i) Gradient a line parallel to X-axis

$l_1 \parallel X$-axis $\quad m = 0$

(i') Gradient of X-axis $= 0$

§ (ii) Gradient of a line $l_2 \parallel Y$-axis.

or ($l_2 \perp X$-axis)



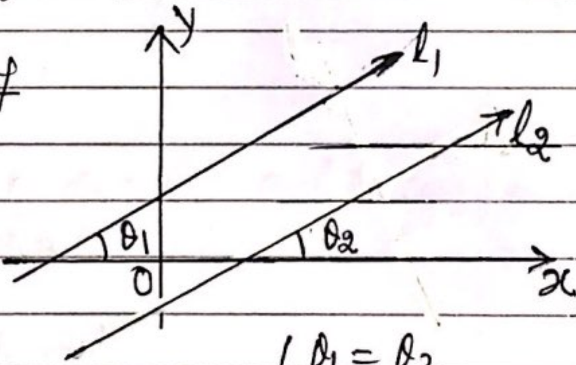
$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{(y_2 - y_1)}{0} \quad (\text{as } x_1 = x_2)$$

$$m = \text{not defined.}$$

(ii') Gradient of Y-axis is not defined.

§ (iii) Relation between the gradients of parallel lines, $l_1 \parallel l_2$



Grad. of $l_1 = m_1$

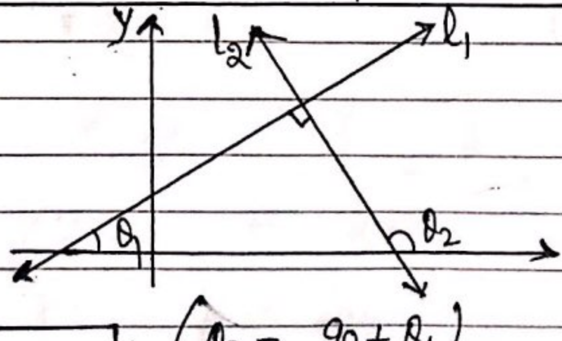
Grad of $l_2 = m_2$

Then $l_1 \parallel l_2 \Leftrightarrow m_1 = m_2$

($\theta_1 = \theta_2$

($\tan\theta_1 = \tan\theta_2$

§ (iv) Relation between the gradients of perpendicular lines $l_1 \perp l_2$.



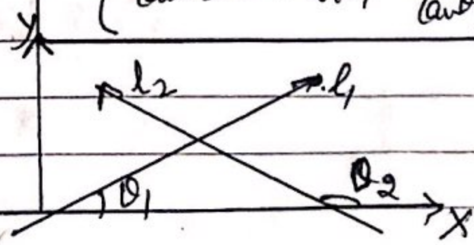
$l_1 \perp l_2 \Leftrightarrow m_1 \times m_2 = -1$

or $\quad m_2 = -\frac{1}{m_1}$

$$\left(\begin{array}{l} \theta_2 = 90 + \theta_1 \\ \tan\theta_2 = -\cot\theta_1 = \frac{1}{\tan\theta_1} \end{array}\right)$$

§ (V) Gradient of a line l_1, m_1 is positive, or $m_1 > 0$ then $0 < \theta_1 < 90$

(VI) Grad. of a line l_2, m_2 is negative $m_2 < 0$ then $90 < \theta_2 < 180$



Straight line

(Coordinate Geometry)

Example 5: Find the gradient of the lines
(i) Passing through the points (3,−2) and (−1,4)
(ii) Passing through the points (3,−2) and (7,−2),
(iii) Passing through the points (3, −2) and (3, 4)

Solution (i) The gradient of the line AB, A(3,−2), (−1,4)

$$m = \frac{4-(-2)}{-1-3} = \frac{6}{-4} = \frac{-3}{2}$$ ✓

(ii) The gradient of the line CD, C(3,−2) and D(7,−2)

$$m = \frac{-2-(-2)}{7-3} = \frac{0}{4} = 0$$ ✓

(iii) The gradient of the line EF, E(3,−2) and (3,4)

$$m = \frac{4-(-2)}{3-3} = \frac{6}{0}$$ ✓ not defined.

Example 6: Prove that the line joining the points A(3,−2) and B(−1,4) is parallel to the line joining to C(4,−1) and D(0,5)

Solution: Gradient of line $AB = \frac{4-(-2)}{-1-3} = \frac{6}{-4} = -\frac{3}{2} = m_1$ (let)

Gradient of line CD, $m_2 = \frac{5-(-1)}{0-(4)} = \frac{6}{-4} = -\frac{3}{2} = m_2$ (let)

We find $m_1 = m_2$ ∴ AB and CD are Parallel.

Example 7: Prove that the line 'l_1' through the points A(−2,6) and B(4,8) is perpendicular to the line l_2 through the points C(8,12) and D(4,24).

Solution: Gradient of line l_1, $m_1 = \frac{8-6}{4-(-2)} = \frac{2}{6} = \frac{1}{3}$

and Gradient of line l_2, $m_2 = \frac{24-12}{4-8} = \frac{12}{-4} = -3$

Now $m_1 \times m_2 = \frac{1}{3} \times (-3) = -1$ ✓ or $m_1 m_2 = -1$

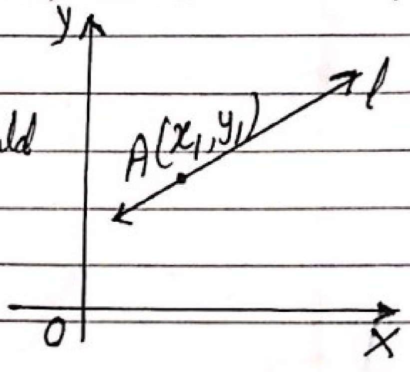
∴ lines l_1 and l_2 are perpendicular to each other.

Straight line

§ Different forms of equation of a line.

§ What do you understand by the equation of a line (or any curve):

By equation of a line we mean that the coordinates of every point on the line should satisfy the equation of the line, and conversely if the coordinates of any point satisfy the equation then the point must lie on the line.



§ (i) General Equation of a line is,

$$ax + by + c = 0 \checkmark$$

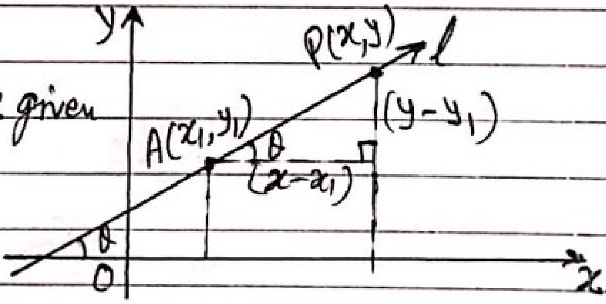
here a and b both are not equal to zero.

Example: (i) $2x - 3y + 10 = 0$

(ii) $3x - 7 = 0$

(iii) $y = 4$

(ii) Equation of line passing through a given point $A(x_1, y_1)$ and gradient m.

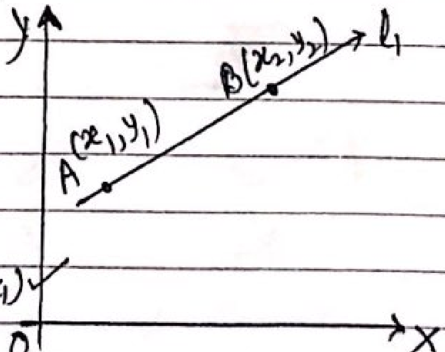


or $\dfrac{y - y_1}{x - x_1} = m$

or $y - y_1 = m(x - x_1) \checkmark$

(iii) Equation of line passing through two given points $A(x_1, y_1)$ and $B(x_2, y_2)$

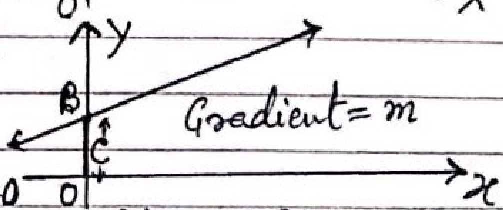
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



∴ Equation of line $y = \left(\dfrac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1) \checkmark$

(iv) Gradient m and y-Intercept c form;

$$y = mx + c$$



(v) To Reduce the general Equation $ax + by + c = 0$ To Gradient form; $y = -\frac{a}{b}x + \left(-\frac{c}{b}\right) \checkmark$

[$by = -ax - c$, Divide by b]

Example 8. Find the equation of the line through $(-2,3)$ with slope -4.

Solution. $m=-4$ and the point is $(-2,3) \leftrightarrow (x_1,y_1)$

Equation of line is

$$y-3=-4(x-(-2)) \qquad [y-y_1=m(x-x_1)]$$
$$y-3=-4x-8$$
$$\text{or} \quad 4x+y+5=0$$

Example 9: Write the equation of line through the points $(1,-1)$ and $(3,5)$.

Solution. Given points $(1,-1)$ and $(3,5)$

Eqn of line is:

$$y-(-1)=\frac{5-(-1)}{3-1}(x-1) \qquad \left[y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)\right]$$
$$y+1=\frac{6}{2}(x-1)$$
$$\text{or} \quad y+1=3x-3$$
$$\text{or} \quad -3x+y+4=0$$

Example 10. Write the equation of a line with gradient $\frac{1}{2}$ and y-intercept $-\frac{3}{2}$.

Solution given $m=\frac{1}{2}$ and y-int $c=-\frac{3}{2}$

$\therefore$ Equation of line, $y=\frac{1}{2}x-\frac{3}{2}$ $\qquad (y=mx+c)$

$$\text{or} \quad 2y-x+3=0$$

Example 11. Find the equation of line passing through $P(-3,5)$ and perpendicular to the line through the points $A(2,5)$ and $B(-3,6)$

Solution: Gradient of line AB, $m_1=\frac{6-5}{-3-2}=-\frac{1}{5}$ — ① $\quad \left[A(2,5), B(-3,6) \right.$

$\therefore$ Gradient of line perpendicular to AB, $m_2=-\frac{1}{m_1}$ $\quad [m_1 \times m_2=-1$

$$\text{or} \quad m_2=-\frac{1}{-1/5}=5$$

$\therefore$ Equation of line passing through $P(-3,5)$ and gradient $m_2=5$

$$y-5=5(x-(-3)) \qquad [y-y_1=m_2(x-x_1)]$$
$$y-5=5x+15$$
$$\text{or} \quad 5y-y+20=0$$

Example 12. Three points have coordinates A(−8,6), B(4,2) and C(−1,7). The line through C perpendicular to AB intersects AB at the point P.

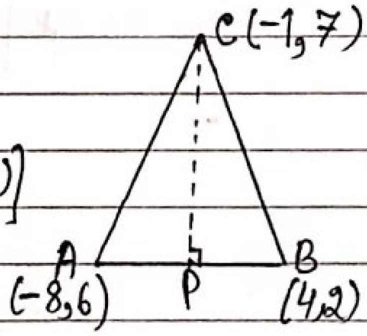
(i) Find the equation of the line AB. ---[2]

(ii) Find the equation of the line CP. ---[2]

(iii) Show that P is the midpoint of AB. ---[3]

(iv) Calculate the length of CP. ---[1]

(v) Hence find the area of the triangle ABC. ---[2] **[S-16/11/Q8]**



Solution (i) A(−8,6), B(4,2)

Equation of line AB,

$y-6=\frac{(2-6)}{(4-(-8))}(x-(-8))$ $\left[y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)\right]$

$y-6=-\frac{4}{12}(x+8)$

or $y=-\frac{1}{3}x+\frac{10}{3}$ ✓ — ①

(ii) Gradient of line AB, $m_1=-\frac{1}{3}$ (fr ① $y=mx+c$, $m=-1/3$)

CP is perp. to AB.

Let the gradient of CP, $m_2=-\frac{1}{m_1}$

$=-\frac{1}{-\frac{1}{3}}=3$

∴ C(−1,7), $m_2=3$

Equation of line CP, $y-7=3(x-(-1))$

or $y=3x+10$ ← ②

(iii) Solving Equation of AB & CP, (① & ②)

P(−2, 4) — ③

Mid point of AB $=\left(\frac{-8+4}{2},\frac{6+2}{2}\right)$

$=(-2,4)$ — ④

fr ③ & ④

P is the mid point of AB.

(iv) P(−2,4), C(−1,7).

Distance CP $=\sqrt{(-1+2)^2+(7-4)^2}=\sqrt{1+9}=\sqrt{10}=3.16$ — ⑤

(V) Area of triangle ABC

$=\frac{1}{2}AB\times CP$ (∵ CP ⊥ AB) — ⑥

Distance AB $=\sqrt{(4+8)^2+(2-6)^2}$

$=\sqrt{144+16}$

$=\sqrt{160}$

$=4\sqrt{10}$ — ⑦

∴ Area of ABC [fr ⑤ & ⑦]

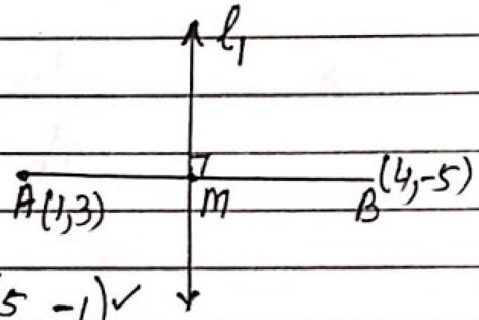
$=\frac{1}{2}\times4\sqrt{10}\times\sqrt{10}$

$=20$ Sq. Units ✓

Straight Line

Example 13. Find the equation of the perpendicular bisector of the line joining the points A(1,3) and B(4,−5). Give your answer in the form $ax+by+c=0$ [S-18 | 11 | Q2] ... [5]

Solution: Perpendicular bisector of line AB passes through the mid point 'M' of AB and perp. to AB.



A(1,3), B(4,−5)

Mid point of AB, $M\left(\frac{1+4}{2}, \frac{3+(-5)}{2}\right) = \left(\frac{5}{2}, -1\right)$ ✓

Gradient of line AB, $m_1 = \frac{-5-3}{4-1} = -\frac{8}{3}$ —— ①

∴ Gradient of Perp. $m_2 = -\frac{1}{m_1} = \frac{-1}{-8/3} = \frac{3}{8}$ ✓

$M\left(\frac{5}{2}, -1\right)$ and $m_2 = 3/8$

∴ Equation of perp. bisector of AB

$y-(-1) = 3/8\left(x-\frac{5}{2}\right)$ $\quad [y-y_1 = m(x-x_1)]$

$y+1 = \frac{3}{8}(x-5/2)$

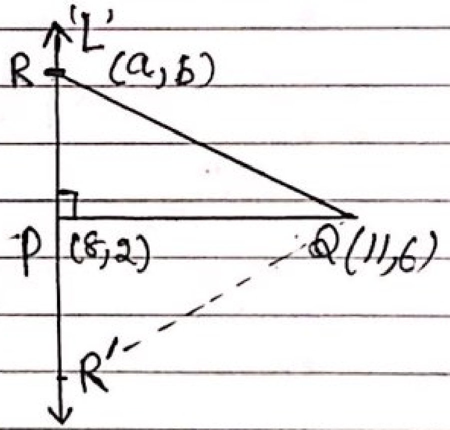
or $8y+8 = 3x-\frac{15}{2}$ or $\underline{6x-16y-31=0}$ ✓

Example 14. P is the point (8,2) and Q is the point (11,6)

(i) Find the equation of line 'L' which passes through 'P' and is perpendicular to the line PQ. ---[3]

The point R lies on 'L' such that the area of triangle PQR is 12.5 units^2.

(ii) Showing all your working, find the coordinates of each of the two possible positions of point R. $\boxed{M-18 | 22 | Q9}$ ---[6]



Solution: P(8,2), Q(11,6)

(i) Gradient of PQ, $m_1 = \frac{6-2}{11-8} = \frac{4}{3}$ —— ①

$\therefore$ Gradient of line 'L' perp to PQ, $m_2 = -\frac{1}{m_1}$

or $m_2 = \frac{-1}{4/3}$ fr ①

or $m_2 = -\frac{3}{4}$

Now P(8,2) and $m_2 = -\frac{3}{4}$

Equation of line 'L' $\quad [y - y_1 = m(x - x_1)$

$y - 2 = -\frac{3}{4}(x-8)$

or $y = -\frac{3}{4}x + 8$ ✓ —— ②

(ii) R lies on line 'L' Let coordinates of R (a,b)

R(a,b) lies on L fr ② $b = -\frac{3}{4}a + 8$ —— ③

Distance $PQ = \sqrt{(11-8)^2 + (6-2)^2}$

$PQ = \sqrt{3^2 + 4^2} = 5$ —— ④

Area of rt angle triangle PQR = 12.5

or $\frac{1}{2} \times PQ \times RP = 12.5$

$\frac{1}{2} \times 5 \times RP = 12.5$

or $RP = \frac{12.5 \times 2}{5} = 5$ ✓ —— ⑤

R(a,b), P(8,2)

$\therefore RP^2 = (a-8)^2 + (b-2)^2 = 5^2$

or $(a-8)^2 + \left(-\frac{3}{4}a + 8 - 2\right)^2 = 5$ fr ③

or simplifying

$a^2 - 16a + 48 = 0$

$(a-4)(a-12) = 0$

$a = 4$ or $a = 12$

fr ③ $\begin{cases} a = 4 \\ b = 5 \end{cases}$ & $\begin{cases} a = 12 \\ b = -1 \end{cases}$

$\therefore$ Required coordinates of R(4,5) or (12,-1) ✓

Example 15. Given points $A(2,-1)$ and $B(6,5)$.

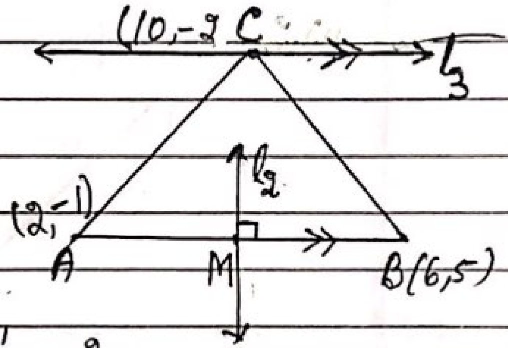
(i) Find the equation of the perp. bisector of AB, giving your answer in the form $ax+by=c$, where a, b and c are integers. ---[4]

The point C has coordinates $(10,-2)$

(ii) Find the equation of the line through C, which is parallel to AB ---[2]

(iii) Calculate the length BC. ---[2]

(iv) Show that triangle ABC is isosceles. [M-15/22/Q8] --[1]



Solution: Given $A(2,-1)$ and $B(6,5)$

(i) Midpoint of AB, $M\left(\frac{2+6}{2}, \frac{-1+5}{2}\right) = (4,2)$ ✓

Gradient of line AB, $m_1 = \frac{5-(-1)}{6-2} = \frac{3}{2}$ ✓

$\therefore$ Gradient of line perp. to AB, $m_2 = -\frac{1}{m_1} = \frac{-1}{3/2} = -\frac{2}{3}$ ✓

$\therefore$ Equation of perp. bisector of AB,

$m_2 = -2/3$, $M(4,2)$,

$y-2 = -\frac{2}{3}(x-4)$ $\qquad y-y_1 = m_2(x-x_1)$

or $2x+3y=14$ ✓

(ii) Given $C(10,-2)$, $\qquad$ Grad. of l_3 = Grad of AB

Let line l_3 is parallel to line AB, $m_3 = m_1 = 3/2$ ✓

$\therefore$ Equation of line l_3, $y+2 = \frac{3}{2}(x-10)$ $\quad [y-y_1 = m_3(x-x_1)]$

or $3x-2y=34$ ✓

(iii) $B(6,5)$, $C(10,-2)$

length $BC = \sqrt{(10-6)^2+(-2-5)^2}$

$= \sqrt{16+49} = \sqrt{65}$

$= 8.062$ ✓

(iv) length $AC = \sqrt{(10-2)^2+(-2+1)^2}$

$= \sqrt{64+1} = \sqrt{65}$ ✓

from (iii) & (iv) $AC = BC = \sqrt{65}$

$\therefore$ Triangle ABC is Isosceles.

(iv) Alternate method.

from (1) Equation of the perp. bisector of AB

$2x+3y=14$ —— (1)

$C(10,-2)$ Put in (1)

$20+3(-2)=14$ True

$\therefore$ C lies on perp. bisect of $AB \Rightarrow CA = CB$

$\therefore \Delta ABC$ is isos.

Straight line

Example 16. The line $y = mx + 4$ meets the lines $x = 2$ and $x = -1$ at the points P and Q respectively. The point R is such that QR is parallel to y-axis and the gradient of RP is 1. The point P has coordinates $(2, 10)$

(i) Find the value of m --- [2]

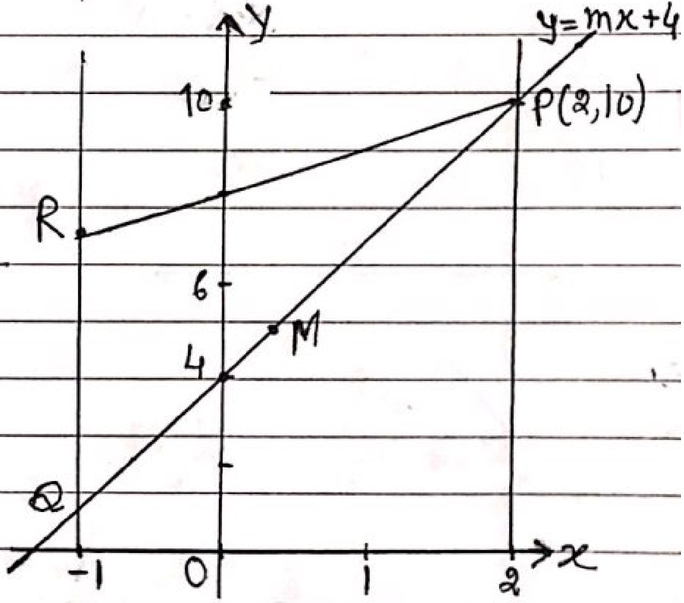
(ii) Find the y-coordinate of Q. -- [1]

(iii) Find the coordinates of R -- [2]

(iv) Find the equation of line through P, perpendicular to PQ, giving your answer in the form $ax + by = c$, where a, b, c are integers. --- [3]

(v) Find the coordinate of the mid point M, of the line PQ. -- [2]

(vi) Find the area of triangle QRM -- [2]



S-15/22/Q9

Solution (i) P $(2, 10)$ lies on the line $y = mx + 4$ — ①

$\therefore \quad 10 = 2m + 4 \Rightarrow \underline{m = 3}$ ✓

(ii) $\therefore$ Equation of PQ fr ① is $y = 3x + 4$ — ②

PQ intersects with line $x = -1$ at Q

$\therefore \quad y = 3 \times (-1) + 4 = 1$

$\therefore \quad \underline{Q(-1, 1)}$ ✓

(iii) R lies on the line $x = -1$

Let Coord. of $R(-1, b)$

Grad of line $PR = \dfrac{10 - b}{2 - (-1)} = 1$ given

or $\quad 10 - b = 3 \Rightarrow b = 7$

$\therefore$ Coord. of $\underline{R(-1, 7)}$ ✓

(iv) Grad of PQ, $m_1 = \dfrac{10 - 1}{2 - (-1)} = 3$

$\therefore$ Grad of line perp to PQ, $m_2 = -\dfrac{1}{m_1}$

$P(2, 10) \qquad = -\dfrac{1}{3}$ ✓

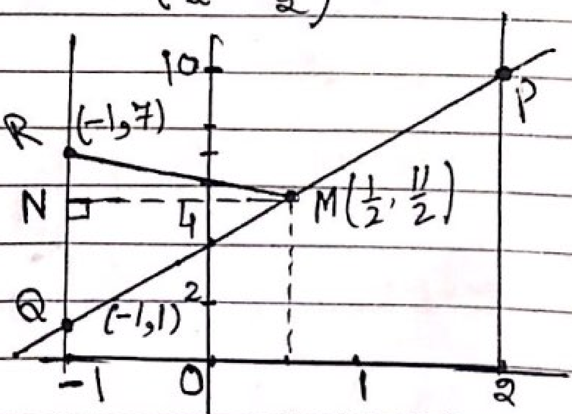
$\therefore$ Equation of line through P, and perp to PQ, $\quad y - 10 = -\dfrac{1}{3}(x - 2)$

or $\quad \underline{3y + x = 32}$ ✓

(v) Mid point of PQ

$M\left(\dfrac{2 + (-1)}{2}, \dfrac{10 + 1}{2}\right)$

$= M\left(\dfrac{1}{2}, \dfrac{11}{2}\right)$



draw $MN \perp QR$

$MN = \dfrac{1}{2} - (-1) = \dfrac{3}{2}$ ✓

$QR = 7 - 1 = 6$

Area of $\Delta QRM = \dfrac{1}{2} QR \times MN$

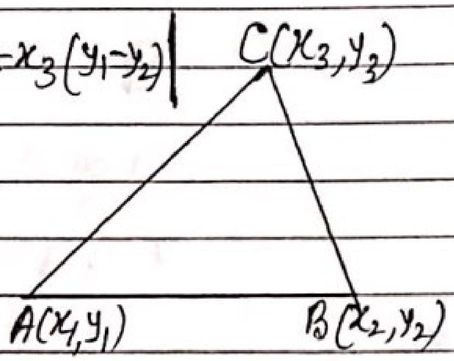
$= \dfrac{1}{2} \times 6 \times \dfrac{3}{2}$

$= \underline{4.5}$ units^2

Straight Line

§(i) Area of triangle $[A(x_1,y_1), B(x_2,y_2), C(x_3,y_3)]$ ABC.

Area of $\Delta ABC = \frac{1}{2}\left| x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$



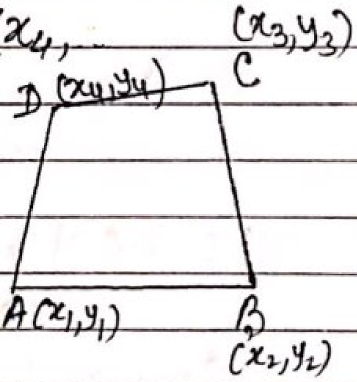
or may be written as:

$\frac{1}{2}\left| \begin{matrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{matrix}\right|$

The Product $\searrow$ are to be taken + and the products $\nearrow$ to be taken −ve.

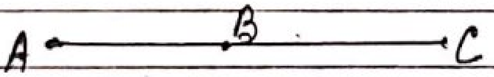
(ii) Area of quadrilateral ABCD.

Given $A(x_1,y_1), B(x_2,y_2), C(x_3,y_3)$ and $D(x_4, y_4)$



Area $= \frac{1}{2}\left| \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{matrix}\right|$

Note:



The points $A(x_1,y_1), B(x_2,y_2)$ and $C(x_3,y_3)$ are collinear if area of $\Delta ABC = 0$.

Example 17. Given the triangle ABC, $A(1,1), B(7,-3), C(12,2)$
$\quad x_1, y_1 \quad x_2\ y_2 \quad x_3\ y_3$

Solution: Area ΔABC.

$= \frac{1}{2}\left| x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$

$= \frac{1}{2}\left| 1\times(-3-2)+7(2-1)+12(1+3)\right|$

$= \frac{1}{2}\left| -5+7+48\right| = \frac{1}{2}\times 50 = 25$ ✓

Alternate method:

$= \frac{1}{2}\left| \begin{matrix} 1 & 7 & 12 & 1 \\ 1 & -3 & 2 & 1 \end{matrix}\right| = \frac{1}{2}\left| -3+14+12-7+36-2\right|$

$= \frac{1}{2}\left| -12+62\right|$

$= \frac{1}{2}\times 50 = 25$ ✓

Example 18. Two points A and B have coordinates (-3,2) and (9,8) respectively.

(i) Find the coordinates of C, the point where AB cuts the Y-axis. ---[3]

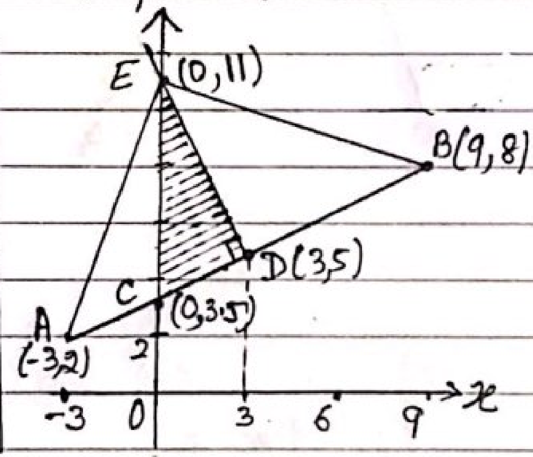
(ii) Find the coordinates of D, the mid point of AB. --- [1] [W-15/21/Q8]

(iii) Find the equation of the perpendicular bisector of AB. --[2]

The perpendicular bisector of AB cuts Y-axis at the point E.

(iv) Find the coordinates of E. ---- [1]

(v) Show that the area of triangle ABE, is four times the area of triangle ECD. --- [3]



Solution: $A(-3,2), B(9,8)$

Gradient of AB, $m_1 = \frac{8-2}{9+3} = \frac{6}{12} = \frac{1}{2}$

∴ Equation of line AB

$y - 2 = \frac{1}{2}(x+3)$

or $y = \frac{1}{2}x + 3.5$ —— (1)

(i) Line AB (1) cuts Y-axis where

$x = 0, \quad y = 3.5, \quad \underline{C(0, 3.5)}$ ✓

(ii) Mid point of AB, $D\left(\frac{-3+9}{2}, \frac{2+8}{2}\right)$

$= \underline{D(3, 5)}$ ✓

(iii) Gradient of perpendicular to AB

$m_2 = -\frac{1}{m_1} = \frac{-1}{\frac{1}{2}} =$

∴ Equation of the perp. bisector of AB,

$D(3,5)$ and $m_2 = 2$

$y - 5 = -2(x-3) \Rightarrow \underline{y = -2x + 11}$ —— (2)

(iv) The perp. bisector (2) cuts Y-axis

where $x = 0$; $y = 11$, $\underline{E(0, 11)}$ ✓

(v) $A(-3,2), B(9,8), E(0,11)$

∴ Area of ΔABE.

$\frac{1}{2}\begin{vmatrix} -3 & 9 & 0 & 3 \\ 2 & 8 & 11 & 2 \end{vmatrix}$

$= \frac{1}{2}\left| -24 + 99 + 0 - 18 - 0 + 33 \right|$

$= 45$ —— (3) Continued→

Continued→

$E(0,11), C(0,3.5), D(3,5)$

area of ΔECD

$= \frac{1}{2}\begin{vmatrix} 0 & 0 & 3 & 0 \\ 11 & 3.5 & 5 & 11 \end{vmatrix}$

$= \frac{1}{2}\left| 0 + 0 + 33 - 0 - 10.5 - 0 \right|$

$= \frac{1}{2} \times 22.5 = 11.25$ —— (4)

Now from (3) &

Area of $\Delta ABE = 45$

$= 4 \times 11.25$

$= 4$ area of ΔECD

from (4)

∴

$\underline{\text{Area of } ABE = 4 \times \text{Area of } \Delta ECD}$

Example 19. The points $A(-3,2)$ and $B(1,4)$ are vertices of an isosceles triangle ABC, where angle $B = 90°$

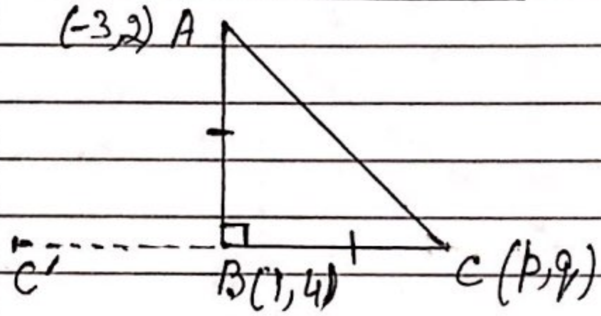
(i) Find the length of the line AB. ---[1]

(ii) Find the equation of line BC. ---[3]

(iii) Find the coordinates of each of the two possible positions of C. ---[6]

[W-13/11/Q10]

Solution: $A(-3,2)$, $B(1,4)$.



(i) Length $AB = \sqrt{(1+3)^2+(4-2)^2}$

$= \sqrt{16+4}$

$= \underline{\sqrt{20}} = 4.47$ — (i)

(ii) Gradient of AB, $m_1 = \dfrac{4-2}{1-(-3)} = \dfrac{2}{4} = \dfrac{1}{2}$

$\therefore$ Gradient of line BC perp. to AB, $m_2 = -\dfrac{1}{m_1} = \dfrac{-1}{\frac{1}{2}} = -2$ $\Bigg\}$ $\angle B = 90°$, $BC \perp AB$

Equation of line BC,

$B(1,4)$ and Gradient $m_2 = -2$

$y - 4 = -2(x-1)$

or $y = -2x + 6$ ——— (ii)

(iii) Let Coordinates of $C(p,q)$.

$C(p,q)$ lies on line BC, from (ii) $q = -2p + 6$ ——— (iii)

and $\triangle ABC$ is isosceles, length $BC = AB = \sqrt{20}$ from (i)

$\therefore$ In rt $\triangle ABC$, $AC^2 = AB^2 + BC^2$

$A(-3,2)$, $C(p,q)$, $(p+3)^2 + (q-2)^2 = 20 + 20$ $\quad [AC^2 = (p+3)^2 + (q-2)^2]$

or $(p+3)^2 + (-2p+6-2)^2 = 40$ $\quad$ from (iii) $q = -2p+6$

or $5p^2 - 10p - 15 = 0$ or $p^2 - 2p - 3 = 0$

$(p-3)(p+1) = 0$

$p = 3, -1$

from (iii) $\begin{cases} p = 3 \\ q = 0 \end{cases}$ $\quad \begin{matrix} p = -1 \\ q = 8 \end{matrix}$ $\quad \therefore C(3,0)$ or $(-1,8)$ ✓

Straight line Graphs

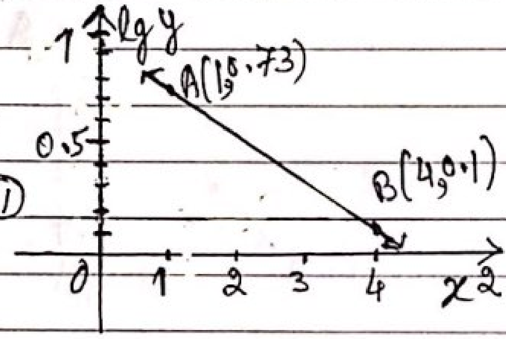
Converting from linear form to a non-linear equation

Example 20. Variables x and y are such that when $\lg y$ is plotted against x^2, a straight line graph passing through the points $(1, 0.73)$ and $(4, 0.10)$ is obtained.

(a) Given that $y = Ab^{x^2}$, find the value of each of the constants A and b. --[4]

(b) Find the value of y when $x = 1.5$ ---[2]

(c) Find the positive value of x when $y = 2$. /SP-20/02/Q7/ --[2]



Solution: Straight line graph, $A(1, 0.73)$, $B(4, 0.1)$

Gradient of line $m = \dfrac{0.1 - 0.73}{4 - 1} = -0.21$ ✓ —— (1)

(a)

Given $y = Ab^{x^2}$ —— (2)

$\Rightarrow \lg y = \lg(Ab^{x^2})$

or $\lg y = \lg b \cdot x^2 + \lg A$ —— (3) Represents line AB on the graph.

$[Y = mX + C]$

Gradient of line (2) from (1) $m = \lg b = -0.21$

or $b = 10^{-0.21}$ $[\because \lg b = \log_{10} b$

or $\underline{b = 0.617}$ ✓

$\therefore$ Y-Int of line (2)

$C = \lg A = 0.94$ from (4)

$\Rightarrow \underline{A = 10^{0.94}} = \underline{8.71}$ ✓

Let Eqn of line AB is

$y = mx + c$

$y = -0.21x + c$

Passes through $B(4, 0.1)$

$\Rightarrow c = 0.94$ Y-Int. —— (4)

(b) from (2)

$y = Ab^{x^2}$

or $y = 8.71\,(0.617)^{x^2}$ —— (5)

when $x = 1.5 \Rightarrow y = 8.71 \times (0.617)^{2.25}$ $[x = 1.5$, $x^2 = 2.25$

$= 8.71 \times 0.3374$

or $y = 2.93$ ✓

(c) Given $y = 2$, $x = ?$

from (3) $\lg y = \lg b\, x^2 + \lg A$

or $\lg 2 = -0.21x^2 + 0.94$

or $0.301 = -0.21x^2 + 0.94$

$\Rightarrow 0.21x^2 = 0.94 - 0.301$

$x^2 = \dfrac{0.639}{0.21}$

$x^2 = 3.042$

or $\underline{x = 1.74}$

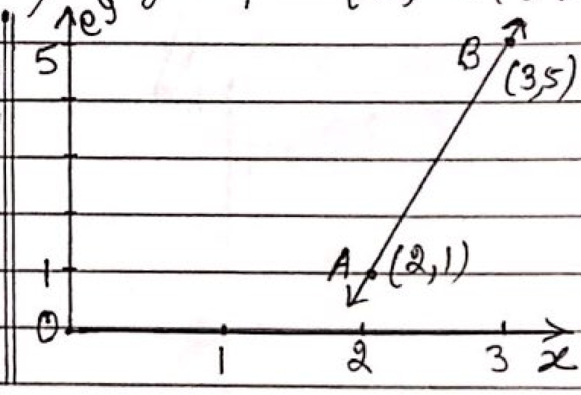
Straight Line Graph.

Example 21. Variables x and y are such that when e^y is plotted against x, a straight line graph is obtained. The diagram shows this straight line graph passing through the points $(2,1)$ and $(3,5)$

(i) Express y in terms of x. --- [4]

(ii) State the value x for which y exists --- [1]

(iii) Find the value of x when $y = \ln 6$ --- [1]



Solution: $e^y = mx + c$ —— ①

Gradient of line AB, $m = \frac{5-1}{3-2} = 4$ ✓

(i) fr ① $e^y = 4x + c$ —— ②

$x = 2$, when $e^y = 1$ fr ② $1 = 4 \times 2 + c \Rightarrow c = -7$ ✓

fr ②

$e^y = 4x - 7$ —— ③

or $\underline{y = \ln(4x-7)}$ —— ④

(ii) fr ④ y exists when $\ln(4x-7)$ exists $\Rightarrow 4x - 7 > 0$

or $\underline{x > \frac{7}{4}}$ ✓

(iii) when $y = \ln 6$

fr ④ $\ln 6 = \ln(4x-7)$

$\Rightarrow 4x - 7 = 6$

$\underline{x = \frac{13}{4}}$ ✓

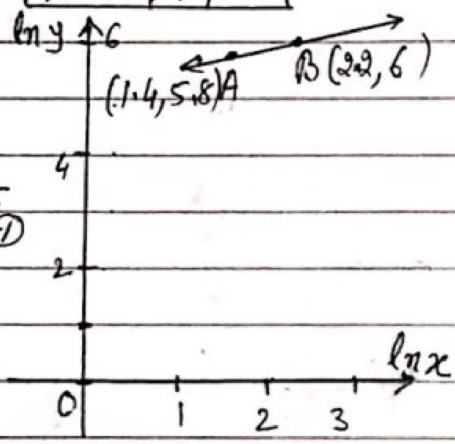
Straight line graph

Example 22. Two variables, x and y are such that $y = Ax^b$, where A and b are constants. When $\ln y$ is plotted against $\ln x$, a straight line graph is obtained, which passes through the points $(1.4, 5.8)$ and $(2.2, 6.0)$

[W-15 | 11 | Q7]

(i) Find the value of A and of b. ---[4]

(ii) Calculate the value of y when $x = 5$ --[2]



Solution: Gradient of AB $m = \frac{6 - 5.8}{2.2 - 1.4} = \frac{0.2}{0.8} = 0.25$ —— ①

(i) Given $y = Ax^b$ —— ②

or $\ln y = b \cdot \ln x + \ln A$ —— ③ line AB

$[Y = mX + C]$

fr ① & ③ Gradient of ③ $m = b = 0.25$ ✓

$\therefore$ fr ③ $\ln y = 0.25 \ln x + \ln A$ —— ④

Passes through $B(2.2, 6)$
$\ln x$ $\ln y$

$\Rightarrow$ $6 = 0.25 \times 2.2 + \ln A$

or $\ln A = 5.45$

or $A = e^{5.45} = 233$ ✓

(ii) $\therefore$ fr ④ $\ln y = 0.25 \ln x + 5.45$ —— ⑤

now when $x = 5$

$\ln y = 0.25 \times \ln 5 + 5.45$

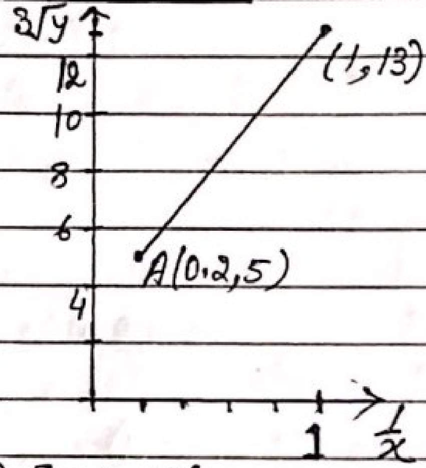
$= 0.4023 + 5.45 = 5.8523$

$y = e^{5.8523}$

or $y = 348$ ✓

Straight line Graph.

Example 23. Variables x and y are such that, when $3\sqrt{y}$ is plotted against $\frac{1}{x}$, a straight line graph passing through the points $(0.2, 5)$ and $(1, 13)$ is obtained.
Express y in terms of x. [S-17/23/Q3] --[4]



Solution. Let Equation of line AB is
$$Y = mX + C \quad \text{—— (1)}$$
Gradient of line AB $= \frac{13-5}{1-0.2} = \frac{8}{0.8} = 10 = m$

AB, Pass through the point B(1,13)

$\therefore$ Equation of line AB, $Y - y_1 = m(X - x_1)$

or $Y - 13 = 10(X - 1)$

or $Y = 10X + 3$ —— (2) [as in (1)]

now put $Y = 3\sqrt{y}$ and $X = \frac{1}{x}$ in (2)

we get $3\sqrt{y} = 10 \times \frac{1}{x} + 3$

or $y = \left(\frac{10}{x} + 3\right)^3$ ✓ is the required relation.

Example 24. When $\lg y$ is plotted against x^2 a straight line a obtained which passes through the points $(4,3)$ and $(12,7)$.

(i) Find the gradient of the line. --[1]

(ii) Use your answer to part (i) to express $\lg y$ in terms of x. --[2]

(iii) Hence express y in terms of x, giving your answer in the form: $y = A(10^{bx^2})$ where A and b are constants. [W-17/11/Q4] --[3]

Solution: (i) gradient of line $m = \frac{7-3}{12-4} = \frac{1}{2}$ ✓

(ii) $Y = mX + C$ —— (1)

$\because$ line passes through $(4,3)$, $m = \frac{1}{2}$

$Y - 3 = \frac{1}{2}(X - 4)$

or $Y = \frac{1}{2}X + 1$

Put $Y = \lg y$ and $X = x^2$

or $\lg y = \frac{1}{2}x^2 + 1$ —— (2)

from (2) $\lg y = \frac{1}{2}x^2 + 1$

or $y = 10^{\left(\frac{1}{2}x^2 + 1\right)}$

or $y = 10 \cdot 10^{\frac{x^2}{2}}$ ✓

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Straight line Graph

Date / Page No P-20

Converting from non-linear form to a linear equation

Example 24: The table shows experimental values of two variables x and y.

x	2	4	6	8
y	9.6	38.4	105	232

It is known that x and y are related by the equation, $y = ax^3 + bx$ where a and b are constants.

(i) A straight line graph is to be drawn for this information with y/x on the vertical axis. State the variable which must be plotted on the horizontal axis. ---[1]

(ii) Draw this straight line graph on the grid. ---[2]

(iii) Use your graph to estimate the value of a and b. ---[3]

(iv) Estimate the value of x for which $2y = 25x$, W-13/21/Q8 ---[2]

Solution: (i) Given $y = ax^3 + bx \Rightarrow \frac{y}{x} = ax^2 + b$, --- ① $\rightarrow (Y = mX + C)$

$\therefore$ x^2 must be plotted along horizontal axis.

(ii)

x^2	4	16	36	64
y/x	4.8	9.6	17.5	29

(iii) Gradient of the line

$\frac{BN}{AN} = m = \frac{29 - 4.8}{64 - 4}$

or $m = 0.4$

Let Equation of line

$Y = mx + C$

$C = OP = 3.4$

$Y = 0.4X + 3.4$ --- ②

from ① & ②

(iii) $a = 0.4$
$b = 3.4$

(iv) $2y = 25x$

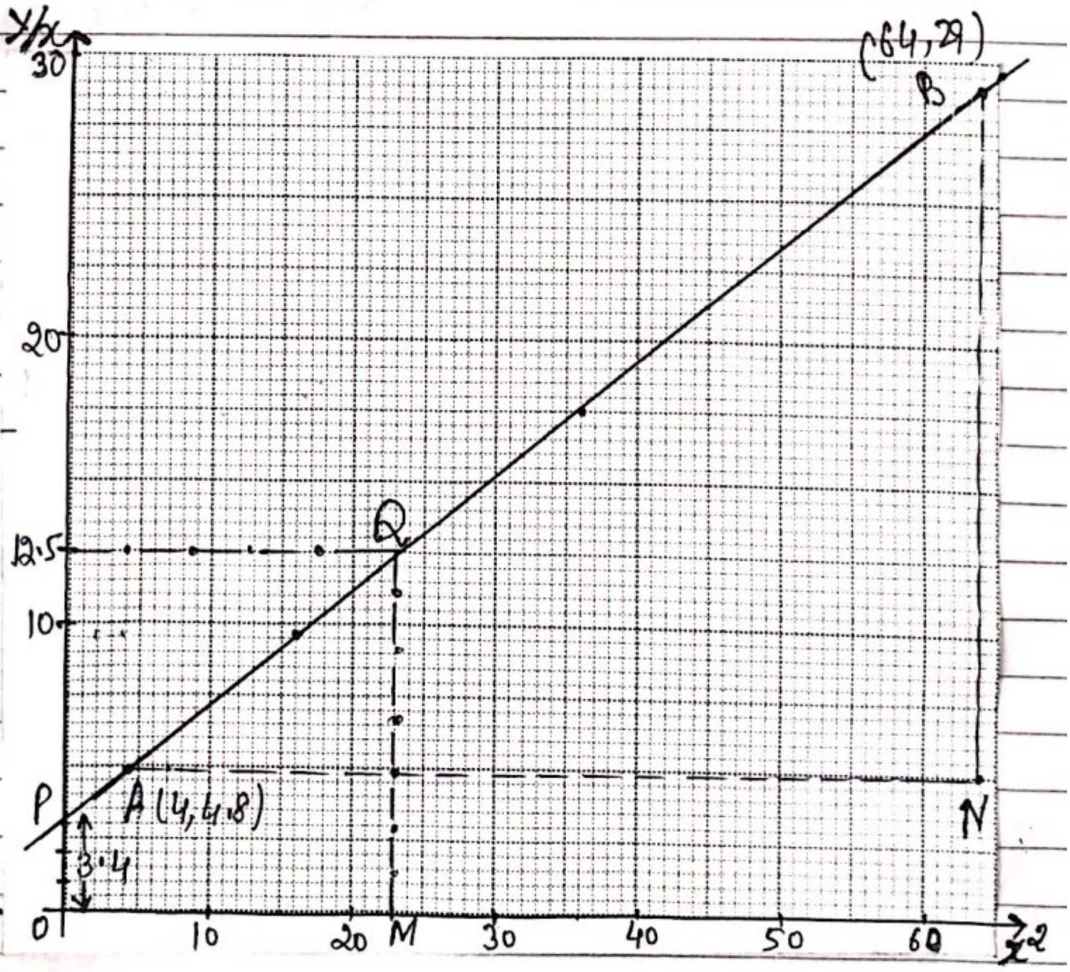
$\Rightarrow \frac{y}{x} = 12.5$

from Graph.

at $y/x = 12.5$

$x^2 = QM = 23$

$\therefore x = \sqrt{23} = 4.8$



Example 25. The table shows values of the variables t and P.

t	1	1.5	2	2.5
P	4.39	8.38	15.8	30.0

(i) Draw the graph of $\ln P$ against t on the grid below. ---[2]

(ii) Use the graph to estimate the value of P when $t=2.2$ ---[2]

(iii) Find the gradient of the graph and state the coordinates of the point where the graph meets the vertical axes. --[2]

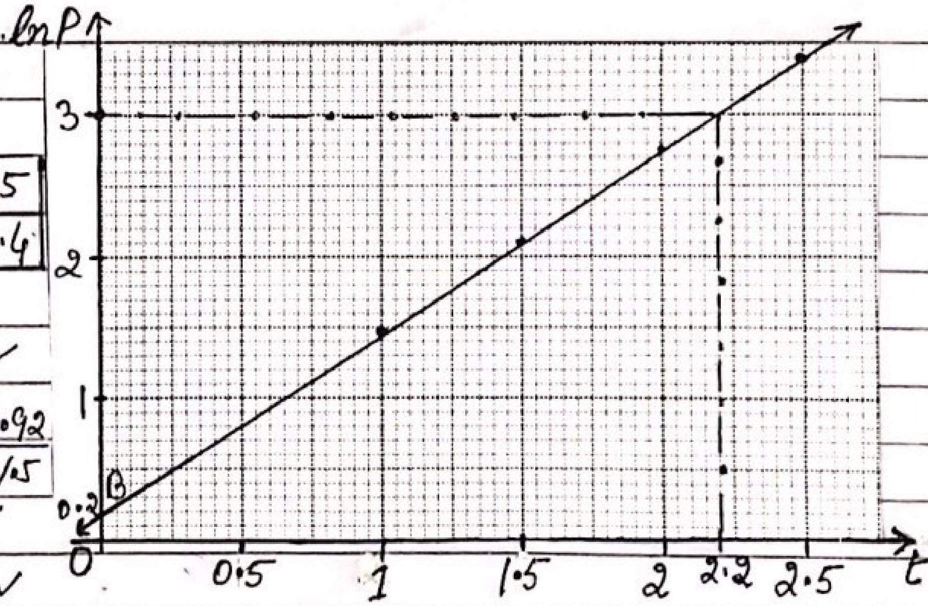
(iv) Using your answers to part (iii), show that $P=ab^t$, where a and are the constants to be found. ---[3]

(v) Given that the equation in part (iv) is valid for values of t upto 10, find the smallest value of t, correct 1 decimal place, for which P is atleast 1000. [S-17/21/Q10] --[2]

Solution:

(i)

t	1	1.5	2	2.5
$\ln P$	1.48	2.12	2.76	3.4



(ii) $t=2.2 \rightarrow \ln P = 3$

$\Rightarrow P = e^3 = 20$ ✓

(iii) Gradient $m = \dfrac{3.4-1.48}{2.5-1} = \dfrac{1.92}{1.5}$

$\Rightarrow m = 1.28$ ✓

y-Intercept $OB = 0.2$ ✓

(iv) Equation of line $\ln P = 1.28t + 0.2$ [as $y = mt + c$]

or $P = e^{(0.2+1.28t)} = e^{0.2} \cdot e^{1.28t}$

or $P = 1.22e^{1.28t}$ ✓

(v) $P \geq 1000 \Rightarrow 1.22e^{1.28t} \geq 1000$

or $e^{1.28t} \geq \dfrac{1000}{1.22} = 819.672$

or $1.28t \geq \ln 819.672 = 6.709$

$t \geq 5.3$

$\therefore$ Smallest value of $t = 5.3$ ✓

Straight Line Graph

Example 26. The trees in a certain forest are dying because of an unknown virus. The number of trees, 'N', surviving 't' years after onset of the virus is shown in the table below.

t	1	2	3	4	5	6
N	2000	1300	890	590	395	260

The relationship between N and t is thought to be of the form $N = Ab^{-t}$

(i) Transform this relationship into straight line form. ---[1]

(ii) Using the given data, draw the straight line on the grid. --[3]

(iii) Use your graph to estimate the value of A and of b. --[3]

If trees continue to die in the same way, find. W-15/23/Q11

(iv) the number of trees surviving after 10 years. --[1]

(v) the number of years until there are only 10 trees surviving --[2]

Solution (i) $N = Ab^{-t}$ ① $\Rightarrow \lg N = (-\lg b)\cdot t + \lg A$ ② $(Y = mX + C)$ $Y = \lg N$, $X = t$

(ii)

t	1	2	3	4	5	6
lg N	3.30	3.11	2.95	2.77	2.60	2.41

(iii) from graph

y-int $= OB = 3.47$

or $\lg A = 3.47$ from ②

or $A = 10^{3.47} = 2950$ ✓

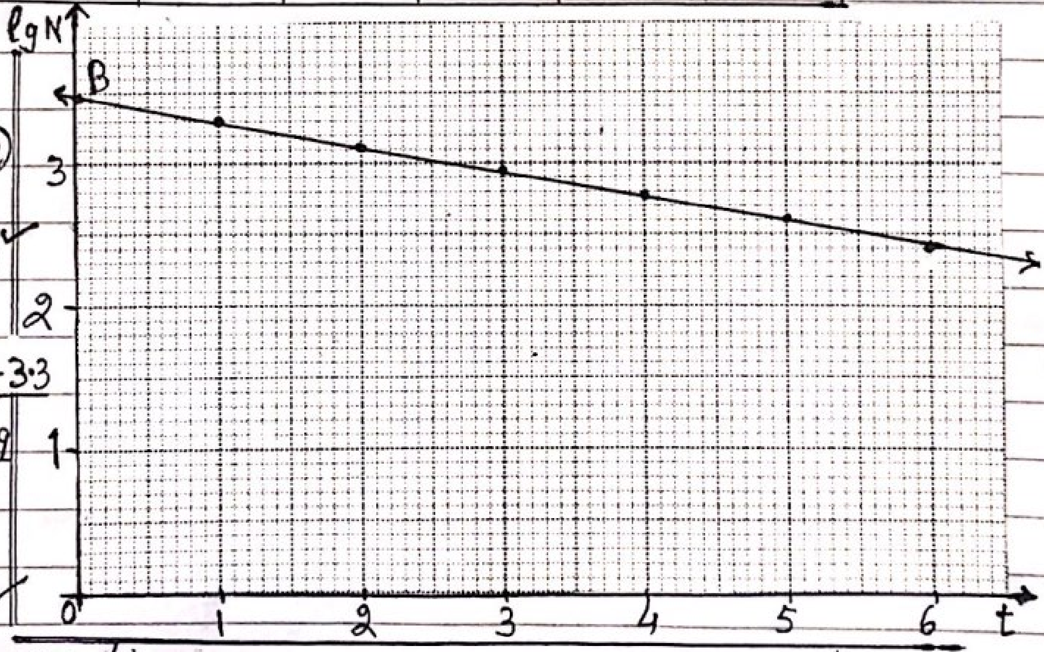
Gradient

$m = -\lg b = \frac{2.41 - 3.3}{5}$

or $-\lg b = \frac{-.89}{5}$

or $\lg b = 0.178$

$b = 10^{0.178} = 1.5$ ✓



(iv) for ① $N = Ab^{-t}$

or $N = 2950(1.5)^{-t}$

when $t = 10$, $\Rightarrow N = 2950(1.5)^{-10}$

$= 51$

$\therefore$ at $t = 10$, $N = 51$ ✓

(v) $N = 2950(1.5)^{-t} = 10$ given

$\Rightarrow (1.5)^t = 295$

or $t \cdot \lg 1.5 = \lg 295$

$t = \frac{\lg 295}{\lg 1.5} = 14$ ✓

Straight Line Graph.

Example 27. The table shows the variables x and y.

x	2	4	6	8	10
y	736	271	100	37	13

The relationship between x and y is thought to be of the form $y = Ae^{bx}$, where A and b are constants.

(i) Transform this relationship into straight line form. --- [1]

(ii) Hence by plotting a suitable graph, show that the relationship $y = Ae^{bx}$ is correct. --- [2]

(iii) Use your graph to find the value of A and b. --- [4]

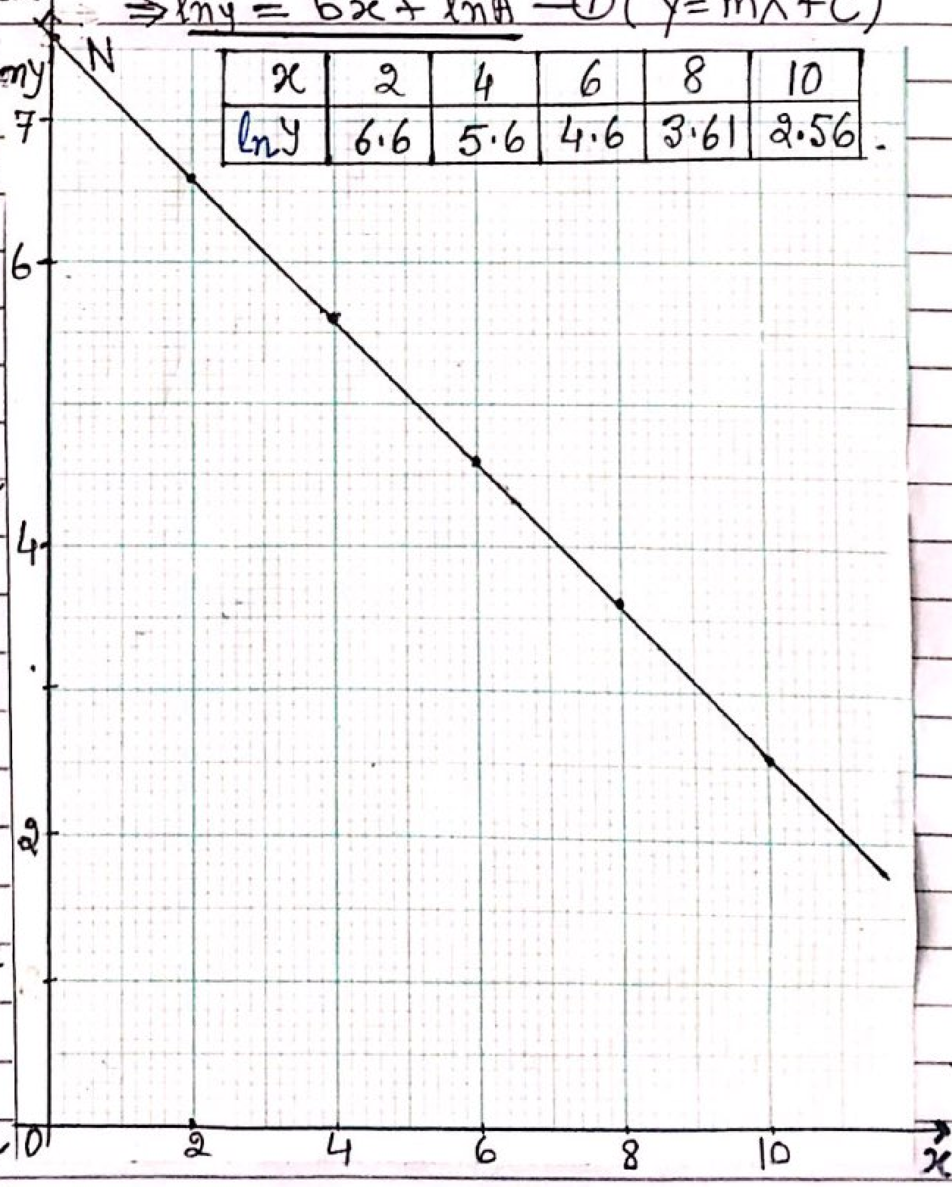
(iv) Estimate the value of x, when $y = 500$ --- [2]

(v) Estimate the value of y when $x = 5$. [M-18/12/Q9] --- [2]

Solution: (i) $y = Ae^{bx}$ $\Rightarrow \underline{\ln y = bx + \ln A}$ — ① $(Y = mX + C)$

x	2	4	6	8	10
$\ln y$	6.6	5.6	4.6	3.61	2.56

(ii) Graph is a straight line.



(iii) Gradient $m = \frac{4.6 - 6.6}{6 - 2} = -\frac{2}{4}$

fr ① $m = b = $ $\underline{b = -0.5}$ ✓

and y-Intercept

$\ln A = 7.6$

or $\underline{A = e^{7.6} = 2000}$ ✓ (approx)

(iv) Given $y = 500$, $x = ?$

fr ① $\ln y = bx + \ln A$

or $\ln 500 = -0.5x + 7.6$

or $6.214 = -0.5x + 7.6$

$\underline{x = 2.77}$ ✓

(v) $y = Ae^{bx}$

$y = 2000e^{-0.5x}$

when $x = 5$

$y = 2000e^{-0.5 \times 5}$

or $\underline{y} = 2000e^{-2.5} = \underline{164}$ ✓