# IG_0606 Additional Maths

# Vectors Notes

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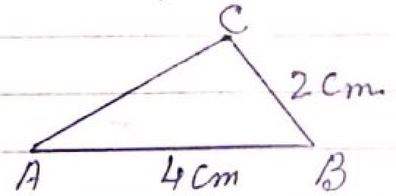
# Vectors Notes

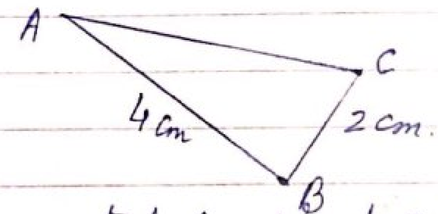
**Scalar Quantities:** A scalar quantity has magnitude (unit and a real number), length, mass, distance, speed, density are all scalar quantities.

Example:




Length of side AB = 4 cm.

Length of side BC = 2 Cm.




(Note: Here direction of the sides) is not considered)

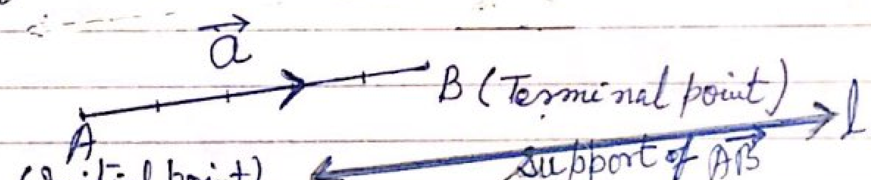
**Vectors:** A vector quantity has magnitude (unit and real no) and direction (support and sense).

Velocity, displacement, force are vector qualities.

**Geometric representation of a Vector:**

Vectors are represented by directed line segments.

Example: A force of 5 newtons is applied, it is denoted by " **a** (bold face letter) or $\vec{a}$ or $\underset{\sim}{a}$ or **AB** or $\vec{AB}$ "




Magnitude of Vector $\vec{AB}$ = $|\vec{AB}|$ or $|\vec{a}|$ = 5 Newton

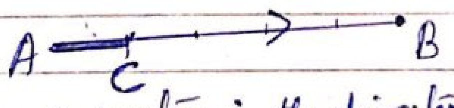
{In this particular example).
Unit - Newton
Real no - 5

**Direction of Vector $\vec{AB}$:**

support AB // line l or // line AB (on $\overleftrightarrow{AB}$)

sense: A to B. (Shown by arrow)

**Unit Vector:**




Given a Vector $\vec{AB}$, then a vector in the direction of $\vec{AB}$ and magnitude = 1

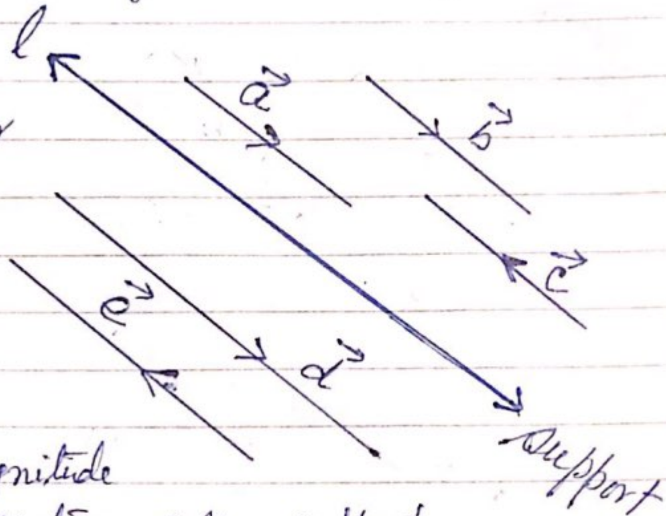
Unit Vector $\vec{AC} = \dfrac{\vec{AB}}{|\vec{AB}|}$.

# Vectors Notes



**Collinear Vectors:** Vectors having the same support.

Vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ and $\vec{e}$ are all collinear vectors as they have the same support (line 'l')




(a) **Equal Vectors:** $\vec{a} = \vec{b}$

(i) same magnitude

and (ii) same direction — same support, same sense

(b) **Parallel Vectors:** $\vec{m} = K\vec{n}$ $\quad K \neq 0$ real no.

all the vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}, \vec{e}$ are parallel vectors

(i) if $K > 0$ same direction ; $\vec{d} = 2\vec{a}$

Vectors $\vec{a}, \vec{b}, \vec{d}$ are in the same direction.

and $\vec{c}$ and $\vec{e}$ are also in the same direction.

(ii) If $K < 0$

Vectors $\vec{a}$ and $\vec{e}$ are in opposite directions

as $\vec{e} = -\frac{3}{2}\vec{a}$

and $\vec{d} = -2\vec{c}$

(c) **Opposite Vectors:**

$\vec{b} = -\vec{c}$

(i) same magnitude but opposite direction (sense) (same magnitude and same support but opp.)

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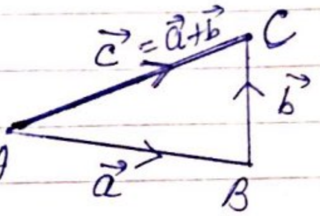
# Vectors Notes

## Addition of Vectors (Triangle Law of Vector addition):

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} \quad [\vec{a}+\vec{b}=\vec{c}]$$

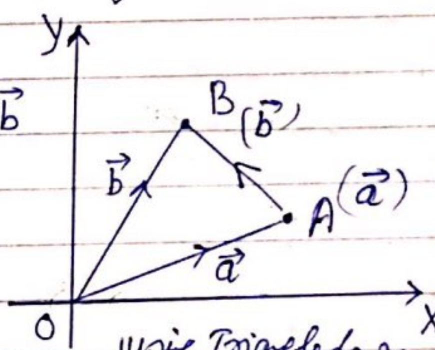



If two vectors are denoted by two sides of a triangle with their magnitude and directions taken in same order then the third side denotes their vector sum by magnitude and direction in opposite order.

[Or The head of vector $\vec{a}$ connects to the tail of vector $\vec{b}$, Then the vector obtained by joining the tail of $\vec{a}$ to the head of vector $\vec{b}$ gives their vector sum)

## Position Vector of a point:

Let $\overrightarrow{OA}=\vec{a}$ and $\overrightarrow{OB}=\vec{b}$



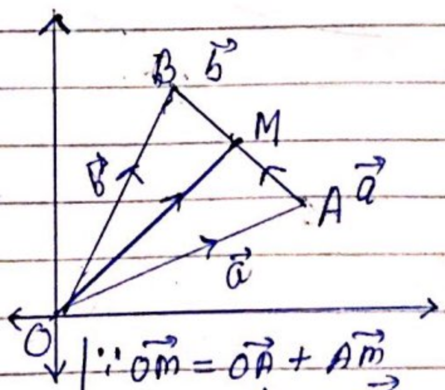

Let the reference point is origin O.
Then the position Vector of point A $= \vec{a}$
and the position Vector of point B $= \vec{b}$

Note: $\overrightarrow{AB} = \vec{b} - \vec{a}$
(Position Vector Terminal point) − (Position vector of Initial point)

Using Triangle Law:
$\because$ In $\Delta OAB$
$\overrightarrow{OA}+\overrightarrow{AB}=\overrightarrow{OB}$
$\therefore \overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$
$=\vec{b}-\vec{a}$

$$\boxed{\overrightarrow{AB}=\vec{b}-\vec{a}}$$

## Position Vector of Mid point:




Given the position Vectors of points A and B (with reference to origin O) are $\vec{a}$ and $\vec{b}$. Let M is the mid point of segment AB.

Then the position Vector of the mid point.

$$\overrightarrow{OM} = \left(\frac{\vec{a}+\vec{b}}{2}\right)$$

$\because \overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$
$= \vec{a} + \frac{1}{2}\overrightarrow{AB}$
$= \vec{a} + \frac{1}{2}(\vec{b}-\vec{a})$
$= \frac{(\vec{a}+\vec{b})}{2}$

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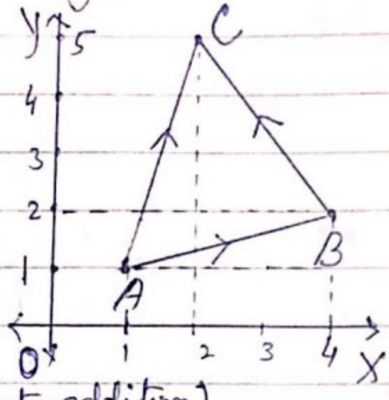
# Vectors (In two Dimensions) Notes

## § Column Vector:

A column vector indicates a Translation (or a shift) in the direction of x-axis and along y-axis.

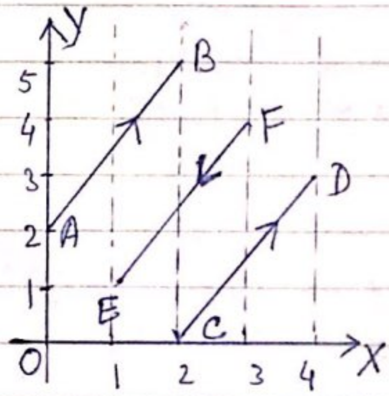
(i) $\vec{AB} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ ; $\vec{BC} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ ; $\vec{AC} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$




Now $\begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

$\therefore$ $\vec{AB} + \vec{BC} = \vec{AC}$ [Triangle Law of Vector addition]

(ii) $\vec{AB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ ; $\vec{CD} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\vec{FE} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$



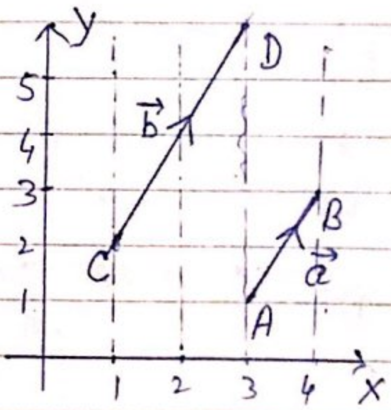

$\therefore$ $\vec{AB} = \vec{CD}$ Equal Vectors

$\vec{FE} = \begin{pmatrix} -2 \\ -3 \end{pmatrix} = -1\begin{pmatrix} 2 \\ 3 \end{pmatrix} = -\vec{AB} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$

or $\vec{AB} = -\vec{FE}$

$\therefore$ $\vec{AB}$ and $\vec{FE}$ are opposite vectors.

## § Scalar Multiplication:




$\vec{AB} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ ; $\vec{CD} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

$2 \cdot \vec{AB} = 2\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \vec{CD}$

$\vec{AB}$ & $\vec{CD}$ are two vectors in the same direction.

[$\vec{a} = k\vec{b}$ and $k > 0$
$\vec{a}$ and $\vec{b}$ are in the same direction]

## § Magnitude of a Column Vector:

$\vec{AB} = \begin{pmatrix} x \\ y \end{pmatrix}$ Then magnitude of $\vec{AB} = |\vec{AB}| = \sqrt{x^2+y^2}$

Example: $\vec{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \Rightarrow |\vec{a}| = \sqrt{1^2+2^2} = \sqrt{5}$

$\vec{b} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \Rightarrow |\vec{b}| = \sqrt{3^2+4^2} = 5$

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# Vectors in two dimensions Notes



**Unit Vector:** A vector whose magitude is one unit.

Given a vector $\vec{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

(i) Find a unit Vector in the direction of $\vec{a}$

(ii) Find a vector $\vec{b}$ in the direction of $\vec{a}$ and $|\vec{b}| = 7$

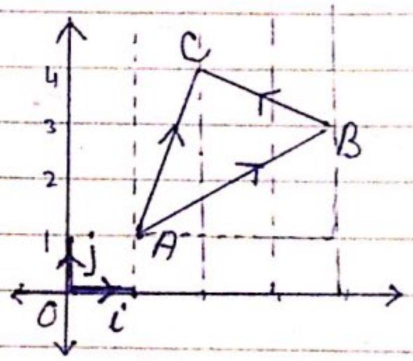
Solution (i) $\vec{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \Rightarrow |\vec{a}| = \sqrt{3^2+4^2} = 5$

$\therefore$ Unit Vector in the direction of $\vec{a} = \dfrac{\vec{a}}{|\vec{a}|} = \dfrac{1}{5}\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

(ii) $\vec{b} = 7 \times$ Unit Vector along $\vec{a}$

$= 7 \times \dfrac{1}{5}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{21}{5} \\ \frac{28}{5} \end{pmatrix}$

**Basic Unit Vectors (in two dimensions)**




$i = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $j = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\left.\begin{aligned} \vec{AB} &= \begin{pmatrix} 3 \\ 2 \end{pmatrix} = 3i+2j \\ \vec{BC} &= \begin{pmatrix} -2 \\ 1 \end{pmatrix} = -2i+j \end{aligned}\right\} \quad \vec{AC} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$\vec{AC} = (i+3j)$

(i) Now $\vec{AB} + \vec{BC}$

$= (3i+2j) + (-2i+j) = (3-2)i + (2+1)j$

$= i + 3j = \vec{AC}$

$\boxed{\vec{AB} + \vec{BC} = \vec{AC}}$ $\longrightarrow$ (Verifies triangle law of Vector addition)

(ii) $\vec{AB} = 3i+2j$

$\therefore |\vec{AB}| = \sqrt{3^2+2^2} = \sqrt{13}$

Unit Vector along $\vec{AB} = \dfrac{1}{\sqrt{13}}\vec{AB} = \dfrac{1}{\sqrt{13}}(3i+2j)$ ✓

(iii) $3\vec{AB} + 2\vec{BC} = 3(3i+2j) + 2(-2i+j)$

$= (9i+6j) + (-4i+2j) = (5i+8j)$

$\left.\begin{aligned} &\text{(iv) Position Vector } B = \vec{OB} = (4i+3j) \\ &\text{(v) Position Vector } A = \vec{OA} = (i+j) \end{aligned}\right\} \quad \vec{AB} = \vec{OB} - \vec{OA}$

$= (4i+3j) - (i+j)$

$= (3i+2j)$ ✓

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# Vectors

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Example 1(a). $\vec{GH} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$

Find (i) $5\vec{GH}$ ---[1]

(ii) $\vec{HG}$ ---[1]

(b) $\begin{pmatrix} 6 \\ 7 \end{pmatrix} + \begin{pmatrix} 2 \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 3 \end{pmatrix}$ find the value of $y$. ---[1]

Solution: (a)(i) $\vec{GH} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$ 0580/S-17/21/Q18

$$\therefore 5\vec{GH} = 5\begin{pmatrix} 6 \\ -4 \end{pmatrix} = \begin{pmatrix} 30 \\ -20 \end{pmatrix} ✓$$

(ii) $\vec{HG} = -\vec{GH} = -\begin{pmatrix} 6 \\ -4 \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \end{pmatrix}$ ✓

(b) $\begin{pmatrix} 6 \\ 7 \end{pmatrix} + \begin{pmatrix} 2 \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 3 \end{pmatrix}$

or $\begin{pmatrix} 6+2 \\ 7+y \end{pmatrix} = \begin{pmatrix} 8 \\ 3 \end{pmatrix} \Rightarrow 7+y=3 \Rightarrow y=-4$ ✓

Example 2(a) D is a point $(2,-5)$ and $\vec{DE} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$.

Find the co-ordinates of the point E. ---[1]

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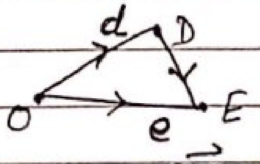
(b) $\mathbf{v} = \begin{pmatrix} t \\ 12 \end{pmatrix}$ and $|\mathbf{v}| = 13$

work out the value of $t$, where $t$ is negative. ---[2]

Solution(a) Let $E(x,y)$, $D=(2,-5)$

$\vec{OE} = \begin{pmatrix} x \\ y \end{pmatrix}$, $\vec{OD} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$




$\vec{DE} = e - d$

$$\vec{DE} = \vec{OE} - \vec{OD} = \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 2 \\ -5 \end{pmatrix}$$

$$= \begin{pmatrix} x-2 \\ y+5 \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \end{pmatrix} \quad \text{Given } \vec{DE} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$$

$\Rightarrow x-2=7 \Rightarrow x=9$ $\therefore E(9,-4)$ ✓

$y+5=1 \Rightarrow y=-4$

(b) $V = \begin{pmatrix} t \\ 12 \end{pmatrix} \therefore |V| = \sqrt{t^2+12^2} = 13$ Given

$\Rightarrow t^2+144=169$

$t^2=25$

$t = \cancel{5}, -5$ ✓ $t$ is negative

$\therefore t=-5$ ✓

# Vectors

Example 3(a) A vector **v** has a magnitude of 102 units and has the same direction as $\begin{pmatrix}8\\-15\end{pmatrix}$. Find **v** in the form $\begin{pmatrix}a\\b\end{pmatrix}$, where a and b are integers. --[2]

(b) Vector $\mathbf{c}=\begin{pmatrix}4\\3\end{pmatrix}$ and $\mathbf{d}=\begin{pmatrix}p-q\\5p+q\end{pmatrix}$ are such that $\mathbf{c}+2\mathbf{d}=\begin{pmatrix}p^2\\27\end{pmatrix}$.

Find the possible values of the constants p and q. [M-17/12/Q7] --[6]

Solution (a) $V=\begin{pmatrix}a\\b\end{pmatrix}$ has the same direction as $\begin{pmatrix}8\\-15\end{pmatrix}$

unit vector in the direction of $\begin{pmatrix}8\\-15\end{pmatrix}$

$\left|\begin{pmatrix}8\\-15\end{pmatrix}\right| = \sqrt{8^2+(-15)^2}$
$=\sqrt{64+225}$
$=\sqrt{289}$
Mangitue $=17$

Unit Vector $=\dfrac{\text{Vector}}{\text{Magnitude}} \Rightarrow = \dfrac{1}{17}\begin{pmatrix}8\\-15\end{pmatrix}$

magnitude of V is 102

Vector = Magnitude x Unit Vector

$\therefore\ V=\begin{pmatrix}a\\b\end{pmatrix} = 102\times\dfrac{1}{17}\begin{pmatrix}8\\-15\end{pmatrix}$

$= 6\begin{pmatrix}8\\-15\end{pmatrix} = \begin{pmatrix}48\\-90\end{pmatrix}$

or $V=\begin{pmatrix}a\\b\end{pmatrix}=\begin{pmatrix}48\\-90\end{pmatrix}$ ✓

(b) $\mathbf{c}=\begin{pmatrix}4\\3\end{pmatrix}$ and $\mathbf{d}=\begin{pmatrix}p-q\\5p+q\end{pmatrix}$

Given $\mathbf{c}+2\mathbf{d}=\begin{pmatrix}p^2\\27\end{pmatrix}$

or $\begin{pmatrix}4\\3\end{pmatrix}+2\begin{pmatrix}p-q\\5p+q\end{pmatrix}=\begin{pmatrix}p^2\\27\end{pmatrix}$

or $\begin{pmatrix}4+2p-2q\\3+10p+2q\end{pmatrix}=\begin{pmatrix}p^2\\27\end{pmatrix}$

$\Rightarrow \begin{cases} 4+2p-2q=p^2 & \text{—(1)}\\ 3+10p+2q=27 & \text{—(2)}\end{cases}$

fm (2) $2q=(24-10p)$ —(3)

or $q=12-5p$ —(4)

Put the value of 2q in (1)

$4+2p-(24-10p)=p^2$

or $p^2-12p+20=0$

$(p-2)(p-10)=0$

$\Rightarrow p=2$ or $p=10$

fm (4) $q=12-5p$

$p=2,\ q=2$ ✓

$p=10,\ q=-38$

$\therefore$ $p=2$ and $q=2$
or $p=10$ and $q=-38$ ✓

# Vectors

**Example 4** Given that $p = 2i - 5j$ and $q = i - 3j$, find the vector in the direction of $3p - 4q$. $\boxed{S-18/11/Q8(a)}$ [4]

Solution:

$$3p - 4q = 3(2i - 5j) - 4(i - 3j)$$
$$= (2i - 3j)$$

$\therefore$ Magnitude of $3p - 4q = |3p - 4q| = \sqrt{2^2 + (-3)^2} = \sqrt{13}$

$\therefore$ Unit vector in the direction of $3p - 4q$

$$= \frac{1}{\sqrt{13}}(2i - 3j) = \frac{2i - 3j}{\sqrt{13}}$$

**Example 5.** Given that $a = 2i + 3j$, $b = i - 5j$ and $c = 3i + 11j$, find

(i) the exact value of $|a + c|$

Solution $a + c = (2i + 3j) + (3i + 11j) = (5i + 14j)$

$$\therefore |a + c| = \sqrt{5^2 + 14^2} = \sqrt{221}$$

(ii) Find the value of constant $m$ such that $a + mb$ is parallel to $j$.

Solution: $a + mb = (2i + 3j) + m(i - 5j)$
$$= (2 + m)i + (3 - 5m)j \quad \text{——— (1)}$$

Given $a + mb$ is parallel to $j$ or $0i + 1j$.

$\therefore$ coeff of $i$ in (1) $= 0$

$$\Rightarrow 2 + m = 0 \Rightarrow m = -2$$

(iii) Find the value of constant $n$, such that $na - b = c$

Solution: $na - b = c$

$$\Rightarrow n(2i + 3j) - (i - 5j) = (3i + 11j)$$

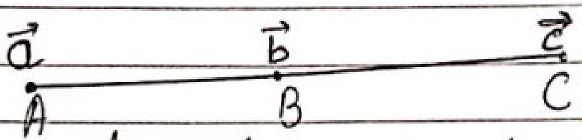
or $(2n - 1)i + (3n + 5)j = (3i + 11j)$ ——— (2)

Equating the coeff. of $i$ in (2) on both the sides.

$$2n - 1 = 3 \Rightarrow n = 2$$

# Vectors

§ To verify that the three points in a plane are collinear, given their position vectors.




Given the position vectors of three points A, B and C are **a**, **b** and **c**.

find vector $\vec{AB} = \vec{b} - \vec{a}$

and $\vec{AC} = \vec{c} - \vec{a}$

Verify that $\vec{AC} = k\,\vec{AB}$ ; $k \in R$, $k \neq 0$

$\Rightarrow \vec{AC}$ is parallel to $\vec{AB}$

and A is the common initial point, proves that the three points A, B and C are collinear.

Example: 6. Given the position vectors of three points A, B and C $a = (2i - j)$, $b = 4i + 3j$ and $c = 3i + j$ respectively, prove that the three points of collinear.

Solution

$\vec{AB} = \vec{b} - \vec{a} = (4i + 3j) - (2i - j) = (2i + 4j)$ — ①

and $\vec{AC} = \vec{c} - \vec{a} = (3i + j) - (2i - j) = (i + 2j)$ — ②

Now $\vec{AB} = (2i + 4j) = 2(i + 2j)$

$= 2\,\vec{AC}$ from ① & ②

as $\vec{AB} = 2\vec{AC}$

vectors $\vec{AB}$ and $\vec{AC}$ are parallel,

but both the vectors have a common initial point,

and $\vec{AB}$ and $\vec{AC}$ lie on the same line or

The three points A, B and C are collinear.

# Vectors



**Example 7:** The vectors a, b and c are such that $a = \begin{pmatrix} 5 \\ -6 \end{pmatrix}$, $b = \begin{pmatrix} 11 \\ -15 \end{pmatrix}$ and $3a + c = b$.

(i) Find c ---[1]

(ii) Find a unit vector in the direction of b. $\boxed{S\text{-}17/23/Q4(a)}$ --[2]

**Solution:** (i) $3a + c = b$

or $c = b - 3a$

$= \begin{pmatrix} 11 \\ -15 \end{pmatrix} - 3\begin{pmatrix} 5 \\ -6 \end{pmatrix}$

or $c = \begin{pmatrix} 11 \\ -15 \end{pmatrix} + \begin{pmatrix} -15 \\ +18 \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$ ✓

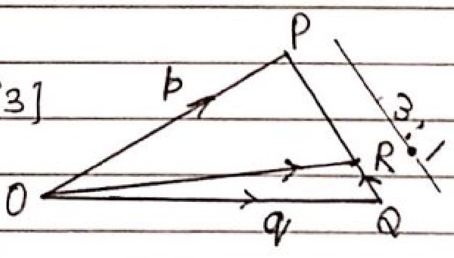
(ii) $b = \begin{pmatrix} 11 \\ -15 \end{pmatrix}$ —— (1)

∴ magnitude of $|b| = \sqrt{(11)^2 + (-15)^2} = \sqrt{121 + 225} = \sqrt{346}$

Now unit vector in the direction of $b = \dfrac{b}{|b|} = \dfrac{1}{\sqrt{346}} \cdot \begin{pmatrix} 11 \\ -15 \end{pmatrix}$ ✓

**Example 8.** In the diagram $\vec{OP} = p$ and $\vec{OQ} = q$. The point R lies on PQ such that $PR = 3RQ$.
Find $\vec{OR}$ in terms of p and q.
Simplify your answer. ----[3]

$\boxed{S\text{-}17/23/Q4(b)}$




**Solution:**

$\vec{QP} = \vec{OP} - \vec{OQ} = p - q$ —— (1)

Now $PR = 3RQ \Rightarrow QR = \frac{1}{4} QP \Rightarrow \vec{QR} = \frac{1}{4}\vec{QP}$

In triangle OQR $= \frac{1}{4}(p - q)$ —— (2)

$\vec{OR} = \vec{OQ} + \vec{QR}$

$= q + \frac{1}{4}(p - q)$ from (2)

$= \frac{1}{4}(3q + p)$

or $\vec{OR} = \frac{3}{4}q + \frac{1}{4}p$ ✓

Example 9. The vector p has a magnitude of 39 units and has the same direction as $-10i+24j$

(i) Find p in terms of i and j. ---[2]

(ii) Find the vector q such that $2p+q$ is parallel to the positive Y-axis and has a magnitude of 12 units. ---[3]

(iii) Hence show that $|q| = k\sqrt{5}$, where k is an integer to be found. ---[2]

S-17/11/Q5(b)

Solution Given a vector $(-10i+24j) = r$ (let)

(i) magnitude of $r = |r| = \sqrt{(-10)^2+(24)^2} = \sqrt{100+576} = \sqrt{676}$
$= 26$ ✓

$\therefore$ Unit Vector along $r = \frac{r}{|r|} = \frac{1}{26}(-10i+24j) = \frac{1}{13}(-5i+12j)$ ✓

Now p is in the direction r and magnitude 39.

$\therefore p = 39 \times$ Unit Vector r

or $p = 39 \times \frac{1}{13}(-5i+12j) = 3(-5i+12j) = (-15i+36j)$ ✓ —①

(ii) Vector q is such that $2p+q$ is parallel to the positive Y-axis and magnitude 12, so the i-component is zero.

or $2p+q = 12j$

or $2(-15i+36j)+q = 12j$ from ①

or $q = 30i-60j$ ✓

(iii) $|q| = \sqrt{(30)^2+(-60)^2} = \sqrt{900+3600} = \sqrt{4500} = \sqrt{900\times5}$

or $|q| = 30\sqrt{5}$ ✓

# Vectors

Example 10. O, P, Q and R are four points such that $\vec{OP} = p$, $\vec{OQ} = q$ and $\vec{OR} = 3q - 2p$.

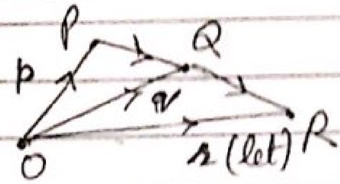
(i) Find in terms of $p$ and $q$, (a) $\vec{PQ}$ (b) $\vec{QR}$ --- [2]

(ii) Justify your answer, what can be said about the positions of the points P, Q, R. --- [2]

(iii) Given that $\vec{OP} = i + 3j$ and $\vec{OQ} = 2i + j$, find the unit vector in the direction of $\vec{OR}$. --- [3]

S-16/21/Q7

(i) Let $\vec{OR} = 3q - 2p = \vec{r}$ (let)




(a) $\vec{PQ} = \vec{OQ} - \vec{OP} = q - p$ —— ①

$\vec{QR} = \vec{OR} - \vec{OQ} = (3q - 2p) - q$

$= (2q - 2p)$

or $\vec{QR} = 2(q - p)$ —— ②

(ii) from ① & ② we find,

$\vec{QR} = 2(q - p) = 2\,\vec{PQ}$

or $\vec{QR} = 2\,\vec{PQ}$

$\therefore$ Vectors $\vec{QR}$ and $\vec{PQ}$ are parallel,

but these two vectors have a common point Q,

$\therefore$ points P, Q, R lie in a line or collinear points

(iii) $\vec{OR} = 3q - 2p$ [Since $\vec{OP} = p = i + 3j$, $\vec{OQ} = q = 2i + j$]

$= 3(2i + j) - 2(i + 3j)$

or $\vec{OR} = (4i - 3j)$ —— ③

and $|\vec{OR}| = \sqrt{4^2 + (-3)^2} = 5$ ✓

$\therefore$ a unit vector along $\vec{OR} = \dfrac{\vec{OR}}{|\vec{OR}|} = \dfrac{1}{5}(4i - 3j)$

$= \left(\dfrac{4}{5}i - \dfrac{3}{5}j\right)$ ✓

**Example 11.** Vectors a, b and c are such that $a=\begin{pmatrix}2\\y\end{pmatrix}$, $b=\begin{pmatrix}1\\3\end{pmatrix}$ and $c=\begin{pmatrix}-5\\5\end{pmatrix}$.

(i) Given that $|a|=|b-c|$, find the possible values of $y$ ---[3]

(ii) Given that $\mu(b+c)+4(b-c)=\lambda(2b-c)$, find the values of $\lambda$ and $\mu$. [S-16/12/Q3] ---[3]

**Solution:**

(i) $a=\begin{pmatrix}2\\y\end{pmatrix} \Rightarrow |a|=\sqrt{2^2+y^2}=\sqrt{4+y^2}$ ——— ①

and $b-c=\begin{pmatrix}1\\3\end{pmatrix}-\begin{pmatrix}-5\\5\end{pmatrix}=\begin{pmatrix}6\\-2\end{pmatrix}$

$\therefore |b-c|=\sqrt{6^2+(-2)^2}=\sqrt{36+4}=\sqrt{40}$ ——— ②

given $|a|=|b-c|$

$\sqrt{4+y^2}=\sqrt{40}$ fm ① & ②

squaring $4+y^2=40$

$y^2=36 \Rightarrow y=\pm 6$ ✓

(ii) $\mu(b+c)+4(b-c)=\lambda(2b-c)$

$\mu\left[\begin{pmatrix}1\\3\end{pmatrix}+\begin{pmatrix}-5\\5\end{pmatrix}\right]+4\left[\begin{pmatrix}1\\3\end{pmatrix}-\begin{pmatrix}-5\\5\end{pmatrix}\right]=\lambda\left[2\begin{pmatrix}1\\3\end{pmatrix}-\begin{pmatrix}-5\\5\end{pmatrix}\right]$

$\mu\begin{pmatrix}-4\\8\end{pmatrix}+4\begin{pmatrix}6\\-2\end{pmatrix}=\lambda\begin{pmatrix}7\\1\end{pmatrix}$

or $\begin{pmatrix}-4\mu+24\\8\mu-8\end{pmatrix}=\begin{pmatrix}7\lambda\\\lambda\end{pmatrix}$

Equating the x-component & y-component

$\left.\begin{aligned}-4\mu+24&=7\lambda\\ \text{or}\quad 8\mu-8&=\lambda\end{aligned}\right\}$ or $7\lambda+4\mu=24$ ——— ③

$\lambda-8\mu=-8$ ——— ④

Solving ③ & ④

$\mu=4/3$ ✓ and $\lambda=\frac{8}{3}$ ✓

**Example 12:** The four points O, A, B and C are such that $\vec{OA}=5a$, $\vec{OB}=15b$ and $\vec{OC}=24b-3a$, Show that B lies on line AC. [S-15/21/Q7(a)] -[3]

**Solution:** $\vec{AB}=\vec{OB}-\vec{OA}=15\vec{b}-5\vec{a}=5(3b-a)$ ——— ①

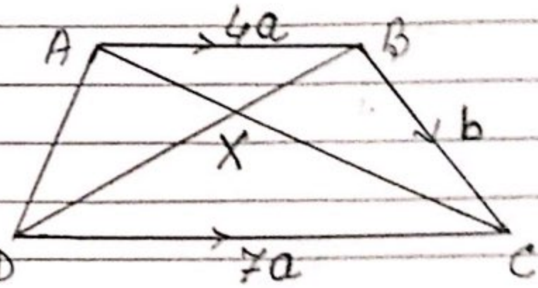
and $\vec{AC}=\vec{OC}-\vec{OA}=(24b-3a)-5a=24b-8a=8(3b-a)$ — ②

fm ① & ② $\vec{AB}=\frac{5}{8}\vec{AC} \Rightarrow$ AB and AC are parallel, but A is a common point.

Collinear points $\therefore$ A, B, C lie in a line or B lies on line AC.

# Vectors

**Example 13:** In the diagram $\vec{AB} = 4a$, $\vec{BC} = b$ and $\vec{DC} = 7a$. The lines AC and DB intersect at the point X.
Find in terms of a and b.




(a) $\vec{DB}$ --- [1]

(b) $\vec{DA}$ --- [1]

Given that $\vec{AX} = \lambda\vec{AC}$, find in terms of a, b and $\lambda$,

(c) $\vec{AX}$ --- [1]

(d) $\vec{DX}$ --- [2]

Given that $\vec{DX} = \mu\vec{DB}$

(e) find the value of $\lambda$ and of $\mu$. SP-2002 Q9 --- [4]

**Solution** (a) $\vec{DB} = \vec{DC} + \vec{CB}$ (In $\Delta DBC$)
$= 7a + (-b)$ $(\because \vec{BC} = \vec{b})$
$= (7a - b)$ ✓ —— ①

(b) $\vec{DA} + \vec{AB} = \vec{DB}$ (In $\Delta DAB$)
$\vec{DA} = \vec{DB} - \vec{AB}$
$= (7a - b) - 4a$
$= 3a - b$ —— ②

(c) $\vec{DA} + \vec{AC} = \vec{DC}$ (In $\Delta DAC$)
$\vec{AC} = \vec{DC} - \vec{DA}$
$= 7a - (3a - b)$ fr ②
$= (4a + b)$ —— ③

Now $\vec{AX} = \lambda\vec{AC}$ (Given)
or $\vec{AX} = \lambda(4a + b)$ fm ③

(d) $\vec{DX} = \vec{DA} + \vec{AX}$ (In $\Delta DAX$)
$= (3a - b) + \lambda(4a + b)$ fm ② & ③ —— ④

(e) Given $\vec{DX} = \mu\vec{DB}$
or $(3a - b) + \lambda(4a + b) = \mu(7a - b)$ from ④ & ①
or $(3 + 4\lambda)a + (\lambda - 1)b = 7\mu a - \mu b$ —— ⑤

fm ⑤ Equating the coefficients of vectors a & b,
$3 + 4\lambda = 7\mu$ & $\lambda - 1 = -\mu$

or $\begin{cases} 4\lambda - 7\mu = -3 & \text{—— ⑥} \\ \lambda + \mu = 1 & \text{—— ⑦} \end{cases}$

Solving ⑥ and ⑦

$\lambda = \frac{4}{11}$ and $\mu = \frac{7}{11}$ ✓

(IG-0606 Maths)

# Vectors



**Example 14.** The diagram shows the quadrilateral OABC, such that $\vec{OA} = \mathbf{a}$, $\vec{OB} = \mathbf{b}$ and $\vec{OC} = \mathbf{c}$. It is given that AM : MC = 2 : 1 and OM : MB = 3 : 2

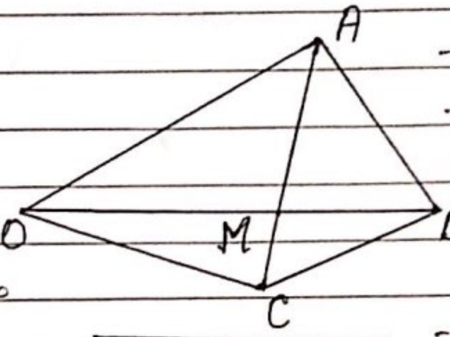
(i) Find $\vec{AC}$ in terms of **a** and **c**. --[1]

(ii) Find $\vec{OM}$ in terms of **a** and **c** --[2]

(iii) Find $\vec{OM}$ in terms of **b** --[1]

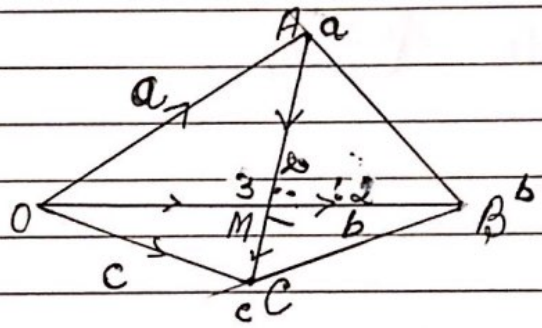
(iv) Find $5\mathbf{a} + 10\mathbf{c}$ in terms of **b** [2]

(v) Find $\vec{AB}$ in terms of **a** and **c**, giving your answer in its simplest form. --[2]




M-18/12/Q6

**Solution** (i) $\vec{AC} = c - a$ —— ①




(ii) $\vec{OM} + \vec{MC} = \vec{OC}$

$\therefore \vec{OM} = \vec{OC} - \vec{MC}$ $\left(\because AM : MC = 2:1,\ \vec{MC} = \frac{1}{3}\vec{AC}\right)$

or $\vec{OM} = \vec{OC} - \frac{1}{3}\vec{AC}$

$= c - \frac{1}{3}(c - a)$ fm ①

$= \left(\frac{2}{3}c + \frac{1}{3}a\right)$ —— ②

(iii) OM : MB = 3 : 2

$\vec{OM} = \frac{3}{5}\vec{OB} = \frac{3}{5}b$ ✓ —— ③

(iv) $5a + 10\vec{c} = 5(a + 2c)$

$= 5 \times 3\vec{OM}$ [fm ② $(a + 2c) = 3\vec{OM}$]

$= 15 \times \frac{3}{5}b$ [$\vec{OM} = \frac{3}{5}b$ fm ③

$= 9b$ —— ④

(v) $\vec{AB} = b - a$

$= \frac{5}{9}(a + 2c) - a$

$= \left(-\frac{4}{9}a + \frac{10}{9}c\right)$ ✓

fm ② & ③ [$\vec{OM} = \frac{3}{5}b = \frac{1}{3}(a + 2c)$

$\Rightarrow b = \frac{5}{9}(a + 2c)$

# Vectors



Example 15. In the diagram $\vec{OA} = 2\mathbf{a}$ and $\vec{OB} = 5\mathbf{b}$. The point M is the mid point of OA and the point N lies on OB such that ON : NB is 3 : 2.

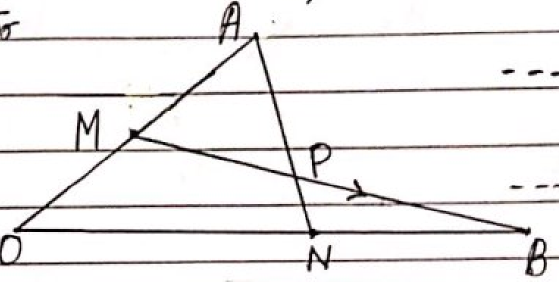
(i) Find an expression for the vector $\vec{MB}$ in terms of $\lambda$, a and b. ---[2]

The point P lies on AN such that $\vec{AP} = \lambda\vec{AN}$

(ii) Find an expression for the vector $\vec{AP}$ in terms of $\lambda$, a and b. ---[2]

(iii) Find an expression for the vector $\vec{MP}$ in terms of $\lambda$, a and b. ---[2]

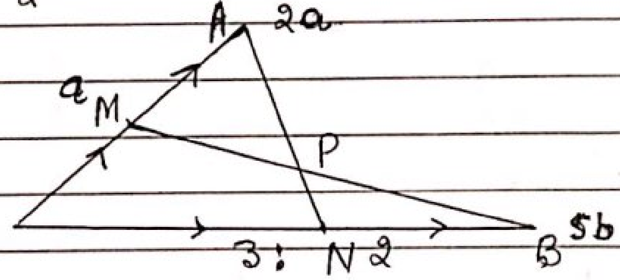
(iv) Given that M, P, B are collinear, find the value of $\lambda$. ---[4]




Solution: Position Vector M, $\vec{OM} = \frac{1}{2}\vec{OA} = \frac{\mathbf{a}}{2}$     [S-14/23/Q11]

(i) $\vec{MB} = \vec{OB} - \vec{OM}$

$= (5\mathbf{b} - \mathbf{a})$ ✓ —— ①




(ii) ON : NB = 3 : 2

$\Rightarrow \vec{ON} = \frac{3}{5}\vec{OB}$

or $\vec{ON} = \frac{3}{5} \times 5\vec{b} = 3\mathbf{b}$ —— ②

Now $\vec{AP} = \lambda\vec{AN}$

$= \lambda(\vec{ON} - \vec{OA})$ (In $\Delta OAN$)

$= \lambda(3\mathbf{b} - 2\mathbf{a})$ ✓ —— ③ ✓

(iii) $\vec{MP} = \vec{MA} + \vec{AP}$ (In $\Delta MAP$)

$= \mathbf{a} + \lambda(3\mathbf{b} - 2\mathbf{a})$ —— ④ ✓

(iv) Given M, P and B are collinear points

$\therefore \vec{MP} = \mu\vec{MB}$ (let)

or $\mathbf{a} + \lambda(3\mathbf{b} - 2\mathbf{a}) = \mu(5\mathbf{b} - \mathbf{a})$ from ① & ④

or $(1 - 2\lambda)\mathbf{a} + 3\lambda\mathbf{b} = -\mu\mathbf{a} + 5\mu\mathbf{b}$

Equating the coefficients of a & b

$1 - 2\lambda = -\mu \Rightarrow \mu = (2\lambda - 1)$ —— ⑤

and $5\mu = 3\lambda \Rightarrow \mu = \frac{3}{5}\lambda$ —— ⑥

from ⑤ & ⑥

$2\lambda - 1 = \frac{3}{5}\lambda \Rightarrow \frac{7}{5}\lambda = 1 \Rightarrow \lambda = \frac{5}{7}$ ✓

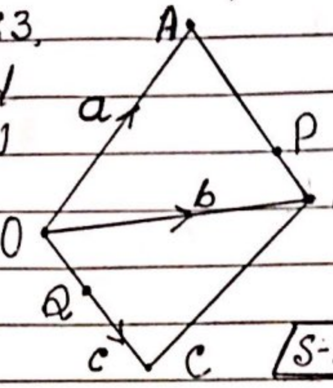
# Vectors

Example 16. The fig. shows points A, B and C with position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively, relative to an origin O. The point P lies on AB such that AP : AB = 3 : 4. The point Q lies on OC such that OQ : QC = 2 : 3.

(i) Express $\vec{AP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$ and hence show that $\vec{OP} = \frac{1}{4}(\mathbf{a}+3\mathbf{b})$ --- [3]

(ii) Find $\vec{PQ}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$. --- [3]

(iii) Given that $5\vec{PQ} = 6\vec{BC}$, find $\mathbf{c}$ in terms of $\mathbf{a}$ and $\mathbf{b}$ --- [2]




S-13/11/Q9

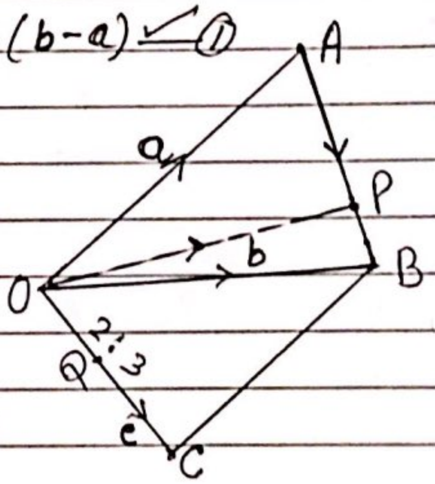
Solution: $\vec{AB} = \mathbf{b} - \mathbf{a}$

(i) as AP : AB = 3 : 4 $\Rightarrow \vec{AP} = \frac{3}{4}\vec{AB} = \frac{3}{4}(\mathbf{b}-\mathbf{a})$ ✓ ①

Now $\vec{OP} = \vec{OA} + \vec{AP}$ (In $\Delta$ OAP)

$= \mathbf{a} + \frac{3}{4}(\mathbf{b}-\mathbf{a})$ frm ①

$= \frac{1}{4}(\mathbf{a}+3\mathbf{b})$ ✓ ②




(ii) OQ : QC = 2 : 3

$\Rightarrow \vec{OQ} = \frac{2}{5}\vec{OC} = \frac{2}{5}\mathbf{c}$ ③

Now $\vec{PQ} = \vec{OQ} - \vec{OP}$

$= \frac{2}{5}\mathbf{c} - \frac{1}{4}(\mathbf{a}+3\mathbf{b})$ ✓ frm ② & ③ ④

(iii) $\vec{BC} = \mathbf{c} - \mathbf{b}$ ... ⑤

Given $5\vec{PQ} = 6\vec{BC}$

$\Rightarrow 5\left[\frac{2}{5}\mathbf{c} - \frac{1}{4}(\mathbf{a}+3\mathbf{b})\right] = 6(\mathbf{c}-\mathbf{b})$ frm ④ and ⑤

$\Rightarrow 2\mathbf{c} - \frac{5}{4}(\mathbf{a}+3\mathbf{b}) = 6\mathbf{c} - 6\mathbf{b}$

$\Rightarrow 4\mathbf{c} = 6\mathbf{b} - \frac{5}{4}(\mathbf{a}+3\mathbf{b})$

$= \frac{9\mathbf{b} - 5\mathbf{a}}{4}$

$\Rightarrow \mathbf{c} = \left(\frac{9\mathbf{b}-5\mathbf{a}}{16}\right)$ ✓

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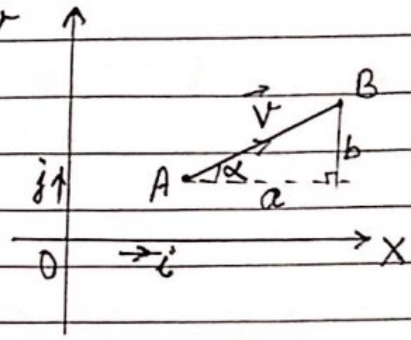
# Vectors



## Compose and resolve velocities

§ Given the velocity of particle $P = v$

$\vec{v}$ is inclined at an angle $\alpha$ with the positive direction of $x$-axis




Let the $x$-component of $\vec{v} = ai$

$$\frac{a}{|\vec{v}|} = \cos\alpha$$

$a = |\vec{v}|\cos\alpha$ and $y$-component of $\vec{v}$, $b = |\vec{v}|\sin\alpha$

and $\boxed{\vec{v} = ai + bj}$

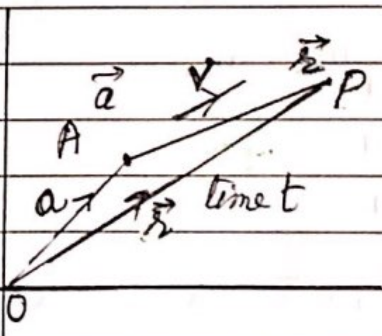
and $|\vec{v}| = \sqrt{a^2+b^2} = \text{speed} = \frac{\text{distance}}{\text{time}}$.

$$\text{velocity } \vec{v} = \frac{\text{displacement}}{\text{time}}$$

§ Position Vector : $\boxed{\vec{r} = \vec{a} + \vec{v}t}$

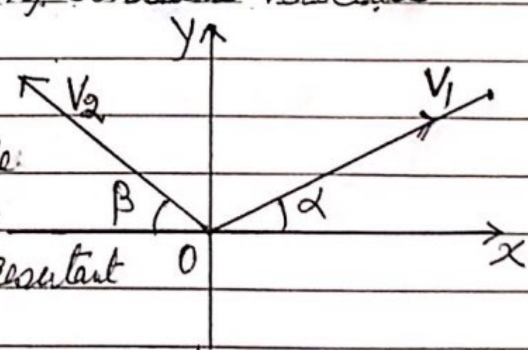
here $a$ is the position vector of initial position,

$r$ is the position vector of terminal point, after the particle moves with a velocity $v$ for $t$ seconds (unit of time).




§ To find the Resultant of two (or more) constant velocities acting upon a particle:

Let R is the resultant of velocities $V_1$ and $V_2$ and directions as shown in the figure.




Let $V_x$ and $V_y$ are components of the resultant along $x$-axis and $y$-axis,

$V_x = (V_1\cos\alpha - V_2\cos\beta)$ and $V_y = (V_1\sin\alpha + V_2\sin\beta)$

Then Resultant velocity $V = \sqrt{V_x^2 + V_y^2}$ ✓

$$\tan\theta = \frac{V_y}{V_x}$$

and if $V$ is inclined at an angle $\theta$, with the +ve direction of $x$-axis

## Compose and Resolve Velocities

Example 17. (a) The vector $v$ has a magnitude of $3\sqrt{5}$ units and has the same direction as $i-2j$. Find $v$ giving your answer in the form $ai+bj$, where $a$ and $b$ are integers. ---[2]

(b) The velocity vector $W$ makes an angle of $30°$ with the positive $x$-axis such that $|W|=2$. Find $W$ giving your answer in the form $\sqrt{c}\,i+dj$, where $c$ and $d$ are integers. ---[2]

S-17/12/Q3

Solution: Direction of $v$ is $i-2j = r$ (let)

$$|r| = \sqrt{1^2+(-2)^2} = \sqrt{5}$$

Unit vector along $r = \frac{r}{|r|} = \frac{1}{\sqrt{5}}(i-2j)$ —— ①

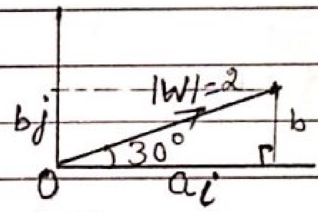
Required vector $v$ is parallel to vector $r$ and magnitude of $v$ is $3\sqrt{5}$

$\therefore\ v = 3\sqrt{5} \times \frac{1}{\sqrt{5}}(i-2j)$ fr ①

or $v = (3i-6j)$ ✓

(b) $|W| = 2$

Let $W = ai+bj$ —— ①




$a = |W|\cos 30° = 2\cos 30° = 2\times\frac{\sqrt{3}}{2} = \sqrt{3}$ —— ②

$b = |W|\sin 30° = 2\times\frac{1}{2} = 1$ —— ③

fr ① $W = \sqrt{3}i + 1\cdot j$ fr ①, ② & ③

or $W = \sqrt{3}i + j$ ✓

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# Vectors



## Compose and Resolve constant velocities

Example 18. The initial position vectors of particles A and B, relative to a fixed point O, are $(2i+4j)$ and $(10i+14j)$ respectively. Particles A and B start moving at the same time. A moves with constant velocity $i+j$ and B moves with constant velocity $-2i-3j$. Find.

(i) the position vector of A after t seconds. ---[1]

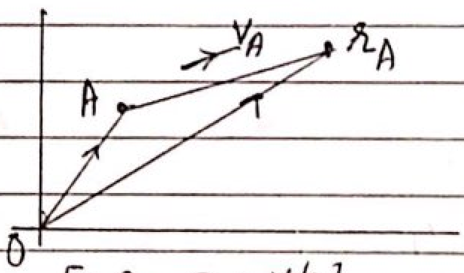
(ii) the position vector of B after t seconds. ---[1]

It is given that X is the distance between A and B after t seconds.

(iii) Show that $X^2=(8-3t)^2+(10-4t)^2$ ---[3]

(iv) Find the value of t for which $(8-3t)^2+(10-4t)^2$ has a stationary value and the corresponding value of X. [W-17/21/Q10] ---[4]

Solution: Given $\vec{OA}=2i+4j$ and $\vec{OB}=(10i+4j)$ and $V_A=(i+j)$ and $V_B=(-2i-3j)$




$\therefore$ Position Vector of A after t sec.

$[\because r = a+vt]$

(i) $r_A=(2i+4j)+t(i+j)$

(ii) and $r_B=(10i+14j)+t(-2i-3j)$

(iii) $r_B-r_A=[(10i+14j)+t(-2i-3j)]-[(2i+4j)+t(i+j)]$

or $r_B-r_A=(8-3t)i+(10-4t)k$ —— (1)

The distance X between A and B after t second,

$X=|r_B-r_A|=\sqrt{(8-3t)^2+(10-4t)^2}$ fr(1)

or $X^2=(8-3t)^2+(10-4t)^2$ —— (2)

(ii) Now $\frac{d}{dt}X^2=2(8-3t)(-3)+2(10-4t)(-4)$

$=-128+50t$ —— (3)

Now for $X^2$ to have a stationary value, $\frac{d}{dt}X^2=0 \Rightarrow -128+50t=0$

$\Rightarrow t=2.56$ ✓

$\therefore$ Value of X at stationary point $=\sqrt{(8-3\times2.56)^2+(10-4\times2.56)^2}$

$=0.4$ ✓

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# Vectors

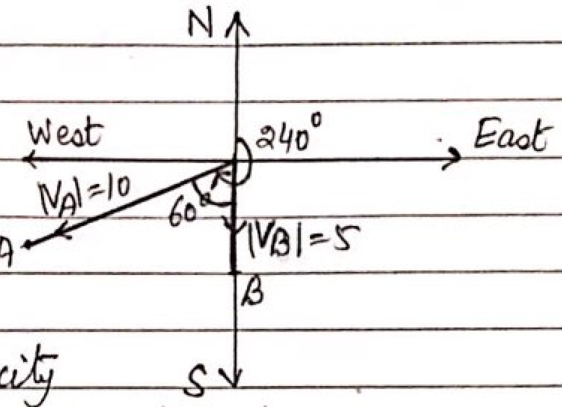


## Compose and resolve constant velocities.

**Example 19.** Calculate the magnitude and bearing of the resultant velocity of $10 ms^{-1}$ on a bearing of $240°$ and $5 ms^{-1}$ due south.

[M-17/22/Q7(a)] ... [5]

**Solution:** Let $|V_A| = 10 ms^{-1}$ is the velocity with bearing of $240°$.
or $V_A$ makes an angle $60°$ with South, and
$|V_B| = 5 ms^{-1}$ in the south direction.




Now let $V$ is the resultant Velocity inclined at an angle '$\theta$' with south toward west.

Resolve part of $V_A$ along 'South' $= 10\cos 60 = 10 \times \frac{1}{2} = 5$ ✓

Resolve part of $V_A$ along West $= 10 \sin 60 = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}$ ✓

$\therefore$ Component of Resolve part of $V$ along South $V_{South} = |V_B| + 10\cos 60$
$= 5 + 5 = 10$ ✓

and Component of Resolve part of $V$ along West $V_{West} = 5\sqrt{3} + 0 = 5\sqrt{3}$ ✓

$\therefore$ magnitude of $V = \sqrt{V_{South}^2 + V_{West}^2} = \sqrt{10^2 + (5\sqrt{3})^2} = \sqrt{175}$

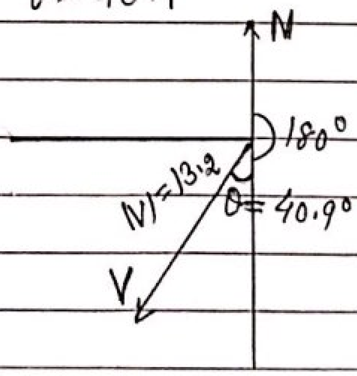
$\therefore |V| = 13.2$ ✓✓

Let the Result makes an angle '$\theta$' with south to west,

$$\tan\theta = \frac{V_{West}}{V_{South}} = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$$

$$\theta = 40.9°$$

$\therefore$ Bearing of resultant Velocity V.
$= 180 + \theta$
$= 180 + 40.9$
$= 220.9°$



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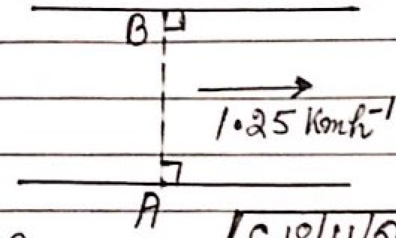
# Vectors



## To compose and resolve constant velocities.

**Example 20.** A river flows between parallel banks at a speed of $1.25\ \text{Km h}^{-1}$. A boy standing at point A on one bank sends a toy boat across the river to his father standing directly opposite at B. The toy boat, which can travel at $v\ \text{Km h}^{-1}$ in still water, crosses the river with resultant speed $2.73\ \text{Km h}^{-1}$ along line AB.




(i) Calculate the value of $v$ --- [2]

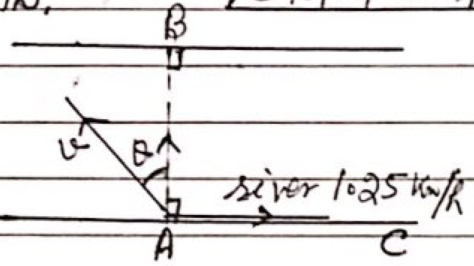
The direction in which the boy points the boat makes an angle $\theta$ with the line AB.

[S-18/11/Q8(b)]

(ii) Find the value of $\theta$.

**Solution:** AB is perpendicular to AC.

river flows along AC with speed $1.25\ \text{Km h}^{-1}$




(i) Component of velocity boat along line AC = $V\sin\theta$ opposite to river

$$\therefore V\sin\theta = 1.25 \quad \text{---} (1)$$

Component of $v$ along AB = $V\cos\theta = 2.73$ --- (2) Given.

Sq. and add (1) & (2) $V^2(\sin^2\theta + \cos^2\theta) = 1.25^2 + 2.73^2$

or $v^2 = 9.01$

$v = 3$

(ii) Now $\tan\theta = \dfrac{V\sin\theta}{V\cos\theta} = \dfrac{1.25}{2.73}$ from (1) & (2)

$= 0.4578$

$\therefore \theta = \tan^{-1} 0.4578 = 24.6°$

# Vectors



## To compose and resolve constant velocities.

**Example 21.** The initial position vectors of particles A and B, relative to a fixed point O, are $i+5j$ and $qi-15j$ respectively. A and B start moving at the same time. A moves with velocity $pi-3j$ and B moves with velocity $3i-j$.

(i) Given that A travels with a speed of $5\,ms^{-1}$, find the value of positive constant $p$. ---[1]

(ii) Find the direction of motion of B as a bearing correct to the nearest degree. ---[2]

(iii) State the position vector of A after $t$ seconds. ---[1]

(iv) State the position vector of B after $t$ seconds. ---[1]

(v) Find the time taken until A and B meet. ---[2]

(vi) Find the position vector of the point where A and B meet. ---[1]

(vii) Find the value of the constant $q$. $\boxed{W-16/23/Q9}$ ---[1]

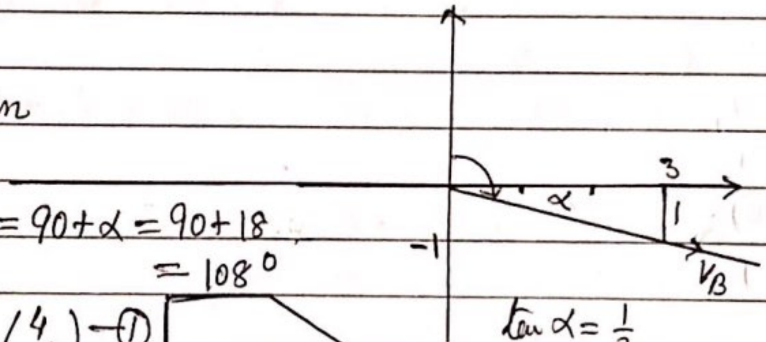
**Solution** (i) $V_A = pi - 3j$

$|V_A| = \sqrt{p^2+9} = 5$ given

$\Rightarrow p = 4$ ✓

(ii) $V_B = 3i - j$; Bearing of $V_B = 90 + \alpha = 90 + 18$

$= 108^\circ$




$\tan\alpha = \frac{1}{3}$

$\alpha = \tan^{-1}\frac{1}{3}$

$= 18.4^\circ$

(iii) $r_A = a + V_A t = \begin{pmatrix}1\\5\end{pmatrix} + t\begin{pmatrix}4\\-3\end{pmatrix}$ — ①

(iv) $r_B = \begin{pmatrix}q\\-15\end{pmatrix} + t\begin{pmatrix}3\\-1\end{pmatrix}$ — ②

(v) $r_A = r_B \Rightarrow \begin{pmatrix}1\\5\end{pmatrix} + t\begin{pmatrix}4\\-3\end{pmatrix} = \begin{pmatrix}q\\-15\end{pmatrix} + t\begin{pmatrix}3\\-1\end{pmatrix}$

Equating the y-components

$5 - 3t = -15 - t \Rightarrow t = 10$ ✓

(vi) fr ① at $t=10$, $r_A = \begin{pmatrix}1\\5\end{pmatrix} + 10\begin{pmatrix}4\\-3\end{pmatrix}$

$= \begin{pmatrix}41\\-25\end{pmatrix}$ ✓

(vii) fr ② at $t=10$

$\begin{pmatrix}q\\-15\end{pmatrix} + 10\begin{pmatrix}3\\-1\end{pmatrix} = \begin{pmatrix}41\\-25\end{pmatrix} \Rightarrow q + 30 = 41 \Rightarrow q = 11$ ✓

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# Vectors

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## To compose and resolve constant velocities.

Example 22. At 1200 hours a ship leaves a port 'P' and travels with 26 $kmh^{-1}$ in the direction of $5i+12j$.

(i) Show that the velocity of the ship is $(10i+24j)\ kmh^{-1}$ ---[2]

(ii) Write down the position vector of the ship, relative to P', at 1600 hours. --[1]

(iii) Find the position vector of the ship, relative to P, t hours after 1600 hours. $\boxed{S\text{-}14/12/Q10}$ ---[2]

Solution: direction of motion $= 5i+12j$ $\qquad \because |5i+12j| = \sqrt{5^2+12^2} = 13$

$\therefore$ Unit vector along the motion $= \frac{1}{13}(5i+12j)$

(i) Given speed $= 26\ kmh^{-1}$

$\therefore$ Velocity $= 26 \times \frac{1}{13}(5i+12j) = 2(5i+24j)$

$= (10i+24j)$ ✓ — ①

(ii) at 1200 hours ship is at P,
at 1600 hours after P, time $t = 4$ hrs

$$r = a + tv$$

$$\therefore\ r = 0 + 4(10i+24j) = (40i+96j)$$ ✓ — ②

(iii) After t hours. $r = a + tv$

$r = (40i+96j) + t(10i+24j)$ fr ② & ①

# Vectors

## To compose and resolve constant Velocities.

**Example 23.** At time $t=0$, boat $A$ leaves the origin $O$ with velocity $(2i+4j)$ Km $h^{-1}$. Also at $t=0$ boat $B$ leaves the point with position vector $(-21i+22j)$ and travels with velocity $(5i+3j)$ Km $h^{-1}$.

(i) Write down the position vectors of boats $A$ and $B$ after $t$ hours. ---[2]

(ii) Show that $A$ and $B$ are 25 Km apart when $t=2$. ---[3]

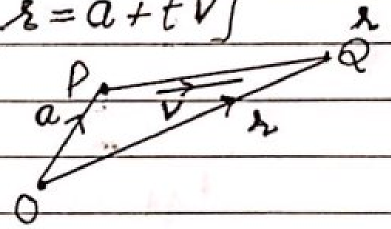
(iii) Find the length of time for which $A$ and $B$ are less than 25 Km apart.

[W-13/23/Q11] ---[5]

**Solution** (i) $r_A = 0 + (2i+4j)t = (2i+4j)t$ ✓

$r_B = (-21i+22j) + (5i+3j)t$

$[r = a + tv]$




(ii) $r_A r_B = r_B - r_A$

$= [(-21i+22j)+(5i+3j)t] - [(2i+4j)t]$

$= (-21+3t)i + (22-t)j$ —— ①

Now at $t=2$, displacement $= (-15i+20j)$

hence at $t=2$ distance between the boat $A$ and $B = |-15i+20j|$

$= \sqrt{(-15)^2+(20)^2}$

$= 25$ ✓

(iii) Distance between the boats $A$ and $B$ at time $t$ is less that 25 Km.

from ①

$|(-21+3t)i+(22-t)j| < 25$

or $(-21+3t)^2 + (22-t)^2 < 25^2$

or $t^2 - 17t + 30 < 0$

$(t-15)(t-2) < 0$

$2 < t < 15$

$a > 0$

$|x| < a$

$\Rightarrow x^2 < a^2$

$\therefore$ Required length of time $= 15 - 2 = 13$ hours.