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IG. 0606 Additional Maths

Trigonometry Notes

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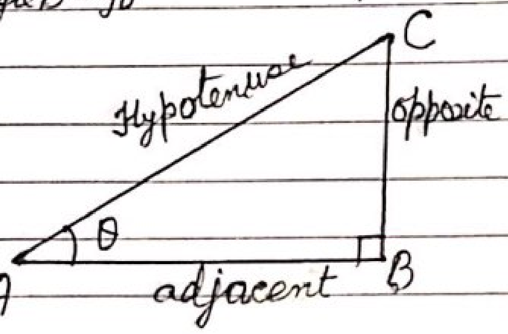
Trigonometry

§ Trigonometric Ratios in a right angle triangle.

In a rt angled triangle ABC, angle $B = 90^\circ$



1. $\sin\theta = \dfrac{opp}{hyp} = \dfrac{BC}{AC}$.
2. $\cos\theta = \dfrac{adj}{hyp} = \dfrac{AB}{AC}$.
3. $\tan\theta = \dfrac{opp}{adj} = \dfrac{BC}{AB}$

Note 1. $\dfrac{\sin\theta}{\cos\theta} = \tan\theta$ ✓ --- (1) $\left[\because \dfrac{\sin\theta}{\cos\theta} = \dfrac{BC}{AC} \Big/ \dfrac{AB}{AC} = \dfrac{BC}{AB} = \tan\theta\right]$

§ Reciprocals of $\sin\theta$, $\cos\theta$ and $\tan\theta$.

4. $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$ or $\sin\theta = \dfrac{1}{\operatorname{cosec}\theta}$.
5. $\sec\theta = \dfrac{1}{\cos\theta}$ or $\cos\theta = \dfrac{1}{\sec\theta}$
6. $\cot\theta = \dfrac{1}{\tan\theta}$ or $\tan\theta = \dfrac{1}{\cot\theta}$

Note 2. $\dfrac{\cos\theta}{\sin\theta} = \cot\theta$ —— (2)

§ Trigonometric Identities:

$[\sin^2\theta = (\sin\theta)^2]$

(1) $\sin^2\theta + \cos^2\theta = 1$	(2) $1 + \tan^2\theta = \sec^2\theta$	(3) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$
or $1 - \sin^2\theta = \cos^2\theta$	or $\sec^2\theta - \tan^2\theta = 1$	$\operatorname{cosec}^2\theta - \cot^2\theta = 1$
or $1 - \cos^2\theta = \sin^2\theta$	or $\sec^2\theta - 1 = \tan^2\theta$	$\operatorname{cosec}^2\theta - 1 = \cot^2\theta$

Proof:

In rt triangle ABC, $AB^2 + BC^2 = AC^2$ (Pythagoras Theorem)

or $\dfrac{AB^2}{AC^2} + \dfrac{BC^2}{AC^2} = 1$ Div. by AC^2

$\left(\dfrac{AB}{AC}\right)^2 + \left(\dfrac{BC}{AC}\right)^2 = 1$

or $\cos^2\theta + \sin^2\theta = 1$ ✓

Trigonometry

Example 1. Show that, $\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta} = 2\sec\theta$. ---[4]

$\boxed{W\text{-}13/21/Q12}$

Solution L.H.S. $\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta}$

$$= \frac{(1+\sin\theta)^2 + \cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{1+2\sin\theta+\sin^2\theta+\cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{1+2\sin\theta+1}{\cos\theta(1+\sin\theta)} \qquad \because \sin^2\theta+\cos^2\theta=1$$

$$= \frac{2\cancel{(1+\sin\theta)}}{\cos\theta\cancel{(1+\sin\theta)}} = \frac{2}{\cos\theta} \qquad \because \frac{1}{\cos\theta}=\sec\theta.$$

$$= 2\sec\theta = R.H.S$$

Example 2. Show that, $\tan^2\theta - \sin^2\theta = \sin^4\theta \cdot \sec^2\theta$. ---[4]

$\boxed{W\text{-}13/13/Q3}$

Solution: L.H.S $\tan^2\theta - \sin^2\theta$

$$= \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$$

$$= \frac{\sin^2\theta - \sin^2\theta\cdot\cos^2\theta}{\cos^2\theta}$$

$$= \frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta} \qquad (1-\cos^2\theta=\sin^2\theta)$$

$$= \frac{\sin^2\theta\cdot\sin^2\theta}{\cos^2\theta} = \sin^4\theta\cdot\sec^2\theta \qquad \left(\frac{1}{\cos^2\theta}=\sec^2\theta\right)$$

$$= R.H.S$$

Example 3. Show that $(1-\cos\theta-\sin\theta)^2 - 2(1-\sin\theta)(1-\cos\theta) = 0$ ---[3]

$\boxed{S\text{-}13/12/Q3}$

Solution: L.H.S $[(1-\cos\theta)-\sin\theta]^2 - 2(1-\cos\theta)(1-\sin\theta)$

$$= (1-\cos\theta)^2 - 2\sin\theta(1-\cos\theta) + \sin^2\theta - 2(1-\cos\theta)(1-\sin\theta)$$

$$= (1-\cos\theta)^2 - 2\sin\theta(1-\cos\theta) + (1-\cos^2\theta) - 2(1-\cos\theta)(1-\sin\theta)$$

$$= (1-\cos\theta)[(1-\cancel{\cos\theta}) - 2\sin\theta + 1+\cancel{\cos\theta}] - 2(1-\cos\theta)(1-\sin\theta)$$

$$= (1-\cos\theta)[2(1-\sin\theta)] - 2(1-\cos\theta)(1-\sin\theta)$$

$$= 2(1-\cos\theta)(1-\sin\theta) - 2(1-\cos\theta)(1-\sin\theta)$$

$$= 0 = R.H.S$$

Trigonometry

Example 4 Prove $\left(\frac{1+\sin\theta}{\cos\theta}\right)^2 + \left(\frac{1-\sin\theta}{\cos\theta}\right)^2 = 2+4\tan^2\theta$ ---[4]

[S-13/21/Q1]

Proof. L.H.S $\left(\frac{1+\sin\theta}{\cos\theta}\right)^2 + \left(\frac{1-\sin\theta}{\cos\theta}\right)^2$

$$= \frac{(1+\sin\theta)^2}{\cos^2\theta} + \frac{(1-\sin\theta)^2}{\cos^2\theta}$$

$$= \frac{1+\sin^2\theta+\cancel{2\sin\theta}+1+\sin^2\theta-\cancel{2\sin\theta}}{\cos^2\theta}$$

$$= \frac{2+2\sin^2\theta}{\cos^2\theta} = \frac{2}{\cos^2\theta} + \frac{2\sin^2\theta}{\cos^2\theta}$$

$$= 2\sec^2\theta + 2\tan^2\theta$$

$$= 2(1+\tan^2\theta) + 2\tan^2\theta$$

$$= 2+2\tan^2\theta+2\tan^2\theta$$

$$= 2+4\tan^2\theta = R.H.S$$

Example 5. Prove that $\frac{\tan\theta+\cot\theta}{\sec\theta+\operatorname{cosec}\theta} = \frac{1}{\sin\theta+\cos\theta}$ ---[3]

Proof: L.H.S. $\frac{\tan\theta+\cot\theta}{\sec\theta+\operatorname{cosec}\theta} = \dfrac{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}}{\frac{1}{\cos\theta}+\frac{1}{\sin\theta}}$ [S-14/23/Q7(a)]

$$= \dfrac{\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}}{\frac{(\sin\theta+\cos\theta)}{\sin\theta\cdot\cos\theta}}$$

$[\because \sin^2\theta+\cos^2\theta=1]$

$$= \frac{1}{\cancel{\sin\theta\cos\theta}} \times \frac{\cancel{\sin\theta\cos\theta}}{(\sin\theta+\cos\theta)}$$

$$= \frac{1}{(\sin\theta+\cos\theta)} = R.H.S$$

Example 6. Show that: $\sqrt{\sec^2\theta-1}+\sqrt{\operatorname{cosec}^2\theta-1} = \sec\theta\cdot\operatorname{cosec}\theta$ ---[5]

[W-15/11/Q3]

Proof: L.HS $\sqrt{\sec^2\theta-1}+\sqrt{\operatorname{cosec}^2\theta-1}$

$$= \sqrt{\tan^2\theta}+\sqrt{\cot^2\theta}$$

$$= \tan\theta+\cot\theta$$

$$= \frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cdot\cos\theta}$$

$$= \frac{1}{\sin\theta\cdot\cos\theta} = \frac{1}{\sin\theta}\times\frac{1}{\cos\theta}$$

$$= \operatorname{cosec}\theta\cdot\sec\theta = R.H.S$$

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Trigonometry

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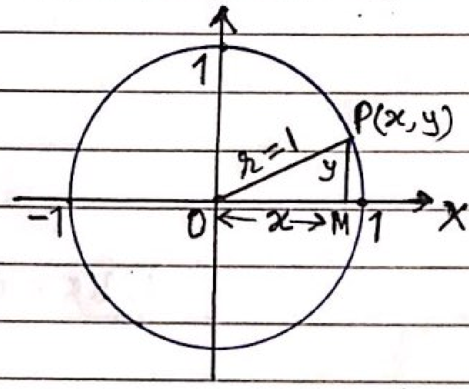
§ Trigonometric functions for any angle (Circular functions):

consider a unit circle ($r=1$)

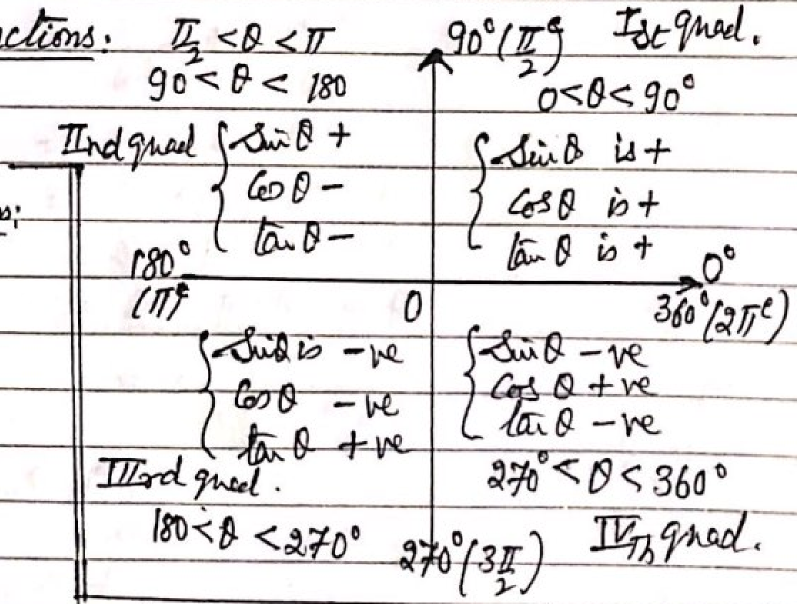
(i) $\sin\theta = \frac{PM}{OP} = \frac{y}{1} = y$

(ii) $\cos\theta = \frac{OM}{OP} = \frac{x}{1} = x$

(iii) $\tan\theta = \frac{PM}{OM} = \frac{y}{x}$



§ Signs of Trigonometric functions:



§ Limits of trigonometric functions:

(i) $-1 \leq \sin\theta \leq 1$

(ii) $-1 \leq \cos\theta \leq 1$

(iii) $-\infty < \tan\theta < \infty$

§

1. $\begin{cases} \sin(90-\theta) = \cos\theta \\ \cos(90-\theta) = \sin\theta \end{cases}$

2. $\begin{cases} \sin(90+\theta) = \cos\theta \\ \cos(90+\theta) = -\sin\theta \end{cases}$

✓3. $\begin{cases} \sin(180-\theta) = \sin\theta \\ \cos(180-\theta) = -\cos\theta \end{cases}$

✓4. $\begin{cases} \sin(180+\theta) = -\sin\theta \\ \cos(180+\theta) = -\cos\theta \end{cases}$

5. $\begin{cases} \sin(270-\theta) = -\cos\theta \\ \cos(270-\theta) = -\sin\theta \end{cases}$

6. $\begin{cases} \sin(270+\theta) = -\cos\theta \\ \cos(270+\theta) = \sin\theta \end{cases}$

✓7. $\begin{cases} \sin(360-\theta) = -\sin\theta \\ \cos(360-\theta) = \cos\theta \end{cases}$

✓8. $\begin{cases} \sin(-\theta) = -\sin\theta \\ \cos(-\theta) = \cos\theta \end{cases}$

✓9. $\begin{cases} \sin(n \times 360+\theta) = \sin\theta & n \in I \\ \cos(n \times 360+\theta) = \cos\theta & n \in I \end{cases}$

$\boxed{\left.\begin{matrix} 90 \pm \theta \\ \text{or } 270 \pm \theta \end{matrix}\right\} \text{Name change } \sin \leftrightarrow \cos}$

Note:
(i) $\sin x = \sin(180-x)$
(ii) $-\sin x = \sin(180+x) = \sin(360-x)$
(iii) $\cos x = \cos(360-x)$
(iv) $\cos(180-x) = \cos(180+x) = -\cos x$

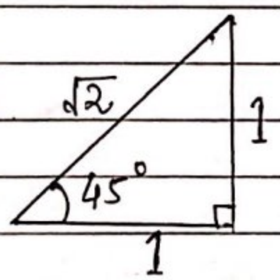
Trigonometry

§ Values of the trig. ratios some particular angles

Angle	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
	0°	30°	45°	60°	90°	180	270°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\sqrt{3}/2$	1	0	-1
$\cos\theta$	1	$\sqrt{3}/2$	$\frac{1}{\sqrt{2}}$	$1/2$	0	-1	0

Angle	0	$\pi/6$	$\pi/4$	$\frac{\pi}{3}$	$\frac{\pi}{2}^-$	$\frac{\pi}{2}^+$	π	$\frac{3}{2}\pi^-$	$\frac{3}{2}\pi^+$
	0°	30°	45°	60°	90^-	90^+	180	270^-	270^+
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$+\infty$	$-\infty$	0	$+\infty$	$-\infty$

§ For 45°

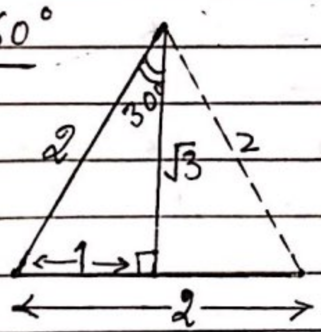


$\sin 45^\circ = \frac{1}{\sqrt{2}}$

$\cos 45^\circ = \frac{1}{\sqrt{2}}$

$\tan 45^\circ = 1$

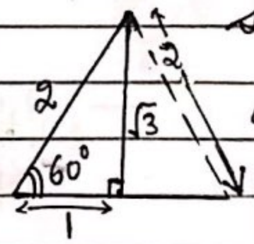
§ for 30° & 60°



$\sin 30^\circ = \frac{1}{2}$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\tan 30^\circ = \frac{1}{\sqrt{3}}$

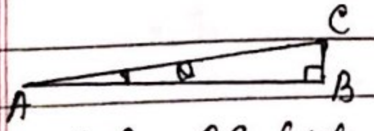


$\sin 60 = \frac{\sqrt{3}}{2}$

$\cos 60^\circ = \frac{1}{2}$

$\tan 60^\circ = \sqrt{3}$

§ for 0° & 90°



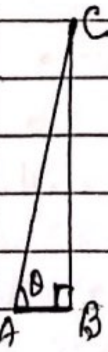
$\sin\theta = \frac{BC}{AC}$

$\therefore \sin 0^\circ = 0$

$\left\{ \begin{array}{l} \text{when } \theta \to 0 \\ BC \to 0 \\ AC \to AB \end{array} \right.$

$\cos\theta = \frac{AB}{AC} \Rightarrow \cos 0^\circ = 1$

and $\tan 0^\circ = 0$



when $\theta \to 90^\circ$

$AB \to 0$

$AC \to BC$

$\therefore \sin 90 = \frac{BC}{AC} = 1$

$\cos 90 = \frac{AB}{AC} = 0$

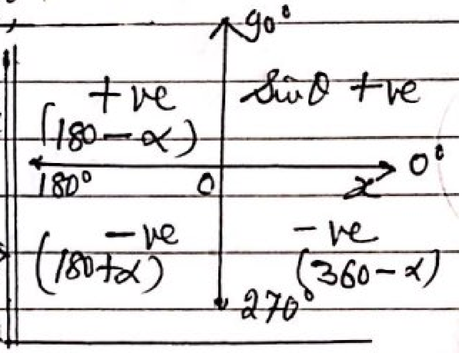
$\tan 90^- = \frac{BC}{AB} = +\infty$

Trigonometry

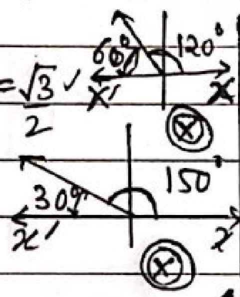
Example 7. Find the values of trigonometric functions, $\sin\theta$, $\cos\theta$ and $\tan\theta$ for the following angles.

(i) $120°$, $150°$, $180°$, $225°$, $240°$, $270°$, $300°$.

Solution:



$\sin 120° = \sin(180-60) = \sin 60 = \frac{\sqrt{3}}{2}$ ✓

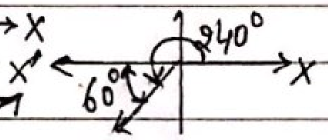


$\sin 150° = \sin(180-30) = \sin 30° = \frac{1}{2}$ ✓

$\sin 180° = \sin(180-0°) = \sin 0° = 0$

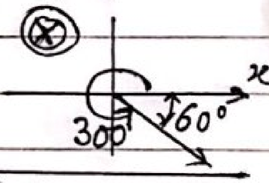
$\sin 225° = \sin(180+45°) = -\sin 45° = -\frac{1}{\sqrt{2}}$

$\sin 240° = \sin(180+60°) = -\sin 60 = -\frac{\sqrt{3}}{2}$



$\sin 270° = \sin(180+90°) = -\sin 90 = -1$

$\sin 300 = \sin(360-60) = -\sin 60 = -\frac{\sqrt{3}}{2}$



$\cos 120° = \cos(180-60) = -\cos 60 = -\frac{1}{2}$

$\cos 150° = \cos(180-30) = -\cos 30 = -\frac{\sqrt{3}}{2}$

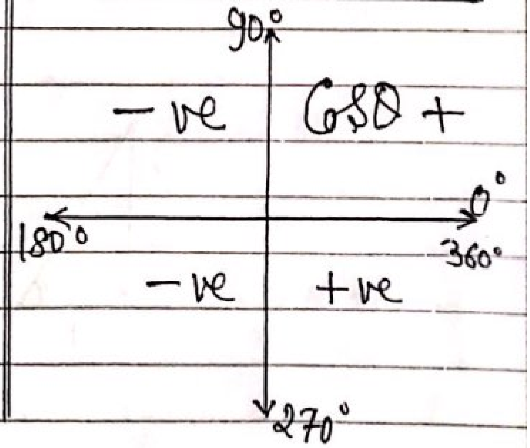
$\cos 180° = \cos(180-0°) = -\cos 0° = -1$

$\cos 225 = \cos(180+45) = -\cos 45 = -\frac{1}{\sqrt{2}}$

$\cos 240 = \cos(180+60) = -\cos 60 = -\frac{1}{2}$

$\cos 270 = \cos(180+90) = -\cos 90 = 0$

$\cos 300 = \cos(360-60) = \cos 60 = \frac{1}{2}$



$\tan 120° = \tan(180-60) = -\tan 60 = -\sqrt{3}$

$\tan 150° = \tan(180°-30°) = -\tan 30° = -\frac{1}{\sqrt{3}}$

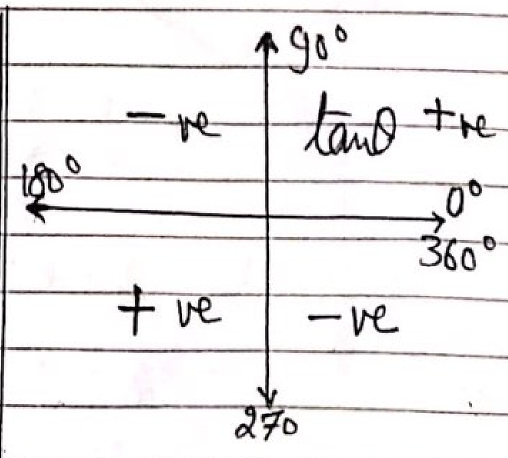
$\tan 180° = \tan(180-0) = -\tan 0° = 0$

$\tan 225° = \tan(180°+45°) = +\tan 45 = +1$

$\tan 240° = \tan(180°+60°) = +\tan 60 = +\sqrt{3}$

$\begin{cases} \tan 270^- = \tan(180°+90^-) = \tan 90° = +\infty \\ \tan 270^+ = \tan(180+90^+) = -\tan 90 = -\infty \end{cases}$

$\tan 300° = \tan(360-60) = -\tan 60 = -\sqrt{3}$



Note: Find that, what is the "acute" angle made by the given angle with the x-axis. Then find value of t-ratio for this, with proper sign.

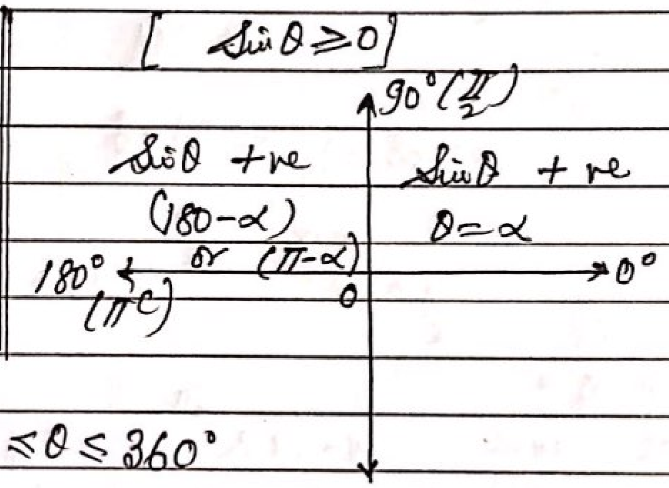
§ **Solution of trigonometric Equations:** $0 \leq \theta \leq 360$ (or $0 \leq \theta \leq 2\pi^c$)

$\boxed{\text{Case I}}$ $a \geq 0$

Solve $\sin\theta = a$
$= \sin\alpha$
α is an acute angle

Then
$\theta = \alpha, \ (180-\alpha)$ or $(\pi - \alpha)$

[$\sin\theta \geq 0$]



Example (8)(i) Solve: $\sin\theta = \frac{1}{2}$ $\quad 0 \leq \theta \leq 360^\circ$
$= \sin 30^\circ$

$\therefore$ Required values of $\theta = 30^\circ$ and $(180-30^\circ)$
$= 30^\circ$ or 150° ✓

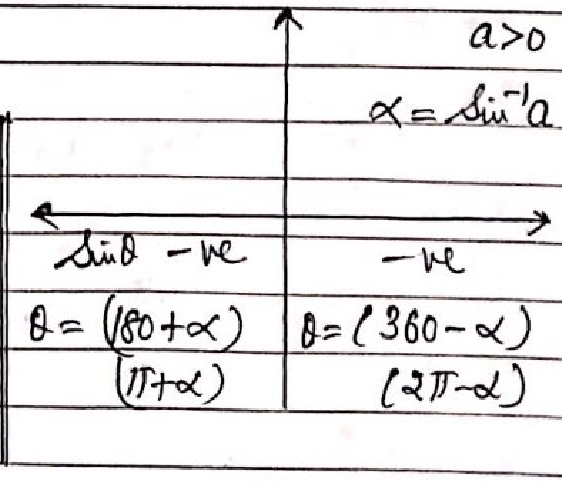
(ii) Solve: $\sin\theta = \frac{1}{\sqrt{2}}$, $\quad 0 \leq \theta \leq 2\pi$
$= \sin\frac{\pi}{4}$

$\therefore$ $\theta = \frac{\pi}{4} : (\pi - \frac{\pi}{4})$
or $\theta = \frac{\pi}{4}^c$ and $\frac{3\pi}{4}^c$ ✓

$\boxed{\text{Case II}}$

Solve $\sin\theta = -a$
$= -\sin\alpha$

$\theta = (180+\alpha)^\circ$ and $(360-\alpha)$
[or $(\pi+\alpha)$ and $(2\pi-\alpha)$].



Trigonometry

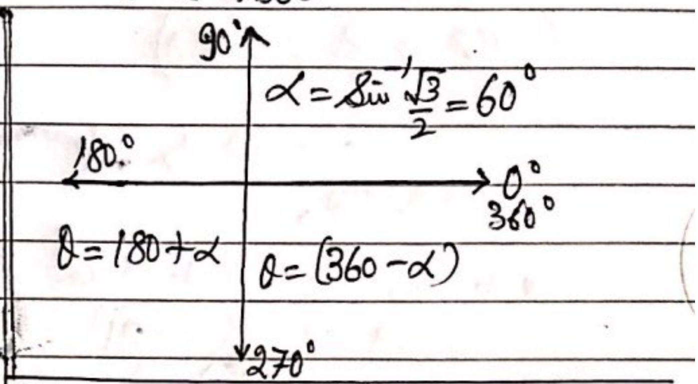
Solution of Trig Equ^n

Example 9 (i) Solve $\sin\theta = -\frac{\sqrt{3}}{2}$ $\quad 0 \le \theta \le 360$

$= -\sin 60^\circ$

$\therefore \theta = 180 + 60$ and $360 - 60$

or $\theta = 240^\circ$ and 300° ✓

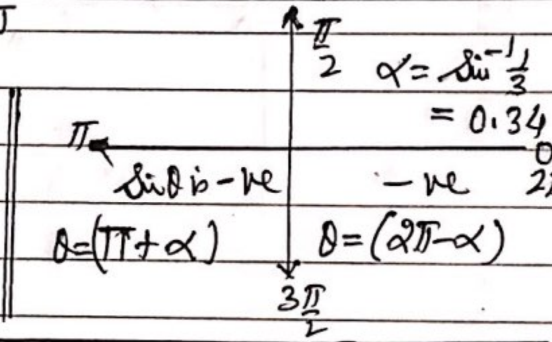


(ii) Solve $\sin\theta = -\frac{1}{3}$, $\quad 0 \le \theta \le 2\pi$

$= -\sin 0.34$

$\therefore \theta = \pi + 0.34$ and $2\pi - 0.34$

or $\theta = 3.48^c$ and 5.943^c ✓

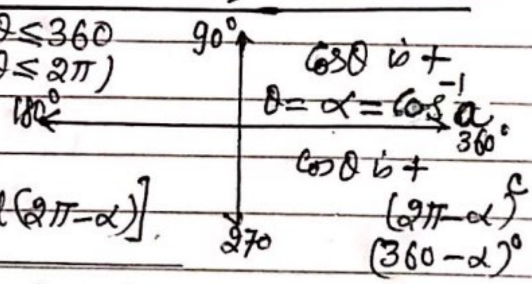


Case III $\quad a \ge 0 \quad$ $0 \le \theta \le 360$ (or $0 \le \theta \le 2\pi$)

Solve $\cos\theta = a$

$= \cos\alpha$

$\therefore \theta = \alpha^\circ$ or $(360 - \alpha)^\circ$ [or α and $(2\pi - \alpha)$]

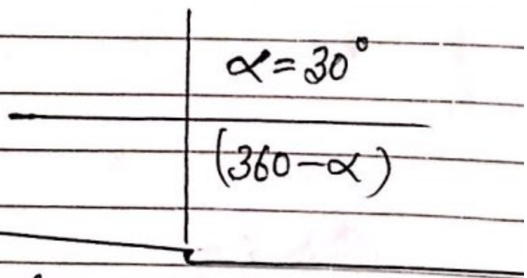


Example 10 (i) Solve $\cos\theta = \frac{\sqrt{3}}{2}$ $\quad 0 \le \theta \le 360$

$= \cos 30^\circ$

$\therefore \theta = 30^\circ$ and $(360 - 30)^\circ$

or $\theta = 30^\circ$ and 330° ✓

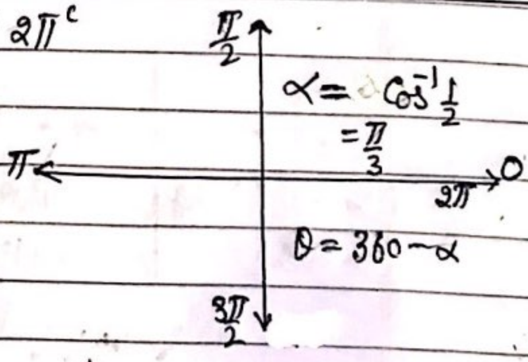


(ii) Solve $\cos\theta = \frac{1}{2}$; $\quad 0 \le \theta \le 2\pi^c$

$= \cos\frac{\pi}{3}$

$\theta = \frac{\pi}{3}$ and $2\pi - \frac{\pi}{3}$

or $\theta = \frac{\pi}{3}$ and $\frac{5\pi}{3}$ ✓



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Trigonometry

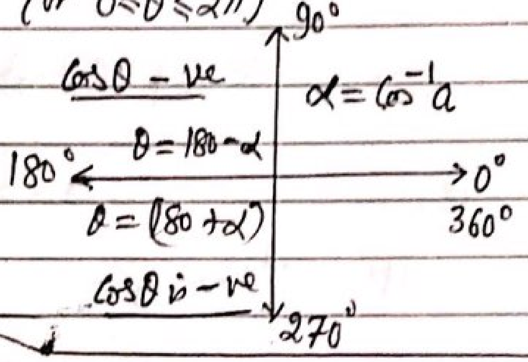
Solution of Trig. Equⁿ

Case IV Solve. $\cos\theta = -a$, $a>0$ $\quad 0 \le \theta \le 360^\circ$ (or $0 \le \theta \le 2\pi$)

$= -\cos\alpha$

$\theta = (180-\alpha)$ and $(180+\alpha)$

[or $\theta = (\pi-\alpha)^c$ and $(\pi+\alpha)^c$]



Example 11 (i) Solve.

$\cos\theta = -\frac{1}{\sqrt{2}}$ $\quad 0 \le \theta \le 360^\circ$

$= -\cos 45^\circ$

$\therefore \theta = (180-45)^\circ$ and $(180+45)^\circ$

or $\underline{\theta = 135^\circ \text{ and } 225^\circ}$

cos θ is – ve
$\theta = (180-\alpha)$
$\alpha = \cos^{-1}\frac{1}{\sqrt{2}} = 45^\circ$

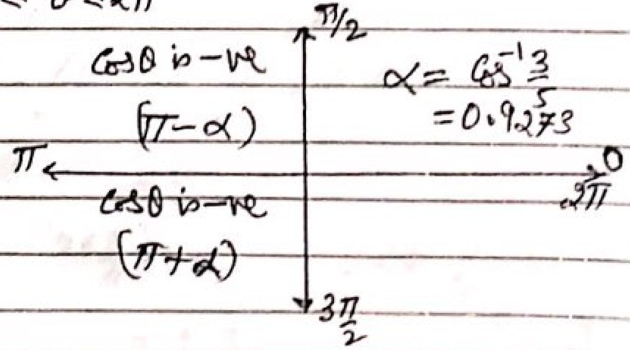
cos θ – ve
$\theta = (180+\alpha)$

(ii) Solve. $\cos\theta = -\frac{3}{5}$; $\quad 0 \le \theta \le 2\pi$

or $\cos\theta = -\cos 0.9273$

$\theta = (\pi - 0.9273)$ and $(\pi + 0.9273)$

or $\theta = 2.214^c$ and 4.069^c ✓

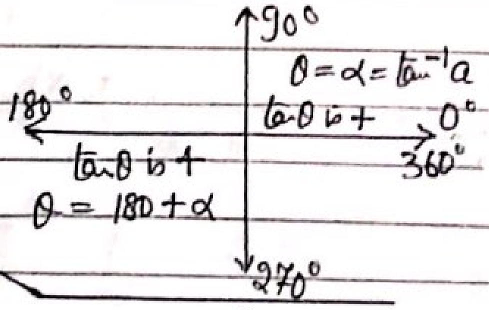


Case (V) Solve $\tan\theta = a$, $a > 0$ $\quad 0 \le \theta \le 360$ (or $0 \le \theta \le 2\pi$)

$= \tan\alpha$

$\theta = \alpha$ and $(180+\alpha)^\circ$

[or α and $\pi+\alpha$]

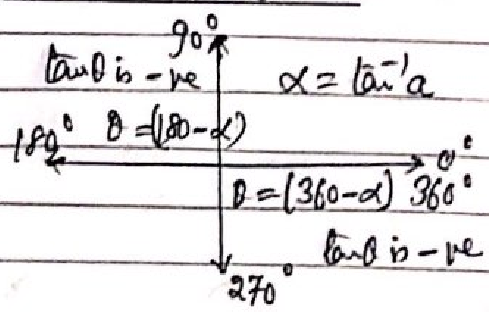


Case VI Solve $\tan\theta = -a$

$= -\tan\alpha$

$\theta = (180-\alpha)$, $(360-\alpha)^\circ$

[or $(\pi-\alpha)$ or $(2\pi-\alpha)$]



Example 12 (a) (i) Show that $\frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - \sin\theta} = \sec^2\theta$ ---[3]

(ii) Hence solve. $\frac{2\operatorname{cosec}\phi}{\operatorname{cosec}\phi - \sin\phi} = 8$ for $0 < \phi < 360°$ ---[3]

(b) Solve, $\sqrt{3}\tan\left(x+\frac{\pi}{4}\right) = 1$ for $0 < x < 2\pi$ ---[3]

Giving your answer in terms of π. $\boxed{\text{SP-20/02/Q11}}$

Solution (a)(i)

L.H.S. $\frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - \sin\theta}$

$= \frac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta} - \sin\theta}$

$= \frac{\frac{1}{\sin\theta}}{\frac{1-\sin^2\theta}{\sin\theta}}$

$= \frac{1}{\cancel{\sin\theta}} \times \frac{\cancel{\sin\theta}}{(1-\sin^2\theta)}$

$= \frac{1}{\cos^2\theta}$

$= \sec^2\theta = \text{R.H.S.}$ ✓

(ii) Solve: $\frac{2\operatorname{cosec}\phi}{\operatorname{cosec}\phi - \sin\phi} = 8$

divide both sides by 2

$\frac{\operatorname{cosec}\phi}{\operatorname{cosec}\phi - \sin\phi} = 4$

from Part (i) $\Rightarrow$ $\sec^2\phi = 4$

or $\cos^2\phi = \frac{1}{4}$

or $\cos\phi = \pm\frac{1}{2}$

$\cos\phi = \frac{1}{2}$ or $\cos\phi = -\frac{1}{2}$

Continued →

(a)(ii) Continued →

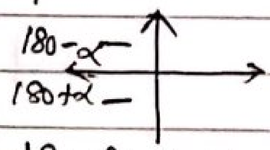
$\cos\phi = \frac{1}{2}$ and $\cos\phi = -\frac{1}{2}$

$= \cos 60°$

$\phi = 60$ and $360 - 60$

$= 60°$ and $300°$ ✓

$\cos\phi = -\cos 60°$



$\phi = 180 - 60, 180 + 60$

$= 120°, 240°$

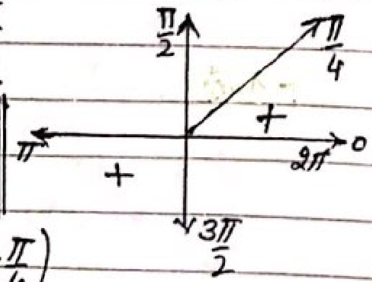
$\therefore \phi = 60°, 120°, 240°$ and $300°$ ✓

(b) Solve $\sqrt{3}\tan\left(x+\frac{\pi}{4}\right) = 1$ $\quad 0 < x < 2\pi$

or $\tan\left(x+\frac{\pi}{4}\right) = \frac{1}{\sqrt{3}}$ $\quad \frac{\pi}{4} < x+\frac{\pi}{4} < \frac{9\pi}{4}$

$= \tan\frac{\pi}{6}$

$\therefore x + \frac{\pi}{4} = \frac{\pi}{6}, \pi+\frac{\pi}{6}, 2\pi+\frac{\pi}{6}$



$x = \left(\frac{\pi}{6}-\frac{\pi}{4}\right), \left(\frac{7\pi}{6}-\frac{\pi}{4}\right), \left(\frac{13}{6}\pi-\frac{\pi}{4}\right)$

$x = -\frac{\pi}{12}$ ✗, $\frac{11\pi}{12}, \frac{23\pi}{12}$ $\quad \because 0 < x < 2\pi$

$\therefore x = \frac{11}{12}\pi$ and $\frac{23}{12}\pi$ ✓

Example 13(a)(i) Show that $\dfrac{(1-\sin A)(1+\sin A)}{\sin A \cdot \cos A} = \cot A$ ---[2]

(ii) Hence Solve, $\dfrac{(1-\sin 3x)(1+\sin 3x)}{\sin 3x \cdot \cos 3x} = \dfrac{1}{2}$ $\quad 0^\circ \leq x \leq 180^\circ$ ---[4]

(b) Solve $10\tan^2 y - \sec y - 1 = 0$ for $0 \leq y \leq 2\pi$ radians. ----[5]

[M-18/22/Q11]

Solution:
(a)(i)

L.H.S $\dfrac{(1-\sin A)(1+\sin A)}{\sin A \cdot \cos A}$

$= \dfrac{1-\sin^2 A}{\sin A \cdot \cos A}$

$= \dfrac{\cos^2 A}{\sin A \cdot \cos A}$

$= \dfrac{\cos A}{\sin A}$

$= \cot A = RHS$ ✓

(ii) Solve $\dfrac{(1-\sin 3x)(1+\sin 3x)}{\sin 3x \cdot \cos 3x} = \dfrac{1}{2}$

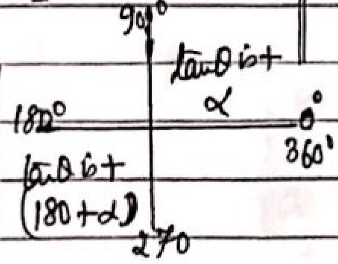
[Using Part(i), using $A = 3x$]

we get $\cot 3x = \frac{1}{2}$; $0 \leq x \leq 180^\circ$

or $\tan 3x = 2$, $0 \leq 3x \leq 540^\circ$

$\tan 3x = 2$
$= \tan 63.4^\circ$

Let $\tan\alpha = 2$
$\alpha = \tan^{-1} 2$
$= 63.4^\circ$



$\therefore 3x = 63.4,\ 180+63.4$ and $360+63.4$

or $3x = 63.4,\ 243.4,\ 423.4$

$x = 21.1^\circ,\ 81.1^\circ,\ 141.1^\circ$ ✓

(b) Solve.

$10\tan^2 y - \sec y - 1 = 0 \qquad 0 \leq y \leq 2\pi$

$10(\sec^2 y - 1) - \sec y - 1 = 0$

or $10\sec^2 y - \sec y - 11 = 0$

$(10\sec y - 11)(\sec y + 1) = 0$

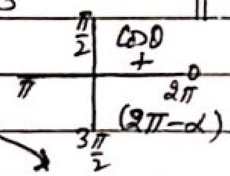
$\sec y = \frac{11}{10}$, $\sec y = -1$

or $\cos y = \frac{10}{11}$ or $\cos y = -1$

$\cos y = \cos 0.43$

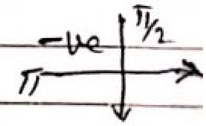
$\cos\alpha = \frac{10}{11}$
$\alpha = \cos^{-1}\frac{10}{11}$
$= 0.43$

$y = 0.43$ and $2\pi - 0.43$



$\cos y = -\cos 0$

$y = \pi - 0 = \pi$ ✓



$y = 0.43^c$ and 5.85^c, π

$\therefore$ $y = 0.43^\circ,\ \pi^c$ and 5.85^c ✓

IG-Maths 0606

Trigonometry

Example 14. Solve for $0 \leq x \leq 360°$, the equations,

(i) $\cot(2x - 10°) = 3/4$ --- [4]

(ii) $\sin^2 x - \cos^2 x = \cos x$. [M-17/22/Q10] --- [5]

Solution: (i) $\cot(2x-10) = \frac{3}{4}$

$0 \leq x \leq 360°$

$\Rightarrow 0 \leq 2x \leq 720$

$\Rightarrow -10° \leq 2x - 10 \leq 710°$ — (1)

or $\tan(2x-10) = \frac{4}{3}$

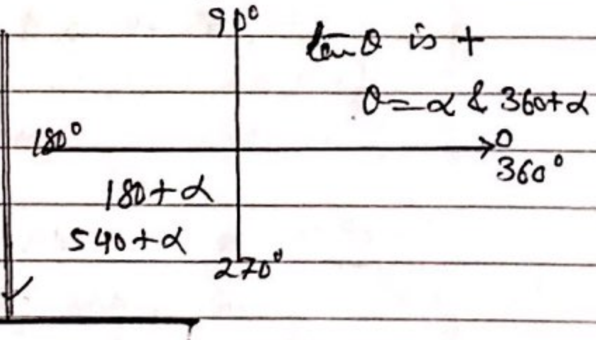
$= \tan\alpha \quad : \alpha = \tan^{-1}\frac{4}{3} = 53°$

$\therefore \tan(2x-10)° = \tan 53°$

$\therefore 2x - 10 = 53,\ 180+53°,\ 360+53°$ and $540+53°$

or $2x - 10 = 53°,\ 233°,\ 413°,\ 593°$

$\therefore x = 31.6°,\ 121.6°,\ 211.6°$ and $301.6°$ ✓



Quadratic trig. equations

(ii) Solve

$\sin^2 x - \cos^2 x = \cos x \qquad 0 \leq x \leq 360°$

or $(1 - \cos^2 x) - \cos^2 x = \cos x$

or $2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$\Rightarrow \cos x = \frac{1}{2}$ or $\cos x = -1$

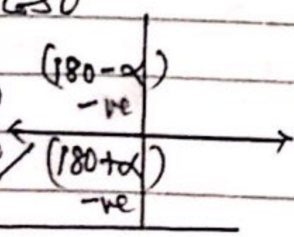
$= \cos 60°$ or $= -\cos 0°$

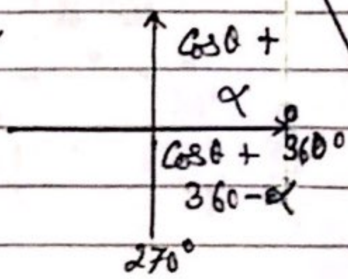
$x = 60, (360 - 60)$

$= 60°, 300°$ ✓

$\therefore x = 180 - 0$

$x = 180°$ ✓





$\therefore x = 60°, 180°$ and $300°$.

Example 15 (i) Prove that: $\sin x(\cot x+\tan x)=\sec x$ --- [4]

(ii) Hence solve the equation.

$|\sin x(\cot x+\tan x)|=2$ for $0\leqslant x\leqslant 360^\circ$ --- [4]

[S-17/21/Q11]

(i) L.H.S. $\sin x(\cot x+\tan x)$

$$=\sin x\left(\frac{\cos x}{\sin x}+\frac{\sin x}{\cos x}\right)$$

$$=\sin x\left(\frac{\cos^2 x+\sin^2 x}{\sin x\cdot\cos x}\right)$$

$$=\cancel{\sin x}\times\frac{1}{\cancel{\sin x}\times\cos x}$$

$$=\sec x = \text{R.H.S. Proved.}$$

(ii) Solve. $|\sin x(\cot x+\tan x)|=2$ for $0^\circ\leqslant x\leqslant 360^\circ$

or $|\sec x|=2$ [for Part(i)]

$\Rightarrow \sec x=\pm 2$

or $\cos x=\frac{1}{2}$ or $\cos x=-\frac{1}{2}$

$=\cos 60^\circ$ | $=-\cos 60$

$\cos\theta$ is + | $\cos\theta$ is -ve

α | $\theta=(180-\alpha)$

$(360-\alpha)$ | -ve

$\cos\theta$ is + | $\theta=(180+\alpha)$

$\therefore x=60^\circ, 360-60$ | $\therefore x=180-60^\circ$ and $180+60$

$=60^\circ, 300^\circ$ | or $x=120; 240^\circ$

$$\therefore \underline{x=60^\circ, 120^\circ, 240^\circ, 300^\circ}$$

Example 16 (a) Solve $4\sin x = \operatorname{cosec} x$ for $0^\circ \le x \le 360^\circ$ ---[3]

(b) Solve $\tan^2 3y - 2\sec 3y - 2 = 0$ for $0 \le y \le 180^\circ$ ---[6]

(c) Solve $\tan\left(z - \frac{\pi}{3}\right) = \sqrt{3}$ for $0 \le z \le 2\pi$ radians ---[3]

S-15 / 11 / Q 10

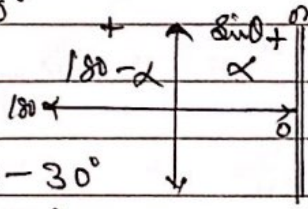
Solution (a) Solve $4\sin x = \operatorname{cosec} x$ for $0^\circ \le x \le 360^\circ$

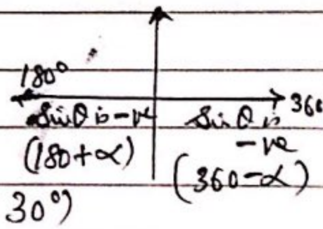
or $4\sin x = \frac{1}{\sin x}$

or $\sin^2 x = \frac{1}{4} \Rightarrow \sin x = \pm\frac{1}{2}$

$\sin x = \frac{1}{2}$ or $\sin x = -\frac{1}{2}$

$= \sin 30^\circ$ or $= -\sin 30^\circ$





$x = 30^\circ$ or $180 - 30^\circ$

$= 30^\circ$ or 150°

$\therefore x = (180+30)$ or $(360 - 30^\circ)$

$= 210^\circ, 330^\circ$

$\therefore x = 30^\circ, 150^\circ, 210^\circ$ and 330° ✓

(b) Solve $\tan^2 3y - 2\sec 3y - 2 = 0$

or $(\sec^2 3y - 1) - 2\sec 3y - 2 = 0$

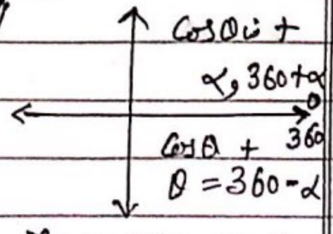
or $\sec^2 3y - 2\sec y - 3 = 0$ $\quad 0 \le y \le 180^\circ$

or $(\sec 3y + 1)(\sec 3y - 3) = 0$; $0^\circ \le 3y \le 540^\circ$

$\cos 3y = -1$ or $\cos 3y = \frac{1}{3}$

$3y = 180^\circ, 540^\circ$ or $\cos 3y = \cos 70.5^\circ$

$y = 60^\circ, 180^\circ$ ✓



$\therefore 3y = 70.5, 360 - 70.5, 360 + 70.5$

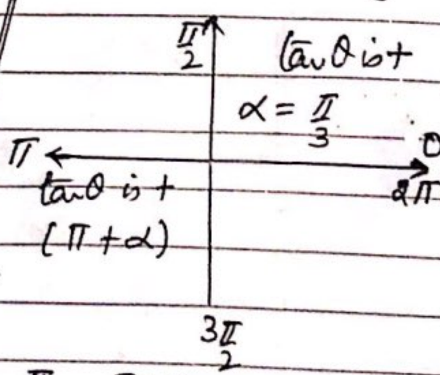
$3y = 70.5, 289.5, 430.5$

$\therefore y = 23.5^\circ, 96.5^\circ, 143.5$

$\therefore x = 60^\circ, 180^\circ, 23.5^\circ, 96.5^\circ$ and 143.5° ✓

(c) Solve $\tan\left(z - \frac{\pi}{3}\right) = \sqrt{3}$, $0 \le z \le 2\pi$

$\therefore \tan\left(z - \frac{\pi}{3}\right) = \tan\frac{\pi}{3}$; $-\frac{\pi}{3} \le z - \frac{\pi}{3} \le \frac{5\pi}{3}$



$\therefore z - \frac{\pi}{3} = \frac{\pi}{3}, \pi + \frac{\pi}{3}$

or $z = \frac{2\pi}{3}, \frac{5\pi}{3}$ ✓

Example 17. (a) Solve the equation $2\operatorname{cosec} x + \frac{7}{\cot x} = 0$ for $0 \leq x \leq 360°$ --- [4]

(b) Solve the equation $7\sin(2y-1) = 5$ for $0 \leq y \leq 5$ radians --- [5]

W-13/21 Q12

Solution (a) Solve $2\operatorname{cosec} x + \frac{7}{\cot x} = 0$ for $0° \leq x \leq 360°$

or $\frac{2}{\sin x} = -\frac{7}{\cot x}$

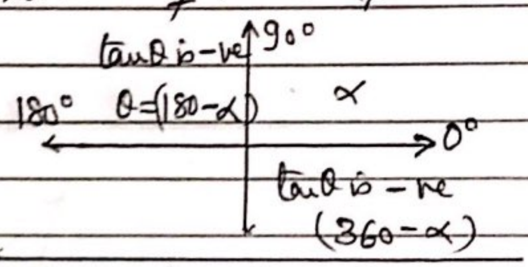
or $\tan x = -\frac{2}{7}$

$= -\tan 15.9°$

Let $\tan\alpha = \frac{2}{7} \Rightarrow \alpha = \tan^{-1}\frac{2}{7} = 15.9°$

$\therefore x = 180 - 15.9$ and $360 - 15.9$

or $x = 164.1°$ and $344.1°$ ✓



(b) Solve, $7\sin(2y-1) = 5$ for $0 \leq y \leq 5$ radian

$0 \leq 2y \leq 10$

$-1 \leq (2y-1) \leq 9$ rad.

or $\sin(2y-1) = \frac{5}{7}$

$= \sin 0.79$

$\alpha = \sin^{-1}\frac{5}{7} = .795$

$\therefore 2y-1 = \alpha,\ \pi-\alpha,\ 2\pi+\alpha$

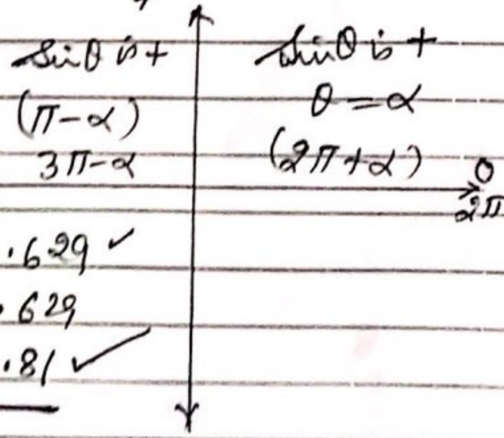
$2y-1 = 0.795, \pi-0.795$ and $(3\pi-0.795)$

$2\pi+0.795$

or $2y-1 = 0.795;\ 2.346,\ 7.078,\ 8.629$ ✓

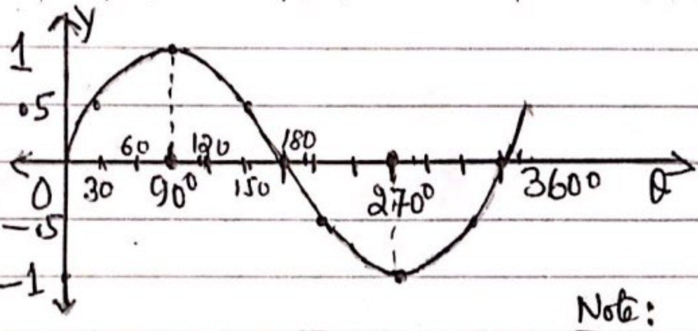
$2y = 1.795,\ 3.346,\ 8.078,\ 9.629$

$y = 0.898;\ 1.67,\ 4.04,\ 4.81$ ✓



§ Graph of $y = \sin\theta$: $0^\circ \le \theta \le 360^\circ$

	θ					$\sin(180-\theta) = \sin\theta$				$\sin(180+\theta) = -\sin\theta$				$\sin(360-\theta) = -\sin\theta$			
θ	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt2}$	$\frac{\sqrt3}{2}$	1	$\frac{\sqrt3}{2}$	$\frac{1}{\sqrt2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt2}$	$-\frac{\sqrt3}{2}$	-1	$-\frac{\sqrt3}{2}$	$-\frac{1}{\sqrt2}$	$-\frac{1}{2}$	0
	0	.5	.7	.86	1	.86	.7	.5	0	$-.5$	$-.7$	$-.86$	-1	$-.86$	$-.7$	$-.5$	0



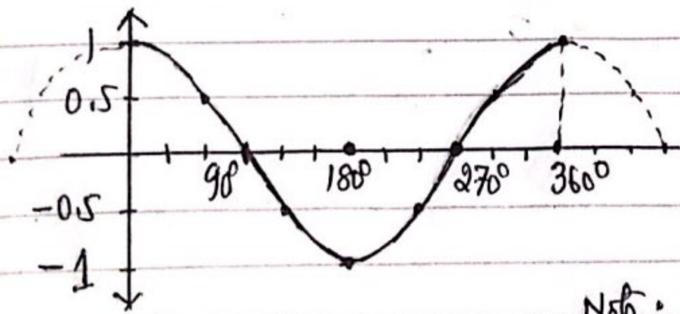
The graph of $\sin\theta$
(i) Period $= 360^\circ$
(ii) Amplitude $= 1$

$-1 \le \sin\theta \le 1$

Note: $y = a\sin b\theta$
(i) Period $= \frac{360^\circ}{b}$
(ii) Amplitude $= a$.

(ii) Graph of $y = \cos\theta$: $0^\circ \le \theta \le 360^\circ$

	θ					$\cos(180-\theta) = -\cos\theta$				$\cos(180+\theta) = -\cos\theta$				$\cos(360-\theta) = \cos\theta$			
θ	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
$\cos\theta$	1	$\frac{\sqrt3}{2}$	$\frac{1}{\sqrt2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt2}$	$-\frac{\sqrt3}{2}$	-1	$-\frac{\sqrt3}{2}$	$-\frac{1}{\sqrt2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{\sqrt2}$	$\frac{\sqrt3}{2}$	1
	1	.86	.7	.5	0	$-.5$	$-.7$	$-.86$	-1	$-.86$	$-.7$	$-.5$	0	.5	.7	.86	1



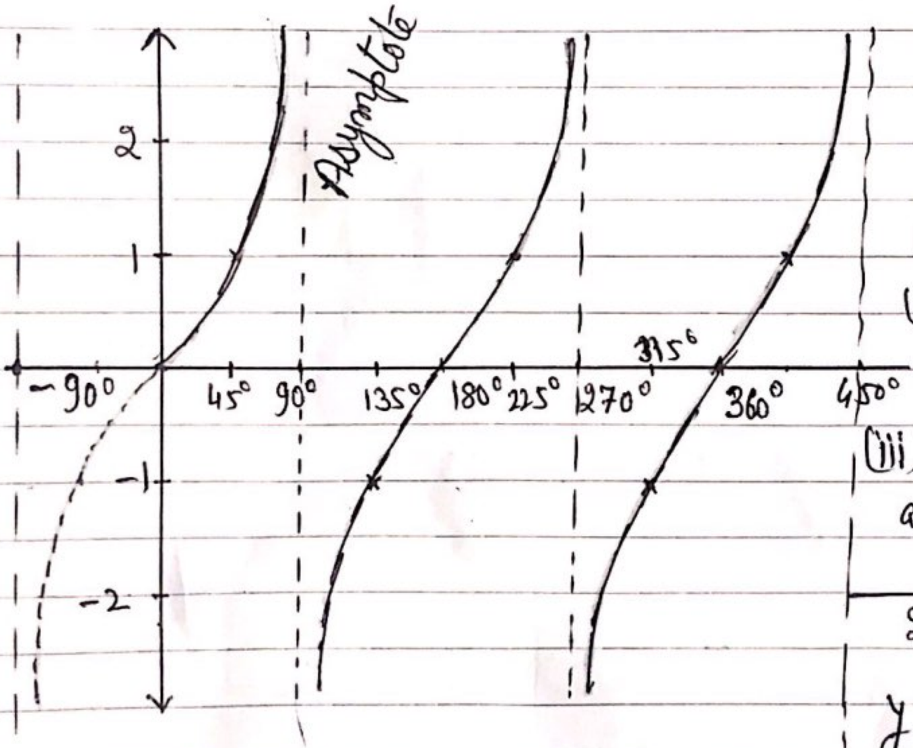
The graph of $y = \cos\theta$
(i) Period $= 360^\circ$
(ii) Amplitude $= 1$

$-1 \le \cos\theta \le 1$

Note: $y = a\cos b\theta$
(i) Period $= \frac{360^\circ}{b}$
(ii) Amplitude $= a$

§ Graph of $y = \tan\theta$ $\quad 0^\circ \leq \theta \leq 360^\circ$

	θ					$\tan(180-\theta) = -\tan\theta$					$\tan(180+\theta) = \tan\theta$				$\tan(360-\theta) = -\tan\theta$				
θ°	0°	30°	45°	60°	90^-	90^+	120°	135°	150°	180°	210°	225°	240°	270^-	270^+	300°	315	330	360°
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\infty$	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞^+	∞^-	$\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0
	0	$.57$	1	1.7	∞	$-\infty$	-1.7	-1	$.57$	0	$.57$	1	1.7	∞^+	∞^-	$-\sqrt{3}$	-1	$-.57$	0



The graph of $y = \tan\theta$

(i) Period $= 180^\circ$

(ii) $-\infty < \tan\theta < \infty$

(iii) Asymptotes: at 90° and 270°

If $y = \tan b\theta$

Period $= \frac{180^\circ}{b}$

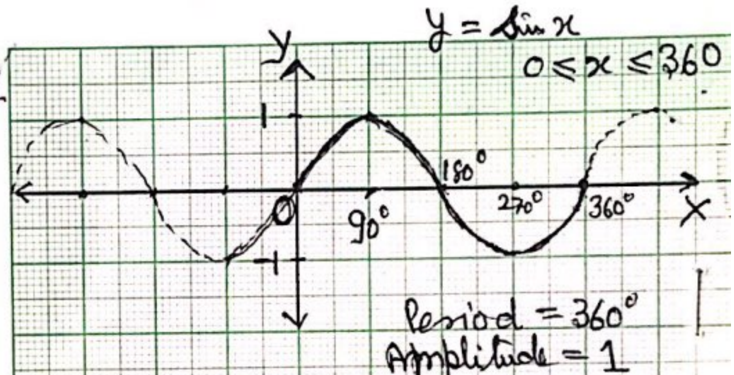
Trigonometry

§ Transforming trig. functions / Amplitude and Period of trig. fn.

1. $y = \sin x, \quad 0 \leq x \leq 360°$

Period $= 360°$

Amplitude $= 1$

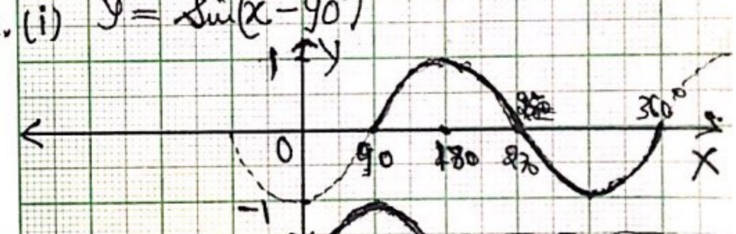


Translation:

2.(i) $y = \sin(x-h)$; $\begin{pmatrix} h \\ 0 \end{pmatrix}$

is obtained by Translation of $y = \sin x$ by h units along x-axis on Right

Example $y = \sin(x - 90°)$ is obtained by Translation of $y = \sin x$ by $90°$ on right

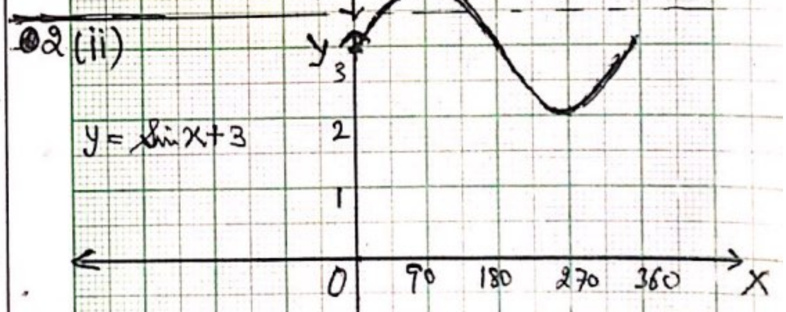


2 (ii) $y = \sin x + k$; $\begin{pmatrix} 0 \\ k \end{pmatrix}$

is obtained by Translation of $y = \sin x$ by k unit along y-axis.

Example $y = \sin x + 3$

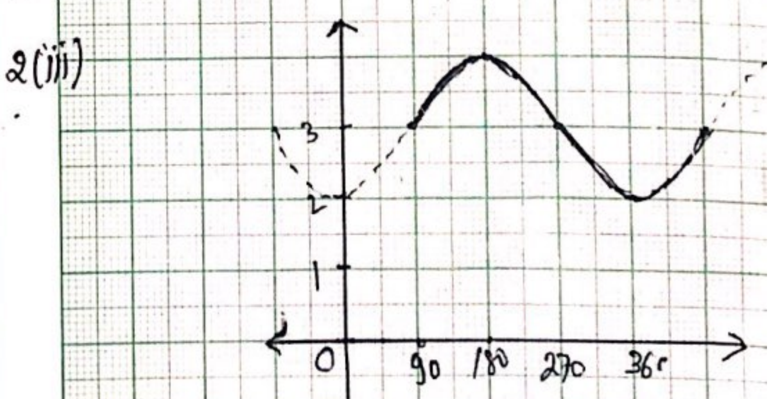
is obtained by translation of $y = \sin x$ by 3 units along y-axis



2 (iii) $y = \sin(x-h) + k$; $\begin{pmatrix} h \\ k \end{pmatrix}$

Example $y = \sin(x-90) + 3$ is obtained by Translation of $90°$ along + x-axis and 3 units along y-axis.

Translation $\begin{pmatrix} 90 \\ 3 \end{pmatrix}$



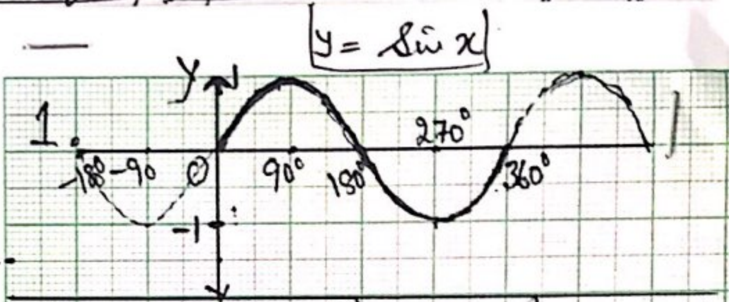
$y = \sin(x-90) + 3$

§.

1 $y = \sin x$, $0 \leq x \leq 360°$

Period $= 360°$

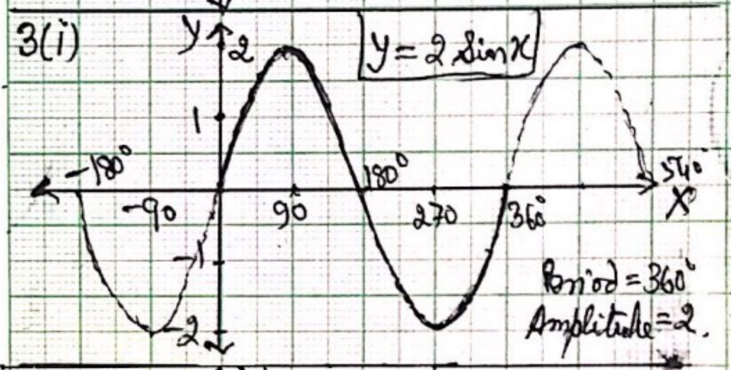
Amplitude $= 1$



3(i) <u>Stretch along Y-axis:</u>

$y = a \sin x$. Amplitude $= a$

<u>Example:</u> $y = 2\sin x$ is obtained with a stretch factor of 2 applied to $y = \sin x$ along Y-axis. $\boxed{\text{Amplitude} = 2}$



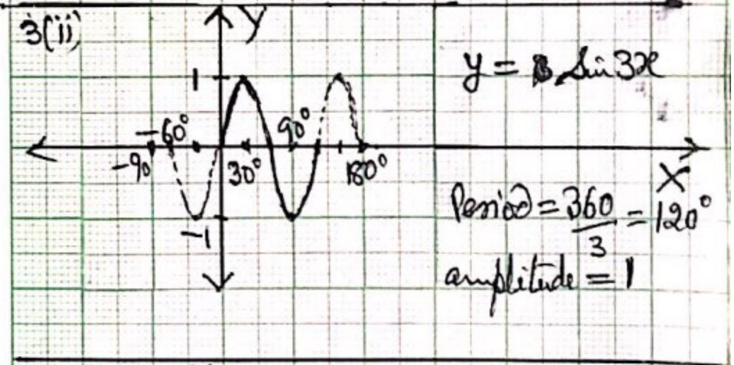
(ii) <u>Stretch along X-axis:</u>

amplitude $= 1$

$y = \sin bx$; Period $= \dfrac{360°}{b}$.

<u>Example:</u> $y = \sin 3x$, is obtained by a stretch of scale factor of $\frac{1}{3}$ along X-axis to $y = \sin x$.

Period $= \dfrac{360°}{3} = 120°$



3 (iii) $y = a\sin bx$

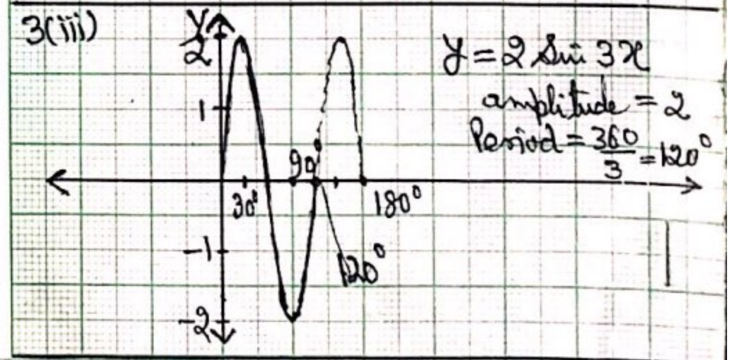
amplitude $= a$

Period $= \dfrac{360}{b}$

<u>Example:</u> $y = 2\sin 3x$

amplitude $= 2$

Period $= \dfrac{360}{3} = 120°$



4. $y = a\sin(bx - h) + k$

<u>Example</u> $y = 2\sin\left(\dfrac{x}{2} + 90°\right) + 3$

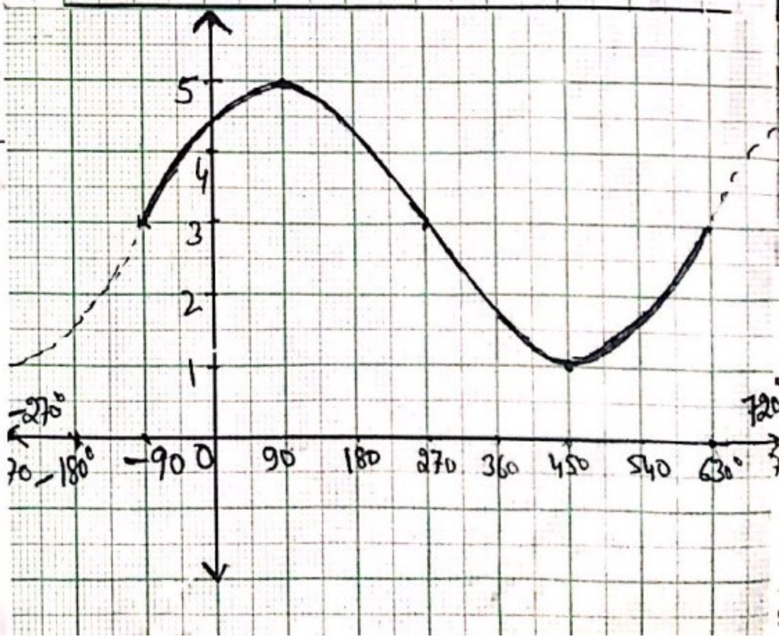
or

$y = 2\sin\left[\dfrac{x}{2} - (-90)\right] + 3$ amplitude $= 2$

Stretch along X-axis of Scale factor 2 Period $= \dfrac{360}{1/2} = 720°$

Translation along X-axis $= -90°$

Translation along Y-axis $= 3$ Units.



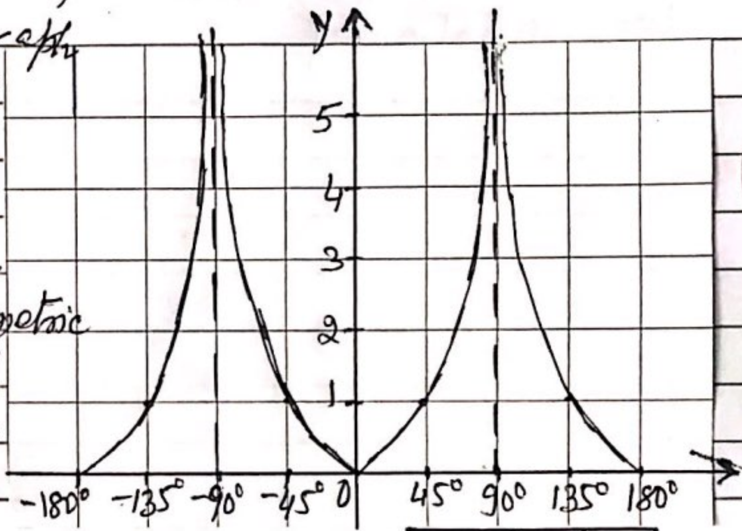
Example 18. (a)(i) State the amplitude of $15\sin 2x - 5$. ---[1]

(ii) State the period of $15\sin 2x - 5$. ---[1]

(b) The diagram shows the graph of $y = |f(x)|$ for $-180° \leq x \leq 180°$ where $f(x)$ is a trigonometric function.

(i) Write down two possible expressions for the trigonometric function $f(x)$. ---[2]

(ii) State the number of solutions of the equation $|f(x)| = 1$ for $-180° \leq x \leq 180°$ ---[1]



M-18/22/Q4

Solution (a)(i) amplitude of $15\sin 2x - 1 = 15$ ✓

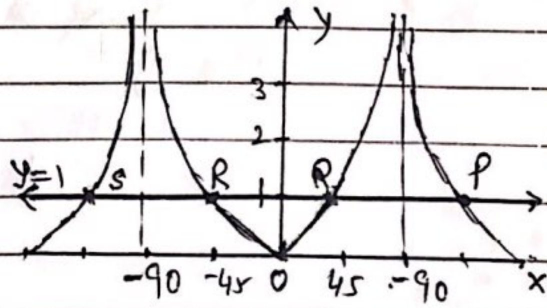
(ii) Period of $15\sin 2x - 1 = \frac{360}{2} = 180°$ ✓ or (π^c)

For $y = a\sin bx + c$
amplitude $= a$
Period $= \frac{360°}{b}$ or $\frac{2\pi^c}{b}$

(b) (i) $f(x) = \tan x$ or $-\tan x$

(ii) 4

$|f(x)| = 1$ at four points P, Q, R and S.



Example 19 Given that $y = 3 + 4\cos 9x$, write down

(i) the amplitude of y. ---[1]

(ii) the period of y. ---[1]

S-17/13/Q2

Solution.

(i) amplitude of $y = 3 + 4\cos 9x$ is 4 ✓

(ii) Period of $y = 3 + 4\cos 9x$ is $\frac{360}{9} = 40°$ ✓ or $\frac{2\pi^c}{9}$

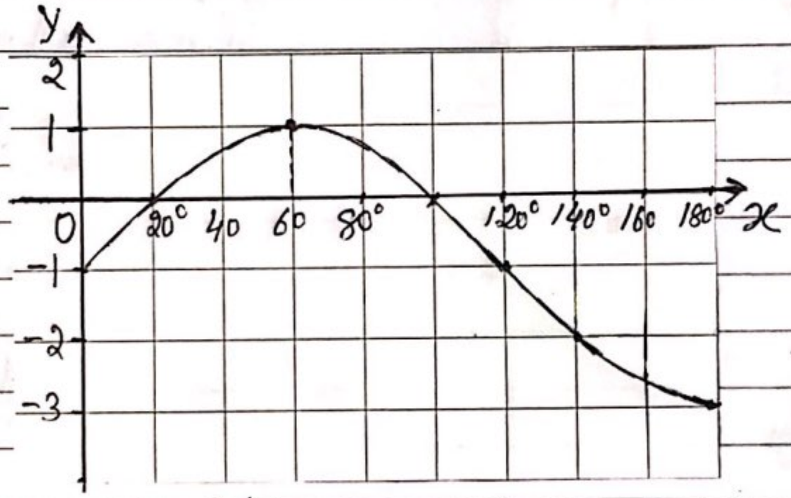
$\because$ for $y = a\cos bx + c$
amplitude $= a$
Period $= \frac{360°}{b}$ or $\frac{2\pi^c}{b}$

Example 20. Sketch the graph of $y = 2\sin\frac{3}{2}x - 1$ for $0 \leq x \leq 180°$, showing the coordinates of the points where the graph meets the coordinate axes. [SP-20/02/Q4] --[4]

Solution.

$y = 2\sin\frac{3x}{2} - 1$

$0 \leq x \leq 180°$



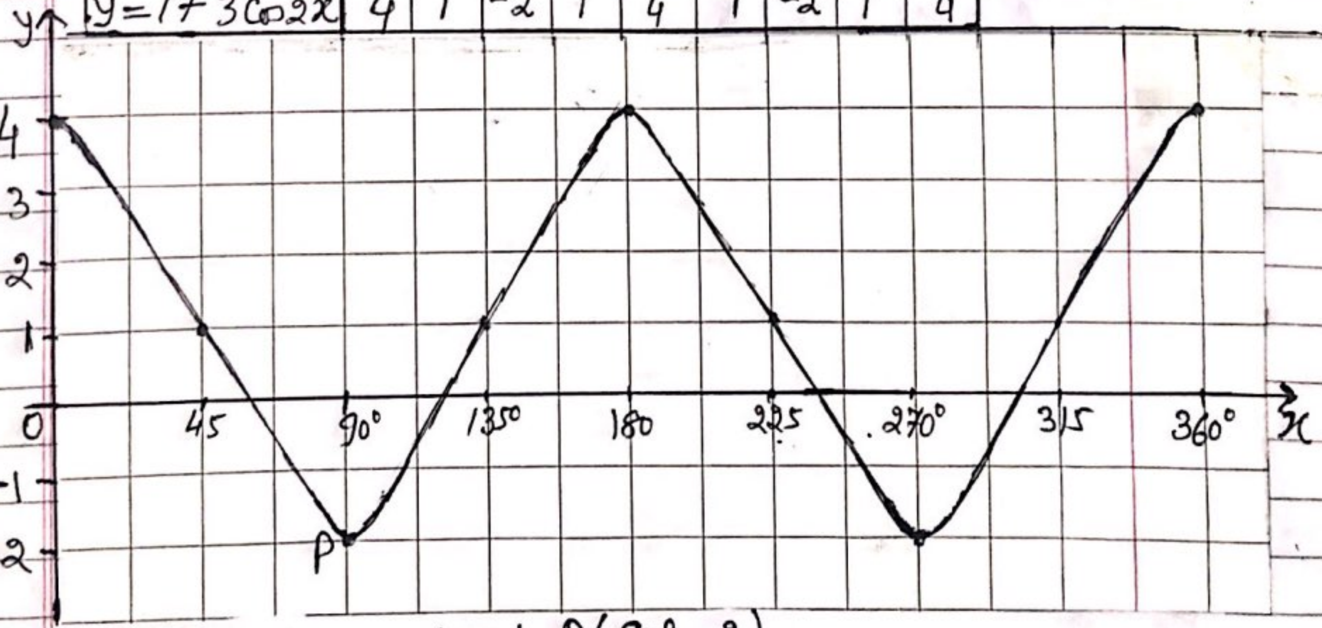
x	0°	20°	60°	100°	120°	140°	180°
y	−1	0	1	0	−1	−2	−3

Example 21. (i) On the axes sketch, for $0° \leq x \leq 360°$, the graph of $y = 1 + 3\cos 2x$. --[3]

(ii) Write down the coordinates of the points, where this graph first has a minimum value. [M-17/12/Q2] --[1]

Solution:

x	0	45°	90	135°	180	225	270	315	360°
$y = 1 + 3\cos 2x$	4	1	−2	1	4	1	−2	1	4



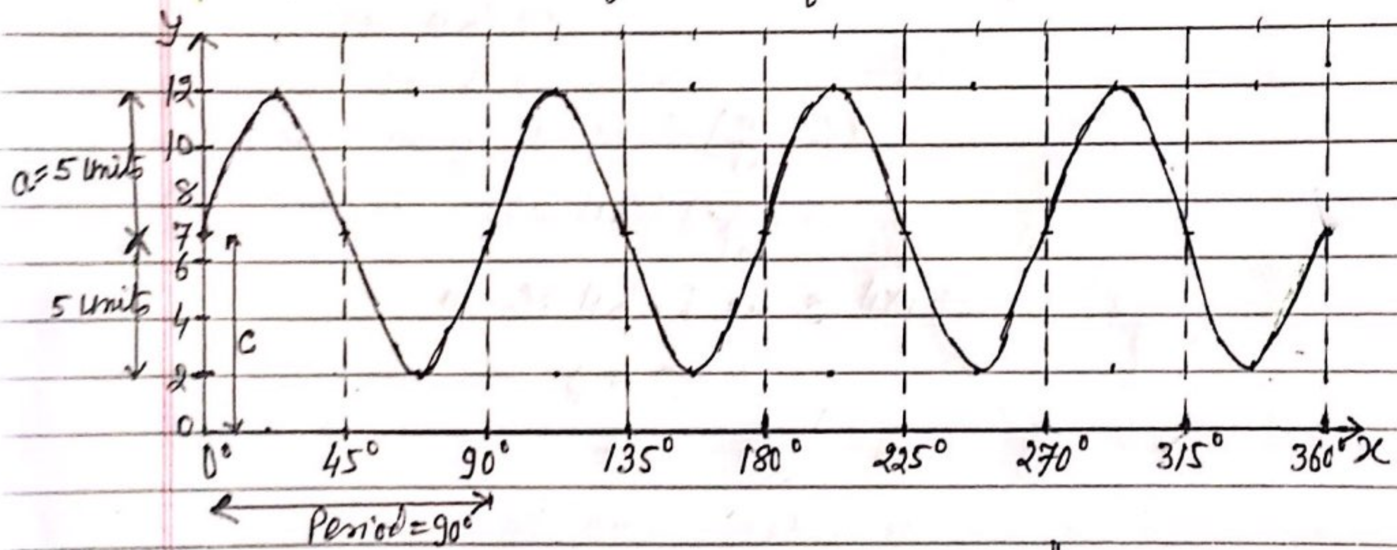
(ii) First minimum point $P(90°, -2)$

Example 22 (a) Given that $y = 7\cos 10x - 3$, where angle x is measured in degrees, state,

(i) the period of y ---[1]

(ii) the amplitude of y. ---[1]

(b) Find the equation of the curve shown, in the form,
$y = a \cdot g(bx) + c$ where $g(x)$ is trigonometric function and a, b, and c are integers to be found. $\boxed{S\text{-}17/12/Q6}$ ---[4]



$y = 7\cos 10x - 3$

Solution: (a) (i) Period of $y = \dfrac{360°}{10} = \underline{36°}$

(ii) the amplitude of $y = 7$

$y = a\cos bx + c$

(i) Period $= \dfrac{360}{b}$

(ii) Amplitude $= a$

(b) $y = a \cdot g(bx) + c$ here $g(x) = \sin x$.

amplitude $a = 5$ ✓

Period $\dfrac{360}{b} = 90° \Rightarrow b = 4$ ✓

Vertical Translation $c = 7$ ✓

$\therefore y = (5\sin 4x + 7)$ ✓

Example 23. The graph of $y = a\sin(bx) + c$ has an amplitude of 4, a period of $\frac{\pi}{3}$ and passes through the point $(\frac{\pi}{12}, 2)$. Find the value of each of the constants a, b and c. $\boxed{W\text{-}17/12/Q2}$ ---[4]

Solution: amplitude $\underline{a = 4}$ ✓

Period $= \frac{2\pi}{b} = \frac{\pi}{3}$ (given) $\Rightarrow \underline{b = 6}$ ✓

$\therefore\ y = a\sin bx + c$

or $y = 4\sin 6x + c$ ——— (1)

It passes through the point $(\frac{\pi}{12}, 2)$

fr (1) $\quad 2 = 4\sin 6 \times \frac{\pi}{12} + c$

or $2 = 4\sin\frac{\pi}{2} + c = 4\times 1 + c$

$\Rightarrow \underline{c = -2}$ ✓

$$\begin{cases} a = 4 \\ b = 6 \\ c = -2 \end{cases}$$

Example 24. $f(x) = a\cos bx + c$, has period $60°$, an amplitude of 10, and is such that $f(0) = 14$, state the values of a, b and c. $\boxed{M\text{-}16/22/Q4}$ ---[2]

Solution: $f(x) = a\cos bx + c$ ——— (1)

Given amplitude $\underline{a = 10}$ ✓

Given period $\frac{360°}{b} = 60° \Rightarrow \underline{b = 6}$ ✓

fr (1) $\quad f(x) = 10\cos 6x + c$ ——— (2)

Given $f(0) = 14$

fr (1) $\quad f(0) = 10\cos 0 + c = 14$

$10 + c = 14$

$\underline{c = 4}$ ✓

$\underline{a = 10,\ b = 6 \text{ and } c = 4}$

Example 25. Given that $y = a\tan bx + c$ has period $\frac{\pi}{4}$ radians and passes through the points $(0,-2)$ and $(\frac{\pi}{16}, 0)$. Find the values of each of the constants a, b and c. [S-16/21/Q9(a)] --[3]

Solution: $y = a\tan bx + c$ —①

Period $\frac{\pi}{b} = \frac{\pi}{4} \Rightarrow \underline{b = 4}$ ✓

fm ① $y = a\tan 4x + c$ —②

Passes through $(0,-2)$

$\Rightarrow -2 = +c \Rightarrow \underline{c = -2}$ ✓

fm ② $y = a\tan 4x - 2$ —③

Passes through $(\frac{\pi}{16}, 0)$

fm ③ $0 = a\tan\frac{\pi}{4} - 2$

or $a - 2 = 0 \Rightarrow \underline{a = 2}$ ✓

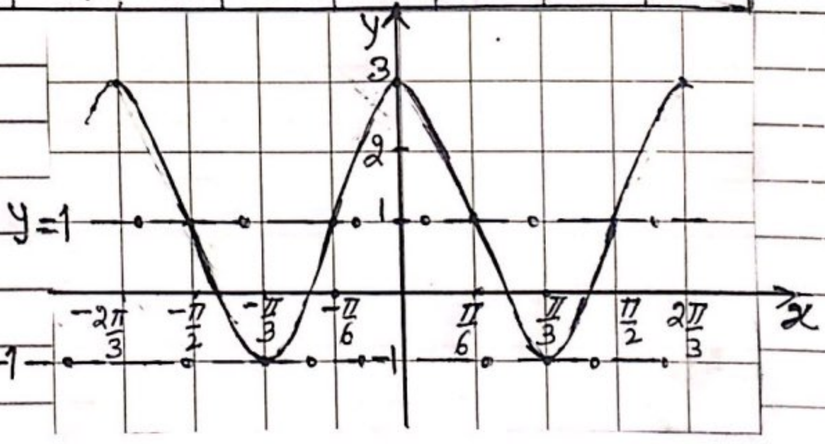
$\therefore\ a = 2,\ b = 4$ and $c = -2$

Example 26 (i) On the axes draw the graph of $y = 2\cos 3x + 1$ for $-\frac{2\pi}{3} \le x \le \frac{2\pi}{3}$. ...[3]

(ii) Using your graph or otherwise, find the exact solution of $(2\cos 3x + 1)^2 = 1$ for $-\frac{2\pi}{3} \le x \le \frac{2\pi}{3}$ [S-16/21/Q 9(b)] ---[2]

Solution (i) $y = 2\cos 3x + 1$ for $-\frac{2\pi}{3} \le x \le \frac{2\pi}{3}$

x	$-\frac{2\pi}{3}$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$
y	3	1	−1	1	3	1	−1	1	3



(ii) $(2\cos 3x + 1)^2 = 1 \Rightarrow 2\cos 3x + 1 = \pm 1$

or $y = 1$ or $y = -1$

$\therefore$ Solutions $x = -\frac{\pi}{2}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{2}$ or $-\frac{\pi}{3}, \frac{\pi}{3}$

or $x = -\frac{\pi}{2}, -\frac{\pi}{3}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$ ✓

Example 27. It is given that $y = 1 + \tan 3x$

(i) State the period of y. --- [1]

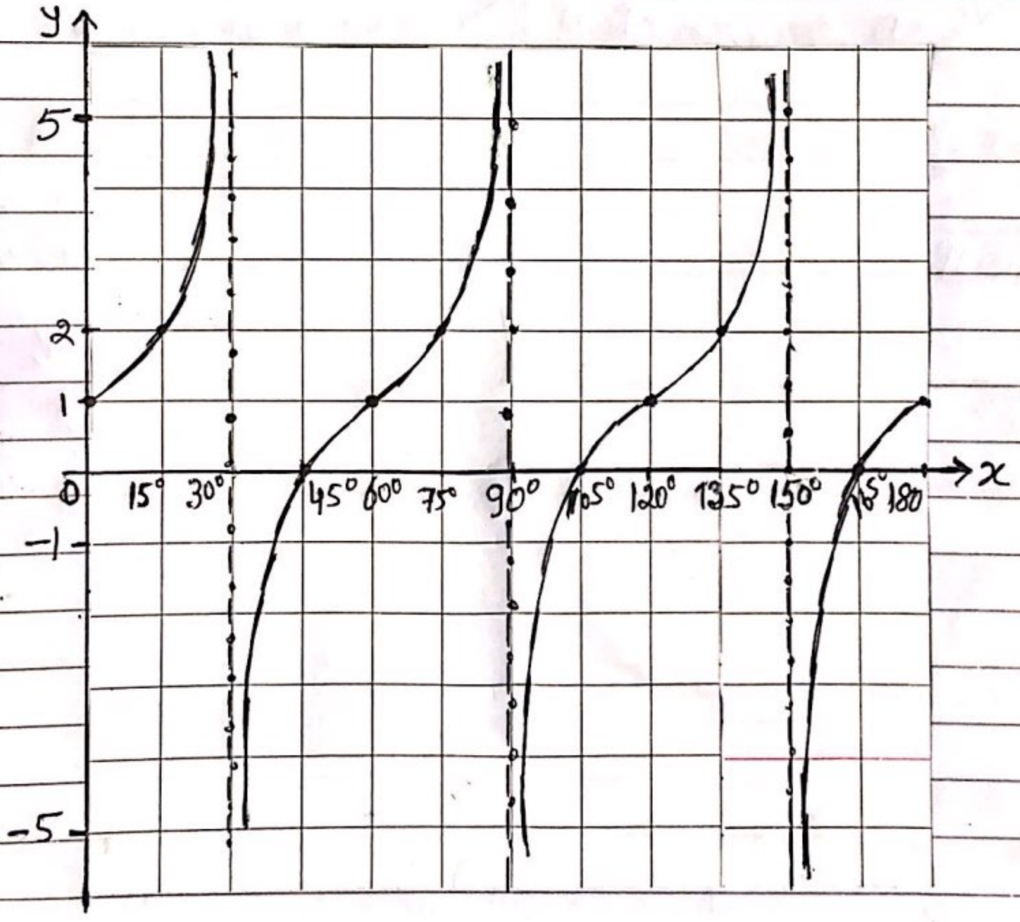
(ii) On the axes below, sketch the graph of $y = 1 + \tan 3x$ for $0^\circ \leq x \leq 180^\circ$

[S-18 | 12 | Q 1] --- [3]

Solution: (i) $y = 1 + \tan 3x$

Period of $y = \frac{180}{3} = 60^\circ$ $\quad \left(\because \text{Period of } \tan bx = \frac{180^\circ}{b} \right)$

(ii) $y = 1 + \tan 3x$ $\quad 0^\circ \leq x \leq 180^\circ$

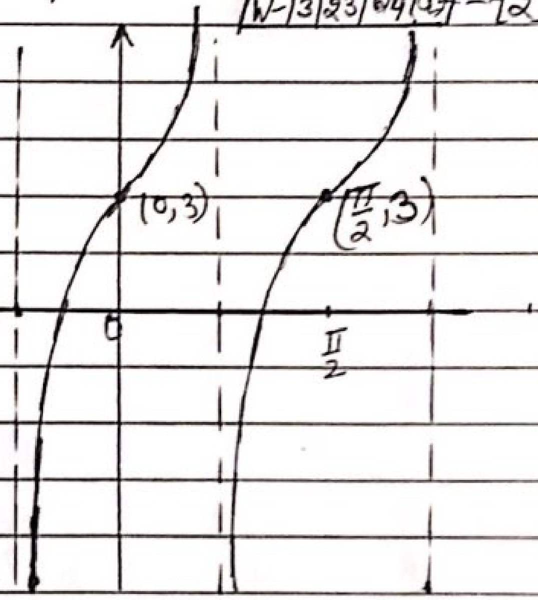


x	0	15	30^-	30^+	45	60	75°	90^-	90^+	105	120	135	150^-	150^+	165	180
y	1	2	∞	$-\infty$	0	1	2	∞	$-\infty$	0	1	2	∞	$-\infty$	0	1

Trigenometry

Example 28 (i) The diagram shows the graph of $y=A+C\tan(Bx)$, passing through the points $(0,3)$ and $(\frac{\pi}{2},3)$, find the value of A and B. [W-13|23|Q4(a)] --[2]

(ii) Given that the point $(\frac{\pi}{8},7)$ also lies on the graph, find the value of C. --[1]



Solution (i) $y=A+C\tan(Bx)$ —— ①

Passes through $(0,3)$

fr ① $3=A+C\tan 0$

$3=A+0 \Rightarrow A=3$ ✓

from the figure we find that the period of the function $=\frac{\pi}{2}$

fr ① or $\frac{\pi}{B}=\frac{\pi}{2}$

$\Rightarrow B=2$

fr ① $y=3+C\tan 2x$ —— ②

Passes through $(\frac{\pi}{8},7)$

fr ② $7=3+C\tan 2\times\frac{\pi}{8}$

or $7=3+C\times 1$ $[\because \tan\frac{\pi}{4}=1]$

or $C=4$

$\therefore$ $A=3$, $B=2$ and $C=4$

(b) Given that $f(x)=8-5\cos 3x$,
State the period and the amplitude of f. [W-13|23|Q4(b)] --[2]

Solution: Period $=\frac{360°}{3}=120°$ or $\frac{2\pi^c}{3}$ ✓

Amplitude $=5$ ✓