1.07.20

M2

Mechanics-2

Equilibrium of a rigid body Notes

Suresh Goel
(Director)
Alliance World School,
Noida. Delhi. NCR.
INDIA

M.2

Equilibrium of a rigid body.

Moments of forces

Notes

- **Moment of a force:**

The moment of a force 'F' about a point 'O' is the product of the magnitude of the the force and the perpendicular distance of the line of action of the force from the point O.

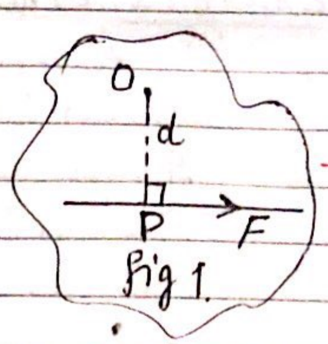


fig 1.

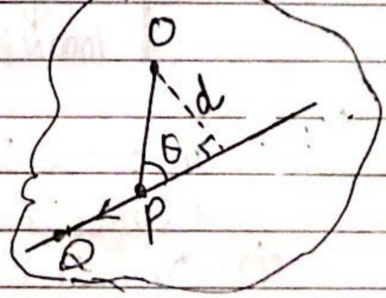
fig1 (i) Moment of force F about the point O is $= \underline{F \times d}$ ✓ where d is the perpendicular distance of O from F's line of action.

fig 2 (ii)

Moment of Force 'F' along $\vec{PQ}$, here angle $OPQ = \theta$.

Moment of Force about 'O' $= F \times d$

$= \underline{F \times OP \sin\theta}$.



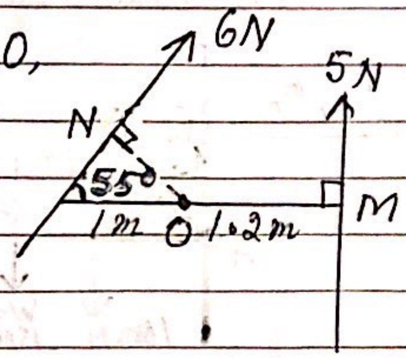
- Unit of Moment of force is Nm

- The moment of force creates a turing effect in the rigit body.

fig 1 → It is anticlockwise moment (+)

fig 2 → Clockwise moment, (−)

Example 1. Find the sum of moments about O, of the forces shown in the figure.



$ON = 1 \times \sin 55°$

Sum $= -6 \times ON + 5 \times OM$

$= -6 \times \sin 55 + 5 \times 1.2$

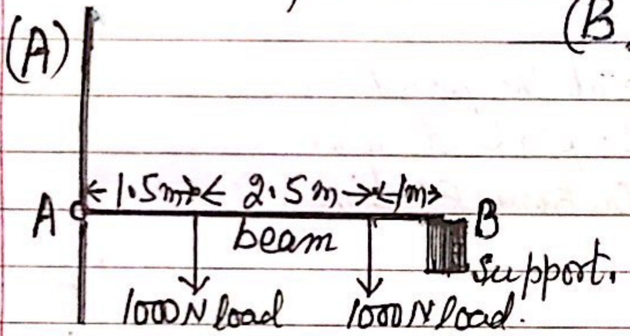
$= -6 \times 0.819 + 6$

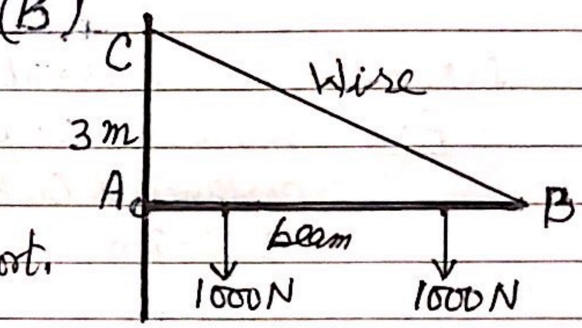
$= -4.91 + 6$

$= +1.09$

∴ The sum of moment is 1.09 Nm anticlockwise.

Example 2: Overhead cables for tramway are supported by uniform, rigid, horizontal beams of weights 1500 N and lengths 5 m. Each beam, AB, is freely pivoted at one end A and supports two cables of which may be modelled by vertical loads, each of 1000 N, one 1.5 m from A and the other at 1 m from B.





In one situation, the beam is held in equilibrium by resting on a small horizontal support at B, as show in figure (A)

(i) Draw a diagram showing all the forces acting of beam AB. Show that the vertical force acting on the beam B is 1850 N.

In other situation, the beam is supported by a wire, instead of the support at B. The wire is light, attached at one end to the beam at B and at the other to the point C which is 3 m vertically above A, as shown in figure (B),

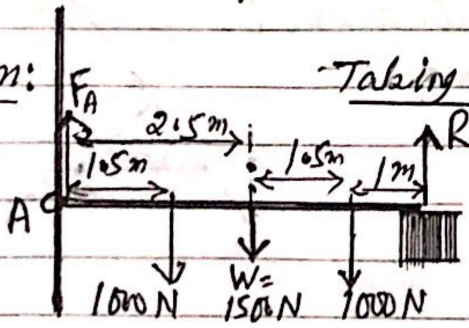
(ii) Calculate the tension in the wire.

(iii) Find the magnitude and direction of the force on the beam at A.

(MEI)

Solution:

(i)



Let the vertical force at B is R N.

Taking moment about A:

$$R \times 5 = 1000 \times 1.5 + 1500 \times 2.5 + 1000 \times 4$$

$$\Rightarrow 5R = 9250$$

$$R = 1850\ N.$$

$\therefore$ Vertical force at B = $\underline{1850\ N}$ ✓

(Continued →)

M-2

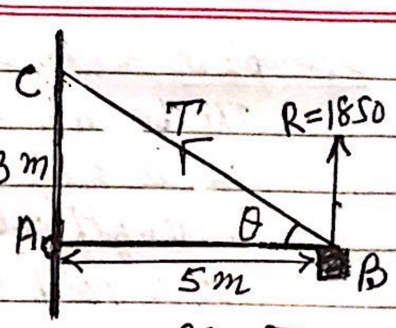
(Continued →)

2(ii) Let the tension in the wire $= T$ N

Let the wire is inclined at angle 'θ'

Angle $ABC = \theta$

Vertical component of $T = T\sin\theta$



$T\sin\theta = R$

$BC = \sqrt{3^2+5^2}$

$\sin\theta = \frac{3}{\sqrt{3^2+5^2}} = \frac{3}{\sqrt{3}}$.

$T \times \frac{3}{\sqrt{34}} = 1850$

$\Rightarrow T = \frac{1850 \times \sqrt{34}}{3}$

$T = 3595.75$

or $T = \underline{3596\ N}$ ✓

(iii) Vertical component of Force at $A = F_y$

$F_y + R = 1000 + 1500 + 1000$

$F_y = 3500 - 1850 = 1650$ N ✓ ($R = 1850$ N)

and the Horizontal component of $F = F_x$

$F_x = T\cos\theta$

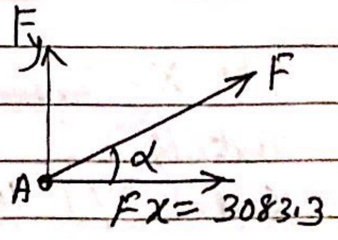
$= 3596 \times \frac{5}{\sqrt{34}} = 3083.3$ N ✓

$\therefore F = \sqrt{1650^2 + 3083.3^2} = \underline{3497\ N}$ ✓

Let the force F at is inclined at an angle α to the horizontal.

$\tan\alpha = F_y/F_x = \frac{1650}{3083.3}$

, $\alpha = \underline{28.15^\circ}$ above the horizontal,



Force at $A = \underline{3497\ N}$ ✓

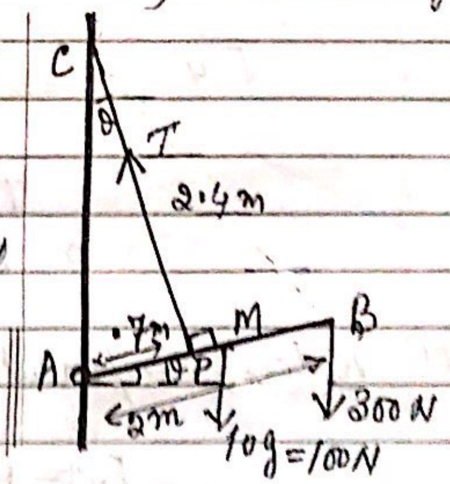
Note: If a body is in equlibrium the sum of the moments of the forces acting on it, about a point is zero.

Example 3: A uniform beam AB has length 2m and mass 10 kg. The beam is hinged at A to a fixed point on a vertical wall, and is held in a fixed position by a light inextensible string of length 2.4 m. One end of the string is attached to the beam at a point 0.7 m from A. The other end of the string is attached to the wall at a point vertically above the hinge. The string is at right angle to AB. The beam carries a load of weight 300 N at B.

(i) Find the tension in the string.

The components of the force exerted by the hinge on the beam are X N horizontally away from the wall and Y N vertically downwards.

(ii) Find the values of X and Y.



[W-07/05/Q3]

Solution: Let the beam is inclined to the horizontal at an angle 'θ'.

(i) angle $ACP = \theta \Rightarrow \tan\theta = 0.7/2.4$, $\sin\theta = \frac{0.7}{2.5}$ and $\cos\theta = \frac{2.4}{2.5}$

Let the tension in string = 'T' N

Taking moment about the point A,

$0.7 \times T = 100\cos\theta \times 1 + 2\cos\theta \times 300$

$\Rightarrow 0.7T = 700 \times \frac{2.4}{2.5}$

$\Rightarrow T = 960 N.$ ✓

(ii) $X = T\sin\theta = 960 \times \frac{0.7}{2.5} = 269N$ ✓ $\rightarrow X = 269 N$ ✓

Vertically

and, $Y + 100 + 300 = T\cos\theta$

$Y = T\cos\theta - 400$

$= 960 \times \frac{2.4}{2.5} - 400$

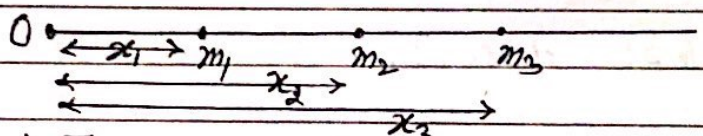
$\therefore Y = 921.6 - 400 = 521.6$

or $Y = 522 N$ ✓

Equilibrium of a rigid body
Centre of Mass
Notes

- **Centre of mass:** This is the point at which the whole mass of the body can be considered to be concentrated. or It is the point through which the line of action of the weight of the body always passes, in whatever postion the body is placed. (or it is the balance point of a body with size and shape)

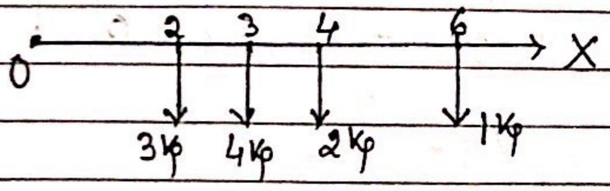
- Centre of mass of the particles of masses m_1, m_2, m_3 --- in a straight line at position x_1, x_2, x_3, --- from one end point O, then $\bar{x}$ is the distance of the centre of mass from O, such that:



$$(m_1+m_2+m_3 ---)\,\bar{x} = m_1x_1 + m_2x_2 + m_3x_3 + ---$$

$$\text{or} \quad M\cdot\bar{x} = \sum_{i=1}^{n} m_i x_i \quad \left[M = \sum_{i=1}^{n} m_i \right].$$

Example 4: Find the position of the centre of mass of four particles of masses 1 kg, 4 kg, 3 kg and 2 kg placed on the x-axis at the points (6,0), (3,0), (2,0) and (4,0) respectively.



Let the 'Centre of mass' G is at $(\bar{x}, 0)$

Sum of all Masses $M = \Sigma m_i = 3+4+2+1 = 10$ kg

$$M\cdot\Sigma\bar{x} = \Sigma m_i x_i$$

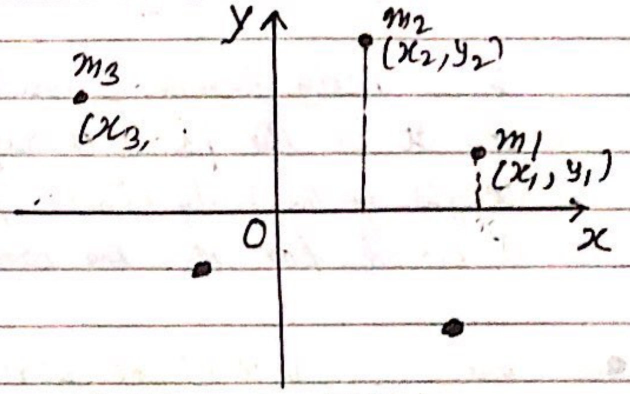
$$\bar{x} = \frac{\Sigma m_i x_i}{M} = \frac{2\times3 + 3\times4 + 4\times2 + 6\times1}{10}$$

$$= 32/10 = 3.2$$

∴ The centre of mass is at $\underline{(3.2, 0)}$ on X-axis ✓

Centre of mass for two dimentral bodies:

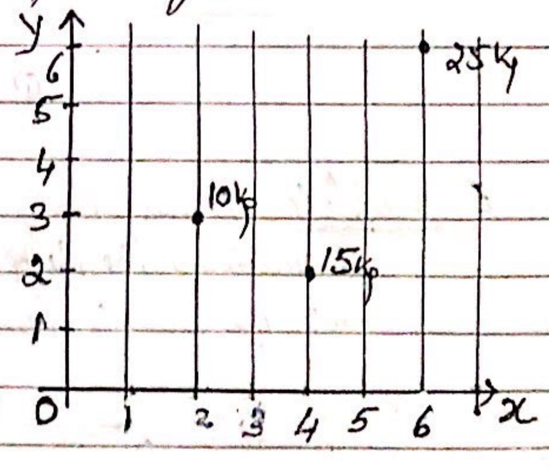
Let the masses m_1, m_2, m_3, --- are placed in a plane, at locations (x_1, y_1), (x_2, y_2), (x_3, y_3)----



Then their centre of Mass will be at $(\bar{x}, \bar{y})$

$$\bar{x} = \frac{\Sigma m_i x_i}{\Sigma m_i} \checkmark, \quad \bar{y} = \frac{\Sigma m_i y_i}{\Sigma m_i} \checkmark$$

Example 5: A system of three particles consists of 10 kg placed at (2,3), 15 kg placed at (4,2) and 25 kg placed at (6,6). Find the coordinates of centre of mass of the system.



Solution: $M = \Sigma m_i = 10+15+25 = 50\,kg$

Let COM G is at $(\bar{x}, \bar{y})$

$$\bar{x} = \frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{10\times2 + 15\times4 + 25\times6}{50}$$

$$\bar{x} = 230/50 = 4.6$$

and $$\bar{y} = \frac{\Sigma m_i y_i}{\Sigma m_i} = \frac{10\times3 + 15\times2 + 25\times6}{50}$$

$$\bar{y} = 210/50 = 4.2$$

$\therefore$ Centre of Mass G (4.6, 4.2) ✓

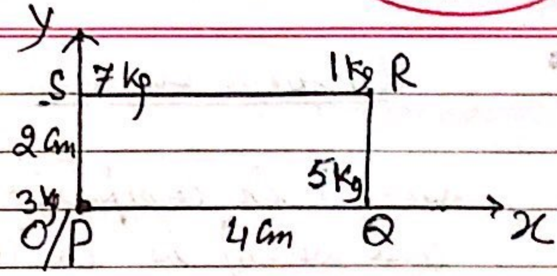
Example 6: A light rectangular metal plate PQRS has PQ = 4 cm and PS = 2 cm. Particles of masses 3 kg, 5 kg, 1 kg and 7 kg are attached respectively to the corners P, Q, R and S of the plate. Find the distance of the centre of mass of the loaded plate from, (a) the side PQ (b) the side PS.

(Continued →)

M.2

(Continued→)

Example 6. Solution:



Rectangular PQRS is place on the axes, such that P is at origin, PQ along x-axis and PS along y-axis.

∴ the points P, Q, R and S has coordinates P(0,0), Q(4,0), R(4,2), S(0,2)

Let the coordinates of Centre of Mass = $(\bar{x}, \bar{y})$

(a) Distance of centre of Mass from PQ = $\bar{y}$

$$\bar{y} = \frac{\Sigma m_i y_i}{\Sigma m_i} = \frac{3\times0 + 5\times0 + 1\times2 + 7\times2}{3+5+1+7}$$

$$\bar{y} = 16/16 = 1$$

$\therefore \underline{\bar{y} = 1}$

(b) Distance of centre of mass from side PS = $\bar{x}$

$$\bar{x} = \frac{\Sigma m_i x_i}{\Sigma x_i} = \frac{3\times0 + 5\times4 + 1\times4 + 7\times0}{3+5+1+7}$$

$$\bar{x} = 24/16 = 1.5$$

$\therefore \underline{\bar{x} = 1.5}$

Centre of Mass

- **Uniform Lamina:**

An object whose thickness is very small in compersision to the plane surface(its length and width) is called a uniform lamina and its weight is proportional to the area.

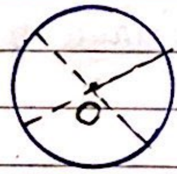
A lamina is uniform if its mass is evenly spread throughout its area.

(i) If a lamina has a axis of symmetry then its then its centre of mass lie on the axis of symmetry.

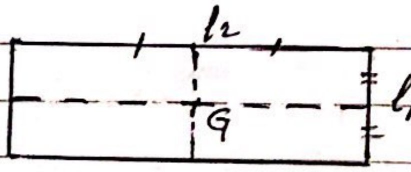
(ii) If the lamina has more than one axis of symmetry, then the centre of mass lie at the point of intersection of the axes of symmetry.

- Uniforom lamina — COM-G

1. Circular disc. — Centre of disc

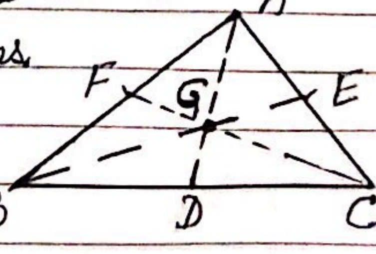


2. Rectangular lamina., — Point of int. of l_1 & l_2



l_1 and l_2 are lines joining the mid points of opp. sides.

3. Triangular lamia; — Centrod of the triangle
Point of intersection of medians.



$$\frac{AG}{GD} = \frac{BG}{GE} = \frac{CG}{GF} = \frac{2}{1}$$

or $AG = \frac{2}{3} AD$

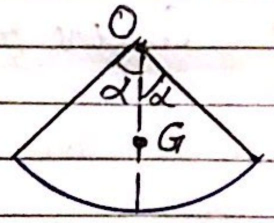
or $GD = \frac{1}{3} AD$

Here if the coordinates of $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$

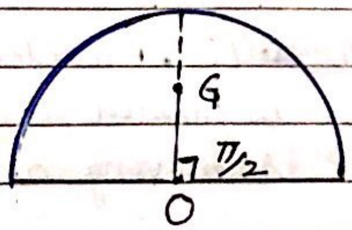
COM- $G\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

Uniform lamina . COM-G

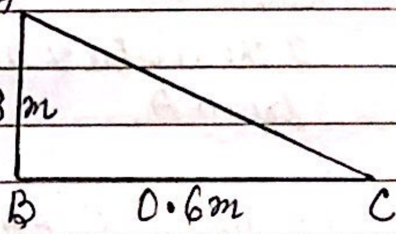
4. Sector of a circle. $\frac{2r}{3} \cdot \frac{\sin\alpha}{\alpha} = OG$



5 Semi circular lamina $OG = \frac{2r \sin\frac{\pi}{2}}{3 \times \frac{\pi}{2}}$



Example 7: ABC is a uniform lamina in the form of a triangle with $AB = 0.3m$, $BC = 0.6m$ and a right angle at B. State the distance of the centre of mass of the lamina from AB and BC.



[S-19/51/Q6] -- [2]

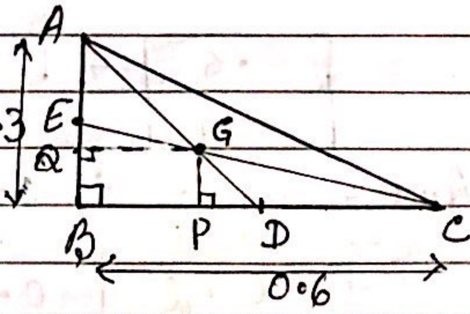
Solution: Let the medians AD and CE intersect at G.

$\frac{AG}{GD} = \frac{CG}{GE} = \frac{2}{1}$

$\Rightarrow GE = \frac{1}{3} CE$

$\Rightarrow GQ = \frac{1}{3} BC = \frac{1}{3} \times 0.6$

$GQ = 0.2m$

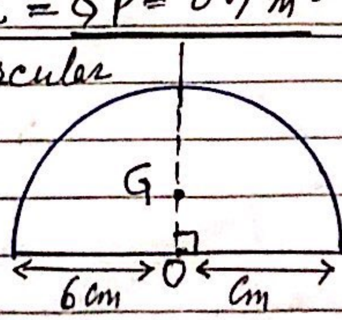


$\therefore$ Distance of COM, G from $AB = GQ = 0.2\,m$

Again $GP = \frac{1}{3} AB = \frac{1}{3} \times 0.3 = 0.1\,m$

$\therefore$ The distance of COM from $BC = GP = 0.1\,m$

Example 8: The diagram shows a uniform semicircular lamina of radius 6cm with centre O. Find the centre of mass of the lamina.



Solution: $OG = \frac{2r \sin\alpha}{3\alpha} = \frac{2 \times 6 \times \sin\frac{\pi}{2}}{3 \times \frac{\pi}{2}} = \frac{8}{\pi}$

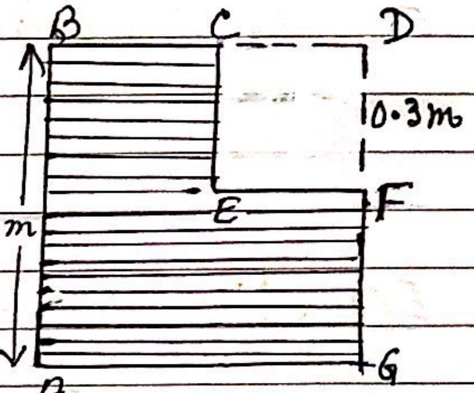
Scanned with CamScanner

Centre of mass of Composite lamina:

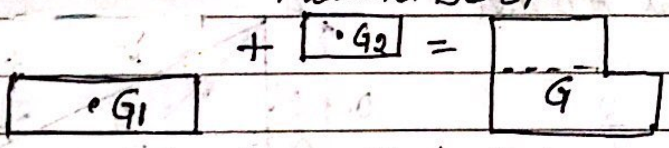
We can use $\bar{x}.\Sigma m_i = \Sigma m_i x_i$
and $\bar{y}.\Sigma m_i = \Sigma m_i y_i$

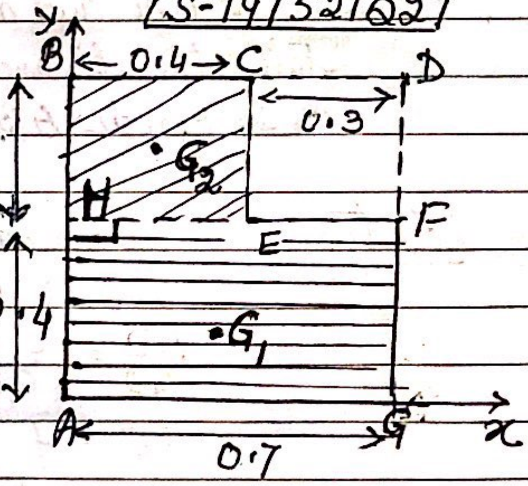
Example 9: A uniform lamina ABCEFG is is formed from a square ABCD by removing a smaller square CDFE from One corner. AB = 0.7 Cm. and DF = 0.3 m. Find the distance of the Centre of mass of the lamina from A. ---[4]

[S-19/52/Q2]



Solution: Draw EH ⊥ AB,
The Given lamina ABCEFG
= lamina AHFG
+ lamina BCEF





Area →	0.28	0.12	0.40
x →	0.35	0.2	$\bar{x}$
y →	0.2	0.55	$\bar{y}$

$\bar{x}.\Sigma m_i = \Sigma m_i x_i \Rightarrow \bar{x} \times 0.40 = 0.35 \times 0.28 + .12 \times .2$
$\Rightarrow 0.4\bar{x} = 0.098 + 0.024 = 0.122$
$\Rightarrow \bar{x} = 0.122/0.4 = 0.305$ ✓

$\bar{y}\Sigma m_i = \Sigma m_i y_i \Rightarrow \bar{y} \times 0.4 = 0.28 \times 0.2 + 0.12 \times 0.55$
$\Rightarrow 0.4\bar{y} = 0.056 + 0.066 = 0.122$
$\bar{y} = 0.122/0.4 = 0.305$

$\therefore AG = \sqrt{0.305^2 + 0.305} = \sqrt{0.18605}$
$= 0.431$ ✓

$\therefore AG = 0.431$ ✓

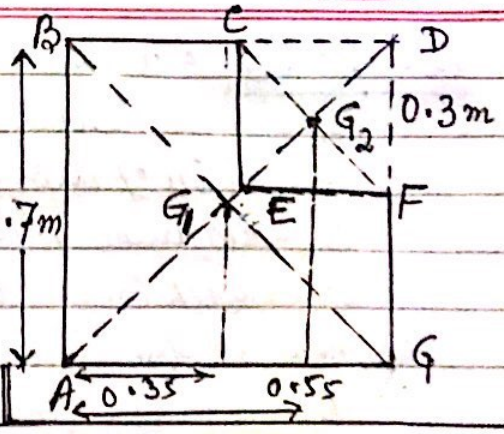
(Continued →)

Centre of Mass

(continued →)

uniform lamina

Example 9. (Alternate method)



Let us consider uncut square ABCD of side of length 0.7.

	Lamina	Area	COM
(i)	Original Sq. ABCD	$0.7^2 = 0.49$	$AG_1 = \sqrt{0.35^2 + 0.35^2} = 0.495$
(ii)	Small Sq. CDFE	$0.3^2 = 0.09$	$AG_2 = \sqrt{0.55^2 + 0.55^2} = \sqrt{0.605} = 0.7778$
(iii)	Given Lamina ABCEFG	$(0.49 - 0.09) = 0.4$	Let the centre of mass at X, $AX = ?$

Now $AX \times (0.4) + 0.09 \times 0.7778 = 0.49 \times 0.495$

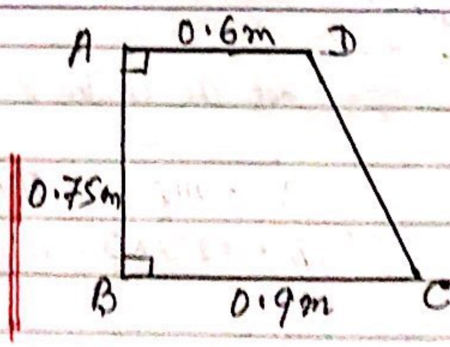
$\Rightarrow 0.4 \times AX = 0.49 \times 0.495 - 0.09 \times 0.7778$

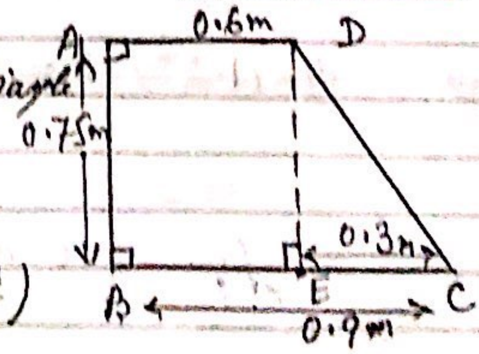
$0.4 \times AX = 0.24255 - 0.07 = 0.172548$

$AX = 0.172548 / 0.4 = 0.43137$

$\therefore \; AX = 0.431$

Example 10: The diagram shows a uniform lamina ABCD with $AB = 0.75m$ and $AD = 0.6m$ and $BC = 0.9m$, angle BAD = angle $ABC = 90°$. Show that the distance of the centre of mass of the lamina from $AB = 0.38m$ and find the distance of the centre of mass from BC. [M-17/ 52/ Q4] ---[5]





Solution: Area of Trap = ar Rect. + ar of Triangle

$A = 0.6 \times 0.75 + \frac{1}{2} \times 0.3 \times 0.75$

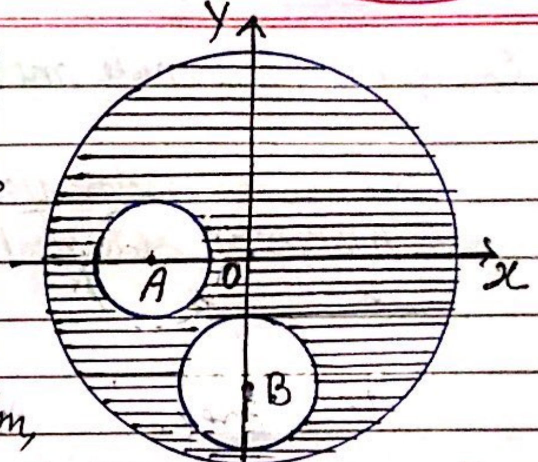
Total $A = 0.5625$

Now $0.5625\,\bar{x} = (0.75 \times 0.6) \times 0.3 + (\frac{1}{2} \times 0.3 \times 0.75)(0.6 + \frac{0.3}{3})$

$\Rightarrow \bar{x} = 0.38$ m from AB.

Now $0.5625 \cdot \bar{y} = (0.75 \times 0.6) \times 0.375 + (\frac{1}{2} \times 0.3 \times 0.75) \times 0.25 \Rightarrow \bar{y} = 0.35$

Centre of mass (Uniform lamina)



Example 11: A uniform circular disc has centre O and radius 1.2 m. The centre of disc is at the origin of x-axis and y-axis. Two circular holes with centres at A and B are made in the disc. The point is on the negative x-axis with OA = 0.5 m. The point B is on the negative y-axis with OB = 0.7 m. The hole with centre A has radius 0.3 m and the whole with centre B has radius 0.4 m. Find the distance of the centre of mass of the object from, W-15/51 Q6

(i) the x-axis --[4]

ii) the y-axis --[3]

Solution:

	Whole Uncut circle	Given Lamina	Circle (B)	circle (A)
Area	$\pi \times 1.2^2$	$\pi(1.2^2 - .4^2 - .3^2)$	$\pi \times 0.4^2$	$\pi \times 0.3^2$
COM from X-axis	0	$\bar{y}$ (let)	-0.7	0
COM from Y axis	0	$\bar{x}$ (let)	0	-0.5

(i) Let the centre of mass from x-axis at distance $\bar{y}$

$$(\pi \times 1.2^2) \times 0 = \pi(1.2^2 - 0.4^2 - 0.3^2) \cdot \bar{y} + (\pi \times 0.4^2) \times (-0.7) + (\pi \times 0.3^2) \times 0$$

$$\Rightarrow \quad 0 = 1.19\bar{y} - 0.112 \Rightarrow \bar{y} = 0.0941 \text{ m}$$

Distance of Centre of mass from x-axis = 0.0941 m

(ii) Again. $(\pi \times 1.2^2) \times 0 = \pi(1.2^2 - 0.4^2 - 0.3^2) \times \bar{x} + (\pi \times 0.4^2) \times 0 + (\pi \times 0.3^2)(-0.5)$

$$\Rightarrow \quad 0 = 1.19\bar{x} - 0.045 \quad \Rightarrow \bar{x} = 0.0378,$$

$\therefore$ distance of COM from Y-axis = 0.0378

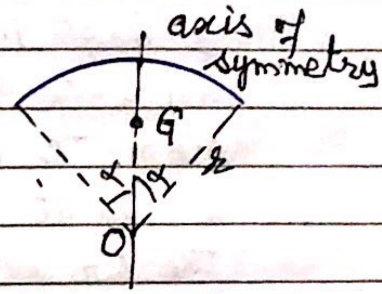
M-2

Centre of Mass

Framework made of Uniform rods (wires)

- Centre of mass of a framework made by rods (or wires) using Centre of mass of these rods (or wires):

(i) Uniform Circular Arc:

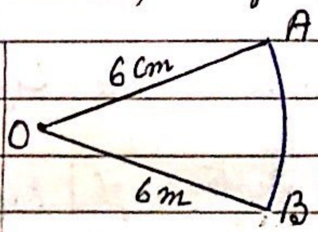


COM is at a distance

$OG = \frac{r \cdot \sin\alpha}{\alpha}$, $\begin{cases} \text{Centre angle} = 2\alpha \\ \text{radius} = r \end{cases}$

(ii) A framework made by joining the rods (or wire) using the COM of each rod (or wire).

Example 12. Find the position of the centre of mass of the framework shown in the diagram which is formed by bending a uniform piece of wire of total length $(12+2\pi)$ cm to form a sector of a circle, Centre O, radius 6 cm.

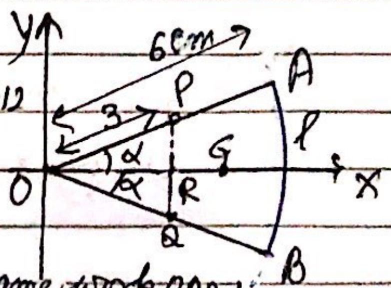


Solution: Mass of wire is proportional to its length.

Let P and Q are the mid points of OA and OB resp.

Let Centre angle $= (2\alpha)$; length of arc $AB = (12+2\pi) - 12 = 2\pi$ ✓

length of arc $AB = r \cdot \theta = 6 \times 2\alpha$ ✓



Let the line of symmetry is along X-axis, O is origin $\left\{ \because\ l = 6 \times 2\alpha = 2\pi \Rightarrow \alpha = \frac{\pi}{6} \right.$

	Arc AB	OA	OB	Frame work OAB
Length	2π	6	6	$12+2\pi$
COM from O: x	$\frac{r\sin\alpha}{\alpha} = \frac{6 \cdot \sin\frac{\pi}{6}}{\frac{\pi}{6}}$	$3\cos\pi/6 = \frac{3\sqrt{3}}{2}$	$3\cos\pi/6 = \frac{3\sqrt{3}}{2}$	$\bar{x}$
COM from O: y	0	$3\sin 30 = \frac{1}{2}$	$-3\sin 30 = -\frac{1}{2}$	$\bar{y}$

Along X-axis $\quad \bar{x} \sum m_i = 2\pi \times \frac{6\sin\frac{\pi}{6}}{\frac{\pi}{6}} + \frac{3\sqrt{3}}{2} \times 6 + \frac{3\sqrt{3}}{2} \times 6$

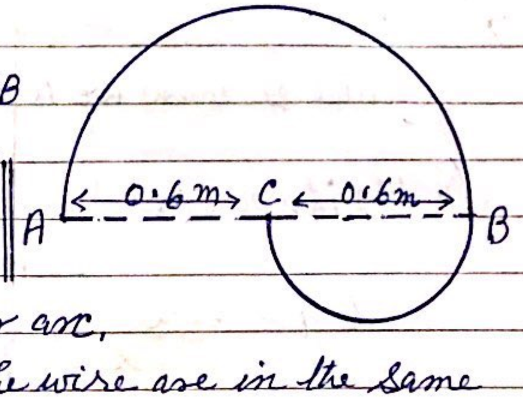
$\bar{x}(12+2\pi) = 36 + 9\sqrt{3} + 9\sqrt{3} = (36 + 18\sqrt{3})$

$\Rightarrow \bar{x} = \frac{(36+18\sqrt{3})}{12+2\pi} = \frac{9(\sqrt{3}+2)}{(6+\pi)}$ ✓

$\bar{y}(12+2\pi) = 0 + 6 \times \frac{1}{2} - 6 \times \frac{1}{2} = 0 \Rightarrow \bar{y} = 0$ ✓

$\therefore$ COM is on the axis of symmetry $OG = \frac{9(\sqrt{3}+2)}{(6+\pi)}$ ✓

Example 13: A uniform wire is bent to form an object which has a semicircular arc with diameter AB of length 1.2m, with a smaller arc with diameter BC of length 0.6m. The end C of the smaller arc is at the centre of the larger arc. The two semicircular arcs of the wire are in the same plane. Show that the distance of the centre of mass of the object from the line ACB is 0.191m, correct to 3 s.f. --[3]



[W-16 | 51 | Q2]

Solution: Mass of wire is proportional to the length.

COM of Arc $= \frac{r \sin\alpha}{\alpha}$

Length of Wire	Bigger Circle $\pi\times0.6$	Smaller Circle $\pi\times0.3$	Object $(\pi\times0.6+\pi\times0.3) = 0.9\pi$
Distance of COM from line ACB	$\frac{0.6\sin\frac{\pi}{2}}{\frac{\pi}{2}} = \frac{1.2}{\pi}$	$-\frac{0.3\sin\frac{\pi}{2}}{\frac{\pi}{2}} = -\frac{0.6}{\pi}$ below line ACB	D

$$D\times\Sigma m_i = 0.6\pi\times\frac{1.2}{\pi} + 0.3\pi\times\left(-\frac{0.6}{\pi}\right)$$

$$D\times(0.9\pi) = 0.72 - 0.18$$

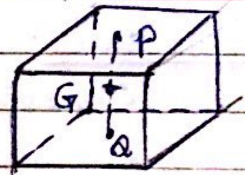
$$\Rightarrow D = \frac{0.54}{0.9\pi} = 0.191\,m$$

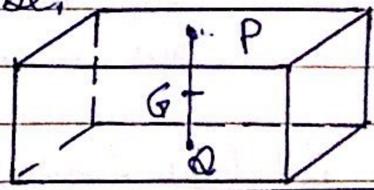
$\therefore$ distance of COM from line ACB = 0.191 m

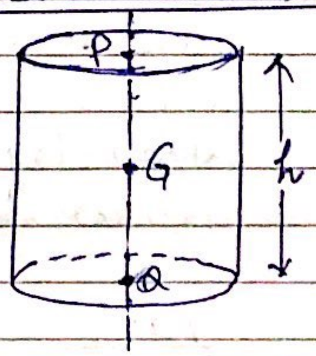
Centre of Mass

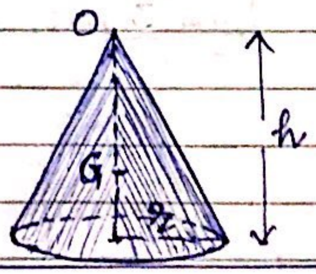
Three dimensional Shapes

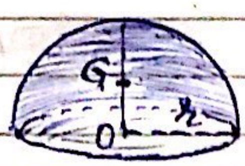
•	Body	Position of centre of Mass.
1.	Cube	G - Mid point of PQ P and Q are the point of int. of diagonals two opp. faces of the cube.
2.	Cuboid.	G - Mid point of PQ.
3.	Cylinder (Solid)	PQ is the line of symmetry joining the centres of the circular bases. G - Mid point of PQ $GQ = h/2$
4.	Solid Cone or Pyramid.	$\frac{3}{4}h$ from vertex. $OG = \frac{3}{4}h$
5.	Solid hemisphere.	$\frac{3}{8}r$ from centre $OG = \frac{3}{8}r$
6.	Hemispherical Shell.	$\frac{1}{2}r$ from centre $OG = \frac{1}{2}r$

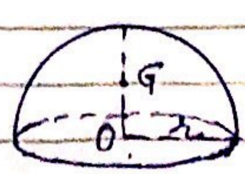






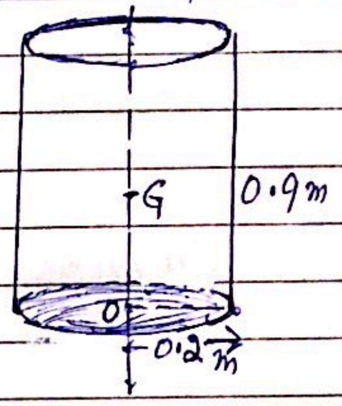






Example 14: A cylinderical container is open at the top. The curved surface and the circular base of the container are both made from from the same thin uniform material. The container has radius 0.2 m and height 0.9 m. --[3]
Show that the centre of mass of the container is 0.405 m from the base.

M-17/52/Q2



Solution: Mass of the surfaces of cylinder are proportial to the area.
Centre of mass will be on the line of symmetry, joining the centres of circular faces.

	Curved Surface	Circular base	Total container
Area	$2\pi r h = 2\pi \times 0.2 \times 0.9$	$\pi r^2 = \pi(0.2)^2$	$(2\pi \times 0.2 \times 0.9) + \pi(0.2)^2$
COM along the line of symmetry above O.	$\frac{0.9}{2}$	0	$OG = \bar{y}$

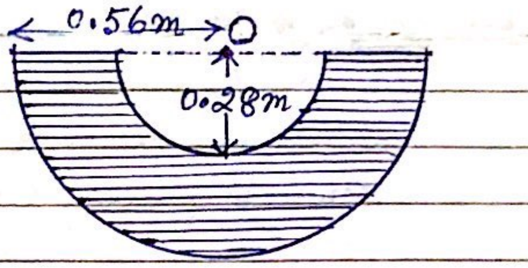
$$\therefore \quad \bar{y}\,\Sigma m_i = \Sigma m_i y_i$$

$$\bar{y}\left(2\pi \times 0.9 \times 0.2 + \pi(0.2)^2\right) = (2\pi \times 0.2 \times 0.9) \times \frac{0.9}{2} + \pi(.2)^2 \times 0$$

$$\bar{y} = \frac{0.162\,\pi}{0.4\pi} = 0.405$$

$\therefore$ Centre of mass is along the line of symmetry at distance $OG = 0.405$ m from base.

Example 15: An object is made from a uniform solid hemisphere of radius 0.56 m and centre O, by removing a hemisphere of radius 0.28 m and centre O.



The diagram shows a cross-section through O of the object.

(i) Calculate the distance of the centre of mass of the object from O. --[4]

The object has weight 24 N. A uniform hemisphere 'H' of radius 0.28 m is placed in the hollow part of the object to create a non-uniform hemisphere with centre O. The centre of mass of the non-uniform hemisphere is 0.15 m from O.

(ii) Calculate the weight of H. [S-17/51/Q3] --[3]

Solution: Centre of mass of solid hemisphere $= \frac{3}{8} r$ from O

Volume of hemisphere $= \frac{2}{3}\pi r^3$

	Full hemisphere of $r = 0.56$	Small hemisph $r = 0.28$	The Object
Volume	$\frac{2}{3}\pi (0.56)^3$	$\frac{2}{3}\pi(.28)^3$	$\frac{2}{3}\pi(0.56^3 - 0.28^3)$
d_i = distance of COM from O	$\frac{3}{8} \times 0.56 = 0.21\, m$	$\frac{3}{8} \times 0.28 = 0.105$	d

$$d \cdot \Sigma m_i = \Sigma m_i \cdot d_i$$

$$0.21 \times \frac{2}{3}\pi(.56)^3 = 0.105 \times \frac{2}{3}\pi(0.28)^3 + \frac{2}{3}\pi(0.56^3 - 0.28^3) \times d$$

$$\Rightarrow 0.21 \times 0.175616 = 0.021952 \times 0.105 +$$

$$\Rightarrow 0.03688 = 0.0023 + 0.153664\, d$$

$$\Rightarrow d = (0.03688 - 0.0023)/0.153664 = \underline{0.225\, m}$$

Again: New total solid hemisph = Previous object + New small hemisphere

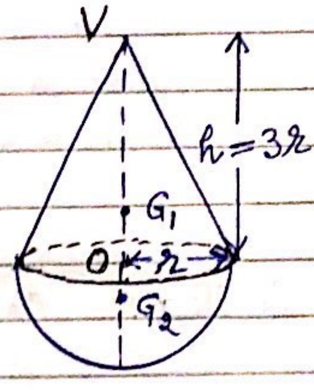
$$(24 + W) \times 0.15 = 24 \times 0.225 + W\left(\frac{3}{8} \times 0.28\right)$$

$$\Rightarrow W = \underline{40\, N.}$$

Scanned with CamScanner

Example 16. A child's toy consists of a uniform solid circular cone, of vertical height $3r$ and radius r, and a uniform solid hemisphere of radius r. The circular base of the cone and the hemisphere are joined together so that they coincide. The cone and the hemisphere are made of the same material. --- [4]

Show that the centre of mass of the toy is at a distance of $\frac{27}{10}r$ from the vertex of the cone. FM|SP-20|03|Q1



Solution:

	Cone	Hemisphere	Combind Toy
Volume	$\frac{1}{3}\pi r^2 \times 3r$	$\frac{2}{3}\pi r^3$	$\frac{5}{3}\pi r^3$
distance of COM from Vertex V	$\frac{3}{4}\times 3r$	$3r+\frac{3}{8}r$	$\bar{x}$

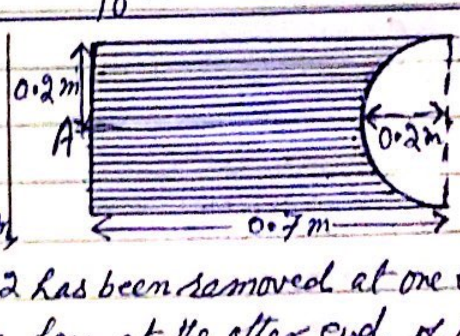
$$\bar{x}\cdot\Sigma m_i = \Sigma m_i \cdot x_i$$

$$\frac{5}{3}\pi r^3 \times \bar{x} = \pi r^3 \times \frac{9}{4}r + \frac{2}{3}\pi r^3 \times \frac{27}{8}r$$

$$\Rightarrow \bar{x} = \frac{27}{10}r$$

distance of COM from vertex $= \frac{27}{10}r$

Example 17: The diagram shows the cross-section through the centre of mass of uniform solid object. The object is a cylinder of radius 0.2 m and of length 0.7 m, from which a hemisphere of radius 0.2 has been removed at one end. The point 'A' is the centre of the plane face at the other end of the object. Find the distance of the centre of mass of object from A. --[5]



S-19|51|Q3

Solution:

	Uncut Cylinder	Hemisphere	Object
Volume	$\pi\times 0.2^2\times 0.7$	$\frac{2}{3}\pi\times 0.2^3$	$(\pi\times 0.2^2\times 0.7-\frac{2}{3}\pi\cdot 0.2^3)$
x-COM from A	$0.7/2$	$(0.7-\frac{3}{8}\times 0.2)$	$\bar{x}$

$$x\cdot\Sigma m_i = \Sigma x_i m_i$$

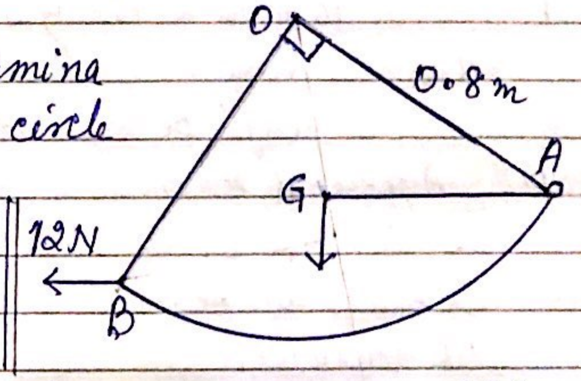
$$\Rightarrow \frac{0.7}{2}\times\pi\times 0.2^2\times 0.7 = (0.7-\frac{3}{8}\times 0.2)\times\frac{2}{3}\pi\times 0.2^3 + \bar{x}\left[\pi\times 0.2^2\times 0.7 - \frac{2}{3}\pi\times 0.2^3\right]$$

$$\Rightarrow \bar{x} = 0.285$$

Equilibrium of a rigid body

"Equilibrium of Lamina"

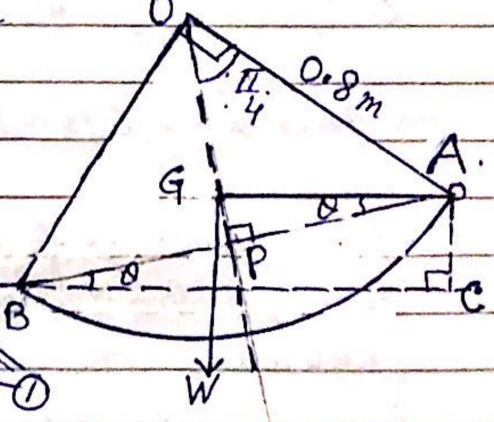
- Equilibrium of a lamina when suspended freely from a fixed point (or pivots freely about a horizontal axis) will rest in equilibrium in a vertical plane with its centre of mass vertically below the point of suspension (or the pivot).



Example 18: OAB is a uniform lamina in the shape of a quadrant of a circle with centre O, and radius 0.8 m. Which has its centre of mass at G. The lamina is smoothly hinged at A to a fixed point and is free to rotate in the vertical plane. A horizontal force of magnitude 12 N acting in the plane of the lamina is applied to the lamina at B. The lamina is in equilibrium with AG horizontal.

(i) Calculate the length of AG --[3]

(ii) Find the weight of lamina --[5] [W-17/52/Q5]



Solution: G lies on the line of symmetry of the sector of circle. angle $AOG = \frac{1}{2} \times \frac{\pi}{2} = \frac{\pi}{4}$

(i) OG produced, $OP \perp AB$

In rt $\triangle AOG$: $OP = 0.8 \cos \frac{\pi}{4}$ and $AP = 0.8 \sin \frac{\pi}{4}$

G: Centre of Mass; $OG = \frac{2}{3} \frac{r \sin \alpha}{\alpha}$

$\therefore OG = \frac{2}{3} \times 0.8 \sin \frac{\pi}{4} / \frac{\pi}{4} = 0.48016$ — (1)

Now in rt $\triangle APG$,

$AG^2 = GP^2 + AP^2$

$= (OP - OG)^2 + AP^2$

$= \left[0.8 \cos \frac{\pi}{4} - 0.48016\right]^2 + \left(0.8 \sin \frac{\pi}{4}\right)^2$

$\Rightarrow AG = \underline{0.572\,m}$ ✓ — (1)

(ii) Let angle $GAP = \theta$ = Angle ABC

$\tan \theta = \frac{GP}{AP} = \frac{(OP - OG)}{AP}$

$= \frac{\left[0.8 \cos \frac{\pi}{4} - 0.48\right]}{0.8 \sin \frac{\pi}{4}}$

$\Rightarrow \theta = 8.6°$ ✓

Now taking moment about A,

$W \times AG = 12 \times AC$

$W \times 0.572 = 12 \times 2AP \cdot \sin \theta$

$W = \frac{12 \times 2 \times 0.8 \sin \frac{\pi}{4} \times \sin 8.6}{0.572}$

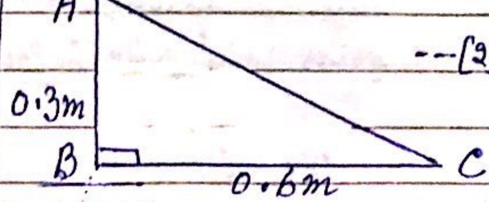
$W = \underline{3.55\,N}$ ✓

Equilibrium of a rigid body.

Equilibrium of a Lamina.

Example 19: ABC is a uniform lamina in the form of a triangle with AB = 0·3m, BC = 0·6m and a right angle at B.

(i) State the distance of the centre of mass of the lamina from AB and from BC. --[2]



The lamina is freely suspended at B and hangs in equilibrium.

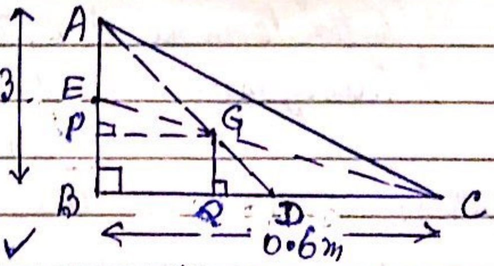
(ii) Find the angle between AB and the horizontal. --[2]

A force of magnitude 12 N is applied along the edge AC of the lamina in the direction from A towards C. The lamina is still suspended at B and is now in equilibrium with AB vertical.

(iii) Calculate the weight of the lamina. [S-19 | 51 | Q6] --[3]

Solution: Let the medians AD and BE

(i) intersect at 'G'; G is the COM.

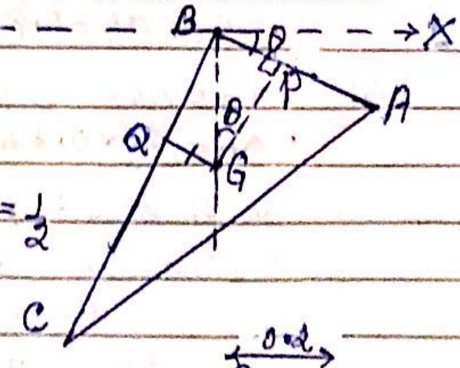


Draw $GP \perp AB$, $GQ \perp BC$.

Distance of G from AB = $GP = \frac{1}{3} \times 0.6$
$= 0.2\text{ m}$ ✓

and the distance of G from BC = $GQ = \frac{1}{3} \times 0.3 = 0.1\text{ m}$ ✓ $\left[\because GP = BQ,\ \frac{BQ}{BC} = \frac{GE}{CE} = \frac{1}{3}\right]$

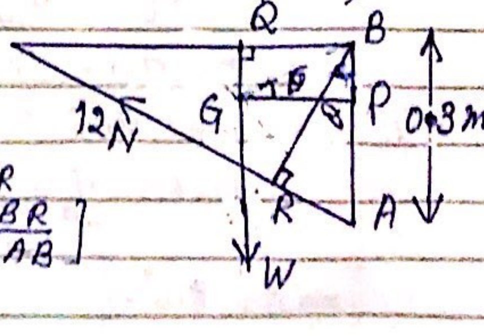
(ii) When lamina is freely suspended from B, ⇒ BG is vertical. Let AB is inclined at angle θ with horizontal BX.



$\therefore \angle BG = \theta \Rightarrow \tan\theta = \frac{BP}{GP} = \frac{GQ}{GP} = \frac{0.1}{0.2} = \frac{1}{2}$

$\therefore \theta = 26.6°$ ✓

(iii) Let the weight of the lamina is W N. passes through G, and GW is vertical.
Draw $BR \perp AC$, angle $ABR = \theta = 26.6°$
Now taking moment about point B,



$W \times BQ = 12 \times BR$

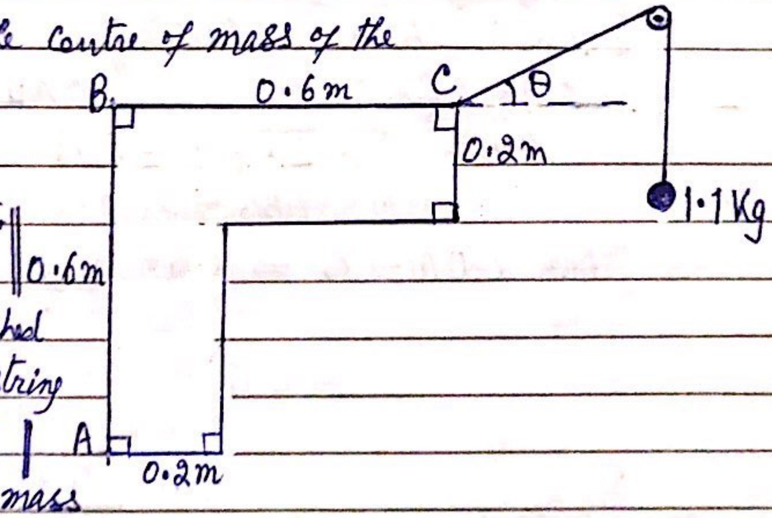
$W \times 0.2 = 12 \times 0.3 \cos 26.6$ $\left[\text{In } \triangle ABR,\ \cos\theta = \frac{BR}{AB}\right]$

$\Rightarrow W = 16.1\text{ N}$ ✓

Example 20. A uniform lamina of weight 15 N has dimensions as shown in the diagram.

(i) Show that the distance of the centre of mass of the lamina from AB is 0.22 m.

The lamina is freely hinged at B to a fixed point. One end of a light inextensible string is attached to the lamina at C. The string passes over a fixed smooth pulley and a particle of mass 1.1 Kg is attached to the other end of the string. The lamina is in equilibrium with BC horizontal. The string is taut and makes an angle of θ° with the horizontal at C, and the particle hangs freely below the pully.

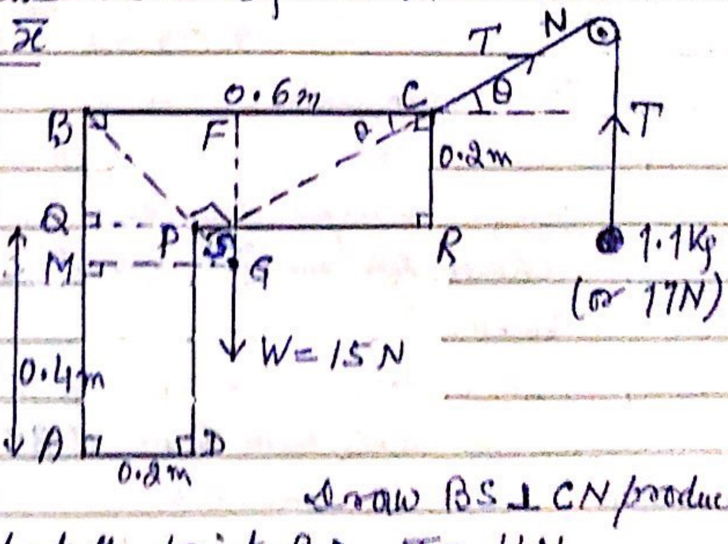


S_06/05/Q5

(ii) Find the value of θ.

Solution: Draw $PQ \perp AB$, we have two rectangles BCRQ and PQAD

Let the COM of object from $AB = \bar{x}$



$\bar{x} \times (0.6 \times 0.2 + 0.4 \times 0.2)$

$= (0.6 \times 0.2) \times 0.3 + (0.4 \times 0.2) \times 0.1$

$\Rightarrow 0.2\bar{x} = .036 + 0.008$

$.2\bar{x} = 0.044$

$\Rightarrow \bar{x} = 0.22\,m$ ✓

Now let G is the COM

$W = 15\,N$ acts at G, $GM \perp AB$

$GM = \bar{x} = 0.22\,m$

Draw $BS \perp CN$ produced.

Now taking moment about the point B; $T = 11\,N$.

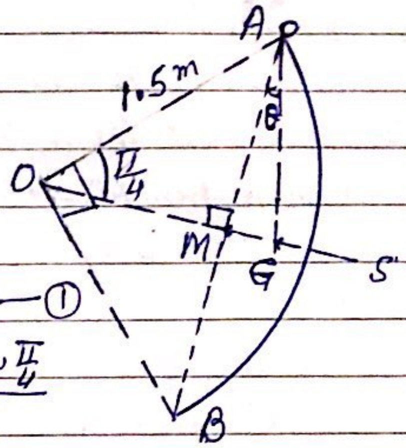
$W \times BF = T \times BS$

$15 \times 0.22 = 11 \times 0.6 \sin\theta$

$\left\{ \begin{array}{l} BS = 0.6 \sin\theta \\ BF = GM = 0.22 \end{array} \right.$

$$\sin\theta = \frac{15 \times 0.22}{0.6 \times 11} = 0.5$$

$\theta = \sin^{-1} 0.5 = 30^\circ$ ✓



Solution 21: Let G is the centre of Mass.
angle $AOB = \frac{\pi}{2}$, line of symmetry bisects $\angle AOB$, $\angle AOS = \frac{\pi}{4}$
$OA = OB = r = 1.5$, $OM \perp AB$.
$AB = \sqrt{1.5^2 + 1.5^2} = \sqrt{4.5} = 2.1213$
$\Rightarrow AM = AB/2 = 2.1213/2 = 1.0606$ — ①
Now COM → G $\Rightarrow OG = \frac{r \sin\alpha}{\alpha} = \frac{1.5 \sin\frac{\pi}{4}}{\frac{\pi}{4}}$

$$\Rightarrow OG = \frac{6 \times 1}{\sqrt{2} \times \pi} = 1.3504$$

$$\therefore OG = 1.35$$

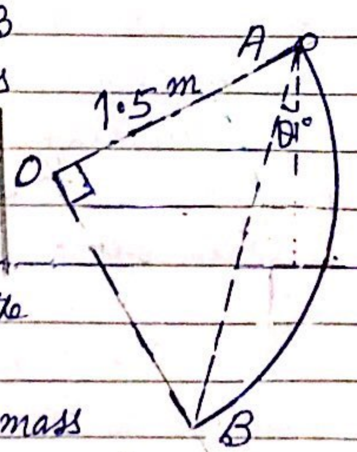
Now $MG = OG - OM = 1.35 - 1.0606$ [$OM = AM = 1.0606$ from ①]
$MG = 0.2898$

In ΔAMG, $\tan\theta = \frac{MG}{OM} = \frac{0.2898}{1.0606} = 0.27324$

(Solution)

$\theta = 15.28° \Rightarrow \theta = 15.3°$

Example 21: A uniform rigid wire AB is in the form a circular arc of radius 1.5 m with centre O. The angle AOB is a right angle. The wire is in equilibrium, freely suspended from the end A. The chord AB makes an angle of θ° with the vertical.



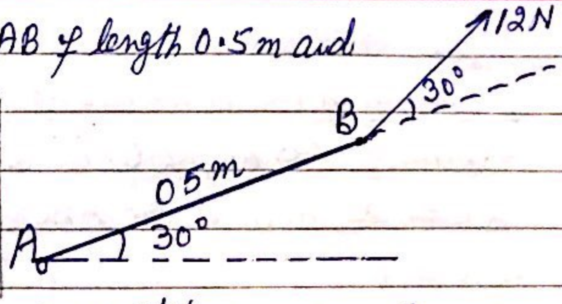
(i) Show that the distance of the centre of mass of the arc from O is 1.35 m, correct to 3 s.f.
(ii) Find the value of θ.

[S-08/05/Q2]

Equilibrium of a rigid body

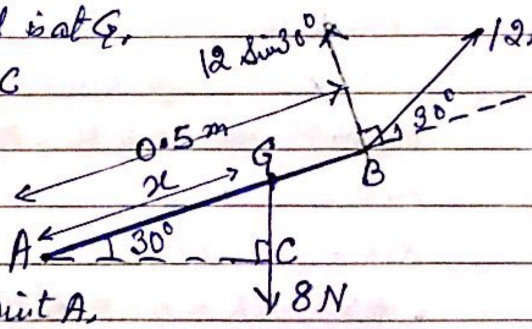
Equilibrium of a rod. P-23

Example 22: A non-uniform rod AB of length 0.5 m and weight 8 N is freely hinged to a fixed point at A. The rod makes an angle of 30° with the horizontal, with B above the level of A. The rod is held in equilibrium by a force of magnitude 12 N acting in the vertical plane containing the rod at an angle of 30° to AB, applied at B. Find the distance of the centre of mass of the rod from A.



[S-18/51/Q2] ---[3]

Solution: Let the centre of mass of rod is at G, and $OG = x$ m, Draw $AC \perp GC$; the weight of rod is vertically at G, the component of Force 12 N perp. to $AB = 12 \sin 30°$



Now taking moment about the point A.

$$W \times AC = AB \times 12 \sin 30$$

$$8 \times x \cos 30° = 12 \sin 30° \times 0.5$$

$$x = \frac{12 \sin 30° \times 0.5}{8 \cos 30} = 0.433$$

[In ΔAGC

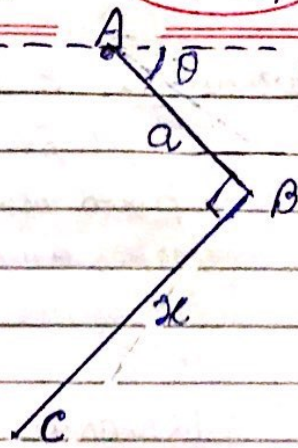
$\frac{AC}{x} = \cos 30°$

$x = x \cos 30°$

∴ COM is at a distance of 0.433 m from A.

Equilibrium of a wire

Example 23: ABC is an object made from a uniform wire consisting of two straight portions AB and BC, in which $AB = a$, $BC = x$ and angle $ABC = 90°$. When the object is freely suspended from A and is in equilibrium, the angle between AB and the horizontal is θ.

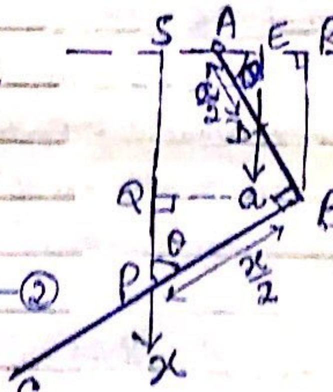


(i) Show that $x^2 \tan\theta - 2ax - a^2 = 0$ --[3]

(ii) Given that $\tan\theta = 1.25$, calculate the length of the wire in terms of a. S-18/52/Q3 --[2]

Solution: Mass is proportional to the lengths.

mass of rod $AB = a$, mass of $BC = x$



(i) Taking moment about A.

$x \times AS = a \times AE$ —— ①

$AS = SR - AR = QB - AR$

$= \frac{x}{2}\sin\theta - a\cos\theta$ —— ②

and $AE = \frac{a}{2}\cos\theta$ —— ③

from ② and ③ in ①

$x\left(\frac{x}{2}\sin\theta - a\cos\theta\right) = a \times \frac{a}{2}\cos\theta$

$\Rightarrow \frac{x^2}{2}\sin\theta = \cos\theta\left(ax + \frac{a^2}{2}\right)$

$\Rightarrow \frac{x^2}{2}\tan\theta = ax + \frac{a^2}{2}$

$\Rightarrow \underline{x^2 \tan x - 2ax - a^2 = 0}$ —— ④

(ii) Given $\tan\theta = 1.25$

from ④ $1.25x^2 - 2ax - a^2 = 0$

$\Rightarrow 5x^2 - 8ax - 4a^2 = 0$

$\Rightarrow (x - 2a)(5x + 2a) = 0$

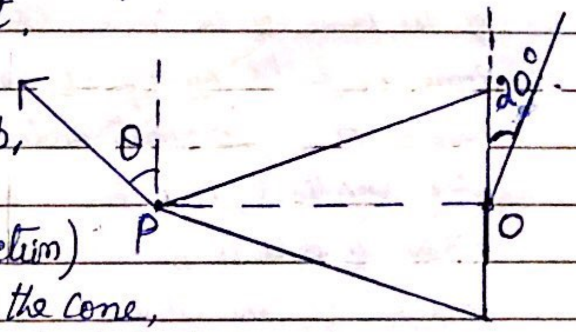
$x = 2a$ or $x = -\frac{2a}{5}$ ×

Now length of wire $= x + a$

$= 2a + a$ $(\because x = 2a)$

$= \underline{3a}$

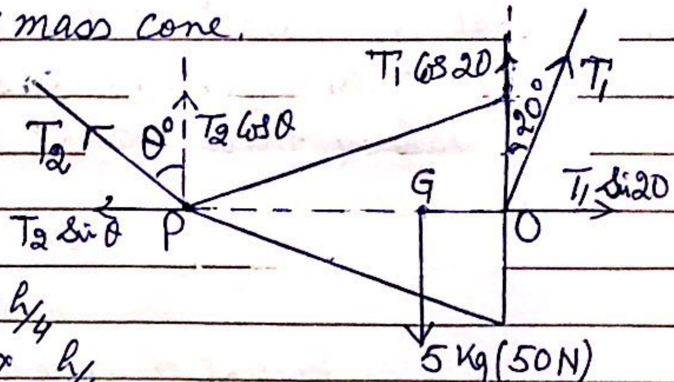
Example 24: P is the vertex of a uniform solid cone of mass 5 kg, and O is the centre of its base. Strings are attached to the cone at P and at O. The hangs in equilibrium with PO horizonal and the string taut.
The strings attached at P and O make angles of $\theta°$ and $20°$ resp, with the vertical.
(diagram shows a cross-section)



(i) By taking moment about P for the cone, find the tension in the string attached at O.
(ii) Find the value of θ and the tension in the string attached at P.

W-09/52/Q6

Solution: Let 'G' is the centre of mass cone.
(i) and h is its height.
Then. $PG = \frac{3}{4}h$ and $OG = \frac{h}{4}$



Taking moment about P.

$$W \times PG = T_1 \cos 20 \times h/4$$

$$50 \times \frac{3}{4}h = T_1 \cos 20 \times h/4$$

$$\Rightarrow T_1 = \frac{3 \times 50}{4 \times \cos 20} = 39.9\ N$$

$$\therefore T_1 = 39.9\ N$$

(ii) As the cone is in equilibrium. the sum of horizontal component be zero or $T_2 \sin\theta = T_1 \sin 20 = 39.9 \sin 20$

$$\Rightarrow T_2 \sin\theta = 13.6466 \quad (1)$$

Now taking moment about O,

$$T_2 \cos\theta \times h = 50 \times h/4$$

$$\Rightarrow T_2 \cos\theta = 12.5 \quad (2)$$

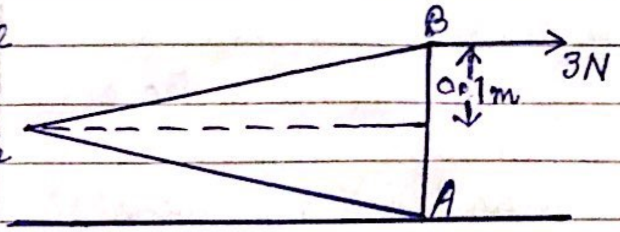
Divide (1) by (2) $\tan\theta = 13.6466/12.5 = 1.0917$

$$\Rightarrow \theta = 47.5°$$

from (2) $T_2 \cos\theta = 12.5 \Rightarrow T_2 \times \cos 47.5 = 12.5$

$$\Rightarrow T_2 = 18.5\ N$$

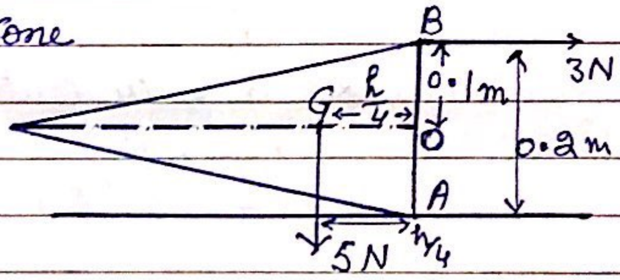
Example 25: A uniform solid cone has weight 5N and base radius 0.1m. AB is a diameter of the base of the cone. The cone is held in equilibrium, with A in contact with a rough horizontal surface and AB vertical, by a force applied at B. The force has magnitude 3N and acts parallel to the axis of the cone. Calculate the height of the cone. [W-19/51/Q1] ...[3]



Solution: Let the height of the cone = h

Let the centre of mass of cone at G, $OG = \frac{h}{4}$ where O is the centre of circular base.

Taking moment about A.



$$5 \times \frac{h}{4} = 3 \times 0.2$$

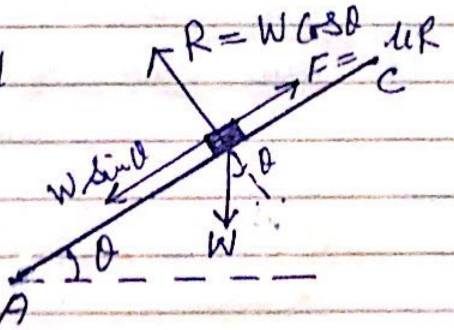
$$\Rightarrow h = 0.48$$

height of cone $= \underline{0.48\,m}$

Equilibrium of a rigid body.
Sliding and Toppling

- The object is on the point of sliding, then the force of friction $F = \mu R$.
  R is normal contact force between the two bodies and μ is the coefficient of friction.
  or $\mu = \frac{F}{R}$.

- Rough inclined plane:
  If a body is placed on a rough inclined plane is on the point of sliding down the plane under the action of its weight and the reactions of the plane only, the angle of inclination of the plane to the horizontal 'θ' is equal to the angle of friction.

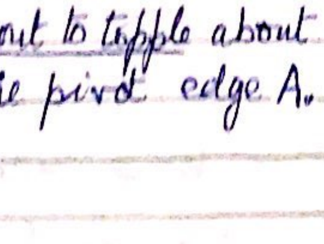


Resolving parallel and perpending to the plane.

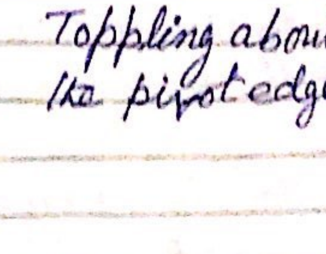
Parallel to plane: $W \sin\theta = \mu R$
Perp to plane: $W \cos\theta = R$

$\Rightarrow$ $\tan\theta = \mu$ coefficient of friction
here 'θ' is called the angle of friction.

- Toppling:



About to topple about the pivot edge A.

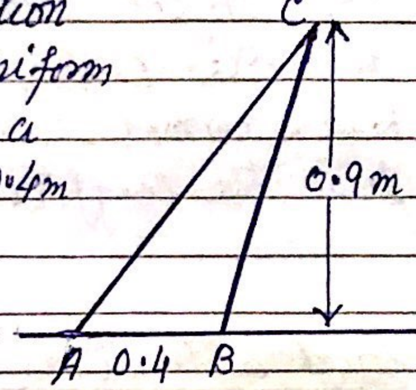


Toppling about the pivot edge A.

Equilibrium of rigid bodies.

Sliding and Toppling

Example 26: ABC is the cross-section through the centre of mass of a uniform prism, which rests with AB on a rough horizontal surface. $AB = 0.4\,m$ and C is $0.9\,m$ above the surface. The prism is on the point of toppling about its edge through B.



(i) Show that angle $BAC = 48.4°$ correct to 3 s.f. --[3]

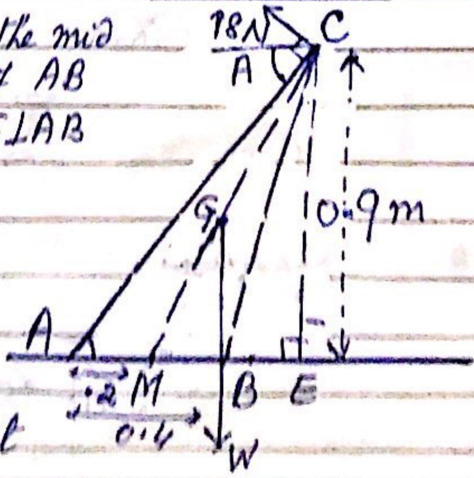
A force of magnitude 18N acting in the plane of cross-section and perpendicular to AC is now applied to the prism at C. The prism is on the point of rotating about its edge through A.

(ii) Calculate the weight of the prism. --[3]

(iii) Given also that the prism is on the point of slipping, calculate the coefficient of friction between the prism and the surface. --[4]

[W-19/51/Q7]

Solution: Let G is the COM of the prism, M is the mid point of AB. Draw $CE \perp AB$



(i) $\frac{GM}{CM} = \frac{GB}{CE} = \frac{1}{3} = \frac{GB}{0.9}$

$\Rightarrow GB = 0.3\,m$ ✓, $MB = 0.2\,m$

Since the prism is on the point of toppling about B

$\therefore \frac{MB}{ME} = \frac{1}{3}$

$\Rightarrow ME = 3 \times MB = 0.6$ ✓

$\Rightarrow AE = AM + ME = 0.8$

GW the vertical through COM G must pass through B. $\Rightarrow \tan A = \frac{CE}{AE} = \frac{0.9}{0.8}$

$\Rightarrow$ angle $A = 48.4°$ ✓

In triangle CAE; $\frac{CE}{AC} = \sin A$

(ii) $AC = \frac{CE}{\sin A} = \frac{0.9}{\sin 48.4} = 1.2041$

Now Taking moment about A

$18 \times AC = W \times AB \Rightarrow 18 \times 1.2041 = W \times 0.4$

$\Rightarrow \underline{W = 54.2\,N}$ ✓

(iii) Horizontal component of force

$H = 18 \sin A = 18 \times \sin 48.4 = 13.46$

Vertical $= W - 18 \cos A$

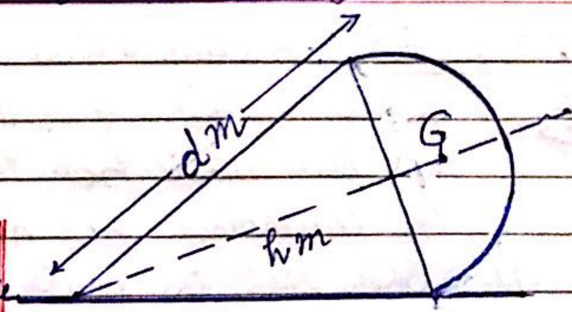
$= 54.2 - 18 \cos 48.4$

$V = 42.35$ ✓

$\therefore \mu = \frac{H}{V} = \frac{13.46}{42.35} = 0.319$

$\Rightarrow \mu = 0.319$ ✓

Example-27: An object is formed by joining a hemisphere shell of radius 0.2m and a solid cone, with base radius 0.2m and height h m, along the circumference.

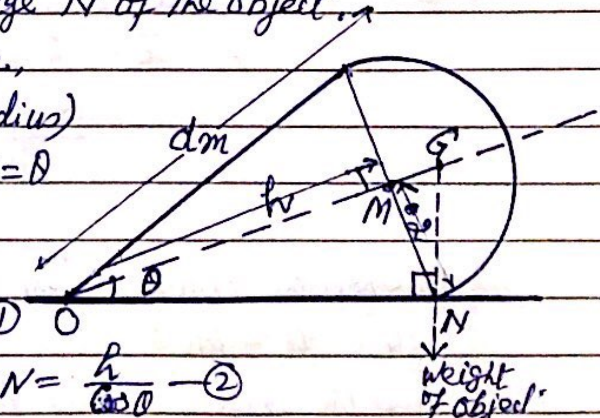


The centre of mass, G, of the object is dm from the vertex of the cone on the axis of symmetry of the object. The object rests in equilibrium on a horizontal plane, with the curved surface of the cone in contact with the plane, the object is on the point of toppling.

(i) Show that: $d = h + \frac{0.04}{h}$ ---[3]

(ii) It is given that the cone is uniform and of weight 4N, and the hemispherical shell is uniform and of weight W N. Given also that h = 0.8, find W. [W-15/53/Q6] --[6]

Solution: As the object is on the point of toppling the vertical through COM 'G' will pass through the edge N of the object.
Let M is the centre of circular base.
GN is vertical, MN = 0.2 (radius)
Let OM = h, OG = d, let ∠GON = θ



(i) In rt∆ GNO $\frac{ON}{OG} = \cos\theta = \frac{ON}{d}$

$\Rightarrow ON = d\cos\theta$ —①

and In rt∆ OMN,

$\frac{OM}{ON} = \cos\theta \Rightarrow \frac{h}{ON} = \cos\theta \Rightarrow ON = \frac{h}{\cos\theta}$ —②

from ① & ② $d\cos\theta = \frac{h}{\cos\theta} \Rightarrow d = \frac{h}{\cos^2\theta}$ —③

Now $ON = \sqrt{h^2 + 0.2^2}$

$\Rightarrow$ from② $\frac{h}{\cos\theta} = \sqrt{h^2 + .2^2}$

$\Rightarrow \cos\theta = \frac{h}{\sqrt{h^2 + 0.04^2}}$ —④

frm ③ & ④ $d = \frac{h}{h^2/(0.04 + h^2)}$

$= \frac{0.04 + h^2}{h}$

$\Rightarrow d = h + \frac{0.04}{h}$ ✓

(ii) COM of Cone from O $= \frac{3}{4}h = \frac{3}{4} \times .8 = .6r$

COM of hemisph $= h + \frac{r}{2} = .8 + .1 = .9$ ✓

$\therefore 0.6 \times 0.4 + 0.9 \times W = d(4 + W)$ —⑤

$d = h + \frac{0.04}{h} = .8 + \frac{.04}{.8}$

$= 0.85$

frm⑤ $2.4 + 0.9W = 0.85(4 + W)$

$\Rightarrow W = 20N$ ✓

Example 28: A cylinderical container is open at the top. The curved surface and the circular base of the container are both made from the same thin uniform material. The container has radius 0.2m and height 0.9 m.

(i) Show that the centre of mass of the container is 0.405m from the base. ---[3]

The container is placed with its base on a rough inclined plane. The container is in equilibrium of the point of slipping down the plane and also on the point of toppling.

(ii) Find the coefficient of friction between the container and the plane. [M-17/52/Q2] --[3]

Solution: $M = 2\pi r h + \pi r^2 = 2\pi \times 0.2 \times 0.9 + \pi \times 0.2^2$

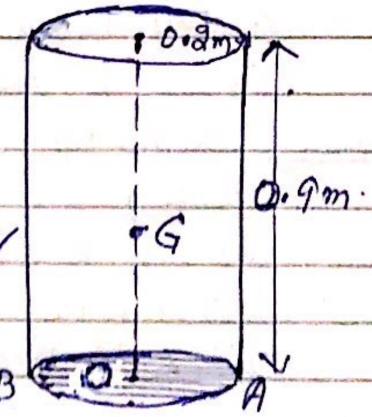
$\Rightarrow (2\pi \times 0.2 \times 0.9 + \pi \times 0.2^2)\,\bar{x}$

$= (2\pi \times 0.2 \times 0.9) \times \frac{0.9}{2} + \pi \times 0.2^2 \times 0$

(distance from O, the centre of circular base)

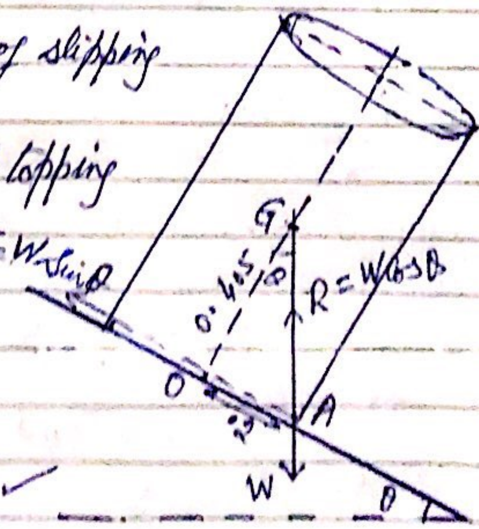
$\Rightarrow \bar{x} = 0.405m \Rightarrow OG = 0.405$ ✓

G is the centre of mass



When the object is the point of slipping then $\mu = \tan\theta$.

and the object at the point of topping GA is vertical.



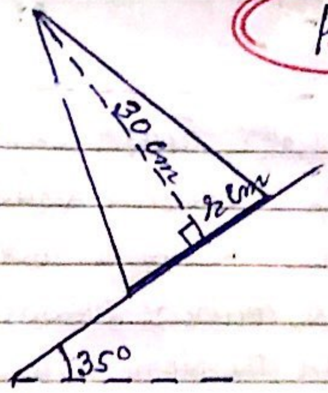
$\mu = \frac{F}{R} = \frac{W \sin\theta}{W \cos\theta} \Rightarrow \mu = \tan\theta = \frac{OA}{OG}$

$= \frac{0.2}{0.405}$

$\mu = 0.494$ ✓

M.2

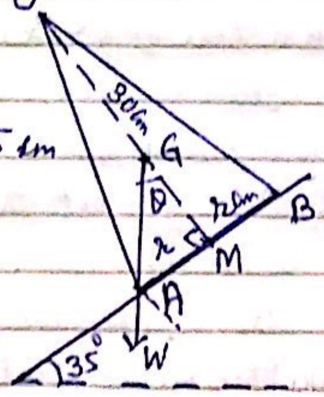
Equilibrium of a rigid body.
Toppling but not sliding



Example 29: A uniform solid cone has eight 30 cm and base radius r cm. The cone is placed with its axis vertical on a rough horzontal plane. The plane is slowly tilted and the cone remains in equilibrium until the angle of inclination of the plane reaches 35°, when the plane topples. The diagram shows a cross-section of the cone.

(i) Find the value of r.

(ii) Show that the coefficient of friction between the cone and the plane is greater than 0.7. | S-10 | 51 | Q2 |



Solution: COM 'G' of cone is at distance $\frac{3h}{4}$ from O

or $GM = h/4 = \frac{30}{4} = 7.5$ cm

as the plane topples when $\theta = 35$

at this moment GA is vertical.

angle $AGM = \theta = 35°$

$\therefore \tan\theta = \frac{AM}{GM} \Rightarrow \frac{r}{7.5} = \tan 35°$

$\Rightarrow r = 7.5 \times \tan 35° = 5.25$ cm

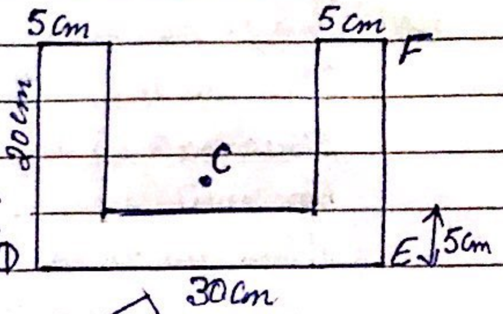
$\therefore$ $r = \underline{5.25 \text{ cm}}$ ✓

(Note: when an object is at the point of sliding and toppling both $\mu = \tan\theta$, GA is vertical)

In this case as the object topples but does not slide $\mu > \tan\theta \Rightarrow \mu > \tan 35$

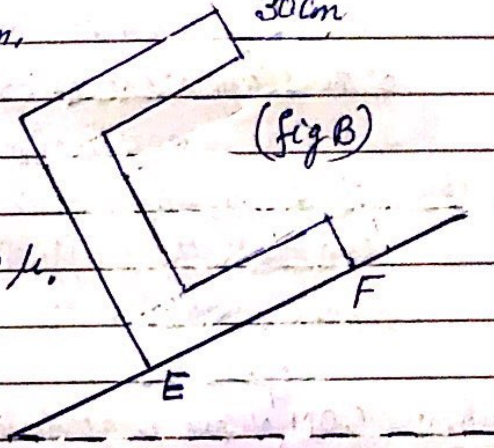
$\Rightarrow \underline{\mu > 0.7}$ ✓

Example 30: Fig (A) shows the cross-section of a uniform solid. The cross-section has the shape and dimensions shown. The centre of mass C of solid lies in the plane of this cross-section. The distance of C from DF is y cm.



(i) Find the value of y.

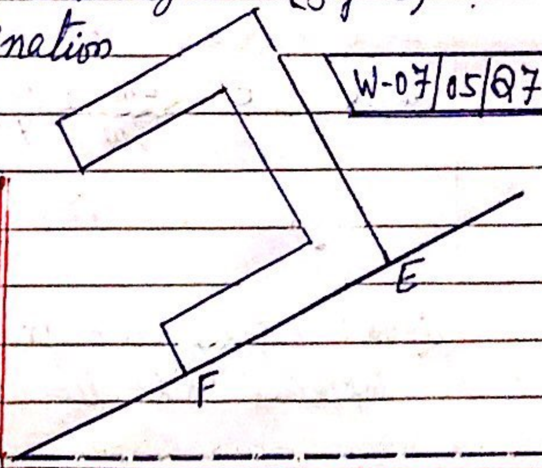
The solid is placed on a rough plane. The coefficient of friction between the solid and the plane is μ. The plane is tilted so that EF lies along a line of greatest slope.



(ii) The solid is placed so that F is (fig B) higher up the plane than E. When the angle of inclination is sufficiently great the solid starts to topple. Show that $\mu > \frac{1}{2}$.

(iii) The solid is now placed so that E is higher (fig: C) up that F. When the angle of inclination is sufficiently great the solid starts to slide (without topping). Show that $\mu < \frac{5}{6}$.

W-07/05/Q7



Solution: BL and QP are produced to meet DE at M and R resp.

(I) Now the rect. ABMD, LMRP and QREL each is of size 20×5 cm, let CG = y, C is the COM all distance measured from DE.

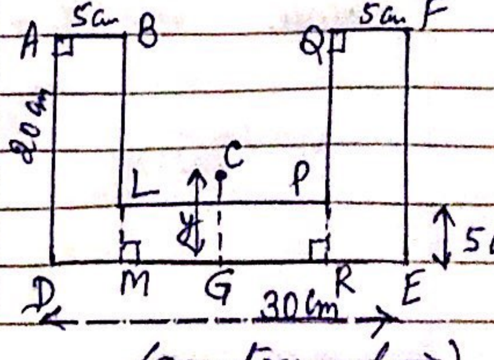
$\therefore\ y(20\times5+20\times5+20\times5)$

$=(20\times5)\times10+(20\times5)\times2.5+(20\times5)\times10$

$\Rightarrow 300y = 2250$

$y = \frac{2250}{300} = 7.5$ cm

or $CG = 7.5$ cm



(continued→)

30(ii) Given that when the plane is inclined at an angle 'θ' to the horizontal the object is at the point of toppling at edge E, CE is Vertical, 'COM' is at C, such that CG = 7.5

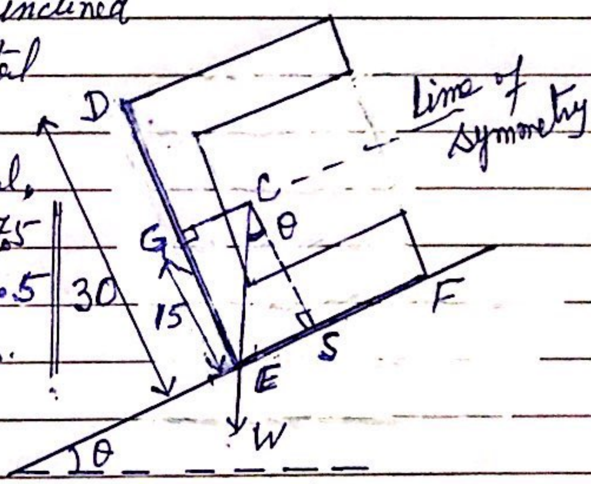
draw CS ⊥ EF ⇒ ES = 7.5

$CS = GF = \frac{30}{2} = 15$ cm.

$\angle ECS = \theta$

In rt triangle CSE

$$\tan\theta = \frac{ES}{CS} = \frac{7.5}{15} = \frac{1}{2}$$



now as for this value of 'θ' the object is at the point of toppling but not sliding ⇒ $\mu > \frac{1}{2}$ ✓

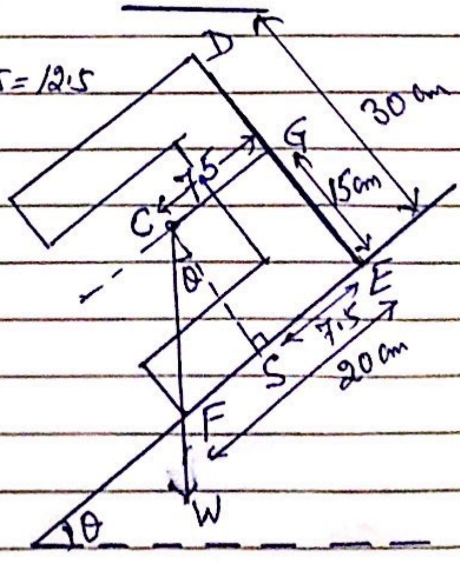
(iii) Now for toppling. $FS = 20 - 7.5 = 12.5$

$\mu = \tan\theta = \frac{FS}{CS} = \frac{12.5}{15}$

(CF is Vertical)

(CS⊥EF) $\tan\theta = \frac{5}{6}$

for this value of $\tan\theta$ the object will topple,



But given that object slides without toppling

⇒ $\mu < \tan\theta$

⇒ $\mu < \frac{5}{6}$ ✓

← × —— × —— × →