IG-0606 Additional Maths

Straight Line Graphs Exercise

Suresh Goel
(Director)
Alliance world School,
Noida, Delhi-NCR, India.

Straight line graphs

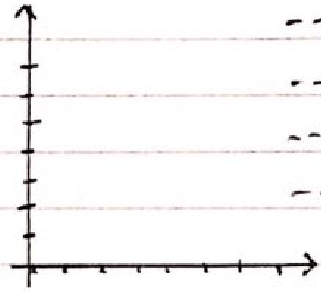
Q1. The table shows the variables x and y.

x	2	4	6	8	10
y	736	271	100	37	13

The relationship between x and y is thought to be of the form $y = Ae^{bx}$, where A and b are constants.

(i) Transform this relationship into streight line form, ---[1]

(ii) Hence, by plotting a suitable graph, show that the relationship $y = Ae^{bx}$ is correct. ---[2]



(iii) Use your graph to find the value of A and of b. ---[4]

(iv) Estimate the value of x when $y = 500$ ---[2]

(v) Estimate the value of y when $x = 5$. ---[2]

M-18/12/Q9

Q2 Solutions to this question by acculate drawing **will not** be accepted.

P is the point (8,2) and Q is the point (11,6)

(i) Find the equation of line L which passes through P and is perpendicular to the line PQ. ---[3]

The point R lies on L such that the area of triangle PQR is 12.5 unit².

(ii) Showing all your working, find the coordinates of each of the two possible positions of point R. ---[6]

M-18/22/Q9

Q3 The points A(3,7) and B(8,4) lie on the line L. The line through the point C(6,−4) with gradient $\frac{1}{6}$ meets the line L at the point D. Calculate,

(i) the coordinates of D. ---[6]

(ii) the equation of line through D perpendicular to the line $3y - 2x = 10$ ---[2]

M-17/22/Q8

IG-Maths 0606

Straight Line Graph.

P-2

Q4 The curve $3x^2+xy-y^2+4y-3=0$ and the line $y=2(1-x)$ intersect at the points A and B.

(i) Find the coordinates of A and B. ---[5]

(ii) Find the equation of perpendicular bisector of the line AB, ---[4] giving your answer in the form $ax+by=c$, where a, b and c are int.

[S-17/21/Q9]

Q5 Solutions to this question by accurate drawing will not be accepted.

The points A and B are $(-8,8)$ and $(4,0)$ respectively.

(i) Find the equation of the line AB. ---[2]

(ii) Calculate the length of AB. ---[2]

The point C is $(0,7)$ and D is the mid point of AB.

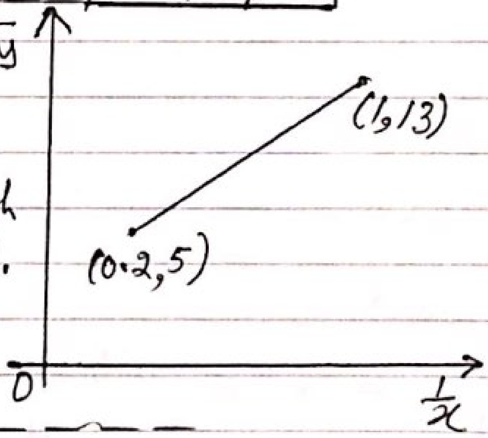
(iii) Show that angle ADC is a right angle. ---[3]

The point E is such that $\vec{AE}=\begin{pmatrix}4\\-7\end{pmatrix}$

(iv) Write down the position vector of the point E. ---(1)

(v) Show that ACBE is parallelogram. [S-17/22/Q8] ---[2]

Q6 Variables x and y are such that when $\sqrt[3]{y}$ is plotted against $\frac{1}{x}$, a straight line graph passing through the points $(0\cdot2, 5)$ and $(1, 13)$ is obtained. Express y in terms of x. ---[4]



[S-17/23/Q3]

Q7 The line $y=2x+1$ intersects the curve $xy=14-2y$ at the points P and Q. The mid point of line PQ is point M.

(i) Show that the point $(-10, \frac{23}{8})$ lies on the perp. bisector of PQ. ---[9]

The line PQ intersects the y-axis at the point R. The perp. bisector of PQ intersects the y-axis at the point S.

(ii) Find the area of the triangle RSM. ---[3]

[W-17/13/Q12]

IG-Maths 0606

Straight Line Graph

Date P-3

Page No.

Q8 Solution to this question by accurate drawing will not be accepted.

Three points have coordinates $A(-8,6)$, $B(4,2)$ and $C(-1,7)$.
The line through C perpendicular to AB intersects AB at point P.

(i) Find the equation of the line AB. --[2]

(ii) Find the equation of the line CP. --[2]

(iii) Show that P is the mid point of AB. --[3]

(iv) Calculate the length CP. --[1]

(v) Hence find the area of the triangle ABC. S-16/11/Q8 --[2]

Q9 The coordinates of three points are $A(-2,6)$, $B(6,10)$ and $C(p,0)$.

(i) Find the coordinates of M, the mid point of AB. --[2]

(ii) Given CM is perpendicular to AB, find the value of the constant p. --[2]

(iii) Find angle MCB. S-16/22/Q5 --[3]

Q10(a) The line $y = kx - 4$, where k is a positive constant, passes through the point $P(0,-4)$ and is a tangent to the curve $x^2 + y^2 - 2y = 8$ at the point T. Find

(i) the value of k. --[5]

(ii) the coordinates of T. --[3]

(iii) the length of TP. W-16/21/Q9 --[2]

Q10(b) The points A and B have coordinates $(2,-1)$ and $(6,5)$ respectively.

(i) Find the equation of the perp. bisector of AB, giving your answer in the form $ax + by = c$, where a, b and c are integers. --[4]

The point C has coordinates $(10,-2)$

(ii) Find the equation of the line through C which is parallel to AB. --[2]

(iii) Calculate the length BC. --[2]

(iv) Show that triangle ABC is isosceles. M-15/22/Q8 --[1]

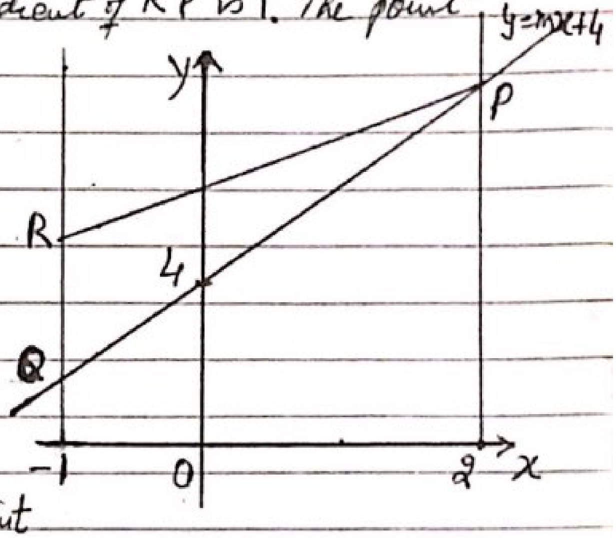
Q11 The curve $y = xy + x^2 - 4$ intersects the line $y = 3x - 1$ at the points A and B. Find the equation of the perpendicular bisector of the line AB. S-15/11/Q5 --[8]

IG-Maths 0606

Date P-4

Straight Line Graph.

Q12 The line $y = mx + 4$ meets the lines $x = 2$ and $x = -1$ at the points P and R respectively. The point R is such that QR is parallel to y-axis and the gradient of RP is 1. The point P has coordinates $(2, 10)$.



(i) Find the value of m. ---[2]

(ii) Find the y-coordinate of Q --[1]

(iii) Find the coordinates of R. ---[2]

(iv) Find the equation of the line through P, perpendicular to PQ, giving your answer in the form $ax + by = c$, where a, b and c are integers. ---[3]

(V) Find the coordinates of the mid point M, of the line PQ. --[2]

(Vi) Find the area of the triangle QRM. -- [2] $\boxed{S\text{-}15/22/Q9}$

Q13 The line $2x - y + 1 = 0$ meets the curve $x^2 + 3y = 19$ at the points A and B. The perpendicular bisector of the line AB meets the x-axis at the point C. Find the area of triangle ABC. $\boxed{W\text{-}15/11/Q12}$ ---[9]

Q14 Two points A and B have coordinates $(-3, 2)$ and $(9, 8)$ respectively.

(i) Find the coordinates of C, the point where line AB cuts the y-axis. ---[3]

(ii) Find the coordinates of D, the mid point of AB. --[1]

(iii) Find the equation of the perpendicular bisector of AB. ---[2]

The perpendicular bisector of AB cuts the y-axis at the point E.

(iv) Find the coordinates of E. --[1]

(v) Show that the area of triangle ABE is four times the area of triangle ECD. $\boxed{W\text{-}15/21/Q8}$ ---[3]

Q15 The line $x - y + 2 = 0$ intersects the curve $2x^2 - y^2 + 2x + 1 = 0$ at the points A and B. The perpendicular bisector of the line AB intersects the curve at the points C and D. Find the length of the line CD in the form $a\sqrt{5}$, where a is an integer. ---[10]

$\boxed{W\text{-}15/13/Q11}$

Straight Line Graph.

Q16 The trees in a certain forest are dying because of an unknown virus. The number of trees, N, surviving t years after the onset of the virus is shown in the table below.

t	1	2	3	4	5	6
N	2000	1300	890	590	395	260

The relationship between N and t is thought to be of the form $N = Ab^{-t}$

(i) Transform this relationship into straight line form. ---[1]

(ii) Using the given data, draw the straight line on the grid. --[3]

(iii) Use your graph to estimate the value of A and of b. --[3]

If the trees continue to die in the same way, find

(iv) the number of trees surviving after 10 years. --[1]

(v) the number of years taken until there are only 10 trees surviving. --[2]

W-15/23/Q11

Q17 The table shows the values of the variables V and p.

V	10	50	100	200
p	95.0	8.5	3.0	1.1

(i) By plotting a suitable straight line graph, show that V and p are related by the equation $p = kV^n$, where k and n are constants. --[4]

Use your graph to find

(ii) the value of n. --[2]

(iii) the value of p when V = 35.

S-14/11/Q8

Q18 The points $A(p,1)$, $B(1,6)$, $C(4,q)$ and $D(5,4)$, where p and q are constants, are the vertices of a kite $ABCD$. The diagonals of the kite, AC and BD, intersect at the point E. The line AC is the perpendicular bisector of BD. Find

(i) the coordinates of E, --- [2]

(ii) the equation of the diagonal AC. --- [3]

(iii) the area of kite $ABCD$. $\boxed{S\text{-}14/21/Q9}$ --- [3]

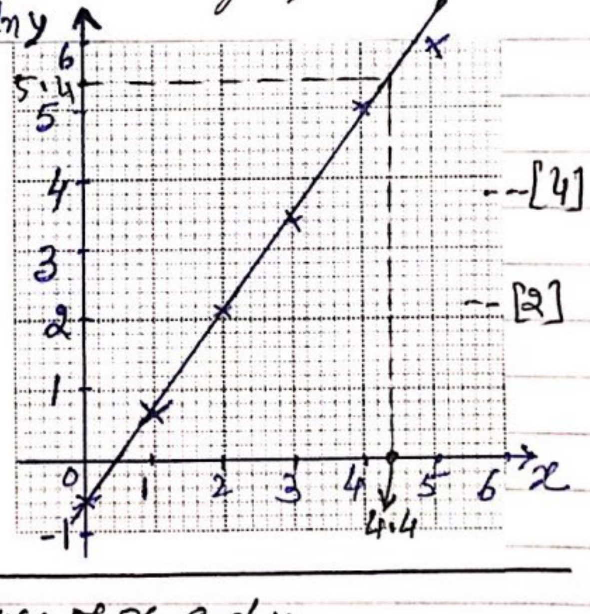
Q19 Two variables x and y are connected by the relationship $y = Ab^x$, where A and b are constants.

(i) Transform the relationship $y = Ab^x$ into a straight line form. --- [2]

An experiment was carried out measuring values of y for certain values of x. The values of $\ln y$ and x were plotted and a line of best fit was drawn. The graph is shown on the grid below.

(ii) Use the graph to determine the value of A and value of b, giving each to one significant figure. --- [4]

(iii) Find x when $y = 220$. --- [2]



$\boxed{S\text{-}14/22/Q10}$

Q20 The table shows experimental values of x and y.

x	1.50	1.75	2.00	2.25
y	3.9	8.3	19.5	51.7

(i) Complete the following table:

x^2				
$\lg y$				

--- [1]

(Continued →)

IG-Maths 0606

Straight Line Graph

P-7

(→ Continued)

Q20 (ii) By plotting a suitable straight line graph on the grid below, show that x and y are related by the equation, $y = Ab^{x^2}$, where A and b are constants. ...[2]

(iii) Use your graph to find the value of A and b. ...[4]

(iv) Estimate the value of y when $x = 1{\cdot}25$ ...[2]

S-14/13/Q10

Q21 The point $A(2, 11)$, $B(-2, 3)$ and $C(2, -1)$ are the vertices of a triangle.

i) Find the equation of the perpendicular bisector of AB. --- [4]

The line through A parallel to BC intersect the perp. bisector AB at the point D.

(ii) Find the area of the quadrilateral $ABCD$. ---[6]

S-14/23/Q9

IG-Maths 0606

Straight Line Graph

Q22 The point P lies on the line joining A(-2,3) and B(10,19) such that AP : PB = 1 : 3

(i) Show that the x-coordinate of P is 1 and find the y-coord. of P. ---[2]

(ii) Find the equation of the line through P which is perpendicular to AB. --[3]

The line through P which is perpendicular to AB, meets the y-axis at the point Q,

(iii) Find the area of the triangle AQB. W-14/11/Q8 ---[3]

Q23 The table shows experimental values of variables x and y.

x	2	2.5	3	3.5	4
y	18.8	29.6	46.9	74.1	117.2

(i) By plotting a suitable straight line graph on the grid below, show that x and y are related by the equation $y = ab^x$, where a and b are constants. ---[4]

(ii) Use your graph to find the value of a and of b. W-14/11/Q9 ---[4]

Straight Line Graph

Q24 The line $4y = x + 8$ cuts the curve $xy = 4 + 2x$ at the point A and B. Find the exact length of AB. [W-14/13/Q2] ---[5]

Q25 Points A and B have coordinates $(-2, 10)$ and $(4, 2)$ respectively. C is the mid point of line AB. Point D is such that $\vec{CD} = \binom{12}{9}$.

(i) Find the coordinates of C and D. ---[3]

(ii) Show that CD is perpendicular to AB. ---[3]

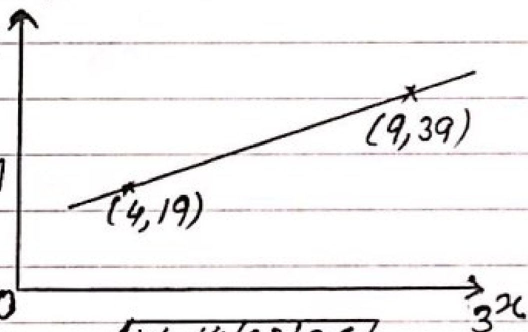
(iii) Find the area of triangle ABD. [W-14/23/Q3] --[2]

Q26 Variables x and y are such that, when $\ln y$ is plotted against 3^x, a straight line graph passing through $(4, 19)$ and $(9, 39)$ is obtained.

(i) Find the equation of this line in the form $\ln y = m3^x + c$, where m and c are the constants to be found. --- [3]

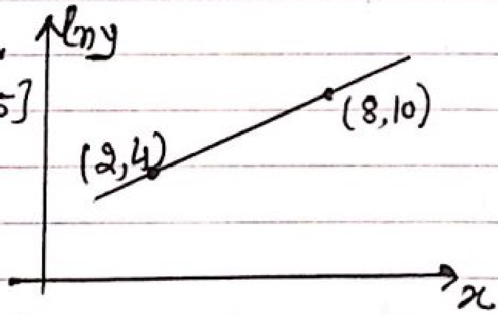
(ii) Find y when $x = 0.5$ ---[2]

(iii) Find x when $y = 2000$ ---[3]



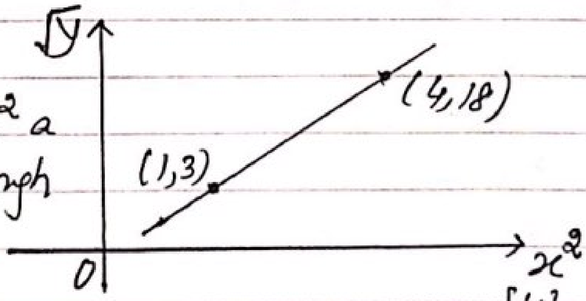
[W-14/23/Q6]

Q27 Variables x and y are such that $y = Ab^x$, where A and b are constants. Find the value of A and of b. ---[5]



[S-13/11/Q2]

Q28 The variables x and y are such that when $\sqrt{y}$ is plotted against x^2 a straight line graph passing through the points $(1, 3)$ and $(4, 18)$ is obtained. Express y in terms of x.



[S-13/22/Q1] ---[4]

Straight Line Graph

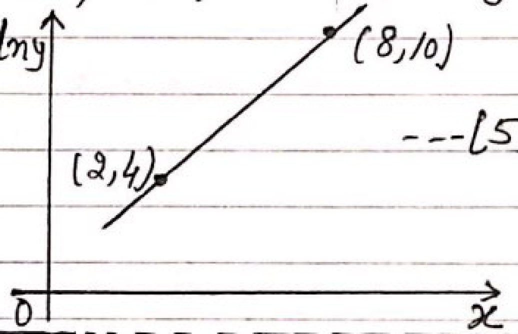
Q29 The points $A(-6,2)$, $B(2,6)$ and C are the vertices of a triangle.

(i) Find the equation of the line AB in the form $y = mx+c$ ---[2]

(ii) Given that angle $ABC = 90°$, find the equation of BC. ---[2]

(iii) Given that the length of AC is 10 units, find the coordinates of each of the two possible positions of point C. S-13/22/Q8 ---[4]

Q30 Variables x and y are such that $y = Ab^x$, where A and b are constants. The diagram shows the graph of $\ln y$ against x, passing through the points $(2,4)$ and $(8,10)$.



Find the value of A and b. ---[5]

S-13/13/Q2

Q31 The points $A(-3,2)$ and $B(1,4)$ are vertices of an isosceles triangle ABC, where angle $B = 90°$.

(i) Find the length of the line AB. W-13/11/Q10 ---[1]

(ii) Find the equation of the line BC. ---[3]

(iii) Find the coordinates of each of the two possible positions of C. --[6]

Q32 The table shows experimental values of two variables x and y.

x	2	4	6	8
y	9.6	38.4	105	232

It is known that x and y are related by the equation $y = ax^3 + bx$, where a and b are constants.

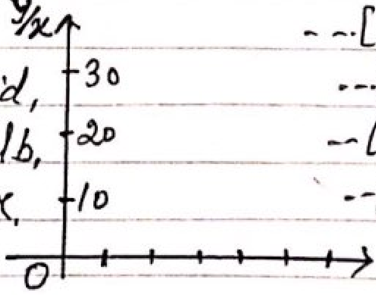
(i) A straight line graph is to be drawn for this information with y/x on the vertical axis. State the variable which must be plotted on the horizontal axis. --[1]

(ii) Draw this straight line graph on the grid. --[2]

(iii) Use your graph to estimate the value of a and b. --[3]

(iv) Estimate the value of x for which $2y = 25x$. --[2]

W-13/21/Q8



IG-Maths 0606

Straight line Graph.

Date P-11

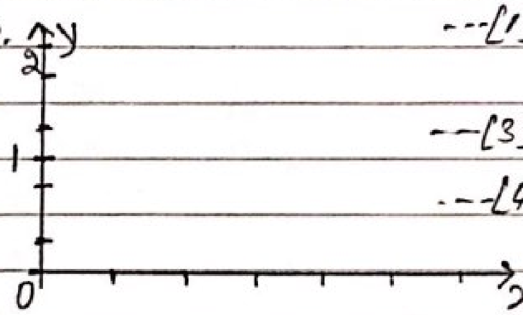
Page No.

Q33 The variables s and t are related by the equation $t = ks^n$, where k and n are constants. The table below shows values of variables s and t.

s	2	4	6	8
t	25.00	6.25	2.78	1.56

(i) A straight line graph is to be drawn for this information with lg t plotted along y-axis. State the variable which must be plotted on the horizontal axis. ---[1]

(ii) Draw this straight line graph on the grid, here. ---[3]



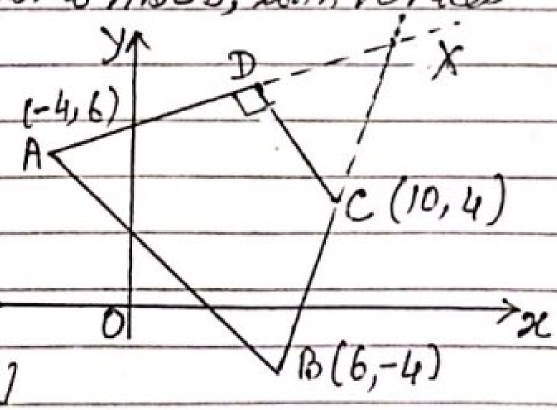
(iii) Use your graph to find the value of k and of n. ---[4]

(iv) Estimate the value of s when t = 4. ---[2]

[W-13/13/Q10]

Q34 The line $4x + y = 16$ intersects the curve $\frac{4}{x} - \frac{8}{y} = 1$ at the points A and B. The x-coordinate of A is less than the x-coordinate of B. Given that the point C lies on the line AB such that AC : CB = 1 : 2, find the coordinates of C. [W-13/23/Q7] ---[8]

Q35 The diagram shows a quadrilateral ABCD, with vertices A(−4, 6), B(6, −4), C(10, 4) and D. The angle ADC = 90°. The lines BC and AD are extended to intersect at the point X.



(i) Given that C is the midpoint of BX, find the coordinates of D. ---[7]

(ii) Hence calculate the area of the quadrilateral ABCD. ---[2]

[W-13/23/Q8]

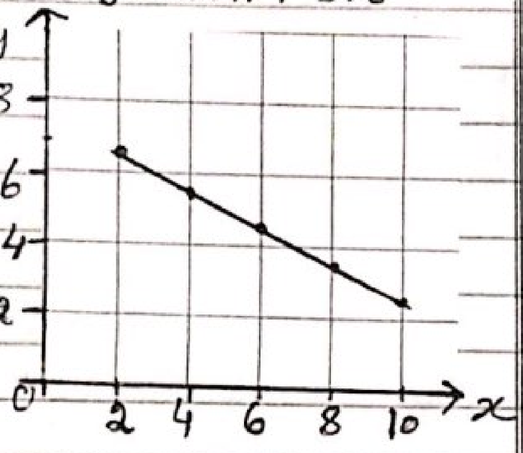
IG - Maths 0606

Straight line Graph

Answers

Q1 (i) $\ln y = \ln A + bx$ ✓

(ii)



(iii) Gradient $= b = -0.5$ ✓

and $A = 2000$ ✓

(iv) $y = 500 \rightarrow x = 2.77$ ✓

(v) $x = 5, \ \ln y = 5.1$

$y = 164$ ✓

Q2 (i) $m_{PQ} = \frac{6-2}{11-8} = \frac{4}{3}$

$m_L = -1/\frac{4}{3} = -\frac{3}{4}$

L: $y - 2 = -\frac{3}{4}(x - 8)$

or $y = -\frac{3}{4}x + 8$ ✓

(ii) $PQ^2 = (11-8)^2 + (6-2)^2$

$\therefore PQ = 5$

$\frac{1}{2} \times PQ \times PR = 12.5 \Rightarrow PR = 5$

$\therefore R \ (8 \pm 4, 2 \pm 3)$

$(4, 5), (12, -1)$ ✓

Q3 line 'L' $y - (-4) = \frac{1}{6}(x - 6)$ —— (1)

$m_{AB} = \frac{7-4}{3-8} = -3/5$

AB: $y - 7 = -3/5(x - 3)$ —— (2)

Solving (1) and (2)

$x = 18, \ y = -2$ ✓

Q4 (i) Substitute $y = 2(1 - x)$

$\Rightarrow 3x^2 - 2x - 1 = 0$

$(3x+1)(x-1) = 0$

$A(-\frac{1}{3}, \frac{8}{3}), B(1, 0)$ ✓

(ii) $m = \frac{1}{2}$, Mid point of $AB = (\frac{1}{3}, \frac{4}{3})$

Perp. bisector $y - \frac{4}{3} = \frac{1}{2}(x - \frac{1}{3})$

or $6y - 3x = 7$ ✓

Q5 (i) $y - 8 = -\frac{8}{12}(x + 8)$

$3y = -2x + 8$ ✓

(ii) $AB^2 = (-8-4)^2 + (8-0)^2$

$AB = \sqrt{208} = 14.42$ ✓

(iii) $D(-2, 4)$, $C(0, 7)$, $A(-8, 8)$

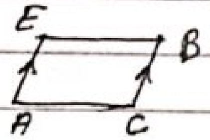
$m_{AD} = -2/3$, $m_{DC} = 3/2$

$m_1 \times m_2 = -\frac{2}{3} \times \frac{3}{2} = -1$

$\therefore \angle ADC$ is rt angle.

(iv) $-4i + j$ or $\begin{pmatrix}-4\\1\end{pmatrix}$

(v) $\overrightarrow{AE} = \begin{pmatrix}4\\-7\end{pmatrix}$



$\overrightarrow{CB} = \begin{pmatrix}4-0\\0-7\end{pmatrix} = \begin{pmatrix}4\\-7\end{pmatrix}$

"ACBE"

$\overrightarrow{AE} = \overrightarrow{CB}$ $\therefore$ Parallelogram.

Q6 $m = \frac{13-5}{1-0.2} = 10$

Eqn of line $Y - 13 = 10(X - 1)$

or $Y = 10X + 3$ —— (1)

or $\sqrt[3]{y} = 10 \cdot \frac{1}{x} + 3$

or $y = \left(\frac{10}{x} + 3\right)^3$ ✓

Straight Line Graph.

Answers

Q7. $2x^2+5x-12=0$

$(2x-3)(x+4)=0$

$x=-4,\ y=-7;\ x=\frac{3}{2},\ y=4$

Mid point $\left(-\frac{5}{4}, -\frac{3}{2}\right)M$

(i) Grad. of $PQ=2$, perp. Grad $=-\frac{1}{2}$

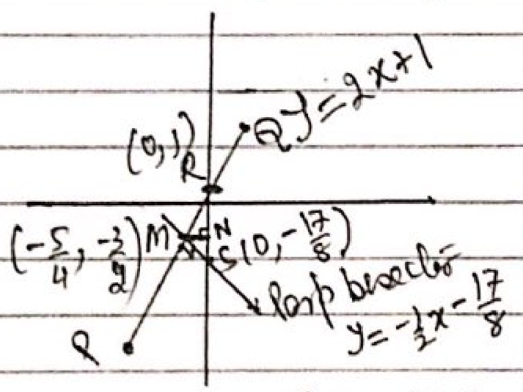
Perp. bisector $y+\frac{3}{2}=-\frac{1}{2}\left(x+\frac{5}{4}\right)$

$y=-\frac{1}{2}x-\frac{17}{8}$ ✓ —— ①

Point $\left(-10, \frac{23}{8}\right)$ satisfy ① ✓

$\frac{23}{8}=-\frac{1}{2}\times-10-\frac{17}{8}$ ✓

(ii)



Area of $\Delta RSM=\frac{1}{2}\times RS\times MN$

$=\frac{1}{2}\left(1+\frac{17}{8}\right)\times\frac{5}{4}$

$=\frac{125}{64}=1.95$ ✓

Q10(b) $M(4,2)$

(i) $m_{AB}=\frac{3}{2} \Rightarrow m_{Perp}=-\frac{2}{3}$

Perp. bisector $y-2=-\frac{2}{3}(x-4)$

$2x+3y=14$ ✓

(ii) m_{AB} is used

$y+2=\frac{3}{2}(x-10)$

(iii) $BC=\sqrt{65}=8.062$ ✓

(iv) $AC=\sqrt{65}$

$AC=BC=\sqrt{65}$

$\therefore$ Isosceles.

Q8

(i) AB: $y-6=-\frac{4}{12}(x+8)$

$3y+x=10$ ✓ grad $=-\frac{1}{3}$

(ii) $CP\perp AB$; CP: $y-7=3(x+1)$

CP: $y=3x+10$ ✓

(iii) Intersection of CP and AB; $P(-2,4)$

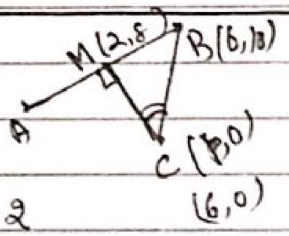
which is mid point of AB. ✓

(iv) $CP=\sqrt{10}$ or 3.16

(v) Area $=\frac{1}{2}\sqrt{10}\times4\sqrt{10}$ $\left[\frac{1}{2}\times CP\times AB\right]$

$=20$ ✓

Q9 (i) $M(2,8)$



(ii) Grad of $AB=\frac{1}{2}$

$CM\perp AB$

Grad of CM $\frac{8-0}{2-p}=-2$

$\Rightarrow p=6$ ✓

(iii) $\tan MCB=\frac{MB}{MC}=\frac{\sqrt{20}}{\sqrt{80}}=\frac{1}{2}$

$\therefore$ angle $MCB=\tan^{-1}\frac{1}{2}=26.56°$ ✓

Q10(a) (i) Intersection of line and curve

$x^2+(kx-4)^2-2(kx-4)=8$

$(1+k^2)x^2-10kx+16=0$

as line is tangent to curve.

$b^2-4ac=0$

$(-10k)^2-4(1+k^2)\times16=0$

$k=\frac{4}{3}$ ✓ [as $k>0$]

(ii) $x=-\frac{b}{2a}=\frac{\frac{4}{3}\times10}{\frac{25}{9}}=\frac{12}{5}$

$x=\frac{12}{5},\ y=-\frac{4}{5}$ $T\left(\frac{12}{5}, -\frac{4}{5}\right)$

(iii) $TP=4$ ✓ Given $P(0,-4)$

Straight Line Graph.

Answers

Q11 $3x-1 = x(3x-1)+x^2-4$

$\Rightarrow 4x^2-4x-3=0$

$(2x-3)(2x+1)=0;$

$x=3/2, \quad x=-\frac{1}{2}$

$y=\frac{7}{2}, \quad y=-\frac{5}{2}$

$M(\frac{1}{2},\frac{1}{2})$

Perp. Grad $= -\frac{1}{3}$

Perp. bisector $\Rightarrow y-\frac{1}{2}=-\frac{1}{3}(x-\frac{1}{2})$

$3y+x-2=0$

Q12 (i) $10=2m+4 \Rightarrow m=3$ ✓

(ii) Equ of line $y=3x+4$

for Q, $x=-1$, $y=1$ ✓ $Q(-1,1)$

(iii) for R, $x=-1$,

grad of PR, $\frac{10-y}{2-(-1)}=1 \Rightarrow y=7$

∴ $R(-1,7)$.

(iv) Grad of PQ $=3$; grad of perp $-\frac{1}{3}$

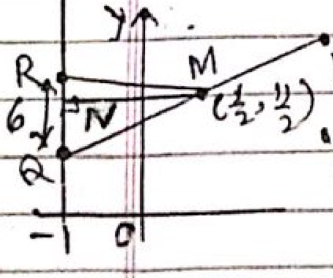
Equ of ⊥ through P $y-10=-\frac{1}{3}(x-2)$

$3y+x=32$

(v) $M(\frac{1}{2},\frac{11}{2})$

(vi) area QRM $=\frac{1}{2}$QR × dis from M

$=\frac{1}{2}\times 6\times\frac{3}{2}=4.5$ ✓



∵ $MN=\frac{1}{2}-(-1)=\frac{3}{2}$

$QR=7-1=6$

area $=\frac{1}{2}$ QR × MN

Q13 $x^2+6x-16=0$

$(x+8)(x-2)=0$

$x=2, y=5$; $x=-8, y=-15$

$A(2,5), B(-8,-15)$

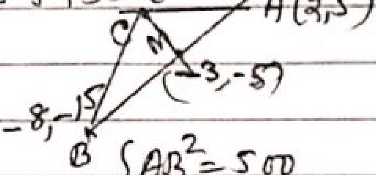
$M(-3,-5)$

Grad AB $=2$; Grad perp $=-\frac{1}{2}$

Perp. bisector $y+5=-\frac{1}{2}(x+3)$

$2y+x+13=0$

Point $C(-13,0)$



$AB^2=500$
$CM^2=125$

∴ Area ABC $=\frac{1}{2}AB\times CM$

$=\frac{1}{2}\sqrt{500}\times\sqrt{125}=125$ ✓

Q14 (i) Grad. AB $=\frac{6}{12}=\frac{1}{2}$

Equation AB, $y-2=\frac{1}{2}(x+3)$

$y=\frac{1}{2}x+3.5$

At C, $x=0 \rightarrow y=3.5$ $C(0,3.5)$ ✓

(ii) $D(3,5)$

(iii) Grad of perp bisect $=-2$

Equ perp bisector $\frac{y-5}{x-3}=-2$

$y=-2x+11$ ✓

(iv) $E(0,11)$

(v) Area ABE $=\frac{1}{2}\left|\begin{matrix}-3 & 9 & 0 & -3\\ 2 & 8 & 11 & 2\end{matrix}\right|$

$=\frac{1}{2}|-24+99-18+33|=45$

Area EDC $=\frac{1}{2}\left|\begin{matrix}3 & 0 & 0 & 3\\ 5 & 3.5 & 11 & 5\end{matrix}\right|$

$=\frac{1}{2}|-10.5+33|=11.25$

area ABE $=45=4\times 11.25$

$=4\times$ area EDC ✓

IG-Maths 0606

Straight Line Graph.

Answers

Q15 $x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$A(3,5)$, $B(-1,1)$; Grad $= 1$

Mid point $M(1,3)$

perp Grad $= -1$

Eqn Perp bisector $y = 4 - x$

meets curve Again if

$x^2 + 10x - 15 = 0$

$(-5 + 2\sqrt{10},\ 9 - 2\sqrt{10})$

$(-5 - 2\sqrt{10},\ 9 + 2\sqrt{10})$

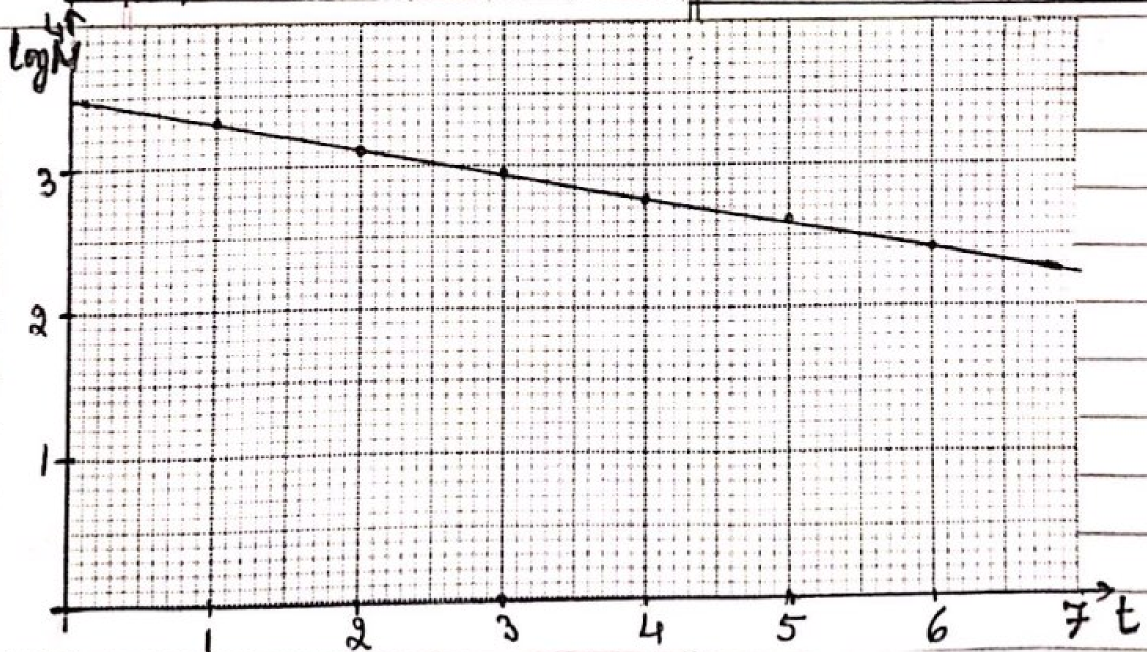
$CD^2 = (4\sqrt{10})^2 + (4\sqrt{10})^2$

$\therefore CD = 8\sqrt{5}$ ✓

Q16(i) $\log N = \log A - t \log b.$

(ii)

t	1	2	3	4	5	6
N	2000	1300	890	590	395	260
$\log_{10} N$	3.30	3.11	2.95	2.77	2.60	2.41



(iii) gradient $-\log b = \frac{2.41 - 3.3}{5}$

$\therefore b = 1.5$ ✓

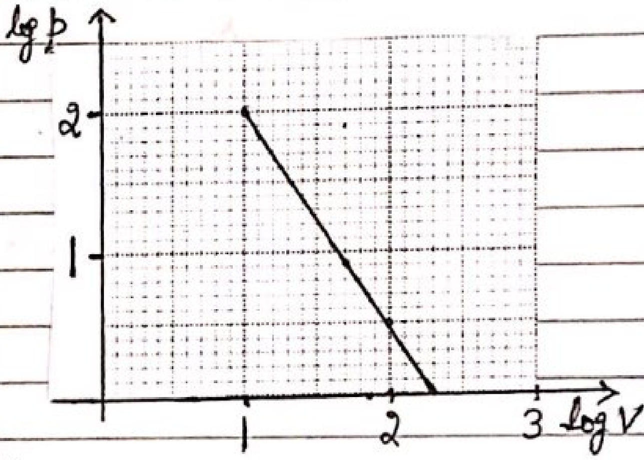
Y-Int $= \log A = 3.47 \rightarrow A = 2950$ ✓

(iv) $t = 10 \Rightarrow N = \frac{2950}{e^{10}} = 51$ ✓

(v) $N = 10 \rightarrow 1.5^t = 295 \rightarrow t = \frac{\log 295}{\log 1.5} = 14$ ✓

Q17 $\log p = n \log V + \log K$ — ①

$\lg V$	1	1.70	2	2.30
$\lg p$	1.98	0.93	0.48	0.04



(ii) $n = \text{Gradient} = \frac{0.04 - 1.98}{2.30 - 1} = -1.49$

(iii) $\log p = n \log V + \log K$

$1.98 = -1.49 \times 1 + \log K \Rightarrow \log K = 3.47$

$\Rightarrow K = 2951$

$\therefore V = 35 \rightarrow p = 14.75$ ✓

16-Maths 0606

Straight Line Graph.

Answers

Q18 (i) $E(3,5)$ [Mid point of BD]

(ii) $m_{BD} = \frac{6-4}{1-5} = -\frac{1}{2}$

$m_{AC} = 2$ $AC \perp BD$

Eqn of AC, Passes through $E(3,5)$

$y - 5 = 2(x-3) \rightarrow y = 2x - 1$ ✓

(iii) $p = 1, q = 7, A(1,1), C(4,7)$

$AC = \sqrt{45},\ BD = \sqrt{20}$

Area $= \frac{1}{2} AC \times BD$

$= \frac{1}{2} \times \sqrt{45} \times \sqrt{20} = 15$ ✓

Q19 (i) $\ln y = \ln A + x \ln b$

(ii) grad of line $= \frac{6-0.7}{5-1} = \frac{5.3}{4}$

$\ln b = 1.325$

$\therefore b = e^{1.325} = 3.76$

or $b = 4$ to one s.f ✓

$x = 0,\ \ln y = \ln A = -0.6$

$A = e^{-0.6} = 0.54$ ✓

(iii) $y = 220$

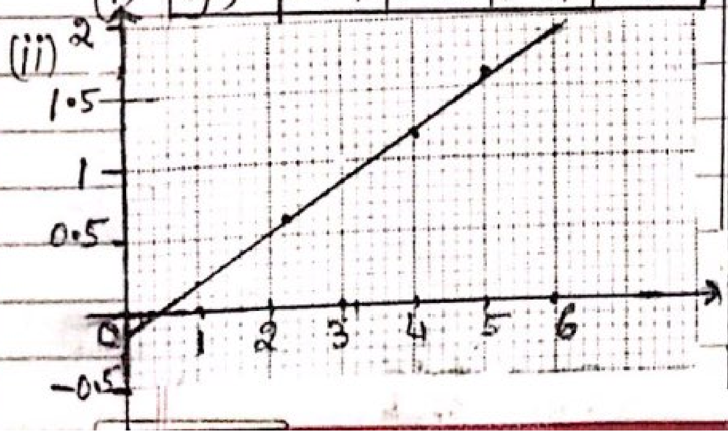
$\ln y = 5.4 \Rightarrow x = 4.4$

using graph.

Q20 (i)

x^2	2.25	3.06	4	5.06
$\lg y$	0.59	0.92	1.29	1.71

(ii)



$y = A b^{x^2}$

(iii) $\lg y = \lg A + x^2 \cdot \lg b$

Gradient $\lg b = \frac{1.71 - 0.59}{5.06 - 2.25} = \frac{1.12}{2.81} = 0.4$

$\Rightarrow b = 10^{0.4} = 2.5$ ✓

Intercept $\lg A = -0.3 \Rightarrow A = 0.5$ ✓

(iv) 2.1

Q21 (i) $M(0,7)$, $m_{AB} = 2$

Perp. grad $= -\frac{1}{2}$; $y = -\frac{1}{2}x + 7$ — ①

(ii) $m_{BC} = -1$, $A(2,11)$

$y = -x + 13$ — ②

solving ① & ② $D(12,1)$

Area $= 84$

Q22 (i) $\vec{AB} = \begin{pmatrix} 12 \\ 16 \end{pmatrix}$, at P $x = -2 + \frac{1}{4} 12$

$x = 1$ ✓

and $y = 3 + \frac{1}{4}(16) = 7$ ✓

(ii) $P(1,7)$, $m_{AB} = \frac{4}{3}$, $m_{Perp} = -\frac{3}{4}$

$\therefore$ Perp line $y - 7 = -\frac{3}{4}(x-1)$

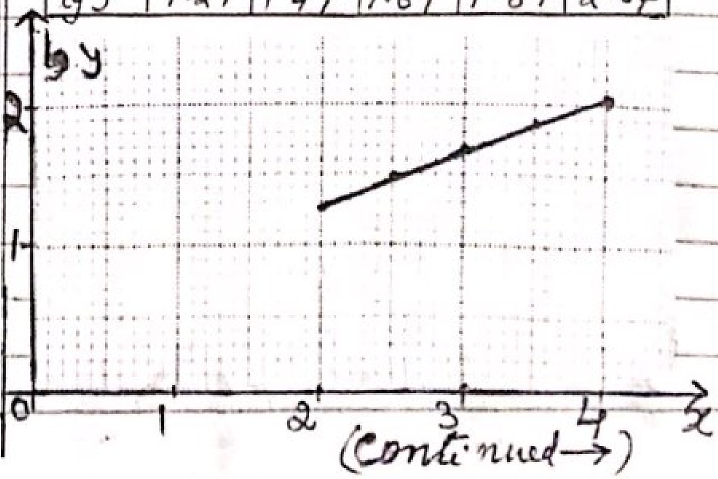
$\Rightarrow 3x + 4y = 31$ ✓

(iii) $Q\left(0, \frac{31}{4}\right)$

Area AQB $= 12.5$ ✓

Q23 (i) $\log y = \log a + x \log b$.

x	2	2.5	3	3.5	4
$\lg y$	1.27	1.47	1.67	1.87	2.07



(Continued →)

Straight Line Graph.

continued →

Answers

Q23(ii) Gradient $= \lg b = \frac{2.07-1.27}{4-2}$

$\lg b = 0.4$

$b = 10^{0.4} = 2.5$ ✓

Intercept $\lg a = 0.47$

$a = 3$ ✓

Q24 $x^2 = 16 \Rightarrow x = \pm 4$, $y = 1, 3$

Point $(-4,1), (4,3)$

$AB = \sqrt{8^2+2^2} = \sqrt{68} = 2\sqrt{17}$ ✓

Q25 (i) $C(1,6)$, D is $(1,6)+(12,9)$
$= (13,15)$

(ii) Grad $CD = \frac{15-6}{13-1} = 3/4$

Grad of $AB = \frac{10-2}{-2-4} = -4/3$

$\frac{3}{4} \times -\frac{4}{3} = -1 \Rightarrow$ lines are perp.

(iii) Area $= \frac{1}{2} AB \times CD = \frac{1}{2} \times 10 \times 15$
$= 75$ ✓

Q26 $m = 4$

(i) Equ^n of line $\frac{\ln y - 39}{3^x - 9} = 4$

$\Rightarrow \ln y = 4 \times 3^x + 3$ ✓

(ii) $x = 0.5 \to \ln y = 4 \times 3^{0.5} + 3 = 9.928$

$\Rightarrow y = 20500$ ✓

(iii) substitute y and rearrange for 3^x:

$3^x = 1.150$

$\Rightarrow x = 0.127$ ✓

Q27 $\ln y = \ln A + x \ln b$

$\ln b = \text{grad} = 1 \Rightarrow b = e^1 = 2.72$ ✓

$\ln A = \text{Intercept} = 2 \Rightarrow A = e^2 = 7.39$ ✓

Q28 $m = \frac{18-3}{4-1} = 5$

Equ^n $Y - 3 = 5(X-1)$

$Y = 5(X-1)+3$

or $\sqrt{y} = 5(x^2-1)+3$

$\sqrt{y} = 5x^2 - 2$

$y = (5x^2-2)^2$ ✓

Q29(i) $y - 2 = \frac{6-2}{2+6}(x+6) \to y = \frac{1}{2}x + 5$ ✓

(ii) Grad of $AB = \frac{1}{2} \to$ Grad of $BC = -2$

Equ^n BC, $y - 6 = -2(x-2) \to y = -2x + 10$

(iii) $\left.\begin{array}{l}(x+6)^2 + (y-2)^2 = 10^2 \\ \text{and } y = -2x+10\end{array}\right\}$

$\Rightarrow$ solve. $C(0,10), (4,2)$.

Q30 $y = Ab^x \Rightarrow \ln y = \ln A + x \ln b$

grad $\ln b = 1 \Rightarrow b = e = 2.72$ ✓

and Int $\ln A = 2 \Rightarrow A = e^2 = 7.39$ ✓

Q31 (i) $\sqrt{20} = 4.47$ ✓

(ii) Grad $AB = \frac{1}{2}$ $\perp$ grad $= -2$

$\perp$ line BC, $y - 4 = -2(x-1) \to y = -2x + 6$ ✓

(iii) $C(x,y)$, $AC^2 = 40$

$\left.\begin{array}{l}\text{or } (x+3)^2 + (y-2)^2 = 40 \\ \text{Intersect } y = -2x+6\end{array}\right\}$

$\Rightarrow 5x^2 - 10x - 15 = 0$

$x = 3, -1$
$y = 0, 8$ $\quad C(3,0), (-1,8)$ ✓

Q32 (i) x^2

(ii) Plot y/x against x^2

x^2	4	16	36	64
y/x	4.8	9.6	17.5	29

(iii) Grad $= 0.4$; $a = 0.4 \pm 0.02$
$b = 3.2 \pm 0.4$

(iv) read $\frac{y}{x} = 12.5$

or substitute in formula.

4.8 ✓

← X —— X →