IG-Math
0606
Additional Maths

Vectors in 2-D
Exercise

Suresh Goel
Director
Alliance World School,
Noida-Delhi NCR India.

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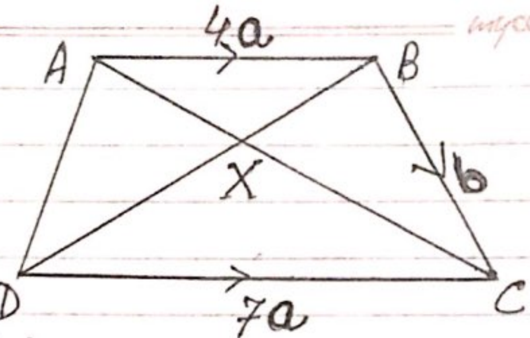
Vectors in 2-D

P-1

Q1 In the diagram, $\vec{AB} = 4\mathbf{a}$, $\vec{BC} = \mathbf{b}$ and $\vec{DC} = 7\mathbf{a}$, The lines AC and DB intersect at the point X.



Find in terms of **a** and **b**.

SP-20/02/Q9

(a) $\vec{DB}$ --- [1]

(b) $\vec{DA}$ -- [1]

Given that $\vec{AX} = \lambda\vec{AC}$ find, in term of $\mathbf{a}$, $\mathbf{b}$ and λ,

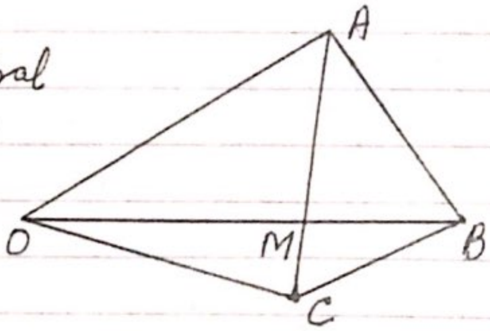
(c) $\vec{AX}$ --- [1]

(d) $\vec{DX}$ -- [2]

Given that $\vec{DX} = \mu\vec{DB}$

(e) find the value of λ and of μ. -- [4]

Q2 The diagram shows the quadrilateral OABC, such that $\vec{OA} = \mathbf{a}$, $\vec{OB} = \mathbf{b}$ and $\vec{OC} = \mathbf{c}$, It is given that AM : MC = 2 : 1 and OM : MB = 3 : 2



(i) Find $\vec{AC}$ in terms of **a** and **c**. --- [1]

(ii) Find $\vec{OM}$ in terms of **a** and **c**. --- [2]

(iii) Find $\vec{OM}$ in terms of **b** --- [1]

(iv) Find $5\mathbf{a} + 10\mathbf{c}$ in terms of **b**. --- [2]

(v) Find $\vec{AB}$ in terms of **a** and **c**, giving your answer in its simplest form. --- [2]

M-18/12/Q6

Q3 (a) A vector **v** has a magnitude of 102 units and has the same direction as $\begin{pmatrix} 8 \\ -15 \end{pmatrix}$. Find **v** in the form $\begin{pmatrix} a \\ b \end{pmatrix}$, where a and b are integers. -- [2]

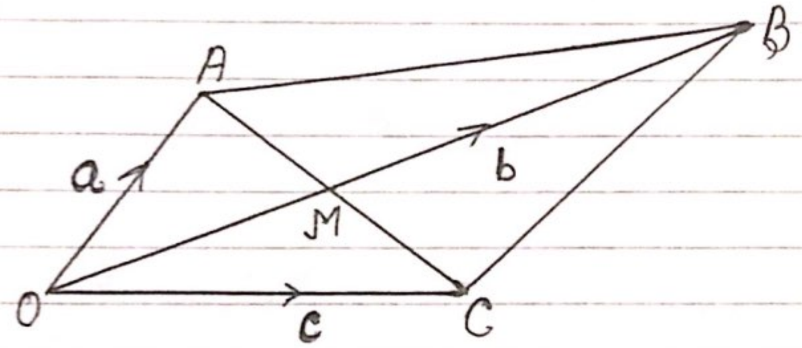
(b) Vector $\mathbf{c} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\mathbf{d} = \begin{pmatrix} p-q \\ 5p+q \end{pmatrix}$ are such that $\mathbf{c} + 2\mathbf{d} = \begin{pmatrix} p^2 \\ 27 \end{pmatrix}$. Find the possible values of the constants p and q. -- [6]

M-17/12/Q7

Q4. Calculate the magnitude and bearing of the resultant velocity of $10\,ms^{-1}$ on a bearing of $240°$ and $5\,ms^{-1}$ due south. --[5]

M-17/22/Q7(a)

Q5 (a)



The diagram shows a figure $OABC$, where $\vec{OA} = \mathbf{a}$, $\vec{OB} = \mathbf{b}$ and $\vec{OC} = \mathbf{c}$. The lines AC and OB intersect at the point M, where M is the mid point of the line AC.

(i) Find, in terms of $\mathbf{a}$ and $\mathbf{c}$, the vector $\vec{OM}$. --[2]

(ii) Given that $OM : MB = 2:3$, find $\mathbf{b}$ in terms of $\mathbf{a}$ and $\mathbf{c}$. ---[2]

(b) Vectors $\mathbf{i}$ and $\mathbf{j}$ are unit vectors parallel to the x-axis and y-axis respectively. The vector $\mathbf{p}$ has a magnitude of 39 units and has the same direction as $-10\mathbf{i} + 24\mathbf{j}$.

(i) Find $\mathbf{p}$ in terms of $\mathbf{i}$ and $\mathbf{j}$. ---[2]

(ii) Find the vector $\mathbf{q}$ such that $2\mathbf{p}+\mathbf{q}$ is parallel to the positive y-axis and has a magnitude of 12 units. ---[3]

(iii) Hence show that $|\mathbf{q}| = k\sqrt{5}$, where k is an integer to be found. ---[2]

S-17/11/Q5

Q6 Vectors $\mathbf{i}$ and $\mathbf{j}$ are unit vectors parallel to the x-axis and y-axis respectively.

(a) The vector $\mathbf{v}$ has a magnitude of $3\sqrt{5}$ unit and has the same direction as $\mathbf{i} - 2\mathbf{j}$. Find $\mathbf{v}$ giving your answer in the form $a\mathbf{i} + b\mathbf{j}$, where a and b are integers. --[2]

(b) The velocity vector $\mathbf{w}$ makes an angle of $30°$ with the positive x-axis such that $|\mathbf{w}| = 2$. Find $\mathbf{w}$ giving your answer in the form $\sqrt{c}\,\mathbf{i} + d\mathbf{j}$, where c and d are integers. --[2]

S-17/12/Q3

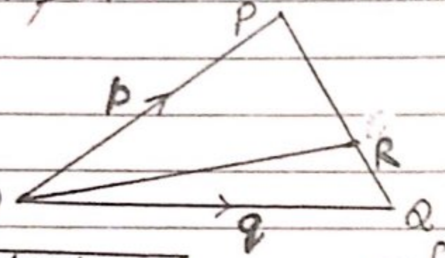
Vectors in 2D

Q7 (a) Vectors **a**, **b**, and **c** are such that $\mathbf{a} = \begin{pmatrix} 5 \\ -6 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 11 \\ -15 \end{pmatrix}$ and $3\mathbf{a} + \mathbf{c} = \mathbf{b}$.

(i) Find **c** --- [1]

(ii) Find a unit vector in the direction of **b**. --- [2]

(b) In the diagram $\vec{OP} = p$ and $\vec{OQ} = q$. The point R lies on PQ such that $PR = 3RQ$. Find $\vec{OR}$ in terms p and q, simplify your answer.



$\boxed{\text{S-17/23/Q4}}$ --- [3]

Q8 In this question i is a unit vector due each and j is a unit vector due north. Units of length and velocity are metres and metres per second respectively.

The initial position vectors of particles A and B, relative to a fixed point O, are $2i + 4j$ and $10i + 14j$ respectively. Particles A and B start moving at the same time. A moves with constant velocity $i + j$ and B moves with constant velocity $-2i - 3j$. Find

(i) the position vector of A after t seconds. [1]

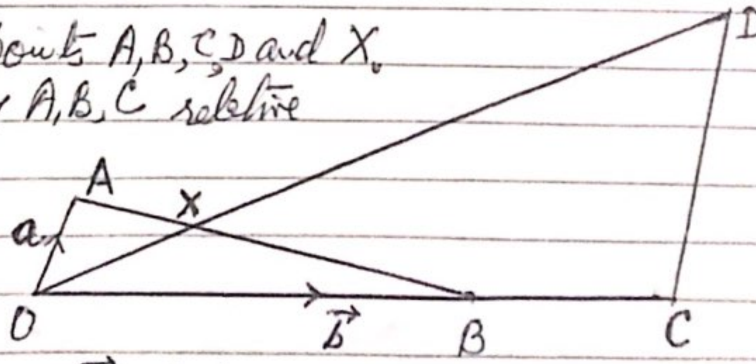
(ii) the position vector of B after t seconds. [1]

It is given that X is the distance between A and B after t seconds.

(iii) Show that $X^2 = (8-3t)^2 + (10-4t)^2$ --- [3]

(iv) Find the value of t for which $(8-3t)^2 + (10-4t)^2$ has a stationary value and the corresponding value of X. $\boxed{\text{W-17/21/Q10}}$ [4]

Q9 The diagram shows points A, B, C, D and X.
The position vectors of A, B, C relative to O are $\vec{OA} = \vec{a}$
$\vec{OB} = \vec{b}$
and $\vec{OC} = \frac{3}{2} b$ and
the vector $\vec{CD} = 3a$



(i) If $\vec{OX} = \lambda \vec{OD}$, Express $\vec{OX}$ in terms of λ, a and b --- [1]

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Q9(ii) If $\vec{AX} = \mu \vec{AB}$ express $\vec{OX}$ in terms of μ, **a** and **b**. ---[2]

(iii) Use your two expressions for $\vec{OX}$ to find the value of λ and of μ. ---[3]

(iv) Find the ratio $\frac{AX}{XB}$ ---[1]

(V) Find the ratio $\frac{OX}{XD}$ ---[1]

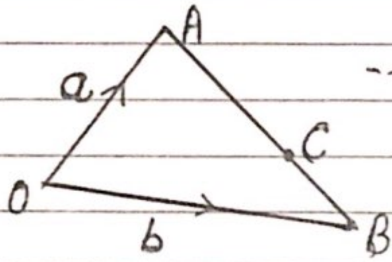
W-17/23/Q5

Q10 (a) The vectors **p** and **q** are such that $p = 11i - 24j$ and $q = 2i + \alpha j$.

(i) Find the value of each of the constants α and β such that $p + 2q = (\alpha + \beta)i - 20j$ ---[3]

(ii) Using the values of α and β found in part (i), find the unit vector in the direction $p + 2q$ ---[2]

(b) The points A and B have position vectors **a** and **b** with respect to an origin O. The point C lies on AB and is such that AB : AC is $1 : \lambda$. Find an expression for $\vec{OC}$ in terms of **a**, **b** and λ. ---[3]



(C) The points S and T have position vectors **s** and **t** with respect to an origin O. The points O, S and T do not lie in a straight line. Given that the vector $2s + \mu t$ is parallel to the vector $(\mu + 3)s + 9t$, where μ is a positive constant, find the value of μ. -- [3]

M-16/22/Q10

Q11 O, P, Q and R are four points such that $\vec{OP} = p$, $\vec{OQ} = q$ and $\vec{OR} = 3q - 2p$

(i) Find in terms of **p** and **q**.

(a) $\vec{PQ}$ (b) $\vec{QR}$ --.[1+1]

(ii) Justifying your answer, what can be said about the positions of the points P, Q and R. ---[2]

(iii) Given that $\vec{OP} = i + 3j$ and $\vec{OQ} = 2i + j$, find the unit vector in the direction $\vec{OR}$. ---[3]

S-16/21/Q7

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Vectors in 2D

Q12 Vectors a, b and c are such that $a=\begin{pmatrix}2\\y\end{pmatrix}$, $b=\begin{pmatrix}1\\3\end{pmatrix}$ and $c=\begin{pmatrix}-5\\5\end{pmatrix}$

(i) Given that $|a|=|b-c|$, find the possible values of y. ---[3]

(ii) Given that $\mu(b+c)+4(b-c)=\lambda(2b-c)$, find the value of μ and λ. ---[3]

S-16/12/Q3

Q13 In this question i is a unit vector due east and j is a unit vector due north. Units of length and velocity are metres and metres per second respectively.

The initial position vectors of particles A and B, relative to a fixed point O, are $i+5j$ and $qi-15j$ respectively. A and B start moving at the same time. A moves with velocity $pi-3j$ and B moves with velocity $3i-j$.

(i) Given that A travels with a speed of $5ms^{-1}$, find the value of positive constant p. ---[1]

(ii) Find the direction of motion of B as a bearing correct to the nearest degree. ---[2]

(iii) State the position vector of A after t seconds. ---[1]

(iv) State the position vector of B after t seconds. ---[1]

(v) Find the time taken until A and B meet. ---[2]

(vi) Find the position vector of the point where A and B meet. ---[1]

(vii) Find the value of the constant q. W-16/23/Q9 ---[1]

Q14 The position vectors of the points A and B relative to an origin O are $-2i+17j$ and $6i+2j$ respectively.

(i) Find the vector $\vec{AB}$ ---[1]

(ii) Find the unit vector in the direction of $\vec{AB}$. ---[2]

(iii) The position vector of the point C relative to the origin O is such that $\vec{OC}=\vec{OA}+m\vec{OB}$, where m is a constant. Given that C lies on the x-axis, find the vector $\vec{OC}$. M-15/22/Q5 ---[3]

Q15 (a) The four points O, A, B and C are such that

$\vec{OA}=5a$ $\vec{OB}=15b$, $\vec{OC}=24b-3a$ ---[3]

Show that B lies on line AC.

(Continued→)

Vectors in 2D

(Continued→)

Q15(b) Relative to an origin O, the position vector of point P is $i-4j$ and the position vector of the point Q is $3i+7j$. Find

(i) $|\overrightarrow{PQ}|$, --- [2]

(ii) the unit vector in the direction $\overrightarrow{PQ}$

(iii) the position vector of M, the mid point of PQ --- [2]

S-15/21/Q7

Q16 In the diagram $\overrightarrow{AB} = 4a$, $\overrightarrow{BC} = b$ and $\overrightarrow{DC} = 7a$. The lines AC and DB intersect at the point X. Find, in terms of a and b,

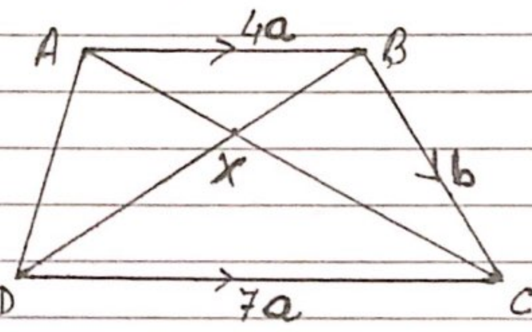
(i) $\overrightarrow{DA}$ --- [1]

(ii) $\overrightarrow{DB}$ --- [1]

Given that $\overrightarrow{AX} = \lambda\overrightarrow{AC}$, find in terms of a, b and λ,

(iii) $\overrightarrow{AX}$ --- [1]

(iv) $\overrightarrow{DX}$ [2]



Given that $\overrightarrow{DX} = \mu\overrightarrow{DB}$

(v) Find the value of λ and of μ S-15/12/Q7 --- [4]

Q17 Relative to an origin O, points A, B and C have position vectors $\begin{pmatrix}5\\4\end{pmatrix}$, $\begin{pmatrix}-10\\12\end{pmatrix}$ and $\begin{pmatrix}6\\-18\end{pmatrix}$ respectively. All distances are measured in kilometres. A man drives at a constant speed directly from A to B in 20 minutes.

(i) Calculate the speed in $km h^{-1}$ at which man drives from A to B. --- [3]

(ii) He now drives directly from B to C at the same speed. Find how long it takes him to drive from B to C. W-15/21/Q3 --- [3]

Q18 Vectors a, b and c are such that $a = \begin{pmatrix}4\\3\end{pmatrix}$, $b = \begin{pmatrix}2\\2\end{pmatrix}$ and $c = \begin{pmatrix}-5\\2\end{pmatrix}$

(i) Show that $|a| = |b+c|$ --- [2]

(ii) Given that $\lambda a + \mu b = 7c$, find the value of λ and of μ. --- [3]

S-14/11/Q2

Q19 In this question i is a unit vector due east and j is a unit vector due north. At 1200 hours, a ship leaves a port 'P' and travels with 26 $km h^{-1}$ in the direction of $5i+12j$. (Continued→)

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P-7

(→ Continued)

Q19 (i) Show that the velocity of the ship is $(10i + 24j)$ km h^{-1}. ---[2]

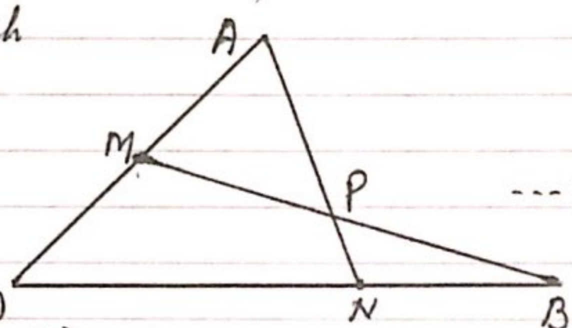
(ii) Write down the position vector of the ship, relative to P, at 1600 hours. ---[1]

(iii) Find the position vector of the ship, relative to P, t hours after 1600 hours. [S-14/12/Q10] ---[2]

Q20 In the diagram $\vec{OA} = 2a$ and $\vec{OB} = 5b$. The point M is the mid point of OA and the point N lies on OB such that ON : NB = 3 : 2

(i) Find an expression for the vector $\vec{MB}$ in terms of a and b ---[2]

The point 'P' lies on AN such that $\vec{AP} = \lambda \vec{AN}$



(ii) Find an expression for the vector $\vec{AP}$ in terms of λ, a and b. ---[2]

(iv) Given that M, P and B are collinear, find the value of λ. ---[4]

(iii) Find an expression for the vector $\vec{MP}$ in terms of λ, a and b ---[2]

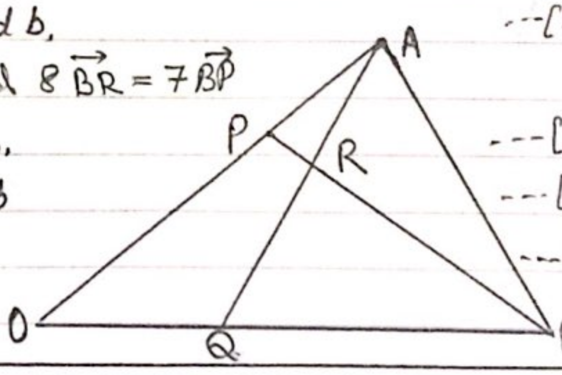
[S-14/23/Q11]

Q21 The position vectors of points A and B relative to an origin O are a and b respectively. The point P is such that $\vec{OP} = \mu \vec{OA}$. The point Q is such that $\vec{OQ} = \lambda \vec{OB}$. The lines AQ and BP intersect at the point R.

(i) Express $\vec{AQ}$ in terms of λ, a and b. ---[1]

(ii) Express $\vec{BP}$ in terms of μ, a and b. ---[1]

It is given that $3\vec{AR} = \vec{AQ}$ and $8\vec{BR} = 7\vec{BP}$



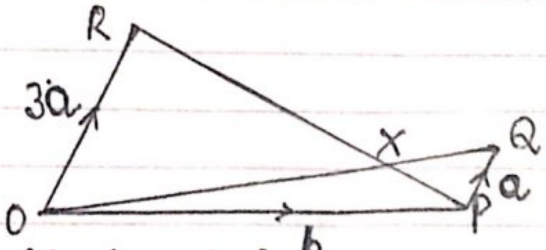
(iii) Express $\vec{OR}$ in terms λ, a and b. ---[2]

(iv) Express $\vec{OR}$ in terms of μ, a and b ---[2]

(v) Hence find the value of μ and of λ. ---[3]

[W-14/11/Q12]

Q22 In the diagram $\vec{OP} = b$, $\vec{PQ} = a$ and $\vec{OR} = 3a$. The lines OQ and PR intersect at X.



(i) Given that $\vec{OX} = \mu \vec{OQ}$, Express $\vec{OX}$ in terms of μ, a and b

(Continued→)

Vectors in 2D.

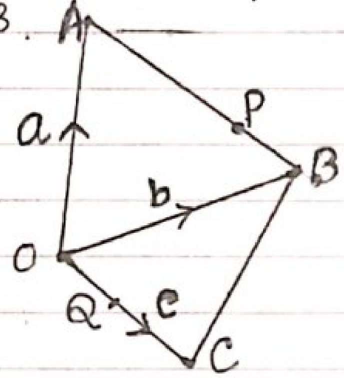
(→ Continued)

Q22 (ii) Given that $\overrightarrow{RX} = \lambda \overrightarrow{RP}$, express $\overrightarrow{OX}$ in terms of λ, a and b. ---[2]

(iii) Hence find the value of μ and λ and state the value of the ratio $\frac{RX}{XP}$ ..[3]

W-14/23/Q5

Q23 The fig. shows points A, B and C with position vectors a, b and c respectively, relative to an origin O. The point P lies on AB such that AP : AB = 3 : 4. The point Q lies on OC such that OQ : QC = 2:3.



(i) Express $\overrightarrow{AP}$ in terms of a and b and hence show that $\overrightarrow{OP} = \frac{1}{4}(a+3b)$ ----[3]

(ii) Find $\overrightarrow{PQ}$ in terms of a, b and c --- [3]

(iii) Given that $5\overrightarrow{PQ} = 6\overrightarrow{BC}$, find c in terms of a and b. ---[2]

S-13/11/Q9

Q24 The position vector of the points A and B, relative to an origin O, are $4i - 21j$ and $22i - 30j$ respectively. The point C lies on AB such that $\overrightarrow{AB} = 3\overrightarrow{AC}$.

(i) Find the position vector of C relative to O. ---[4]

(ii) Find the unit vector in the direction $\overrightarrow{OC}$ S-13/22/Q4 ---[2]

Q25 In this question i is a unit vector due each and j is a unit vector due north.
At time $t=0$ boat A leaves the origin O with velocity $(2i+4j)$ Kmh^{-1}. Also at time $t=0$ boat B leaves the point with positive vector $(-21i+22j)$ and travels with velocity $(5i+3j)$ Kmh^{-1}.

(i) Write down the position vectors of boats A and B after t hours. ...[2]

(ii) Show that A and B are 25 Km apart when $t=2$. --[3]

(iii) Find the length of time for which A and B are less than 25 Km apart. W-13/23/Q11 ---[5]

Vectors in 2-D

Answers

Q1 (a) $\vec{DB} = 7a - b$

(b) $\vec{DA} = 3a - b$

(c) $\vec{AX} = \lambda(4a + b)$

(d) $\vec{DX} = 3a - b + \lambda(4a+b)$

(e) $\lambda = \frac{4}{11}$; $\mu = \frac{7}{11}$

Q2 (i) $\vec{AC} = c - a$

(ii) $\vec{OM} = a + \frac{2}{3}(c-a) = \left(\frac{2}{3}c + \frac{1}{3}a\right)$

(iii) $\vec{OM} = \frac{3}{5}b$

(iv) $\frac{3}{5}b = \frac{2}{3}c + \frac{1}{3}a$ from (ii) & (iii)

$\Rightarrow 10a + 5c = 9b$

(v) $\vec{AB} = b - a$

$= \frac{5}{9}c + \frac{10}{9}a - a$

$= -\frac{4}{9}a + \frac{10}{9}c$

Q3 (a) $v = \frac{102}{17}\begin{pmatrix}8\\-15\end{pmatrix}$

(b) $\begin{pmatrix}2p-2q+4\\10p+2q+3\end{pmatrix} = \begin{pmatrix}p^2\\27\end{pmatrix}$

$\Rightarrow \left.\begin{matrix}2p-2q+4 = p^2\\ \text{and } 10p+2q+3=27\end{matrix}\right\}$

$\Rightarrow p^2 - 12p + 20 = 0$

$\Rightarrow \left\{\begin{matrix}p = 2, \ q = 2\\ p = 10, \ q = -38\end{matrix}\right.$

Q4 $r^2 = 15^2 + 10^2 - 2\times5\times10\cos120$

$\Rightarrow r = 13.2$

$\frac{\sin x}{5} = \frac{\sin 120}{13.2} \Rightarrow x = 19.1$

∴ bearing of resultant Vel

$= (360° - 120° - 19.1)$

Q5 b)(i) $\vec{OM} = \frac{1}{2}(a+c)$

(ii) $b = \frac{5}{2}\vec{OM} = \frac{5}{2}\times\frac{1}{2}(a+c)$

$\therefore b = \frac{5}{4}(a+c)$

(b)(i) $|-10i + 24j| = 26$

$\therefore p = \frac{39}{26}(-10i + 24j)$

or $p = -15i + 36j$

(ii) If parallel to y-axis i-component is zero.

$\therefore 2p + q = 12j$

or $2(-15i + 36j) + q = 12j$

or $q = 30i - 60j$

(iii) $|q| = 30\sqrt{1^2 + (-2)^2} = 30\sqrt{5}$

Q6 (a) $v = 3\sqrt{5}\times\frac{1}{\sqrt{5}}(i - 2j)$

$= 3i - 6j$

(b) $w = 2\cos30° i + 2\sin30° j$

$= \sqrt{3}i + j$

Q7 (a)(i) $c = \begin{pmatrix}-4\\3\end{pmatrix}$

(ii) $|b| = \sqrt{11^2 + (-15)^2} = \sqrt{346}$

unit vector $\hat{b} = \frac{1}{\sqrt{346}}\begin{pmatrix}11\\-15\end{pmatrix}$

(b) $\vec{OR} = \vec{OP} + \frac{3}{4}\vec{PQ}$

$= p + \frac{3}{4}(q - p)$

$\therefore \vec{OR} = \frac{1}{4}p + \frac{3}{4}q$

Q8 (i) $r_A = (2i + 4j) + t(i + j)$

(ii) $r_B = (10i + 14j) + t(-2i - 3j)$

(iii) $r_B - r_A = (8i + 10j) + t(-3i - 4j)$

$X^2 = (8 - 3t)^2 + (10 - 4t)^2$

$\frac{d}{dt}X^2 = 2(8-3t)(-3) + 2(10-4t)(-4)$

$\frac{dX^2}{dt} = 0 \Rightarrow t = 2.56$ and $X = 0.4$

Vectors in 2D.

Answers

Q9 (i) $\vec{OX} = \lambda(1.5b + 3a)$

(ii) $\vec{OX} = a + \mu(b - a)$ [$\because \vec{AB} = b - a$]

(iii) fr ① & ② $1.5\lambda = \mu$ & $3\lambda = 1 - \mu$

$\Rightarrow \mu = 1/3,\ \lambda = 2/9$ ✓

(iv) $\frac{AX}{XB} = \frac{1}{2}$ ✓

(v) $\frac{OX}{XD} = \frac{2}{7}$ ✓

Q10 (a) (i) $(\alpha+\beta)i - 20j = p + 2q$

$= 15i + (2\alpha - 24)j$

$\Rightarrow \alpha = 2;\ \beta = 13$ ✓

(ii) $p + 2q = 15i - 20j$

$\therefore |p + 2q| = |15i - 20j| = \sqrt{15^2 + (-20)^2}$

$= 25$

$\therefore$ Unit Vector along $p+2q = \frac{1}{25}(15i - 20j)$

(b) $\vec{OC} = \vec{OA} + \lambda AB,$

$= a + \lambda(b - a)$

$= (1-\lambda)a + \lambda b$ ✓

(c) $\frac{2}{\mu+3} = \mu/9$

$\Rightarrow \mu^2 + 3\mu - 18 = 0$

$\therefore \mu = 3$ ($\because \mu > 0$)

Q11 (i) (a) $\vec{PQ} = q - p$ ✓

(b) $\vec{QR} = \vec{OR} - \vec{OQ} = 3q - 2p - q$

$= 2q - 2p$ ✓

(ii) The point as collinear as $\vec{PQ} = \frac{1}{2}\vec{QR}$ P Q R

$\Rightarrow PQ \parallel QR$ and Q is a common point.

(iii) $\vec{OR} = 4i - 3j$

$|\vec{OR}| = \sqrt{(4)^2 + (-3)^2} = 5$

$\therefore$ Unit Vector along $\vec{OR}$

$= \frac{1}{5}(4i - 3j)$

Q12 (i) $b - c = \binom{6}{-2}$

$4 + y^2 = 36 + 4 \Rightarrow y = \pm 6$ ✓

(ii) $\mu + 4 = 2\lambda$ and $-4\mu + 24 = 7\lambda$

$\Rightarrow \mu = \frac{4}{3}$ and $\lambda = \frac{8}{3}$ ✓

Q13 (i) $V_A = pi - 3j$

$|V_A| = \sqrt{p^2 + 9} = 5$ given

$\Rightarrow p = 4$ ✓

(ii) $\tan\alpha = \pm\frac{1}{3}$ or ± 3 or $18.4°$ or $71.6°$ or $108°$

(iii) $r_A = \binom{1}{5} + t\binom{p}{-3}$

(iv) $r_B = \binom{9}{-15} + t\binom{3}{-1}$

(v) $5 - 3t = -15 - t \Rightarrow t = 10$ ✓

(vi) $\binom{41}{-25}$ only

(vii) $q = 11$

Q14 (i) $\vec{AB} = (6i + 2j) - (-2i + 17j)$

$= 8i - 15j$

(ii) $|\vec{AB}| = 17$

$\therefore$ Unit Vector along $\vec{AB} = \frac{1}{17}(8i - 15j)$

(iii) $(-2i + 17j) + m(6i + 2j)$ lies on x-axis

$\therefore$ y component $= 0$

$\Rightarrow 17 + 2m = 0 \Rightarrow m = -8.5$

$\therefore \vec{OC} = -53i$

Q15 (a) $\vec{AB} = 5(3b - a)$

$\vec{BC} = 3(3b - a)$

$\Rightarrow \vec{AB} = \frac{5}{3}\vec{BC} \Rightarrow \vec{AB} \parallel \vec{BC}$

and B is a common point

$\therefore$ A, B, C lie on a line.

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Vectors in 2D

Answers

Q15 (b)

(i) $\overrightarrow{PQ} = 2i + 11j$

$|\overrightarrow{PQ}| = 5\sqrt{5}$ ✓

(ii) $\frac{1}{5\sqrt{5}}(2i + 11j)$

(iii) $\frac{(i - 4j) + (3i + 7j)}{2} = (2i + 1.5j)$ ✓

Q16 (i) $\overrightarrow{DA} = 3a - b$

(ii) $\overrightarrow{DB} = 7a - b$

(iii) $\overrightarrow{AX} = \lambda(4a + b)$

(iv) $\overrightarrow{DX} = 3a - b + \lambda(4a + b)$

(v) $3a - b + \lambda(4a+b) = \mu(7a - b)$

$\Rightarrow \begin{cases} 3 + 4\lambda = 7\mu \\ -1 + \lambda = -\mu \end{cases}$

$\Rightarrow \lambda = \frac{4}{11}, \; \mu = \frac{7}{11}$

Q17 (i) $\overrightarrow{AB} = \begin{pmatrix} -15 \\ 8 \end{pmatrix}$

$|\overrightarrow{AB}| = 17$

Speed $= \frac{17}{1/3} = 51$ Km/hr ✓

(ii) $\overrightarrow{BC} = \begin{pmatrix} 16 \\ -30 \end{pmatrix}$

$|\overrightarrow{BC}| = 34$

Time taken $= \frac{\text{Dis}}{\text{time}} = \frac{34}{51}$ h

$= 40$ minutes

Q18 (i) $|a| = \sqrt{4^2 + 3^2} = 5$

$|b + c| = \sqrt{(-3)^2 + 4^2} = 5$

$\therefore |a| = |b + c|$ ✓

(ii) $\lambda\begin{pmatrix} 4 \\ 3 \end{pmatrix} + \mu\begin{pmatrix} 2 \\ 2 \end{pmatrix} = 7\begin{pmatrix} -5 \\ 2 \end{pmatrix}$

$\Rightarrow \left.\begin{array}{l} 4\lambda + 2\mu = -35 \\ \text{and } \; 3\lambda + 2\mu = 14 \end{array}\right\}$

Solving

$\lambda = -49$ and $\mu = 80.5$ ✓

Q19 Direction of motion $= 5i + 12j$

$\therefore$ Unit vector in the motion $= \frac{1}{13}(5i + 12j)$

(i) speed $= 26$ Km h^{-1}

$\therefore$ Velocity $= 26 \times \frac{1}{13}(5i + 12j)$

$= (10i + 24j)$ ✓

(ii) Position Vector $= 4(10i + 24j)$

or $= (40i + 96j)$ ✓

(iii) $(40i + 96j) + t(10i + 24j)$ ✓

Q20 (i) $\overrightarrow{OM} = a$

$\overrightarrow{MB} = 5b - a$ ✓

(ii) $\overrightarrow{ON} = 3b$

$\overrightarrow{AP} = \lambda(3b - 2a)$ ✓

(iii) $\overrightarrow{MP} = \overrightarrow{MA} + \overrightarrow{AP} = a + \lambda(3b - 2a)$

(iv) let $\overrightarrow{MP} = \mu\overrightarrow{MB}$

equate components and solve

$\lambda = 5/7$

Q21 (i) $\overrightarrow{AQ} = \lambda b - a$ ✓

(ii) $\overrightarrow{BP} = \mu a - b$ ✓

(iii) $\overrightarrow{OR} = a + \frac{1}{3}(\lambda b - a)$

$= \frac{2}{3}a + \frac{1}{3}\lambda b$ ✓

(iv) $\overrightarrow{OR} = \vec{b} + \frac{7}{8}(\mu a - b)$

$= \frac{1}{8}b + \frac{7}{8}\mu a$ ✓

(v) from (iii) and (iv)

$\frac{2}{3}a + \frac{1}{3}\lambda b = \frac{1}{8}b + \frac{7}{8}\mu a$

$\Rightarrow \frac{7}{8}\mu = \frac{2}{3} \Rightarrow \mu = \frac{16}{21}$ ✓

and $\frac{1}{3}\lambda = \frac{1}{8} \Rightarrow \lambda = \frac{3}{8}$ ✓

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Answers

Q22 (i) $\vec{OX} = \mu(a+b)$

(ii) $\vec{RP} = b-3a \Rightarrow \vec{RX} = \lambda(b-3a)$

$\vec{OX} = 3a+\lambda(b-3a)$

(iii) $\vec{OX} = \mu(a+b) = 3a+\lambda(b-3a)$ (from (i) & (ii))

$\Rightarrow \mu = 3-3\lambda$ and $\mu = \lambda$

$\Rightarrow \mu = \lambda = 0.75$ ✓

$\frac{RX}{XP} = 3$ or $3:1$ ✓

Q23 (i) $\vec{AP} = \frac{3}{4}(b-a)$

$\vec{OP} = a+\frac{3}{4}(b-a) = \frac{1}{4}(a+3b)$ ✓

(ii) $\vec{OQ} = \frac{2}{5}c$, $\vec{QC} = \frac{3}{5}c$

or $\vec{CQ} = -\frac{3}{5}c$

$\therefore \vec{PQ} = \vec{OQ}-\vec{OP}$

$= \frac{2}{5}c - \frac{1}{4}(a+3b)$ ✓

(iii) $5\vec{PQ} = 6\vec{BC}$

$\Rightarrow 5\left[\frac{2}{5}c - \frac{1}{4}(a+3b)\right] = 6(c-b)$

$\Rightarrow c = \frac{9b-5a}{16}$ ✓

Q24 $\vec{OC} = \vec{OA}+\vec{AC}$ — (i)

$\vec{AB} = 3\vec{AC}$

$\Rightarrow \vec{OB}-\vec{OA} = 3(\vec{OC}-\vec{OA})$

$\Rightarrow (18i-9j) = 3(\vec{OC}-\vec{OA})$

$\Rightarrow \vec{OC}-\vec{OA} = 6i-3j$

$\Rightarrow \vec{OC} = \vec{OA}+6i-3j$

$= 4i-21j+6i-3j$

$= (10i-24j)$ ✓

Q25 (a) (i) $r_A = (2i+4j)t$ and $r_B = (-21i+22j) + (5i+3j)t$

(ii) $\vec{r_A r_B} = (-21+3t)i + (22-t)j$

at $t=2$, $dis = -15i+20j$

$dis = \sqrt{(-15)^2+20^2} = 25$ ✓

(iii) $(-21+3t)^2+(22-t)^2 = 25^2$

$\Rightarrow t^2-17t+30=0$

$t = 15$ or 2

$\therefore$ 13 hours. ✓

← X — X →