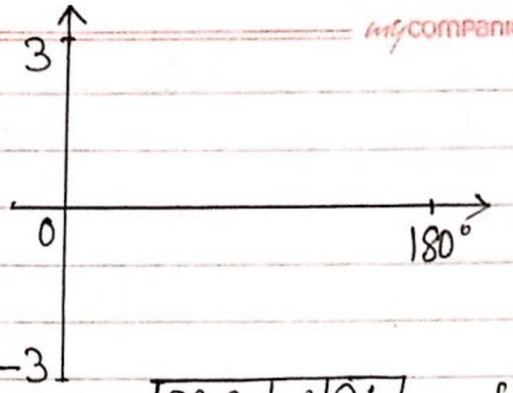
Trigonometry

Q1. On the axes, sketch the graph of $y = 2\sin\frac{3}{2}x - 1$ for $0° \leqslant x \leqslant 180°$, showing the coordinates of the points where the graph meets the coordinate axes.



SP-20 | 02 | Q4 ---[4]

Q2. (a)(i) Show that $\frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - \sin\theta} = \sec^2\theta$ ---[3]

(ii) Hence solve $\frac{2\operatorname{cosec}\phi}{\operatorname{cosec}\phi - \sin\phi} = 8$ for $0 < \phi < 360°$ ---[3]

(b) Solve:

$\sqrt{3}\tan\left(x + \frac{\pi}{4}\right) = 1$ for $0 < x < 2\pi$ ---[3]

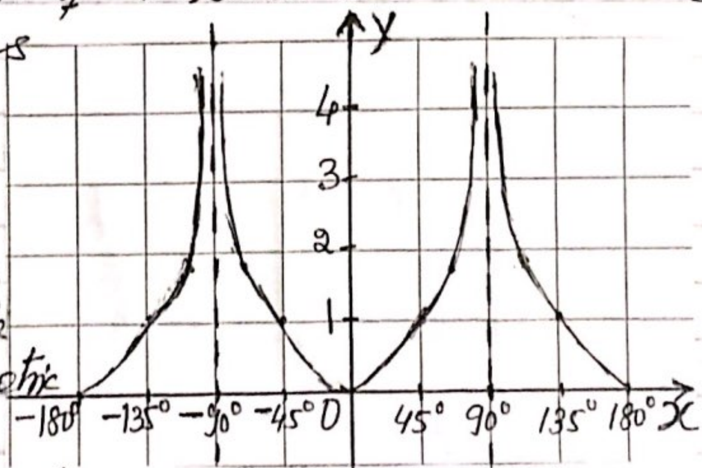
Giving your answers in terms of π.

SP-20 | 02 | Q11

Q3 (a) (i) State the amplitude of $15\sin 2x - 5$ ---[1]

(ii) State the period of $15\sin 2x - 5$ ---[1]

(b) The diagram shows the graph of $y = |f(x)|$ for $-180° \leqslant x \leqslant 180°$, where $f(x)$ is a trigonometric function.



(i) Write down two possible expressions for the trigonometric function $f(x)$. ----[2]

(ii) State the number of solutions of the equation $|f(x)| = 1$ for $-180° \leqslant x \leqslant 180°$ ---[1]

M-18 | 22 | Q4

Trigonometry

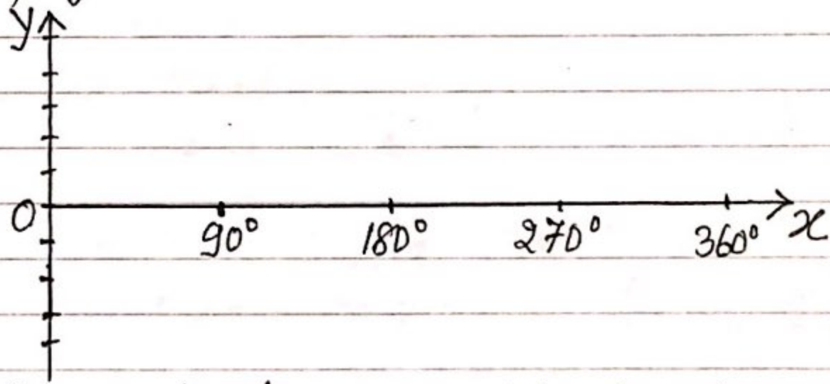
Q4 (a) (i) Show that $\dfrac{(1-\sin A)(1+\sin A)}{\sin A \cdot \cos A} = \cot A$ --- [2]

(ii) Hence solve, $\dfrac{(1-\sin 3x)(1+\sin 3x)}{\sin 3x \cdot \cos 3x} = \dfrac{1}{2}$, $0 \leqslant x \leqslant 180^\circ$ ---[4]

(b) Solve $10\tan^2 y - \sec y - 1 = 0$ for $0 \leqslant y \leqslant 2\pi$ radians. ---[5]

M-18/22/Q11

Q5 (i) On the axes below sketch, for $0^\circ \leqslant x \leqslant 360^\circ$, the graph of $y = 1 + 3\cos 2x$. ---[3]



(ii) Write down the coordinates of the point where this graph first has a minimum value. M-17/12/Q2 ---[1]

Q6 (i) Show that $\operatorname{cosec}\theta - \sin\theta = \cot\theta \cdot \cos\theta$ ---[3]

M-17/12/Q5

(ii) Hence solve the equation:

$\operatorname{cosec}\theta - \sin\theta = \frac{1}{3}\cos\theta$, $0 \leqslant \theta \leqslant 2\pi$ radians. ---[4]

Q7 Solve, for $0^\circ \leqslant x \leqslant 360^\circ$, the equation:

(i) $\cot(2x - 10^\circ) = \dfrac{3}{4}$ ---[4]

(ii) $\sin^2 x - \cos^2 x = \cos x$. M-17/22/Q10 ---[5]

Q8 (i) Show that $5 + 4\tan^2\left(\frac{x}{3}\right) = 4\sec^2\left(\frac{x}{3}\right) + 1$ ---[1]

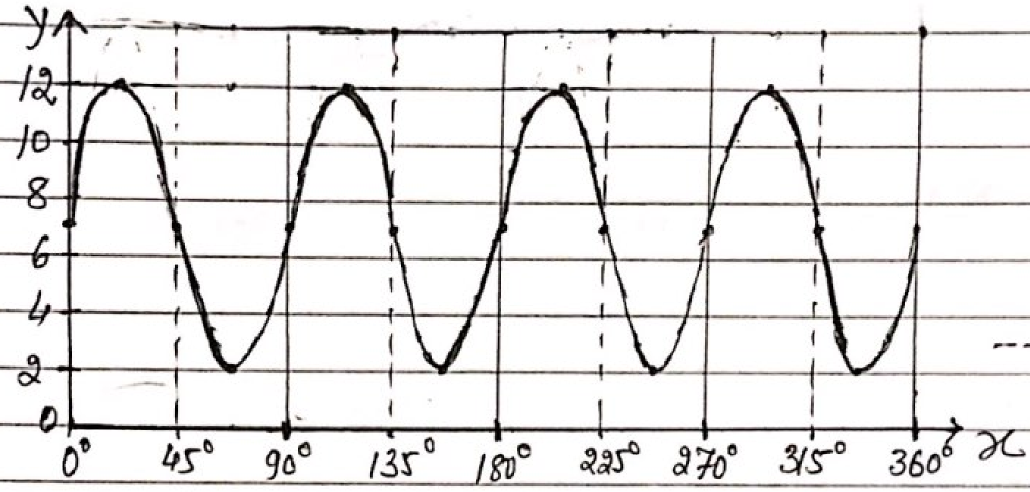
S-17/12/Q9(i)

Q9 (a) Given that $y = 7\cos 10x - 3$, where angle x is measured in degrees, state,

(i) the period of y. ---[1]

(ii) the amplitude of y. ---[1]

(b)



----[4]

Find the equation of the curve shown, in the form,

$y = a\,g(bx) + c$ where $g(x)$ is a trigonometric function and a, b and c are integers to be found. S-17/21/Q4

Q10 (i) Prove that $\sin x(\cot x + \tan x) = \sec x$ ---[4]

(ii) Hence solve the equation:

$|\sin x(\cot x + \tan x)| = 2$ for $0 \leq x \leq 360°$ --[4] S-17/21/Q11

Q11 (i) Show that $\frac{\operatorname{cosec}\theta}{\cot\theta + \tan\theta} = \cos\theta$ ---[4]

(ii) It is given that $\int_0^a \frac{\operatorname{cosec} 2\theta}{\cot 2\theta + \tan 2\theta}\,d\theta = \frac{\sqrt{3}}{4}$ $0 < a < \frac{\pi}{4}$

Using your answer in part (i) find the value of a, giving your answer in terms of π. S-17/12/Q6 ---[4]

Q12 Solve the equation: --[4]

(i) $4\sin\left(3x - \frac{\pi}{4}\right) = 3$ $0 \leq x \leq \frac{\pi}{2}$ radians

(ii) $2\tan^2 y + \sec^2 y = 4\sec y + 3$ for $0° \leq y \leq 360°$ --- [5]

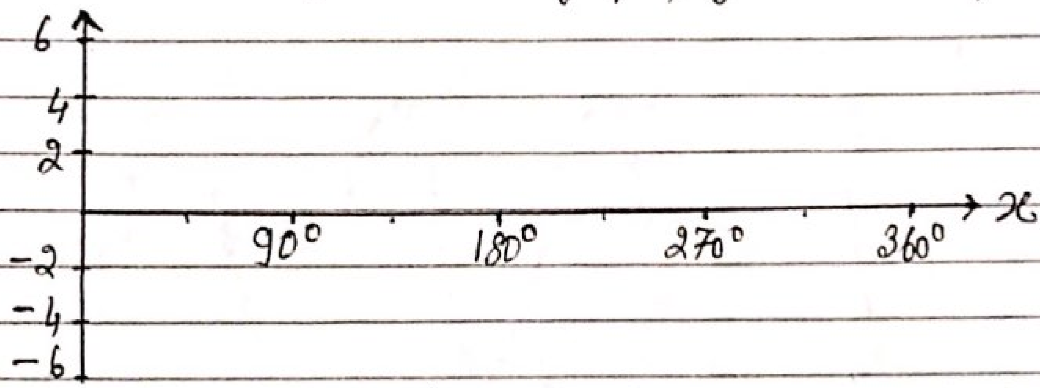
S-17/22/Q10

Q13 Given that $y = 3 + 4\cos 9x$, write down

(i) the amplitude of y, [S-17/13/Q2] ---[1]

(ii) the period of y. ---[1]

Q14 (i) On the axes below, sketch the graph of $y = 3\sin x - 2$ for $0° \leq x \leq 360°$ ---[3]



(ii) Given that $0 \leq |3\sin x - 2| \leq k$ for $0° \leq x \leq 360°$, write down the Value of k. [S-17/13/Q3] ---[1]

Q15 (a) Show that, $\dfrac{\tan^2\theta + \sin^2\theta}{\cos\theta + \sec\theta} = \tan\theta\sin\theta$ --- [4]

(b) Given that $x = 3\sin\phi$ and $y = \dfrac{3}{\cos\phi}$, find the numerical Value of, $9y^2 - x^2y^2$. ---[3]

[S-17/13/Q7]

Q16 Solve the equation

(a) $2|\sin x| = 1$ for $-\pi \leq x \leq \pi$ radian ---[3]

(b) $3\tan(2y + 15°) = 1$ for $0° \leq y \leq 180°$ ---[4]

(c) $3\cot^2 z = \operatorname{cosec}^2 z - 7\operatorname{cosec} z + 1$ for $0° \leq z \leq 360°$ ---[5]

[S-17/23/Q10]

Q17 (a) Solve $3\operatorname{cosec} 2x - 4\sin 2x = 0$ for $0° \leq x \leq 180°$ ---[4]

(b) Solve $3\tan\left(y - \frac{\pi}{4}\right) = \sqrt{3}$ for $0 \leq y \leq 2\pi$ radians, giving your answers in terms of π. ---[4]

[W-17/11/Q10]

Q18 Show that: $\dfrac{1}{1 - \sin\theta} - \dfrac{1}{1 + \sin\theta} = 2\tan\theta\sec\theta$. [W-17/21/Q2] ---[4]

Q19 The graph of $y = a\sin(bx) + c$ has an amplitude of 4, a period of $\frac{\pi}{3}$ and passes through the point $\left(\frac{\pi}{12}, 2\right)$. Find the value of each of the constant a, b and c. [W-17/12/Q2] --[4]

IG-Maths 0606

Trigonometry

Q20 (a) Solve $2\cot(\phi + 35^\circ) = 5$ for $0^\circ \leq \phi \leq 360^\circ$ --- [4]

(b)(i) Show that $\dfrac{\sec\theta}{\cot\theta + \tan\theta} = \sin\theta$ ---- [3]

(ii) Hence solve $\dfrac{\sec 3\theta}{\cot 3\theta + \tan 3\theta} = -\dfrac{\sqrt{3}}{2}$ for $-\dfrac{\pi}{2} \leq \theta \leq \dfrac{\pi}{2}$, giving your answers in terms of π. [W-17/12/Q11] --- [4]

Q21 Given that $y = 2\sec^2\theta$ and $x = \tan\theta - 5$, express y in terms of x. -- [2] [W-17/13/Q1]

Q22 The graph of $y = a\cos(bx) + c$ has an amplitude of 3, a period of $\dfrac{\pi}{4}$ and passes through the point $\left(\dfrac{\pi}{12}, \dfrac{5}{2}\right)$. Find the value of each of the constants a, b and c. [W-17/13/Q4] -- [4]

Q23 (a) Show that $\dfrac{\sin x}{1+\cos x} + \dfrac{(1+\cos x)}{\sin x} = 2\operatorname{cosec} x$ -- [3]

(b) Solve the equations:

(i) $\cot^2 y + \operatorname{cosec} y - 5 = 0$ for $0 \leq y \leq 360^\circ$ --- [5]

(ii) $\cos\left(2z + \dfrac{\pi}{4}\right) = -\dfrac{\sqrt{3}}{2}$ for $0 \leq z \leq \pi$ radians --- [4]

[W-17/23/Q10]

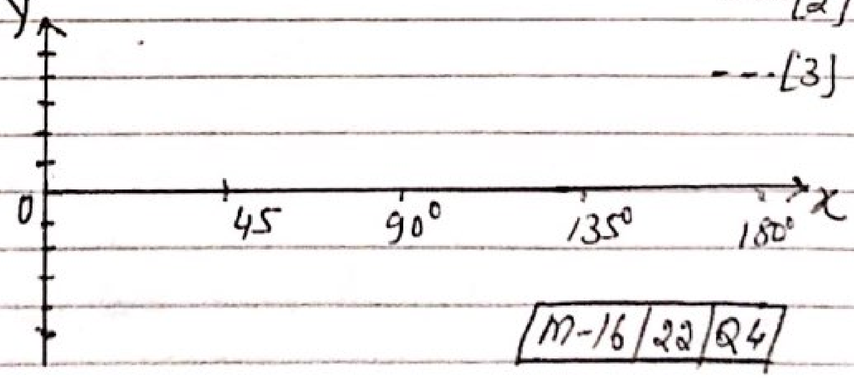
Q24 (i) Show that $\dfrac{1}{\operatorname{cosec}\theta - 1} - \dfrac{1}{\operatorname{cosec}\theta + 1} = 2\tan^2\theta$ -- [4]

(ii) Hence solve; $\dfrac{1}{\operatorname{cosec}\theta - 1} - \dfrac{1}{\operatorname{cosec} + 1} = 6 + \tan\theta$ for $0 < \theta < 360^\circ$ --- [4]

[M-16/12/Q11]

Q25 (a) $f(x) = a\cos bx + c$ has period of 60°; an amplitude of 10 and is such that $f(0) = 14$. State the values of a, b and c. -- [2]

(b) Sketch the graph of $y = 3\sin 4x - 2$ for $0^\circ \leq x \leq 180^\circ$ on the axes shown. --- [3]



[M-16/22/Q4]

Scanned with CamScanner

Trigonometry

Q26 (i) Show that $2\cos x \cot x + 1 = \cot x + 2\cos x$ can be written in the form $(a\cos x - b)(\cos x - \sin x) = 0$, where a and b are constants to be found. ---[4]

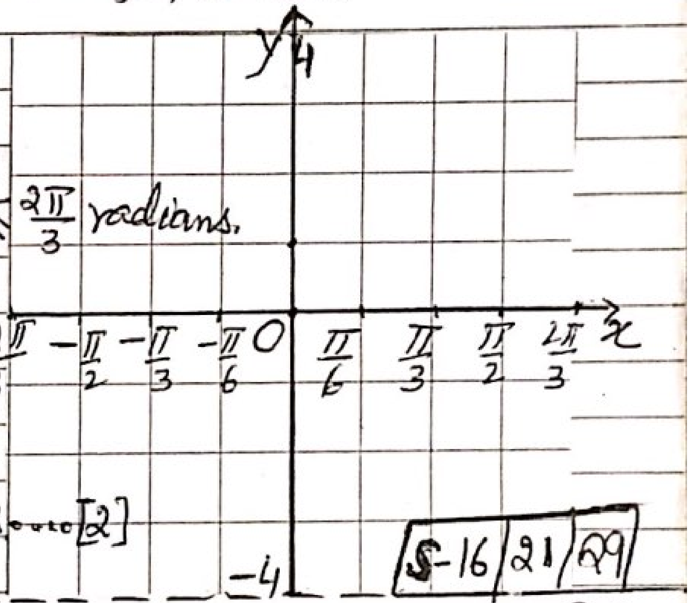
(ii) Hence or otherwise, solve $2\cos x \cot x + 1 = \cot x + 2\cos x$ for $0 < x < \pi$. ---- [3] [S-16/11/Q9]

Q27 (a) Given that $y = a\tan bx + c$ has period $\frac{\pi}{4}$ radians and passes through the point $(0, -2)$ and $(\frac{\pi}{16}, 0)$, find the value of each of the constants a, b, and c. ---[3]

(b) (i) On the given axes, draw the graph of,

$y = 2\cos 3x + 1$ for $-\frac{2\pi}{3} \leq x \leq \frac{2\pi}{3}$ radians. ---[3]

(ii) Using your graph or otherwise, find the exact solutions of $(2\cos 3x+1)^2 = 1$ for $-\frac{2\pi}{3} \leq x \leq \frac{2\pi}{3}$ radians ......[2]



[S-16/21/Q9]

Q28 (i) Show that $(1-\cos\theta)(1+\sec\theta) = \sin\theta\tan\theta$. ---[4]

(ii) Hence solve the equation $(1-\cos\theta)(1+\sec\theta) = \sin\theta$ for $0 \leq \theta \leq \pi$ radians. [S-16/12/Q5] ---[3]

Q29 Solve the equation:

(i) $8\sin^2 A + 2\cos A = 7$ for $0 \leq A \leq 180°$ ---[4]

(ii) $\operatorname{Cosec}(3B+1) = 2.5$ for $0 \leq B \leq \pi$ radians. ---[4]

[S-16/22/Q12]

Q30 (i) Prove that $\frac{\cos x}{1+\tan x} - \frac{\sin x}{1+\cot x} = \cos x - \sin x$ ---[4]

(ii) Hence solve the equation:

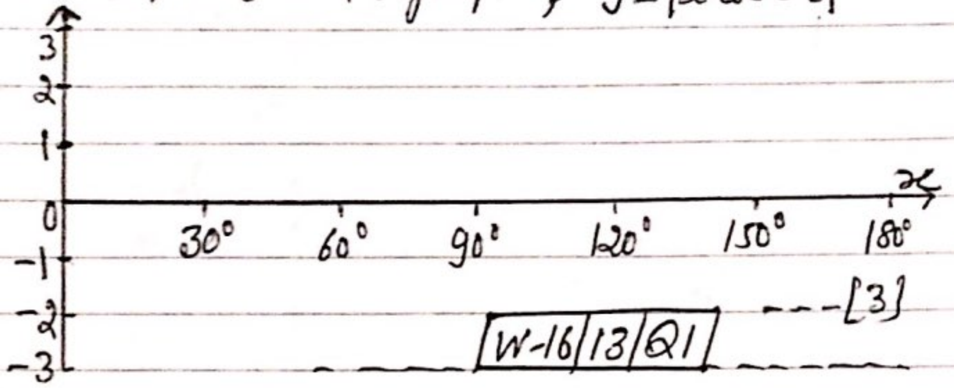
$\frac{\cos x}{1+\tan x} - \frac{\sin x}{1+\cot x} = 3\sin x - 4\cos x$ for $-180° < x < 180°$ ---[4]

[W-16/21/Q6]

IG-Maths 0606

Trigonometry

Q31 On the axes below, sketch the graph of $y = |2\cos 3x|$ for $0^\circ \leqslant x \leqslant 180^\circ$



---[3]

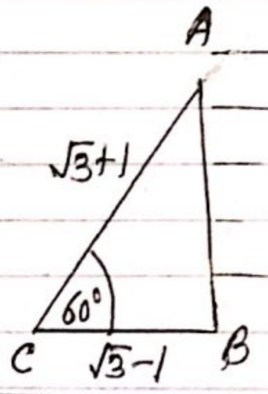
W-16/13/Q1

Q32 (a)(i) Show that $\dfrac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - \sin\theta} = \sec^2\theta$ ---[3]

(ii) Hence solve $\dfrac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - \sin\theta} = 4$ for $0 < \theta < 360^\circ$ ---[3]

(b) Solve $\sqrt{3}\tan\left(x + \frac{\pi}{4}\right) = 1$ for $0 < x < 2\pi$, ---[3]
giving your answer in terms of π. W-16/13/Q8

Q33 In this question all lengths are in centimetres.
In this triangle ABC, $AC = \sqrt{3}+1$, $BC = \sqrt{3}-1$ and angle $ACB = 60^\circ$.



(i) Without using a calculator,
Show that the length of $AB = \sqrt{6}$ ---[3]
(ii) Show that angle $CAB = 15^\circ$ ---[2]
(iii) Without using a calculator,
find the area of triangle ABC. ---[2] W-16/23/Q5

Q34 (a)(i) Show that $\dfrac{\operatorname{cosec} x}{\cot x + \tan x} = \cos x$ ---[3]

(ii) Hence solve $\dfrac{\operatorname{cosec} 3y}{\cot 3y + \tan 3y} = 0.5$ for $0 \leqslant y \leqslant \pi$ radians,
giving your answer in terms of π. ---[3]

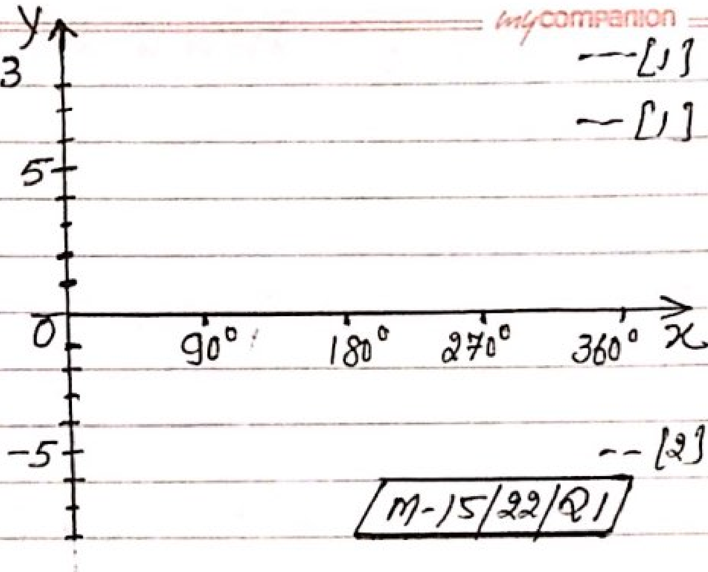
(b) Solve $2\sin z + 8\cos^2 z = 5$ for $0^\circ < z < 360^\circ$ ---[4]

M-15/12/Q11

Q35 (i) State the amplitude of $4\cos x - 3$ --- [1]

(ii) State the period of $4\cos x - 3$. --- [1]

(iii) The function f is defined, for $0 \leqslant x \leqslant 360$, by $f(x) = 4\cos x - 3$, Sketch the graph of $y = f(x)$ on the axes. --- [2]



M-15/22/Q1

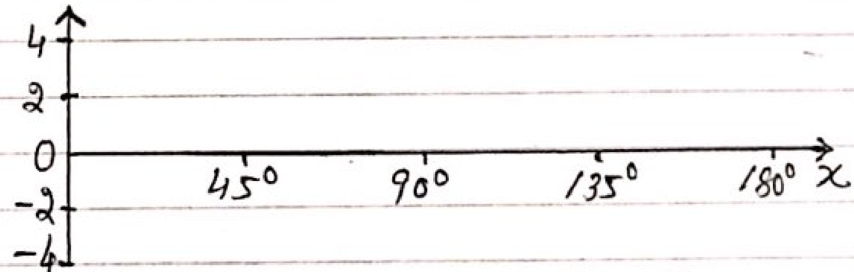
Q36 (i) State the period of $\sin 2x$ --- [1]

(ii) State the amplitude of $1 + 2\cos 3x$ --- [1]

(iii) On the axes below, sketch the graph of,

(a) $y = \sin 2x$ for $0° \leqslant x \leqslant 180$ --- [1]

(b) $y = 1 + 2\cos 3x$ for $0° \leqslant x \leqslant 180°$ --- [2]



(iv) State the number of solutions of $\sin 2x - 2\cos 3x = 1$ for $0 \leqslant x \leqslant 180°$ --- [1]

S-15/11/Q1

Q37 (a) Solve $4\sin x = \operatorname{cosec} x$ for $0 \leqslant x \leqslant 360$ --- [3]

(b) Solve $\tan^2 3y - 2\sec 3y - 2 = 0$ for $0 \leqslant y \leqslant 180°$ --- [6]

(c) Solve $\tan\left(z - \frac{\pi}{3}\right) = \sqrt{3}$ for $0 \leqslant z \leqslant 2\pi$ radians --- [3]

S-15/11/Q10

Q38 Show that $\dfrac{\tan\theta + \cot\theta}{\operatorname{cosec}\theta} = \sec\theta$ S-15/12/Q2 --- [4]

Q39 (a) Solve $2\cos 3x = \sec 3x$ for $0 \leqslant x \leqslant 120°$ --- [3]

(b) Solve $3\operatorname{cosec}^2 y + 5\cot y - 5 = 0$ for $0° \leqslant y \leqslant 360°$ --- [5]

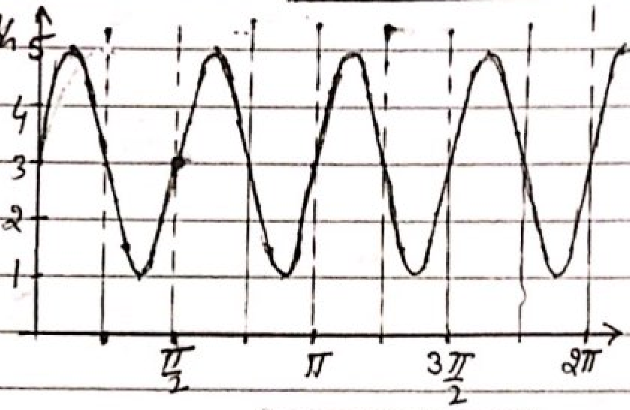
(c) Solve $2\sin\left(z + \frac{\pi}{3}\right) = 1$ for $0 \leqslant z \leqslant 2\pi$ radians --- [4]

S-15/12/Q10

Trigonometry

Q40 Show that $\sqrt{\sec^2\theta - 1} + \sqrt{\operatorname{cosec}^2\theta - 1} = \sec\theta \cdot \operatorname{cosec}\theta$ ---[5]
W-15/11/Q3

Q41 The figure shows part of the graph of $y = a + b\sin cx$.



(i) Find the value of each of the integers a, b and c. ---[3]
Using your values of a, b and c, find
(ii) $\frac{dy}{dx}$ ---[2]
(iii) the equation of the normal to the curve at $\left(\frac{\pi}{2}, 3\right)$. ---[3]
W-15/21/Q6

Q42 Solve the following equations.
(i) $4\sin 2x + 5\cos 2x = 0$ for $0^\circ \leq x \leq 180^\circ$ ---[3]
(ii) $\cot^2 y + 3\operatorname{cosec} y = 3$ for $0^\circ \leq y \leq 360^\circ$ ---[5]
(iii) $\cos\left(z + \frac{\pi}{4}\right) = -\frac{1}{2}$ for $0 \leq z \leq 2\pi$ radians, giving each answer as a multiple of π. W-15/21/Q9 --[4]

Q43 Solve $2\cos^2\left(3x - \frac{\pi}{4}\right) = 1$ for $0 \leq x \leq \frac{\pi}{3}$ W-15/13/Q2 ---[4]

Q44 (i) Prove that $\sec^2 x + \operatorname{cosec}^2 x = \sec^2 x \cdot \operatorname{cosec}^2 x$. W-15/23/Q8 -[4]
(ii) Hence or otherwise, solve:
$\sec^2 x + \operatorname{cosec}^2 x = 4\tan^2 x$ for $90^\circ < x < 270^\circ$ --[4]

Q45 Show that $\tan\theta + \frac{\cos\theta}{1+\sin\theta} = \sec\theta$ S-14/11/Q1 ---[4]

Q46. (a) Solve $5\sin 2x + 3\cos 2x = 0$ for $0 \leq x \leq 180^\circ$ ---[4]
(b) Solve $2\cot^2 y + 3\operatorname{cosec} y = 0$ for $0 \leq y \leq 360^\circ$ --[4]
(c) Solve $3\cos(z + 1\cdot 2) = 2$ for $0 \leq z \leq 6$ radians --[4]
S-14/11/Q11

Q47 Show that $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$ can be written in the form $p\sec A$, where p is an integer to be found. ---[4]
S-14/12/Q1

IG-0606 Maths

Trigonometry

Date P-10
Page No.

Q48 (a) Solve $\tan^2 x + 5\tan x = 0$ for $0^\circ \leq x \leq 180^\circ$ ---[3]

(b) Solve. $2\cos^2 y - \sin y - 1 = 0$ for $0^\circ \leq y \leq 360^\circ$ ---[4]

(c) Solve $\sec\left(2z - \frac{\pi}{6}\right) = 2$ for $0 \leq z \leq \pi$ radians ---[4]

S-14/12/Q11

Q49. Solve: (i) $3\sin x\cos x = 2\cos x$ for $0 \leq x \leq 180^\circ$ ---[4]

(ii) $10\sin^2 y + \cos y = 8$ for $0 \leq y \leq 360^\circ$ ---[5]

S-14/13/Q9

Q50 (a) Prove that $\dfrac{\tan\theta + \cot\theta}{\sec\theta + \operatorname{cosec}\theta} = \dfrac{1}{\sin\theta + \cos\theta}$ ---[3]

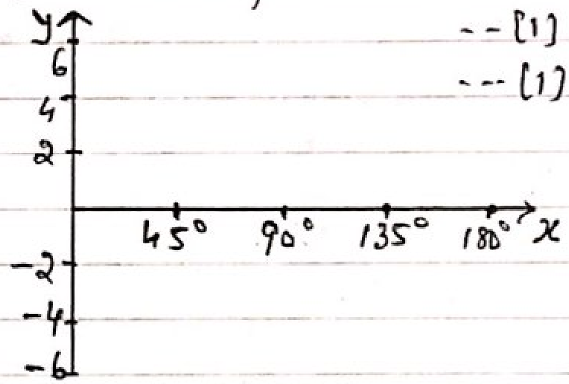
(b) Given that $\tan x = -\frac{5}{12}$ and $90^\circ < x < 180^\circ$, find the exact value of $\sin x$ and of $\cos x$, giving your answer as a fraction. ---[3]

S-14/23/Q7

Q51 (a) On the axes below, sketch the curve $y = 3\cos 2x - 1$ for $0^\circ \leq x \leq 180^\circ$ ---[3]

(b) (i) State the amplitude of $1 - 4\sin 2x$. ---[1]

(ii) State the period of $5\tan 3x + 1$ ---[1]



W-14/11/Q2

Q52 (a) Solve $2\cos 3x = \cot 3x$ $0 \leq x \leq 90^\circ$ ---[5]

(b) Solve $\sec\left(y + \frac{\pi}{2}\right) = -2$ for $0 \leq y \leq \pi$ radians ---[4]

W-14/11/Q11

Q53 (i) Prove that $\sec x \cdot \operatorname{cosec} x - \cot x = \tan x$ ---[4]

(ii) Use the result from part (i) to solve the equation,

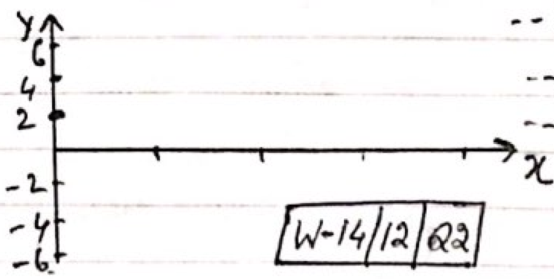
$\sec x \operatorname{cosec} x = 3\cot x$ for $0^\circ < x < 360^\circ$ W-14/21/Q10 ---[4]

Q54 (a) On the axes, sketch the curve: $y = 3\cos 2x - 1$ for $0^\circ \leq x \leq 180^\circ$ ---[3]

(b) (i) State the amplitude of $1 - 4\sin 2x$ ---[1]

(ii) State the period of $5\tan 3x + 1$ ---[1]

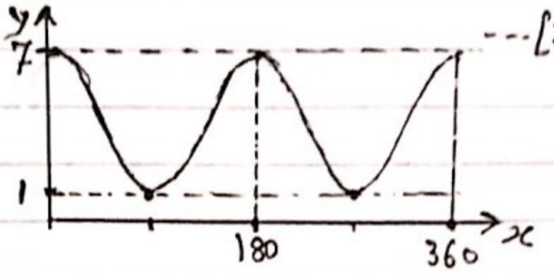
---[1]



W-14/12/Q2

Trigonometry

Q55. The diagram shows the graph of $y = a\cos bx + c$ for $0° \leqslant x \leqslant 360°$, where a, b and c are positive integers.
State the value of each of a, b and c. ---[3]



W-14/13/Q1

Q56 (a) Solve $3\sin x + 5\cos x = 0$ for $0 \leqslant x \leqslant 360°$ ---[3]

(b) Solve $\operatorname{cosec}\left(3y + \frac{\pi}{4}\right) = 2$ for $0 \leqslant y \leqslant \pi$ radians ---[5]

W-14/13/Q4

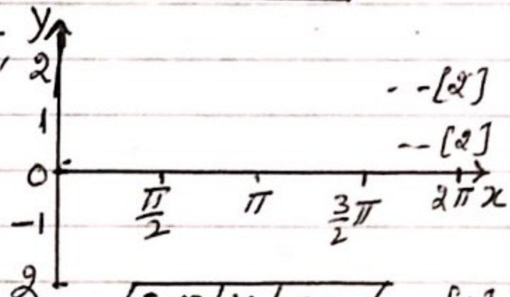
Q.57 (i) Prove that $\frac{1}{1-\cos x} + \frac{1}{1+\cos x} = 2\operatorname{cosec}^2 x$ ---[3]

(ii) Hence solve the equation $\frac{1}{1-\cos x} + \frac{1}{1+\cos x} = 8$ for $0° < x < 360°$ ---[4]

W-14/23/Q10

Q58 On the axes below sketch, for $0 \leqslant x \leqslant 2\pi$, the graph of (i) $y = \cos x - 1$ --[2]

(ii) $y = \sin 2x$ --[2]



(iii) State the number of solutions of the equation $\cos x - \sin 2x = 1$ for $0 \leqslant x \leqslant 2\pi$

S-13/11/Q1 ---[1]

Q59 (a) Solve $2\sin\left(x + \frac{\pi}{3}\right) = -1$ for $0 \leqslant x \leqslant 2\pi$ radians ---[4]

(b) Solve $\tan y - 2 = \cot y$ for $0° \leqslant y \leqslant 180°$ S-13/11/Q11 --[6]

Q60 Prove that $\left(\frac{1+\sin\theta}{\cos\theta}\right)^2 + \left(\frac{1-\sin\theta}{\cos\theta}\right)^2 = 2 + 4\tan^2\theta$ --[4]

S-13/21/Q1

Q61 Show that $(1-\cos\theta-\sin\theta)^2 - 2(1-\sin\theta)(1-\cos\theta) = 0$ S-13/12/Q3 --[3]

Q62 (a) Solve $\cos 2x + 2\sec 2x + 3 = 0$ for $0° \leqslant x \leqslant 360°$ --[5]

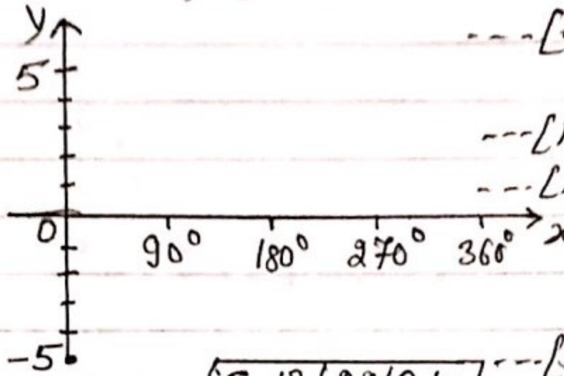
(b) Solve $2\sin^2\left(y - \frac{\pi}{6}\right) = 1$ for $0 \leqslant y \leqslant \pi$ --[4]

S-13/12/Q11

Trigonometry

Q63 (a) The function f is defined, for $0^\circ \leq x \leq 360^\circ$, by $f(x) = 1 + 3\cos 2x$.

(i) Sketch the graph of $y = f(x)$ on the axes. ---[4]



(ii) State the amplitude of f. ---[1]

(iii) State the period of f. ---[1]

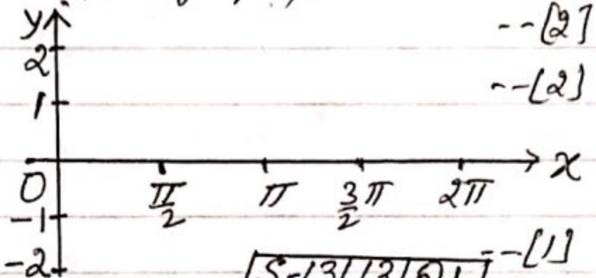
(b) Given that $\cos x = p$, where $270^\circ < x < 360^\circ$, find $\operatorname{cosec} x$ in terms of p. ---[3]

[S-13/22/Q10]

Q64 On the axes sketch, for $0 \leq x \leq 2\pi$, the graph of

(i) $y = \cos x - 1$ --[2]

(ii) $y = \sin 2x$ --[2]



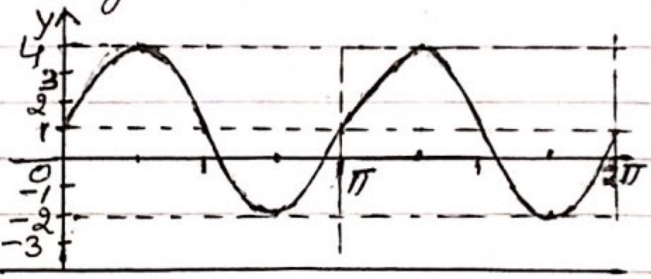
(iii) State the number of solutions of the equation, for $0 \leq x \leq 2\pi$, $\cos x - \sin 2x = 1$ --[1]

[S-13/13/Q1]

Q65 The diagram shows the graph of $y = a\sin(bx) + c$ for $0 \leq x \leq 2\pi$, where a, b and c are positive integers.

State the value of a, of b and of c. ---[3]

[W-13/11/Q1]



Q66 Show that $\dfrac{1+\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{1+\sin\theta} = 2\sec\theta$ [W-13/11/Q3] --[4]

Q67 (a) Solve the equation $2\operatorname{cosec} x + \dfrac{7}{\cos x} = 0$ for $0 \leq x \leq 360$ --[4]

(b) Solve the equation $7\sin(2y-1) = 5$ for $0 \leq y \leq 5$ radians --[5]

[W-13/21/Q12]

Q68 Show that $\tan^2\theta - \sin^2\theta = \sin^4\theta \cdot \sec^2\theta$ [W-13/13/Q3] --[4]

Trigonometry

Q69 (a)(i) Solve $6\sin^2 x = 5 + \cos x$ for $0° < x < 180°$ ---[4]

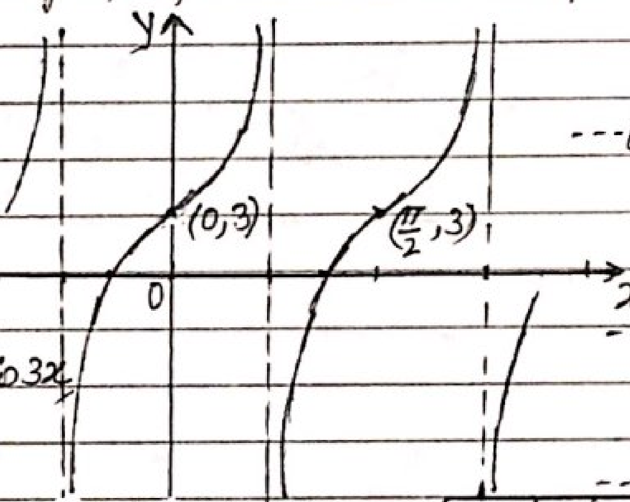
(ii) Hence, or otherwise, solve $6\cos^2 y = 5 + \sin y$ for $0° < y < 180°$ ---[3]

(b) Solve $4\cot^2 z - 3\cot z = 0$ for $0 < z < \pi$ radians. ---[4]

W-13/13/Q9

Q70 (a)(i) The diagram shows the graph of $y = A + C\tan(Bx)$, passing through the point (0,3) and $(\frac{\pi}{2}, 3)$. Find the value of A and of B. ---[2]

(ii) Given that the point $(\frac{\pi}{8}, 7)$ also lies on the graph, find the value of C. ---[1]



(b) Given that $f(x) = 8 - 5\cos 3x$, state the period and the amplitude of f. ---[2]

W-13/23/Q4

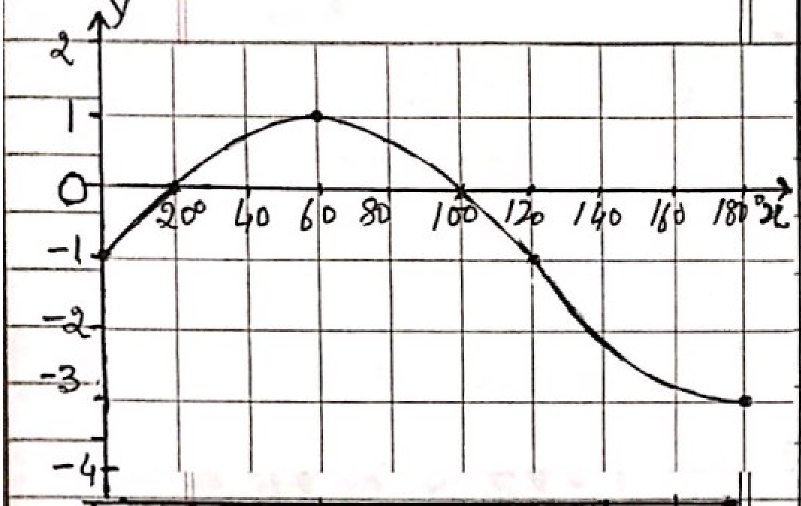
IG-Maths 0606

Trigonometry

Date P-14

Answers

Q1. Sketch the graph of $y = 2\sin\frac{3}{2}x - 1$; $0^\circ \le x \le 180^\circ$



x	0	20	100	120	180
y	-1	0	0	-1	-3

Q2 (a)(i) $\dfrac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - \sin\theta} = \dfrac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta} - \sin\theta}$

$= \dfrac{1}{1-\sin^2\theta} = \dfrac{1}{\cos^2\theta} = \sec^2\theta$ ✓

(ii) Solve $\dfrac{\operatorname{cosec}\phi}{\operatorname{cosec}\phi - \sin\phi} = 8$; $0 < \phi < 360^\circ$

from (i); $2\sec^2\phi = 8 \Rightarrow \cos^2\phi = \frac{1}{4}$

$\Rightarrow \cos\phi = \frac{1}{2}$ or $-\frac{1}{2}$

$\phi = 60^\circ$, $360 - 60^\circ$ | $180 - 60$, $180 + 60$

$\phi = 60^\circ, 120^\circ, 240^\circ, 300^\circ$

(b) Solve; $\sqrt{3}\tan\left(x + \frac{\pi}{4}\right) = 1$ $\quad 0 < x < 2\pi$

$\Rightarrow \tan\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{3}}$, $\frac{\pi}{4} < x + \frac{\pi}{4} < 2\pi + \frac{\pi}{4}$

$= \tan\frac{\pi}{6}$

$\therefore x + \frac{\pi}{4} = \pi + \frac{\pi}{6}, 2\pi + \frac{\pi}{6}$

$x + \frac{\pi}{4} = \frac{7\pi}{6}, \frac{13\pi}{6} \Rightarrow x = \frac{11\pi}{6}, \frac{23\pi}{6}$

Q3(a)(i) 15

(ii) 180° (or π rad)

3(b)(i) $\tan x$; $-\tan x$

(ii) 4

Q4(i) $\dfrac{(1-\sin A)(1+\sin A)}{\sin A \cdot \cos A}$

$= \dfrac{1-\sin^2 A}{\sin A \cdot \cos A} = \dfrac{\cos^2 A}{\sin A \cdot \cos A}$

$= \cot A$ ✓

(ii) Solve; using Part (i)

$\cot 3A = \frac{1}{2}$ $\quad 0 \le x \le 180$, $\quad 0 \le 3x \le 540^\circ$

$\Rightarrow \tan 3A = 2$

$\Rightarrow 3x = 63.4^\circ, 243.4, 423.4^\circ$

$\Rightarrow x = 21.1^\circ, 81.1^\circ, 141.1^\circ$

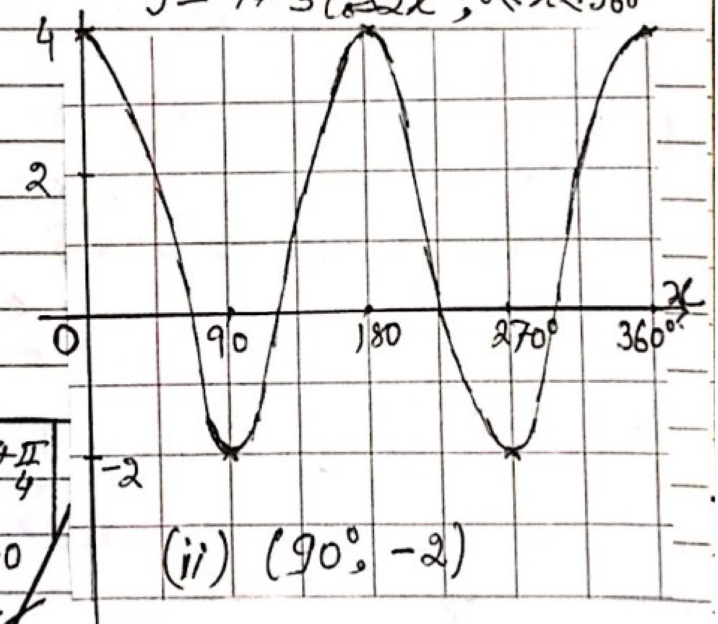
(b) $10(\sec^2 y - 1) - \sec y - 1 = 0$ $\quad 0 \le y \le 2\pi$

$\Rightarrow (10\sec y - 1)(\sec y + 1) = 0$

$\cos y = \frac{10}{11}$, $\cos y = -1$

$y = \pi, 0.43, 5.85$ ✓

Q5(i) Sketch the graph of $y = 1 + 3\cos 2x$; $0^\circ \le x \le 360^\circ$



(ii) $(90^\circ, -2)$

IG-Maths 0606

Trigonometry

Date P-15

Answers

Q6(i) $\operatorname{cosec}\theta - \sin\theta$

$= \frac{1}{\sin\theta} - \sin\theta$

$= \frac{1-\sin^2\theta}{\sin\theta} = \frac{\cos^2\theta}{\sin\theta}$

$= \frac{\cos\theta \cdot \cos\theta}{\sin\theta} = \cot\theta \cdot \cos\theta$ ✓

(ii) Solve: $\operatorname{cosec}\theta - \sin\theta = \frac{1}{3}\cos\theta \quad 0 \le \theta \le 2\pi$

$\cot\theta \cdot \cos\theta = \frac{1}{3}\cos\theta$ from Part (i)

$\cos\theta(\cot\theta - \frac{1}{3}) = 0$

$\Rightarrow \cos\theta = 0$ or $\tan\theta = 3$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}; \theta = 1.25, 4.39$

Q7 Solve $0^\circ \le x \le 360^\circ$

(i) $\cot(2x - 10^\circ) = 3/4$

$\Rightarrow \tan(2x - 10^\circ) = 4/3$

$2x - 10 = \tan^{-1} 4/3 = 53^\circ, 233, 413^\circ, 593^\circ$

$\Rightarrow x = 31.6^\circ, 121.6^\circ, 211.6^\circ, 301.6^\circ$

(ii) $\sin^2 x - \cos^2 x = \cos x$

$\Rightarrow 2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$\Rightarrow x = 60^\circ, 300^\circ; 180^\circ$

Q8 (i) $5 + 4\tan^2(x/3)$

$= 5 + 4[\sec^2 x/3 - 1]$

$= 4\sec^2(x/3) + 1$ ✓

Q9(a) (i) $\frac{360}{10} = 36^\circ$ (ii) 7

(b) $y = 5\sin 4x + 7$

Q10 (i) $\sin x(\cot x + \tan x)$

$= \sin x\left[\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}\right]$

$= \sin x\left[\frac{\cos^2 x + \sin^2 x}{\sin x \cdot \cos x}\right]$

$= \sec x.$ ✓

(continued→)

Q10(ii) solve: $0 \le x \le 360^\circ$

$|\sin x(\cot x + \tan x)| = 2$

$\Rightarrow |\sec x| = 2$ ∵ from Part (i)

$\Rightarrow \cos x = \pm\frac{1}{2}$

$\cos x = \frac{1}{2}$; $\cos x = -\frac{1}{2}$

$60^\circ, 300^\circ$; $120^\circ, 240^\circ$

Q11 (i) $\frac{\operatorname{cosec}\theta}{\cot\theta + \tan\theta}$

$= \frac{1}{\sin\theta} \times \frac{1}{\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}} = \frac{1}{\sin\theta} \times \frac{\sin\theta\cos\theta}{(\cos^2\theta + \sin^2\theta)}$

$= \cos\theta$ ✓

(ii) $\int_0^a \frac{\operatorname{cosec}2\theta}{\cot 2\theta + \tan 2\theta}\, d\theta = \frac{\sqrt{3}}{4}$

$\Rightarrow \int_0^a \cos 2\theta\, d\theta = \frac{\sqrt{3}}{4}$ [from Part (i)

$\Rightarrow \frac{1}{2}\sin 2a = \frac{\sqrt{3}}{4} \quad 0 < a < \frac{\pi}{4}$

$\Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2} \Rightarrow 2a = \frac{\pi}{3}$

$\Rightarrow a = \frac{\pi}{6}$ ✓

Q12 Solve the equation:

(i) $3x - \frac{\pi}{4} = \sin^{-1}\left(\frac{3}{4}\right) \quad 0 \le x \le \frac{\pi}{2}$

$0 \le 3x \le \frac{3\pi}{2}$

$3x = \frac{\pi}{4} + \sin^{-1}\frac{3}{4} = \left(\frac{\pi}{4} + 0.848\right), \left(\frac{\pi}{4} + 2.29\right)$

$x = 0.544; 1.03$ ✓

(ii) $2\tan^2 y + \sec^2 y = 4\sec y + 3$

$\Rightarrow (3\sec y + 1)(\sec y - 5) = 0$

$\Rightarrow \cos y = -3^{\times}$ or $\cos y = \frac{1}{5}$

$\because 0 \le y \le 360^\circ$

$y = 78.5; 281.53$

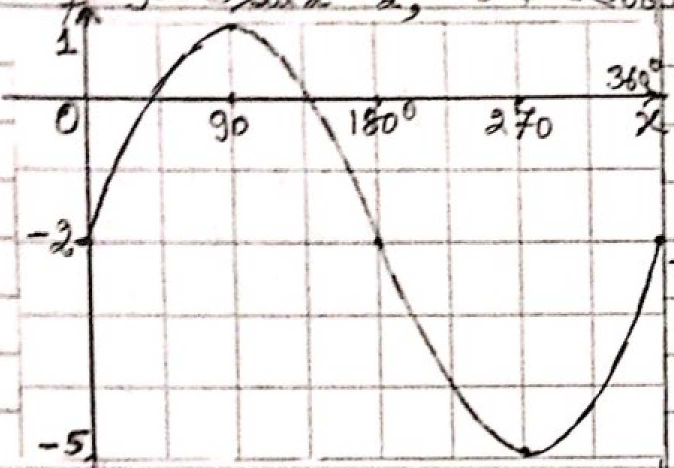
Q13 (i) 4

(ii) 40° or $\frac{2\pi}{9}$

Trigonometry

Answers

Q14 (i) Sketch the graph of $y = 3\sin x - 2$, $0 \le x \le 360^\circ$



(ii) 5

Q15 (a) $\dfrac{\tan^2\theta + \sin^2\theta}{\cos\theta + \sec\theta}$

$= \dfrac{\dfrac{\sin^2\theta}{\cos^2\theta} + \sin^2\theta}{\cos\theta + \dfrac{1}{\cos\theta}}$

$= \dfrac{\sin^2\theta(1+\cos^2\theta)}{\cos\theta(1+\cos^2\theta)} = \tan\theta \cdot \sec\theta$ ✓

(b) $9y^2 - x^2y^2 = y^2(9 - x^2)$

$= \dfrac{9}{\cos^2\phi}(9 - 9\sin^2\phi)$

$= \dfrac{81(\cos^2\phi)}{\cos^2\phi}$

$= 81$ ✓

Q16 Solve:

(a) $2|\sin x| = 1$ $\quad -\pi \le x \le \pi$

$\Rightarrow \sin x = \frac{1}{2}$ or $-\frac{1}{2}$

$\Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}; -\frac{\pi}{6}, -\frac{5\pi}{6}$ ✓

(b) $2y + 15 = \tan^{-1}\frac{1}{3}$ $\quad 0 \le y \le 180$

$= 18.43; 198.43$

$\Rightarrow y = 1.7; 91.7$ ✓

(continued→)

Q16(c) $0^\circ \le z \le 360^\circ$

$3\cot^2 z = \operatorname{cosec}^2 z - 7\operatorname{cosec} z + 1$

$\Rightarrow 2\operatorname{cosec}^2 z + 7\operatorname{cosec} z - 4 = 0$

$(2\operatorname{cosec} z - 1)(\operatorname{cosec} z + 4) = 0$

$\Rightarrow \sin z = 2^x$ or $-\frac{1}{4}$

$\Rightarrow z = 194.5^\circ; 345.5^\circ$ ✓

Q17 Solve: $\quad 0 \le x \le 180^\circ$

(a) $3\operatorname{cosec} 2x - 4\sin 2x = 0$

$\Rightarrow \sin^2 2x = \frac{3}{4}$

$\sin 2x = \pm\frac{\sqrt{3}}{2}$

$2x = 60^\circ, 120^\circ, 240^\circ, 300^\circ$

$x = 30^\circ, 60^\circ, 120^\circ, 150^\circ$ ✓

(b) $\tan\left(y - \frac{\pi}{4}\right) = \frac{1}{\sqrt{3}}$

$y - \frac{\pi}{4} = \frac{\pi}{6}, \frac{7\pi}{6}$

$y = \frac{5\pi}{12}; \frac{17\pi}{12}$ ✓

Q18 $\dfrac{1}{1-\sin\theta} - \dfrac{1}{1+\sin\theta}$

$= \dfrac{1+\sin\theta - (1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$

$= \dfrac{2\sin\theta}{\cos^2\theta} = 2\tan\theta \cdot \sec\theta$ ✓

Q19 $a = 4$, $b = 6$, $c = -2$

Q20 (a) $\tan(\phi + 35^\circ) = \frac{2}{5}$

$\Rightarrow \phi + 35 = 21.8^\circ, 201.8^\circ, 381.8^\circ$

$\Rightarrow \phi = 166.8^\circ, 346.8^\circ$ ✓ $0 \le \phi \le 360^\circ$

(b)(i) $\dfrac{\dfrac{1}{\cos\theta}}{\dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta}} = \dfrac{1}{\cos\theta}\left(\dfrac{\sin\theta\cos\theta}{\cos^2\theta + \sin^2\theta}\right)$

$= \sin\theta$ ✓

(ii) $\sin 3\theta = -\frac{\sqrt{3}}{2}$ $\quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$3\theta = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{4\pi}{3}$

$\theta = -\frac{2\pi}{9}, -\frac{\pi}{9}, \frac{4\pi}{9}$ ✓

IG-Maths 0606

Trigonometry

Answers

Q21 $y = 2(\tan^2\theta + 1)$

$\Rightarrow y = 2[(x+5)^2 + 1]$ ✓

Q22 $a = 3,\ b = 8,$ ✓

$\frac{5}{2} = 3\cos\left(8 \times \frac{\pi}{12}\right) + c$

$\Rightarrow c = 4.$ ✓

Q23(a) $\frac{\sin x}{1+\cos x} + \frac{(1+\cos x)}{\sin x}$

$= \frac{\sin^2 x + (1+\cos x)^2}{\sin x(1+\cos x)}$

$= \frac{2(1+\cos x)}{\sin(1+\cos x)} = 2\operatorname{cosec} x$ ✓

b(i) $\operatorname{cosec}^2 y + \operatorname{cosec} y - 6 = 0$

$(\operatorname{cosec} y - 2)(\operatorname{cosec} y + 3) = 0$

$\Rightarrow \sin y = \frac{1}{2}\ ;\ \sin y = -\frac{1}{3},\quad 0 \le y \le 360^\circ$

$y = 30^\circ, 150^\circ, 199.5^\circ, 340.5^\circ$ ✓

(ii) $2z + \frac{\pi}{4} = \frac{5\pi}{6}$ or $\frac{7\pi}{6}$

$z = \frac{7\pi}{24}, \frac{11\pi}{24}\qquad 0 \le z \le \pi$

Q24 (i) $\frac{1}{\operatorname{cosec}\theta - 1} - \frac{1}{\operatorname{cosec}\theta + 1} = \frac{2}{\operatorname{cosec}^2\theta - 1}$

$= \frac{2}{\cot^2\theta} = 2\tan^2\theta$ ✓

(ii) Solve $2\tan^2\theta = 6 + \tan\theta$

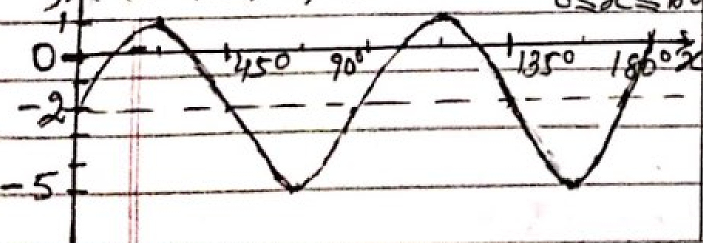
or $(2\tan\theta + 3)(\tan\theta - 2) = 0\quad 0 < \theta < 360^\circ$

$\tan\theta = -3/2,\ \tan\theta = 2$

$\theta = 63.4^\circ, 123.7^\circ, 243.4^\circ, 303.7^\circ$

Q25(a) $a = 10,\ b = 6,\ c = 4$

(b) Graph of $y = 3\sin 4x - 2$, $0 \le x \le 180^\circ$



Q26(i)

$2\cos x\cot x + 1 = \cot x + 2\cos x$

$\Rightarrow 2\cos x \times \frac{\cos x}{\sin x} + 1 = \frac{\cos x}{\sin x} + 2\cos x$

$\Rightarrow 2\cos^2 x - 2\cos x\sin x - \cos x + \sin x = 0$

$2\cos x(\cos x - \sin x) - 1(\cos x - \sin x) = 0$

$(2\cos x - 1)(\cos x - \sin x) = 0$ ✓

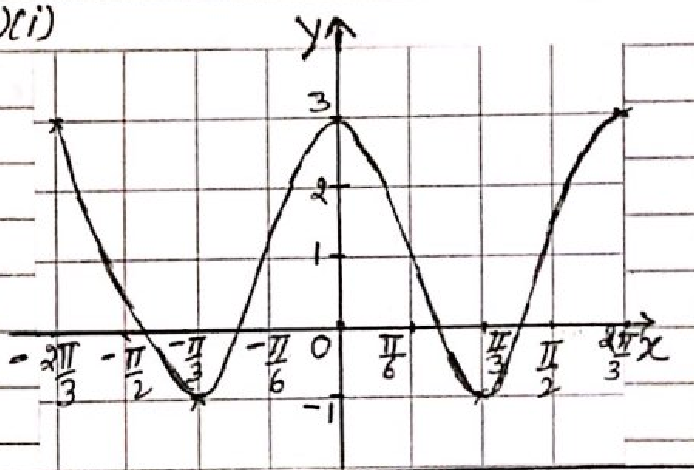
(ii) To Solve $(2\cos x - 1)(\cos x - \sin x) = 0$

$\cos x = \frac{1}{2}$ and $\tan x = 1,\quad 0 < x < \pi$

$\Rightarrow x = \frac{\pi}{3}, \frac{\pi}{4}$

Q27(a) $a = 2,\ b = 4,\ c = -2$

(b)(i)



Q28(i) $(1 - \cos\theta)(1 + \sec\theta)$

$= (1 - \cos\theta)\frac{(1 + \cos\theta)}{\cos\theta}$

$= \frac{1 - \cos^2\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta}$

$= \sin\theta \cdot \tan\theta$ ✓

(ii) Solve $(1 - \cos\theta)(1 + \sec\theta) = \sin\theta$

or $\sin\theta \cdot \tan\theta = \sin\theta$ from part (i)

or $\sin\theta(\tan\theta - 1) = 0\quad : 0 \le \theta \le \pi$

$\Rightarrow \sin\theta = 0\ ;\ \tan\theta = 1$

$\theta = 0, \pi, \frac{\pi}{4}$ ✓

Trigonometry

Answers

Q29 Solve the equation:

(i) $8(1-\cos^2 A)+2\cos A=7$

$\Rightarrow (2\cos A-1)(4\cos A+1)=0$

$\cos A=\frac{1}{2}$, $\cos A=-\frac{1}{4}$ $\quad 0\le A\le 180°$

$A=60°\ ;\ 104.5°$

(ii) $\sin(3B+1)=0.4$; $0\le B\le \pi$

$\Rightarrow B=0.577,\ 1.9,\ 2.67$ ✓

Q30 (i) $\frac{\cos x}{1+\tan x}-\frac{\sin x}{1+\cot x}$

$=\frac{\cos^2 x}{\cos x+\sin x}-\frac{\sin^2 x}{\cos x+\sin x}$

$=\frac{(\cos x-\sin x)(\cos x+\sin x)}{(\cos x+\sin x)(\cos x-\sin x)}$

$=\cos x-\sin x$ ✓

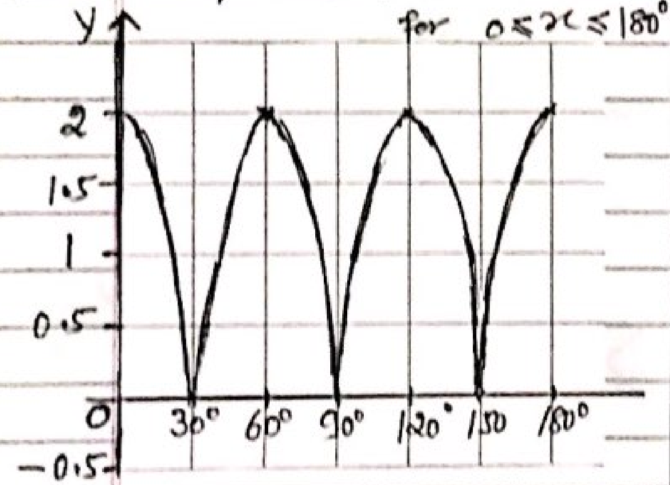
(ii) Solve $\quad -180°<x<180°$

$\cos x-\sin x=3\sin x-4\cos x$

$\Rightarrow \tan x=5/4$

$x=51.3°,\ -128.7°$

Q31 Graph. $y=|2\cos 3x|$ for $0\le x\le 180°$



Q32 (a)(i) $\frac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta}-\sin\theta}$

$=\frac{1}{1-\sin^2\theta}$

$=\frac{1}{\cos^2\theta}=\sec^2\theta$ ✓

Q32 (a)(ii) Solve

$\cos^2\theta=\frac{1}{4}$ $\quad 0<\theta<360°$

$\cos\theta=\pm\frac{1}{2}$

$\theta=60°,\ 120°,\ 240°,\ 300°$

(b) Solve:

$\tan\left(x+\frac{\pi}{4}\right)=\frac{1}{\sqrt3}$ $\quad 0<x<2\pi$

$x=\frac{\pi}{6}-\frac{\pi}{4},\ \frac{7\pi}{6}-\frac{\pi}{4},\ \frac{13\pi}{6}-\frac{\pi}{4}$

$x=\cancel{-\frac{\pi}{12}},\ \frac{11\pi}{12},\ \frac{23\pi}{12}$ ✓

Q33 (i) $AB^2=(\sqrt3+1)^2+(\sqrt3-1)^2-2(\sqrt3+1)(\sqrt3-1)\cos 60° = 6$

$\therefore AB=\sqrt6$ ✓

(ii) $\frac{\sin A}{\sqrt3-1}=\frac{\sin 60}{\sqrt6}$

$\Rightarrow \sin A=\frac{(\sqrt3-1)\sin 60°}{\sqrt6}=0.259$

$\therefore A=\sin^{-1}0.259=15°$ ✓

(iii) Area of Triangle $=\frac{1}{2}(\sqrt3+1)(\sqrt3-1)\sin 60°$

$=\frac{\sqrt3}{2}$ ✓

Q34 (a)(i) $\frac{\frac{1}{\sin x}}{\frac{\cos x}{\sin x}+\frac{\sin x}{\cos x}}=\frac{1}{\sin x}\times\frac{\sin x\cos x}{\sin^2 x+\cos^2 x}$

$=\cos x$ ✓

(ii) Solve, $\cos 3y=0.5$ $\quad 0\le x\le\pi$

$3y=\frac{\pi}{3},\ \frac{5\pi}{3},\ \frac{7\pi}{3}$

$\Rightarrow y=\frac{\pi}{9},\ \frac{5\pi}{9},\ \frac{5\pi}{9}$ ✓

(b) Solve $2\sin z+8(1-\sin^2 z)=5$

$8\sin^2 z-2\sin z-3=0$

$(4\sin z-3)(2\sin z+1)=0$

$\sin z=\frac{3}{4},\ \sin z=-\frac{1}{2},\quad 0°<z<360°$

$z=48.6°,\ 131.4°,\ 210°,\ 330°$

0606 IG-Maths

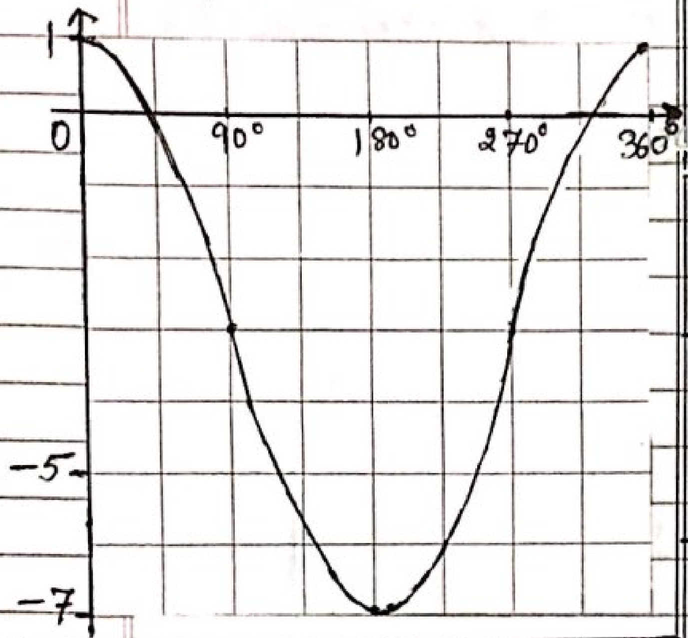
Trigonometry

Answers

Q35(i) 4 (ii) 360°

(iii) Sketch the graph. $0° \leq x \leq 360°$

$f(x) = 4\cos x - 3$

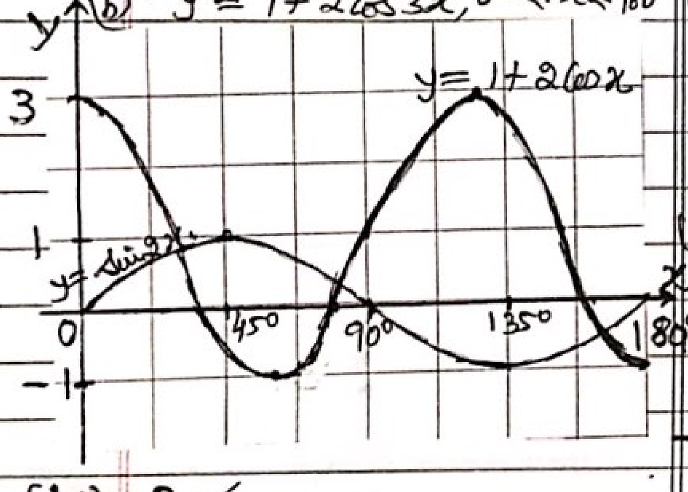


Q36(i) 180° or π rad (ii) 2

(iii) Sketch the graph of:

(a) $y = \sin 2x$, $0° \leq x \leq 180°$

(b) $y = 1 + 2\cos 3x$, $0° \leq x \leq 180$



(iv) 3 ✓

Q37(a) $\sin^2 x = \frac{1}{4}$ $0 \leq x \leq 360°$

$\sin x = \pm\frac{1}{2}$

$x = 30°, 150°, 210°, 330°$ ✓

(Continued →)

Q37(b) $(\sec^2 3y - 1) - 2\sec 3y - 2 = 0$

$\sec^2 3y - 2\sec 3y - 3 = 0$

$(\sec 3y + 1)(\sec 3y - 3 = 0$

$\cos 3y = -1$, $\cos 3y = \frac{1}{3}$ $0 \leq y \leq 180°$

$3y = 180°, 540°, 70.5, 289.5°, 430.5°$

$y = 60°, 180°, 23.5°, 96.5°, 143.5°$ ✓

Q37(C) $z - \frac{\pi}{3} = \frac{\pi}{3}, \frac{4\pi}{3}$ $0 \leq z \leq 2\pi$

$z = \frac{2\pi}{3}, \frac{5\pi}{3}$ ✓

Q38 $\frac{\tan\theta + \cot\theta}{\csc\theta} = \frac{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}$

$= \frac{1}{\cos\theta} = \sec\theta$ ✓

Q39(a) Solve $\cos^2 3x = \frac{1}{2}$ $0 \leq x \leq 120°$

or $\cos 3x = \pm\frac{1}{\sqrt{2}}$

$3x = 45°, 135°, 225°, 315°$

$x = 15, 45°, 75°, 105°$ ✓

(b) $3(\cot^2 y + 1) + 5\cot y - 5 = 0$; $0° \leq y \leq 360°$

$(3\cot y - 1)(\cot y + 2) = 0$

$\tan y = 3$, $\tan y = -\frac{1}{2}$

$y = 71.6°, 251.6°, 153.4°, 333.4°$ ✓

(c) $\sin(z + \frac{\pi}{3}) = \frac{1}{2}$ $0 \leq z \leq 2\pi$

$z + \frac{\pi}{3} = \cancel{\frac{\pi}{6}}, \frac{5\pi}{6}, \frac{13\pi}{6}$

$z = \frac{\pi}{2}, \frac{11\pi}{6}$ ✓

Q40 $\tan\theta + \cot\theta$

$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{1}{\sin\theta \cdot \cos\theta}$

$= \sec\theta \cdot \csc\theta$ ✓

Scanned with CamScanner

IG-Maths 0606

Trigonometry

Answers

Q41 (i) $a = 3, b = 2, c = 4$ ✓

(ii) $\frac{dy}{dx} = 8\cos 4x$

(iii) $\left(\frac{dy}{dx}\right)_{\frac{\pi}{2}} = 8\cos 2\pi = 8$

Eqn of normal,

$$\frac{y-3}{x-\frac{\pi}{2}} = -\frac{1}{8}$$

$\Rightarrow y = -\frac{1}{8}x + 3.20$ ✓

Q42 Solve:

(i) $\tan 2x = -\frac{5}{4}$ $\quad 0 \le x \le 180°$

$2x = 128.7°, 308.7°$

$x = 64.3°, 154.3°$ ✓

(ii) $\csc^2 y + 3\csc y - 4 = 0$

$\Rightarrow 4\sin^2 y - 3\sin y - 1 = 0$

$(4\sin y + 1)(\sin y - 1) = 0$

$\sin y = -\frac{1}{4}, \sin y = 0; 0° \le y \le 360°$

$y = 194.5°, 345.5°, 90°$ ✓

(iii) $\cos\left(z + \frac{\pi}{4}\right) = -\frac{1}{2}; \ 0 \le z \le 2\pi$

$\Rightarrow z + \frac{\pi}{4} = \frac{2\pi}{3}, \frac{4\pi}{3}$

$\Rightarrow z = \frac{5\pi}{12}, \frac{13\pi}{12}$ ✓

Q43 Solve $\cos^2\left(3x - \frac{\pi}{4}\right) = \frac{1}{2}$

$\cos\left(3x - \frac{\pi}{4}\right) = \pm\frac{1}{\sqrt{2}}$ $\quad 0 \le x \le \frac{\pi}{3}$

$3x - \frac{\pi}{4} = -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$

$3x = 0, \frac{\pi}{2}, \pi$

$x = 0, \frac{\pi}{6}, \frac{\pi}{3}$ ✓

Q44 (i) $\sec^2 x + \csc^2 x = \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}$

$= \frac{1}{\sin^2 x \cdot \cos^2 x}$

$= \sec^2 x \cdot \csc^2 x$

(Continued →)

(ii) Solve $\quad 90° < x < 270°$

$\sec^2 x + \csc^2 x = 4\tan^2 x$

$\Rightarrow \sec^2 x \csc^2 x = 4\tan^2 x$

$\Rightarrow \frac{1}{\sin^2 x \cdot \cos^2 x} = \frac{4\sin^2 x}{\cos^2 x}$

$\Rightarrow 4\sin^4 x = 1$

$\Rightarrow \sin x = \pm\frac{1}{\sqrt{2}}$

$x = 135°, 225°$

Q45. $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta}$

$= \frac{\sin\theta + \sin^2\theta + \cos^2\theta}{\cos\theta(1+\sin\theta)}$

$= \frac{\sin\theta + 1}{\cos\theta(1+\sin\theta)} = \frac{1}{\cos\theta} = \sec\theta$ ✓

Q46 (a) Solve:

$\tan 2x = -0.6$ $\quad 0 \le x \le 180°$

$\Rightarrow 2x = 149°, 329°$

$x = 74.5°, 164.5°$

(b) $2(\csc^2 y - 1) + 3\csc y = 0$

$(2\csc y - 1)(\csc y + 2) = 0$

$\csc y = \frac{1}{2}$ ✗, $\csc y = -2$

$\Rightarrow \sin y = -\frac{1}{2}$ $\quad 0 \le y \le 360°$

$y = 210°, 330°$

(c) $\cos(z + 1.2) = 2/3$

$\Rightarrow z + 1.2 =$ $\quad 0 \le z \le 6$

$0.8411, 5.442, 7.142$

$z = 4.24, 5.92$

Q47 $\frac{\cos^2 A + (1+\sin A)^2}{\cos A(1+\sin A)}$

$= \frac{2(1+\sin A)}{\cos A(1+\sin A)}$

$= 2\sec A \Rightarrow p = 2$ ✓

IG-Maths 0606

Trigonometry

Date P-21

Answers

Q48(a) $\tan x(\tan x + 5) = 0$

$\Rightarrow \tan x = 0\ ;\ \tan x = -5,\ 0° \le x \le 180°$

$\Rightarrow x = 0, 180°, 101.3°$ ✓

(b) $2(1 - \sin^2 y) - \sin y - 1 = 0$

$(2\sin y - 1)(\sin y + 1) = 0$

$\sin y = \frac{1}{2},\ \sin y = -1,\ 0 \le y \le 360°$

$y = 30°, 150°, 270°$ ✓

(c) $\cos\left(2z - \frac{\pi}{6}\right) = \frac{1}{2} \quad 0 \le z \le \pi$

$2z - \frac{\pi}{6} = \frac{\pi}{3},\ \frac{5\pi}{3}$

$z = \frac{\pi}{4},\ \frac{11\pi}{12}$ ✓

Q49(i) $\cos x(3\sin x - 2) = 0$

$\cos x = 0,\ \sin x = 2/3\ ;\ 0 \le x \le 180°$

$x = 90°, 41.8°, 138.2°$ ✓

(ii) $10(1 - \cos^2 y) + \cos y = 8$

$\Rightarrow (2\cos y - 1)(5\cos y + 2 = 0$

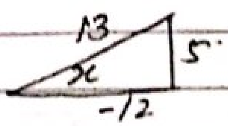
$\cos y = \frac{1}{2},\ \cos y = -\frac{2}{5},\ 0 \le y \le 360°$

$y = 60°, 300°, 113.6°, 246.4°$

Q50(a) $\dfrac{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}{\frac{1}{\cos\theta} + \frac{1}{\sin\theta}}$

$= \dfrac{1}{\sin\theta + \cos\theta}$ ✓

(b) $\tan x = -\frac{5}{12} \quad 90 < x < 180$

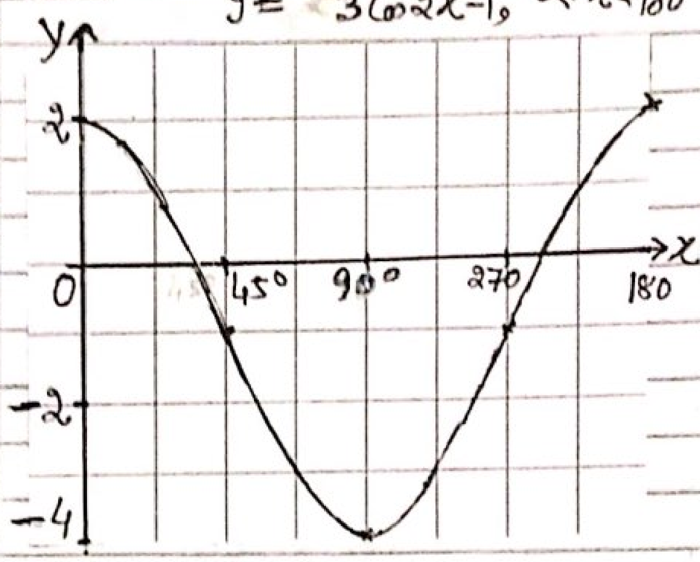


$\sin x = 5/13$ ✓

$\cos x = -\frac{12}{13}$ ✓

Q51(a) Sketch the graph

$y = 3\cos 2x - 1,\ 0 \le x \le 180°$



(b)(i) 4 (ii) $60°$ or $\pi/3$

Q52(a) Solve $2\cos 3x - \frac{\cos 3x}{\sin 3x} = 0$

$\cos 3x\left(2 - \frac{1}{\sin 3x}\right) = 0$

$\Rightarrow \cos 3x = 0\ ;\ \sin 3x = \frac{1}{2},\ 0 \le x \le 90°$

$3x = 90°, 270°, 30°, 150°$

$x = 30°, 90°, 10°, 50°$ ✓

(b) $\cos\left(y + \frac{\pi}{2}\right) = -\frac{1}{2} \quad 0 \le y \le \pi$

$y + \frac{\pi}{2} = \frac{2\pi}{3},\ \frac{4\pi}{3}$

$\Rightarrow y = \frac{\pi}{6},\ \frac{5\pi}{6}$ ✓

Q53(i) $\sec x \operatorname{cosec} x - \cot x$

$= \frac{1}{\cos x \cdot \sin x} - \frac{\cos x}{\sin x}$

$= \frac{1 - \cos^2 x}{\cos x \cdot \sin x} = \frac{\sin^2 x}{\cos x \cdot \sin x}$

$= \tan x$ ✓

(ii) $3\cot x - \cot x = \tan x \Rightarrow 2\cot x = \tan x$

$\rightarrow \tan^2 x = 2 \rightarrow \tan x = \pm\sqrt{2}$

$x = 54.7, 125.3, 234.7, 305.3$ ✓

Trigonometry

Answers

Q54(a) Same as Q 51.

Q55 $a=3,\ b=2,\ c=4$

Q56(a) $\tan x = -\frac{5}{3}$ $\quad 0 \le x \le 360°$

$x = 121.0°,\ 301.0°$

(b) $\sin\left(3y + \frac{\pi}{4}\right) = \frac{1}{2}$ $\quad 0 \le y \le \pi$

$3y + \frac{\pi}{4} = \frac{\pi}{6},\ \frac{5\pi}{6},\ \frac{13\pi}{6},\ \frac{17\pi}{6}$

$3y = -\frac{\pi}{12}\times,\ \frac{7\pi}{12},\ \frac{23\pi}{12},\ \frac{31\pi}{12}$

$y = \frac{7\pi}{36},\ \frac{23\pi}{36},\ \frac{31\pi}{36}$ ✓

Q57(i) $\frac{1+\cos x + 1 - \cos x}{(1-\cos x)(1+\cos x)}$

$= \frac{2}{1-\cos^2 x} = \frac{2}{\sin^2 x}$

$= 2\operatorname{cosec}^2 x$ ✓

(ii) $2\operatorname{cosec}^2 x = 8$

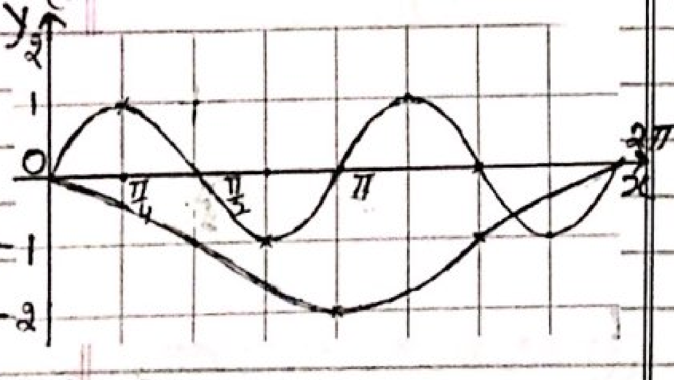
$\Rightarrow \sin^2 x = \frac{1}{4} \Rightarrow \sin x = \pm\frac{1}{2}$

$\Rightarrow x = 30°,\ 150°,\ 210°,\ 330°$

Q58 Sketch the graph of

(i) $y = \cos x - 1$ $\quad 0 \le x \le 2\pi$

(ii) $y = \sin 2x$



(iii) 3 ✓

Q59(a) $\sin\left(x + \frac{\pi}{3}\right) = -\frac{1}{2}$; $0 \le x \le 2\pi$

$x + \frac{\pi}{3} = \frac{7\pi}{6},\ \frac{11\pi}{6}$

$x = \frac{5\pi}{6},\ \frac{3}{2}\pi$ ✓

Q59(b) $\tan y - 2 = \frac{1}{\tan y}$

$\tan^2 y - 2\tan y - 1 = 0$; $0 \le y \le 180°$

$\tan y = 1 \pm \sqrt{2}$

$y = 67.5°,\ 157.5°$

Q60 $\frac{(1+\sin\theta)^2 + (1-\sin\theta)^2}{\cos^2\theta}$

$= \frac{2 + 2\sin^2\theta}{\cos^2\theta}$

$= 2\sec^2\theta + 2\tan^2\theta$

$= 2(1+\tan^2\theta) + 2\tan^2\theta$

$= 2 + 4\tan^2\theta$ ✓

Q61 Consider

$[(1-\cos\theta) - \sin\theta]^2$

$= (1-\cos\theta)^2 + \sin^2\theta - 2\sin\theta(1-\cos\theta)$

$= (1-\cos\theta)^2 + (1-\cos^2\theta) - 2\sin\theta(1-\cos\theta)$

$= (1-\cos\theta)[1-\cos\theta + 1 + \cos\theta - 2\sin\theta]$

$= (1-\cos\theta)(2 - 2\sin\theta)$

$= 2(1-\cos\theta)(1-\sin\theta)$ ✓

Q62(a) $\cos 2x + \frac{2}{\cos 2x} + 3 = 0$

$\Rightarrow \cos^2 2x + 3\cos 2x + 2 = 0$, $0° \le x \le 360°$

$(\cos 2x + 2)(\cos 2x + 1) = 0$

$\cos 2x = -2\ \times$ $\quad \cos 2x = -1$

$\Rightarrow 2x = 180°,\ 540°$

$x = 90°,\ 270°$ ✓

(b) $\sin^2\left(y - \frac{\pi}{6}\right) = \frac{1}{2}$ $\quad 0 \le y \le \pi$

$\sin\left(y - \frac{\pi}{6}\right) = +\frac{1}{\sqrt{2}}$

$y - \frac{\pi}{6} = \frac{\pi}{4},\ \frac{3\pi}{4}$

$y = \frac{5\pi}{12},\ \frac{11\pi}{12}$ ✓

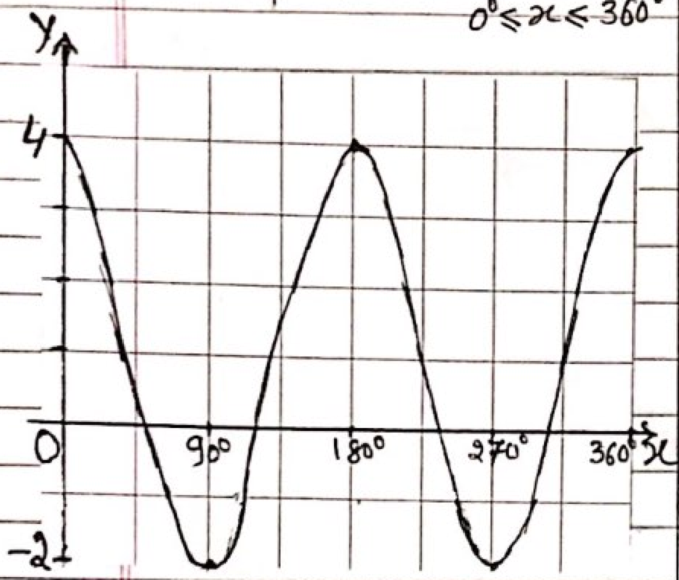
Trigonometry

Answers

Q63 (a)(i) Sketch the graph.

$y = f(x) = 1 + 3\cos 2x$

$0^\circ \leq x \leq 360^\circ$



(ii) 3, (iii) 180°

(iv) $\operatorname{Cosec} x = \frac{1}{\sin x} = \frac{1}{\sqrt{1-\cos^2 x}}$

$\operatorname{Cosec} x = \frac{-1}{\sqrt{1-p^2}}$; $270^\circ < x < 360^\circ$

Q64 Same as Q 58

Q65 $a = 3,\ b = 2,\ c = 1$ ✓

Q66 $\frac{(1+\sin\theta)^2 + \cos^2\theta}{\cos\theta(1+\sin\theta)}$

$= \frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$

$= 2\sec\theta$ ✓

67 (a) $\tan x = -2/7$

$x = 164.1,\ 344.1$ ✓

(b) $\sin(2y-1) = 5/7,\ 0 \leq y \leq 5$ rad

$2y - 1 = \sin^{-1} 5/7$

$= 0.79$ or 2.34

$y = 0.898, 1.67, 4.04, 4.81$ ✓

Q68 $\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$

$= \frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta} = \sin^4\theta \cdot \sec^2\theta$ ✓

Q69 (i) $6(1-\cos^2 x) = 5 + \cos x$

$(3\cos x - 1)(2\cos x + 1) = 0$; $0 < x < 180^\circ$

$x = 70.5^\circ,\quad x = 120^\circ$

(ii) $\cos x = \sin y$

$\therefore \sin y = \frac{1}{3},\ -\frac{1}{2}$ ✗ $0 < y < 180^\circ$

$y = 19.5^\circ, 160.5^\circ$

(b) $\cot z(4\cot z - 3) = 0$ $0 < z < \pi$ ✓

$\Rightarrow \cot z = 0$ or $\cot z = 3/4$

$z = \frac{\pi}{2}$, ✓ or $\tan z = 4/3$

or $z = 0.927$ ✓

Q70 (i) $A = 3,\ B = 2$ ✓

(ii) $C = 4$ ✓

(b) Period $= 120^\circ$ or $\frac{2\pi}{3}$ ✓

Amplitude $= 5$

← X — X →

Scanned with CamScanner