12.07.20

M.2

Mechanics-2

Hook's Law Exercise.

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Hook's Law

1. A light elastic string has natural length a and modulus of elastisity $24mg$. One end of the string is attached to a fixed point A. The other end of the string is attached to a particle of mass $2m$.

(a) Find, in terms of a, the extension of the string when the particle hangs freely in equilibrium below A. --[2]

(b) The particle is released from rest at A.
Find, in terms of a, the distance of the particle below A when it first comes to instantaneous rest [9231, SP-20/03/Q2] --[6]

2. A particle P of mass 0.3 Kg is attached to one end of a light elastic string of natural length 0.6 m and modulus of elastisity 9N. The other end of the string is attached to fixed point O on a smooth plane inclined at 30° to the horizontal. OA is a line of greatest slope of the plane with A below the level of O and OA = 0.8m. The particle P is released from rest at A.

(i) Find the initial acceleration of P. ---[4]

(ii) Find the greastest speed of P. [W-19/51/Q5] --[5]

3. A particle of mass 0.3 Kg is attached to one end of a light elastic string of natural length 0.6m and modulus of elastisity 9N. The other end of the string is attached to a fixed point O on a smooth horizontal surface. The particle is projected horizontally from O with speed $4ms^{-1}$. Find the greatest distance of the particle from O. --[3]
[W-19/52/Q1]

4. A particle P of mass 0.5 kg is attached to one end of a light elastic string of natural length 0.6m and modulus of elasticity 12N. The other end of the string is attached to a fixed point O. The particle P is projected vertically downwards with speed $2ms^{-1}$ from the point 0.5 m vertically below O. For an instant when the acceleration of P is $4ms^{-2}$ downwards, find the extension of the string and the speed of P. --[6]
[W-19/52/Q3]

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Hook's Law

5. A particle P of mass 0.4 kg is attached to one end of a elastic string of natural length 0.5 m and modulus of elasticity 6N. The other end of the string is attached to a fixed point O. The particle P is released from rest at the point $(0.5+x)$ m vertically below O. The particle P comes to instantaneous rest at O.

(i) Find x ---[3]

(ii) Find the greatest speed of P. [S-19/51/Q5] ---[5]

6. A light elastic string has natural length a m and modulus of elasticity λ N. When the length of the string is 1.6 m the tension is 4N. When the length of the string is 2 m the tension is 6N.

(i) Find the value of a and λ. ---[5]

One end of the string is attached to a fixed point O on a smooth horizontal surface. The other end of the string is attached to a particle P of mass 0.2 kg. The particle P moves with constant speed on the surface in a circle with centre O and radius 1.9 m.

(ii) Find the speed of P. [S-19/52/Q5] ---[3]

7. A particle P of mass 0.3 kg is attached to one end of a light elastic string of natural length 0.6 m and modulus of elasticity 24 N. The other end of the string is attached to a fixed point O. The particle P is released from the rest at the point 0.4 m vertically below O.

(i) Find the greatest speed of P. [M-19/52/Q5] ---[5]

(ii) Calculate the greatest distance of P below O. ---[3]

8. A small ball B is connected to one end of light elastic string of natural length 0.4 m and modulus of elasticity 12 N. The other end of the string is attached to a fixed point A. The ball is projected with speed $1\,ms^{-1}$ vertically downwards from a position 0.4 m vertically below A. (Continued →)

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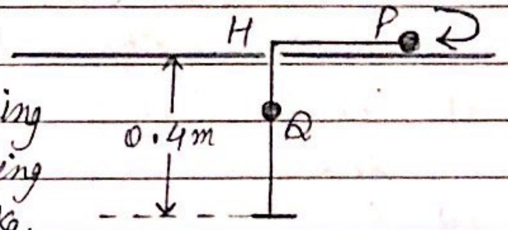
(Continued→)

8. and reaches its greatest speed at the point 0·7m below A.
(i) Show that the mass of B is 0·9kg. ---[2]
(ii) Calculate the greatest speed of B. [M-18/52/Q3] --[4]

9. A particle P of mass 0·2 Kg is attached to one end of a light inextensible string of length 0·6m. The other end of the string is attached to a particle Q of mass 0.3kg.



The string passes through a small hole H in a small horizontal surface. A light elastic string of natural length 0.3m and modulus of elasticity 15N joins Q to a fixed point A which is 0.4m vertically below H. The particle P moves on the surface in horizontal circle with centre H.
(i) Calculate the greatest possible speed of P for which the elastic string is not extended. ---[4]
(ii) Find the distance HP given that the angular speed of P is 8 rad s⁻¹. ---[5]
[S-18/51/Q6]

10. One end of a light elastic string is attached to a fixed to a fixed point O. The other end of the string is attached to a particle P of mass 0·4kg. The string has natural length 0·6m and modulus of elasticity 24N. The particle is released from rest at O. Find the two possible values of the distance OP for which the particle has speed 1·5ms⁻¹. [S-18/52/Q2] ---[6]

11. A particle P of mass 0·4kg is attached to a fixed point O by a light elastic string of natural length of 0·5m and modulus of elasticity 20N. The particle P is released from rest at O.
(i) Find the greatest speed of P in the subsequent motion. ---[4]
(ii) Find the distance below O of the point at which P comes to instantaneous rest. ---[3]
[W-18/51/Q3]

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12. A particle P of mass 0.7 Kg is attached to a fixed point O by a light elastic string of natural length 0.6 m and modulus of elasticity 15 N. The particle P is projected vertically downwards from the point A, 0.8 m vertically below O. The initial speed of P is $2 ms^{-1}$.

(i) Find the distance below A of the point at which P comes to instantaneous rest. ---[4]

(ii) Find the greatest speed of P in motion. [W-18/52/Q5] ---[4]

13. One end of a light elastic string of natural length 0.6 m and modulus of elasticity 24 N is attached to a fixed point O. The other end of the string is attached to a particle P of mass 0.4 kg which hangs in equilibrium vertically below O.

(i) Calculate the extension of the string. ---[2]

P is projected vertically downwards from the equilibrium position with speed $5 ms^{-1}$.

(ii) Calculate the distance P travels before its is first at instantaneous rest. [M-17/52/Q7] --- [4]

14. One end of a light inextensible string is attached to a fixed point A. The other end of the string is attached to a particle P of mass m Kg which hangs vertically below A. The particle is also attached to one end of a light elastic string of natural length 0.25 m. The other end of this string is attached to a point B which is 0.6 m from P and on the same horizontal level as P. Equilibrium is maintained by a horizontal force of magnitude 7N applied to P.

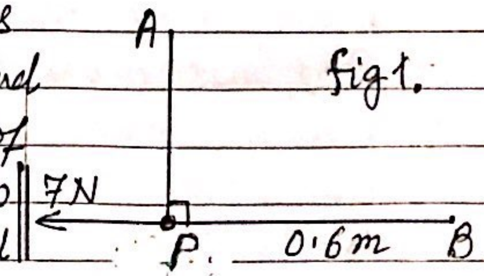

fig 1.

(i) Calculate the modulus of elastity of the elastic string. ---[2]

P is released from rest by removing the 7N force. In its subsequent motion P first comes to ..... rest at a point where BP = 0.3 m and the elastic string makes an angle of 30° with the horizontal.

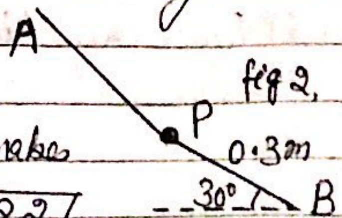

fig 2.

(ii) Find the value of m. ---- [4] [S-17/51/Q2]

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Hook's Law

15. A particle P of mass 0.15 kg is attached to one end of a light elastic string of natural length 0.4 m and modulus of elasticity 12 N. The other end of the string is attached to a fixed point A. The particle P moves in a horizontal circle which has it centre vertically below A, with the string inclined $\theta°$ to the vertical and AP = 0.5 m.

(i) Find the angular speed of P and the value of θ. ---[5]

(ii) Calculate the difference between the elastic potential energy stored in the string and the kinetic energy of P [S-17/51/Q6] -[4]

16. A particle P of mass 0.2 kg moves with speed $4 ms^{-1}$ and angular speed $5 rad s^{-1}$ in a horizontal circle on a smooth surface. P is attached to one end of a light elastic string of natural length 0.6 m. The other end of the string is attached to the point on the surface which is the centre of the circular motion of P

(i) Find the radius of this circle. --[1]

(ii) Find the modulus of elasticity of the string. [S-17/52/Q1] ---[4]

17. A particle of mass 0.3 kg is attached to one end of a light elastic string of natural length 0.8 m and modulus of elasticity 6 N. The other end of the string is attached to a fixed point O. The particle is projected vertically downwards from O with initial speed $2 ms^{-1}$.

(i) Calculate the greatest speed of the particle during its descent. --[5]

(ii) Find the greatest distance of the particle below O. [S-17/52/Q5] --[3]

18. One end of a light elastic string of natural length 0.8 m and modulus of elasticity 24 N is attached to a fixed point O. The other end of the string is attached to a particle P of mass 0.3 kg. P is projected vertically upwards with speed $4 ms^{-1}$ from a position 1.2 m vertically below O.

(i) Calculate the speed of the particle at the position where it is moving with zero acceleration. --[5] (ii) Show that the particle moves 1.2 m while moving upwards with constant deceleration. [W-17/51/Q5] --[3]

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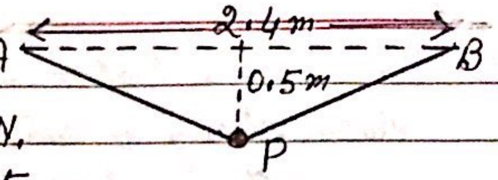
19. A light elastic string has natural length 2 m and modulus of elasticity 39 N.



The ends of the string are attached to fixed points A and B which are at the same horizontal level and 2.4 m apart. A particle P of mass m kg is attached to the mid-point of the string and hangs in equilibrium at a point 0.5 m below AB.

(i) Show that $m = 0.9$ --[4]

P is projected vertically downwards from the equilibrium position, and comes to instantaneous rest at a point 1.6 m below AB.

(ii) Calculate the speed of projection of P. [W-17/52/Q 4] --[5]

20. One end of a light elastic string of natural length 0.4 m and modulus of elasicity 8 N is attached to a fixed point O on a smooth horizontal plane. The other end of the string is attached to a particle P of mass 0.2 kg which moves on the plane in a circular path with centre O. The speed of P is $v\ ms^{-1}$ and the extension of the string is x m.

(i) Given that $v = 2.5$, find x. ---[4]

It is given instead that the kinetic energy of P is twice the elastic potential energy stored in the string. [W-17/52/Q6]

(ii) Form two simultaneous equations and hence find x and v ---[5]

21. A particle P of mass 0.6 kg is attached to one end of a light elastic string of natural length 0.8 m and modulus of elasticity 24 N. The other end of the string is attached to a fixed point A, and P hangs in equilibrium. [M-16/52/Q5]

(i) Calculate the extension of the string. ---[2]

P is projected vertically downwards from the equilibrium position with speed $4.5\ ms^{-1}$.

(ii) Find the distance AP when the speed of P is $3.5\ ms^{-1}$ and P is below the equilibrium position. --[4]

(iii) Calculate the speed of P when it is 0.5 m above the equilibrium position. [3]

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22. A particle P is attached to one end of a light elastic string of natural length 1·2 m and modulus of elasticity 12N. The other end of the string is attached to a fixed point O on a smooth plane inclined at an angle of 30° to the horizontal. P rests in equilibrium on the plane, 1·6 m from O.

(i) Calculate the mass of P.

A particle Q, with mass equal to mass of P, is projected up the plane along a line of greatest slope. When Q strikes P, the two particles coalesce. The combined particle remains attached to the string and move up the plane, coming to instantaneous rest after moving 0·2 m.

(ii) Show that the initial kinetic energy of the combined particle is 1J. --[4]

The combined particle subsequently move down the plane.

(iii) Calculate the greatest speed of the combined particle in the subsequent motion. [S-16/51/Q7] --[5]

23. One end of a light elastic string of natural length 0·4 m is attached to a fixed point O. The other end of the string is attached to a particle of weight 5N which hangs in equilibrium 0·6 m vertically below O.

(i) Find the modulus of elasticity of the string. --[2]

The particle is projected vertically upwards from the equilibrium position and comes to instantaneous rest after travelling 0·3 m upwards.

(ii) Calculate the speed of projection of the particle. --[3]

(iii) Calculate the greatest extension of the string in the subsequent motion. [S-16/52/Q2] --[3]

24. A particle P of mass 0·3 kg moves in a circle with centre O on a smooth horizontal surface. P is attached to O by a light elastic string of modulus of elasticity 12N and natural length l m. The speed of P is $4\,ms^{-1}$, and the radius of the circle in which it moves is $2l$ m.

Calculate l. [W-16/51/Q1] --[4]

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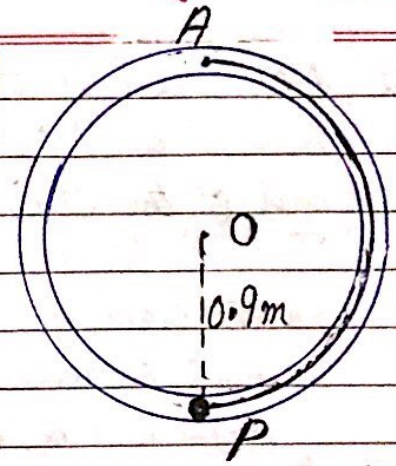

25. The diagram shows a smooth narrow tube formed into a fixed vertical circle with centre O and radius 0.9m. A light elastic string with modulus of elasticity 8N and natural length 1.2m has one end attached to the highest point A on the inside of the tube. The other end of the string is attached to a particle P of mass 0.2kg. The particle is released from rest at the lowest point on the inside of the tube. By considering energy, calculate

(i) the speed of P when it is at the same horizontal level as O. --[4]

(ii) the speed of P at the instant when the string becomes slack. --[3]

[W-16/51/Q6]

26. A particle P of mass 0.5kg is attached to one end of a elastic string with modulus of elasticity 24N and natural length 0.6m. The other end of the string is attached to a fixed point A. The particle P hangs in equilibrium vertically below A.

(i) Find the distance AP. --[2]

The particle P is raised to A and released from rest.

(ii) Calculate the greatest speed of P in the subsequent motion. --[3]

[W-16/52/Q2]

27. One end of a light elastic string of natural length 0.7m is attached to a fixed point A on a smooth horizontal surface. The other end of the string is attached to a particle P of mass 0.3kg which is held at a point B on the horizontal surface, where AB = 1.2m. It is given that P is released from rest at B, and that when AP = 0.9m, the particle has speed 4 ms^{-1}. Calculate the modulus of elasticity of the string. --[3]

[S-15/51/Q1]

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28. A particle P of mass 0.3 kg is attached to one end of a light elastic string of natural length 0.9 m and modulus of elasticity 12N. The other end of the string is attached to a fixed point O which is 3 m above the ground.

(i) Find the extension of string when P is in the equilibrium position. --[2]

P is projected vertically downwards from the equilibrium position with initial speed $6 ms^{-1}$. At the instant when the tension in the tension in the string is 12N the string breaks. P continues to descend vertically.

(ii) (a) Calculate the height of P above the ground at the instant when the string breaks. ---[2]

(b) Find the speed of P immediately before it strikes the ground. --[4]

S-15/51/Q5

29. One end of a light elastic string of natural length 0.5 m and modulus of elasticity 30N is attached to a fixed point O. The other end of the string is attached to a particle P which hangs in equilibrium vertically below O, with OP = 0.8 m.

(i) Show that the mass of P is 1.8 kg. --[2]

The particle is pulled vertically downwards and released from rest from the point where OP = 1.2 m.

(ii) Find the speed of P at the instant when the string first becomes slack. S-15/52/Q2 --[3]

30. One end of a light elastic string of natural length 0.4 m and modulus of elasticity 20 N is attached to a fixed point A on a smooth plane inclined at 30° to the horizontal. The other end of the string is attached to a particle P of mass 0.5 kg which rests in equilibrium on the plane.

(i) Calculate the extension of the string. ---[2]

P is projected down the plane from the equilibrium position with speed $5 ms^{-1}$. The extension of the string is e m when the speed of the particle is $2 ms^{-1}$ for the first time.

(ii) Find e. S-15/53/Q3 --[4]

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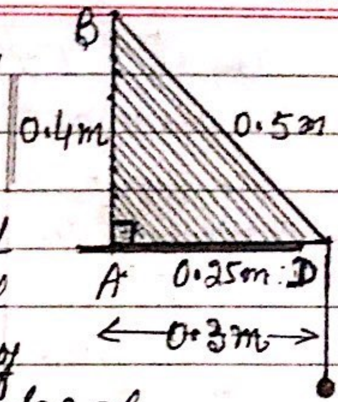
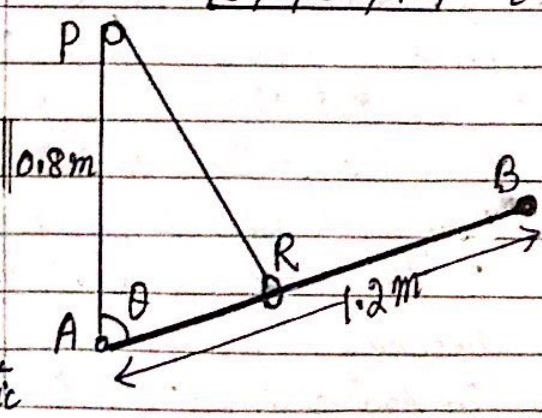
31. A uniform triangular prism of weight 20N rests on a horizontal table. ABC is the cross-section through the centre of mass of the prism, where BC = 0.5m, AB = 0.4m and AC = 0.3m and angle BAC = 90°. The vertical plane ABC is perpendicular to the edge of the table. The point D on AC is at the edge of the table, and AD = 0.25m. One end of a light elastic string of natural length 0.6m and modulus of elasticity 48N is attached to C and a particle of mass 2.5 kg is attached to the other end of the string. The particle is released from rest at C and falls vertically.



(i) Show that the tension in the string is 60N at the instant when the prism topples. ---[3]

(ii) Calculate the speed of the particle at the instant when the prism topples. [S-15/53/Q5] --[5]

32. A uniform rigid rod AB of length 1.2m and weight 8N has a particle of weight 2N attached at the end B. The end A of the rod is freely hinged to a fixed point. One end of a light elastic string of natural length 0.8m and modulus of elasticity 20N is attached to the hinge. The string passes over a small smooth pully P fixed 0.8m vertically above the hinge. The other end of the string is attached to small light smooth ring R which can slide on the rod. The system is in equilibrium with the rod inclined at an angle θ to the vertical.



(i) Show that the tension in the string is 20 sin θ N. --(1)

(ii) Explain why the part of the string attached to the ring is perp. to the rod. --[1]

(iii) Find θ. [W-15/51/Q2] --[3]

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33. A particle P of mass 0·2 kg is attached to one end of a light elastic string of natural length 0·75 m and modulus of elasticity 21 N. The other end of the string is attached to fixed point A which is 0·8 m vertically above a smooth horizontal surface. P rest in equilibrium on the surface.

(i) Find the magnitude of the force exerted on P by the surface. ---[2]

P is now projected horizontally along the surface with speed $3 ms^{-1}$.

(ii) Calculate the extension of the string at the instant when P leaves the surface. ---[3]

(iii) Hence find the speed of P at the instant when it leaves the surface. --[3]

$\boxed{W-15 | 51 | Q5}$

34. A particle 'P' of mass M kg is attached to one end of a light elastic string of natural length 0·8 m and modulus of elasticity 12·5 N. The other end of the string is attached to a fixed point A. The particle is released from rest at A and falls vertically until it comes to instantaneous rest at the point B. The greatest speed of P during its descent is $4{\cdot}4 ms^{-1}$ when extension of string is e m.

(i) Show that $e = 0{\cdot}64M$ --[2]

(ii) Find a second equation in e and M, and hence find M. ---[6]

(iii) Calculate the distance AB. ---[3]

$\boxed{W-15 | 53 | Q7}$

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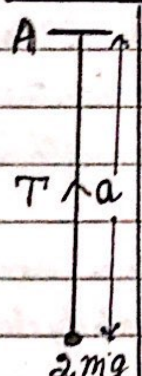
Answers

1. (a) $T = \frac{\lambda}{l_0} \times x$



or $2mg = \frac{24mg \cdot x}{a}$

$\Rightarrow x = \frac{a}{12}$ ✓

(b) Let d be the distance below A,

loss in gravitional potential energy (GPE) for particle $= 2mgd$ —(1)

and Gain in elastic potential energy (EPE) of string $= \frac{1}{2} \times \frac{\lambda}{l_0} \times x^2$

$= \frac{1}{2} \times \frac{24mg}{a}(d-a)^2$ —(2)

fr (1)&(2), No change in Kinetic energy (K.E)

$\Rightarrow 2mgd = \frac{1}{2} \times \frac{24mg}{a}(d-a)^2$

$\Rightarrow 6(d-a)^2 = ad$

$\Rightarrow d = \frac{3}{2}a$ or $\frac{2}{3}a$

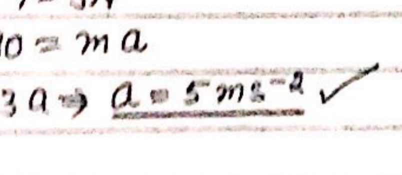
as $d > a$

$\therefore$ Required distance $= \underline{\frac{3}{2}a}$ ✓

2. $m = 0.3\,kg$

$l_0 = 0.6\,m$

$\lambda = 9\,N$



(i) $T = \frac{\lambda}{l_0} x = \frac{9 \times (0.8 - 0.6)}{0.6} = 3$

$\rightarrow T = 3N$

$T - 0.3g \sin 30 = ma$

$3 - \frac{3}{2} = 0.3a \Rightarrow \underline{a = 5\,ms^{-2}}$ ✓

(ii) $0.3g \sin 30 = \frac{9}{0.6} e \Rightarrow e = 0.1$

(Continued ⟶)

K.E = EPE

3. $\frac{1}{2}mv^2 = \frac{\lambda}{2l_0} \times e^2$

or $\frac{1}{2} \times 0.3 \times 4^2 = \frac{9e^2}{2 \times 0.6}$

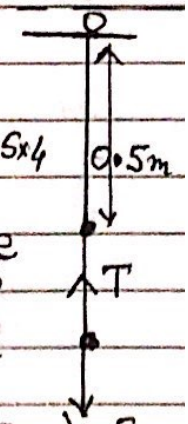
$\Rightarrow e = 0.566\,m$

$\therefore$ distance of the particle from O: $d = 0.6 + 0.566$

$= 1.166$ ✓

$= \underline{1.17\,m}$

4. $l_0 = 0.6\,m$, $\lambda = 12\,N$



Vertically

$mg - T = ma \Rightarrow 0.5g - T = 0.5 \times 4$

$\Rightarrow T = 3$ —(1)

Also $T = \frac{\lambda}{l_0} e \Rightarrow 3 = \frac{12e}{0.6}$

$\Rightarrow e = 0.15\,m$

$EPE = \frac{1}{2}\frac{\lambda}{l_0} e^2 = \frac{1}{2} \times \frac{12}{0.6} \times 0.15^2$

and distance fallen $= (0.6 - 0.5 + .15)\,0.5g$

$\frac{.5}{2} v^2 = \frac{0.5 \times 2^2}{2} + 0.5g(.6 - .5 + .15) - 12 \times .15^2 / 2 \times 0.6$

$\Rightarrow \underline{V = 2.85\,ms^{-1}}$ ✓

2(ii) Continued.

$EPE = \frac{9(0.8-0.6)^2}{2 \times 0.6}$ and $\frac{9 \times 0.1^2}{2 \times 0.6}$

KE gain = Loss in P.E

$\frac{1}{2} \times 0.3v^2 = \frac{9(.8-.6)^2}{2 \times 0.6} - \frac{9 \times 0.1^2}{2 \times 0.6} - 0.3g \times 0.1 \sin 30°$

$\Rightarrow v = \underline{0.707\,ms^{-1}}$ ✓

Hooks Law

Answers

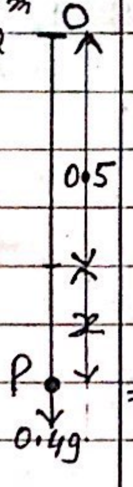
5. (i) $m=0.4kg,\ \lambda=6N,\ l_0=0.5m$

$P.E = E.P.E$ or $mgh = \frac{\lambda x^2}{2l_0}$

$0.4g(0.5+x) = 6x^2/2\times0.5$

$\Rightarrow 6x^2-4x-2=0$

$\Rightarrow x=1 \quad (\cancel{x=-\frac{1}{3}})$



(ii) $T=\frac{\lambda}{l_0}e \Rightarrow 0.4g=\frac{6e}{0.5}$

$\Rightarrow e=\frac{1}{3}$

$P.E\ change = 0.4g(0.5+\frac{1}{3})$

$\frac{0.4v^2}{2} = 0.4g(0.5+\frac{1}{3}) - \frac{6(\frac{1}{3})^2}{2\times0.5}$

$\Rightarrow \underline{V=3.65ms^{-1}}$ ✓

6. $\lambda=?\ ;\ l_0=a\ m,\ T_1=4N, e_1=(1.6-a)$

(i) $T=\frac{\lambda}{l_0}e \Rightarrow 4=\frac{\lambda(1.6-a)}{a}$ — ①

also, $T_2=6,\ e_2=(2-a)$

$\Rightarrow 6=\frac{\lambda(2-a)}{a}$ — ②

fm ① & ② $1.5=\frac{(2-a)}{(1.6-a)}$

$\Rightarrow a=0.8$ ✓

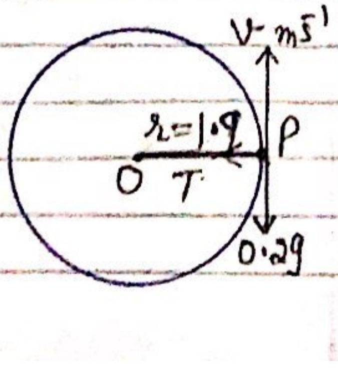
fm ① $4=\frac{\lambda(1.6-.8)}{0.8} \Rightarrow \underline{\lambda=4}$ ✓

(ii) $T=\frac{\lambda}{l_0}e \Rightarrow T=\frac{4}{0.8}(1.9-0.8)=5.5$

$\Rightarrow T=5.5$ ✓

also $T=\frac{mv^2}{r} \Rightarrow 5.5=\frac{0.2V^2}{1.9}$

$\Rightarrow \underline{V=7.23ms^{-1}}$ ✓



7. $\lambda=24N,\ m=0.3kg,\ l_0=0.6m$

$T=\frac{\lambda e}{l_0} \Rightarrow 0.3g=\frac{24e}{0.6} \Rightarrow e=0.075m.$

$P.E\ change = 0.3g\times(0.6-0.4+0.75)$

$=0.3g\times0.275$ ✓

KE

$\frac{1}{2}\times0.3V^2 = 0.3g\times0.275 - \frac{\lambda\times e^2}{2l_0}$

$\Rightarrow \frac{1}{2}\times0.3V^2 = 0.3g\times0.275 - \frac{24\times0.075^2}{2\times0.6}$

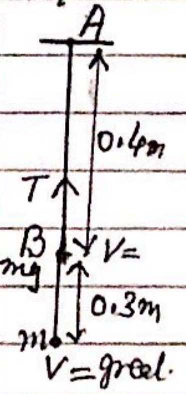
$\Rightarrow \underline{V=2.18ms^{-1}}$ ✓

8. $l_0=0.4m,\ \lambda=12N,\ m=?$

(i) $T=mg=\frac{\lambda e}{l_0}$

$\Rightarrow m.g=\frac{12\cdot(0.7-0.4)}{0.4}$

$\Rightarrow m=0.9kg$ ✓



(ii) $EPE=\frac{\lambda\cdot e^2}{2\times l_0}=\frac{12\cdot(.7-.4)^2}{2\times0.4}$

Now

$\frac{0.9V^2}{2} = 0.9g(0.7-0.4) + \frac{0.9\times1^2}{2} - \frac{12(0.7-0.4)^2}{2\times0.4}$

$\Rightarrow \underline{V=2ms^{-1}}$ ✓

9(i) $r=0.6-(0.4-0.3)=0.5$

$T=0.3g\ ;\ T=mv^2/r$

$\Rightarrow 0.3g=0.2V^2/0.5 \Rightarrow$

$\Rightarrow \underline{V=2.74ms^{-1}}$ ✓

(ii) $r=(0.5+e);\ T=\frac{15e}{0.3}=50e$

$0.2\times8^2\times(.5+e)=50e+0.3g$

$\Rightarrow e=\frac{(6.4-3)}{(50-12.8)}=0.0914$

$\therefore HP=0.5+0.0914=0.5914$

or $\underline{HP=0.591m}$ ✓

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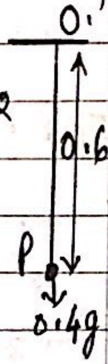
Answers

10. $\lambda = 24N, l_0 = 0.6m, m = 0.4kg$

$EPE = \frac{\lambda e^2}{2l_0} = \frac{24(x-0.6)^2}{2\times 0.6}$



$\frac{0.4\times 1.5^2}{2} = 0.4gx - \frac{24(x-0.6)^2}{2\times 0.6}$

$\Rightarrow 20x^2 - 28x + 7.65 = 0$

$\Rightarrow OP = 1.0279, 0.372m$ (reject)

$\frac{0.4\times 1.5^2}{2} = 0.4gx$

$\Rightarrow x = 0.1125m$

or $OP = 0.1125m$ ✓

11. $m = 0.4kg, l_0 = 0.5, \lambda = 20$

(i) $T = \frac{\lambda e}{l_0} = mg \Rightarrow 0.4g = \frac{20e}{0.5}$

$\Rightarrow e = 0.1$

$\frac{0.4V^2}{2} = 0.4g(.5+.1) - \frac{20\times(0.1)^2}{2\times 0.5}$

$\Rightarrow V = \sqrt{11} = 3.32ms^{-1}$ ✓

(ii)

$0.4g(5+x) = \frac{20x^2}{2\times 0.5}$

$\Rightarrow 20x^2 - 4x - 21 = 0$

$\Rightarrow x = 0.432$

$\therefore$ distance below O $= 0.5 + 0.432$

$= 0.932m$ ✓

12(i) $m = 0.7kg, l_0 = 0.6m, \lambda = 15m$

$EPE = \frac{\lambda e^2}{2l_0} = \frac{15(0.2+x)^2}{2\times 0.6}$

or $\frac{15\times 0.2^2}{2\times 0.6}$

$\Rightarrow \frac{15(0.2+x)^2}{2\times 0.6} = \frac{15\times 0.2^2}{2\times 0.6} + 0.7gx + \frac{0.7\times 2^2}{2}$

$\Rightarrow x = 0.424m$

(Contined)

13. $l_0 = 0.6m, \lambda = 24N, m = 0.4$

(i) $T = \frac{\lambda e}{l_0} \Rightarrow 0.4g = \frac{24e}{0.6} \Rightarrow e = 0.1m.$

(ii) Initial Elastic energy (EE)

$= \frac{\lambda e^2}{2l_0} = \frac{24\times 0.1^2}{2\times 0.6} = 0.2J.$

Initial speed $v = 5ms^{-1}$

KE + PE = E.P.E − Intial EE

$\frac{1}{2}mv^2 + mgh = \frac{1}{2}\frac{\lambda x^2}{l_0} - \frac{\lambda e^2}{2l_0}$

$\Rightarrow \frac{1}{2}\times 0.4\times 5^2 + 0.4gd = \frac{24(0.1+d)^2}{2\times 0.6} - \frac{24\times 0.1^2}{2\times 0.6}$

$\Rightarrow d = 0.5m$ ✓

14(i). $l_0 = 0.25, e = (0.6-0.25) = 0.35, \lambda = ?$

$F = T = \frac{\lambda e}{l_0} \Rightarrow 7 = \frac{\lambda\times 0.35}{0.25}$

$\Rightarrow \lambda = 5N$ ✓

(ii) $EE = \frac{0.35^2\times 5}{2\times 0.25}$ or $\frac{0.05^2\times 5}{2\times 0.25}$

$PE = mg\times 0.3\sin 30^\circ$

$mg\times 0.3\sin 30^\circ = \frac{0.35^2\times 5}{2\times 0.25} + \frac{0.05^2\times 5}{2\times 0.25}$

$\Rightarrow m = 0.8$ ✓

12(ii) $T = mg = \lambda e/l_0 \Rightarrow 0.7g = \frac{15e}{0.6}$

$\Rightarrow e = 0.28m,$

$(0.7\times 2^2)/2 + 0.7g(0.28-0.2) + \frac{15\times 0.2^2}{2\times 0.6} = \frac{0.7V^2}{2} + \frac{15\times 0.28^2}{2\times 0.6}$

$\Rightarrow v = 2.06ms^{-1}$ ✓

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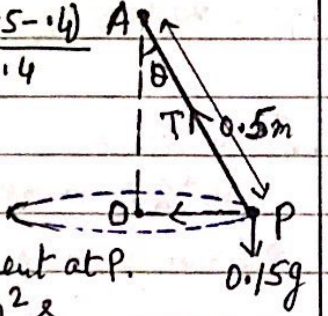
Answers

15 $m=0.15\,kg,\ l_0=0.4\,m,\ \lambda=12N$

(i) $T=\dfrac{\lambda\times e}{l_0}=\dfrac{12\times(.5-.4)}{0.4}$

$\Rightarrow T=3N.$

$r=0.5\sin\theta$



Horizontal component at P.

$T\sin\theta=m\omega^2 r$

$3\sin\theta=0.15\times\omega^2\cdot(0.5\sin\theta)$

$\Rightarrow \underline{\omega=6.32\ rad\,s^{-1}}$ ✓

Vertically at P.

$T\cos\theta=mg\Rightarrow 3\cos\theta=0.15g$

$\Rightarrow \cos\theta=0.5$

$\Rightarrow \underline{\theta=60^\circ}$ ✓

(ii) $v=r\omega=0.5\sin60^\circ\times6.32$

$K.E=\frac{1}{2}mv^2=\frac{1}{2}\times0.15\times(6.32\times0.5\sin60)^2$

$=0.5625\,J$

$EPE=\dfrac{\lambda e^2}{2\times l}=12\times0.1^2/2\times0.4$

Difference $=0.5625-\dfrac{12\times0.1^2}{2\times0.4}$

Diff. $=\underline{0.4125\,J.}$

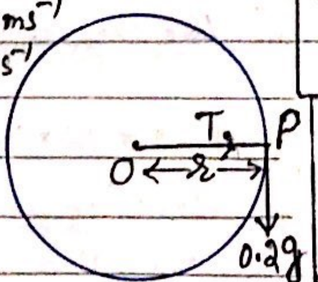
16(i) $m=0.2\,kg,\ v=4ms^{-1}$

$\omega=5\,rad\,s^{-1}$

$v=r\omega$

$4=r\times5$

$\Rightarrow \underline{r=0.8m}$ ✓



(ii) $l_0=0.6m$

$T=m\omega^2r=0.2\times5^2\times0.8$

$\Rightarrow T=4N$ ✓

$T=\dfrac{\lambda e}{l_0}\Rightarrow 4=\lambda\dfrac{(0.8-0.6)}{0.6}$

$\Rightarrow \underline{\lambda=12N}$ ✓

17(i) $m=0.3\,kg,\ l_0=0.8m,\ \lambda=6N$

$v=2ms^{-1}$

$T=mg=\dfrac{\lambda e}{l_0}\Rightarrow0.3g=\dfrac{6\times e}{0.8}\Rightarrow e=0.4m$

$EE=\dfrac{\lambda e^2}{2l_0}=\dfrac{6\times0.4^2}{2\times0.8}$

$0.3\dfrac{v^2}{2}-0.3\times\dfrac{2^2}{2}=0.3g(0.8+0.4)-6\times0.4^2/2\times0.8$

$\Rightarrow v=4.9ms^{-1}$ ✓

(ii) $\dfrac{0.3\times2^2}{2}+0.3gL=6(L-0.8)^2/2\times0.8$

$\Rightarrow \underline{L=2.18m.}$

18.(i) $l_0=0.8m,\ \lambda=24N,\ m=0.3$ | $u=4ms^{-1}$

$T=mg=\dfrac{\lambda e}{l_0}\Rightarrow0.3g=\dfrac{24e}{0.8}$

$\Rightarrow e=0.1m$ ✓

$EE=\dfrac{\lambda x^2}{2L}=\dfrac{24(1.2-0.8)}{2\times0.8}$ or $\dfrac{24\times0.1^2}{2\times0.8}$

$0.3v^2/2=0.3\times4^2/2+24(1.2-0.8)^2/2\times0.8$

$-24\times0.1^2/2\times0.8-0.3g(1.2-0.8)$

$\Rightarrow \underline{v=5ms^{-1}}$ ✓

(ii) $0.3\times5^2/2+24\times0.1^2/2\times0.8$

$=0.3(x+0.9)\times10$

$\Rightarrow x=0.4$

distance moved $=0.8+0.4$

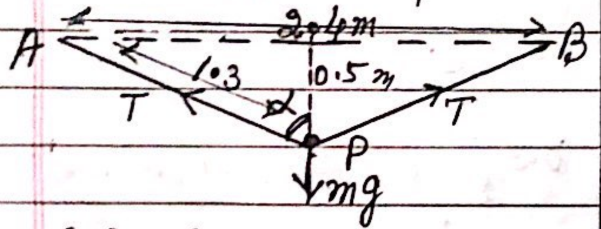
$=\underline{1.2m}$ ✓

Answers

19. $l_0 = 2m$, $\lambda = 39N$, $m = ?$

(i) $e = \sqrt{0.5^2 + 1.2^2} - 1 = 0.3$

$T = \lambda e / l_0 = \frac{39 \times 0.3}{1}$



$mg = 2T\cos\alpha$

$= 2 \times \left(\frac{39 \times 0.3}{1}\right) \times 0.5/1.3$

$\Rightarrow m = 0.9$ ✓

(ii) $E = \sqrt{1.6^2 + 1.2^2} - 1 = 1m$

$EE = \frac{39 \times 1^2}{2 \times 1}$ or $\frac{39 \times 0.3^2}{2 \times 1}$

$0.9\, V^2/2 + 0.9g(1.6 - 0.5)$

$= 2\left[39 \times 1^2/2\times1 - 39 \times 0.3^2/2\times1\right]$

$\Rightarrow \underline{V = 7.54\, ms^{-1}}$ ✓

20 (i) $l_0 = 0.4$, $v = 2.5$, $x = ?$, $m = 0.2\,kg$

$\lambda = 8$; $T = \frac{mv^2}{r} = 0.2 \times 2.5^2/(0.4 + e)$ — ①

also $T = \frac{\lambda e}{l_0} = \frac{8e}{0.4} = 20e$ — ②

from ① & ② $1.25/(0.4+e) = 20e$

$\Rightarrow e^2 + 8e - 1.25 = 0$

$\Rightarrow e = \underline{0.12\,m}$ ✓

(ii) $K.E = 2[E.P.E]$

$\frac{1}{2}mV^2 = 2\left[8x^2/(2\times0.4)\right]$

$\Rightarrow \frac{1}{2}\times 0.2 \times V^2 = 2\left[8x^2/2\times0.4\right]$

and $\frac{0.2V^2}{(0.4+x)} = 8x/0.4$

$\Rightarrow x = 0.4$ and $v = 5.66\, ms^{-1}$

21 (i) $l_0 = 0.8m$, $\lambda = 24$, $m = 0.6\,kg$

$T = mg = \lambda e/l_0 \Rightarrow 0.6g = 24e/0.8$

$\Rightarrow \underline{e = 0.2}$ ✓

(ii) $u = 4.5\, ms^{-1}$

$\lambda e^2/2l_0 = 24 \times 0.2^2/2\times0.8 = 0.6$

$0.6 \times 4.5^2/(2 + 0.6gd) + 24\times0.2^2/2\times0.8$

$= 0.6 \times 3.5^2/2 + 24(0.2+d)^2/2\times0.8$

$\Rightarrow d = 0.4$ so $AP = 0.8 + 0.2 + 0.4$

$\Rightarrow \underline{AP = 1.4m}$ ✓

(iii) $24\times0.2^2/2\times0.8 + 0.6\times4.5^2/2$

$= 0.6\,V^2/2 + 0.6g\times0.5$

$\Rightarrow V = \underline{3.5\, ms^{-1}}$ ✓

22 (i) $l_0 = 1.2m$, $\lambda = 12N$, $l_0 + e = 1.6m$

$T = \frac{\lambda e}{l_0} \Rightarrow mg\sin30° = 12(1.6-1.2)/1.2$

$\Rightarrow \underline{m = 0.8\,kg}$ ✓

(ii) PE change $= 1.6$

$IKE + 12\times0.4^2/2.4$

$= 1.6 \times 0.2\, g\sin30 + 12\times0.2^2/2.4$

$\Rightarrow \underline{I.K.E = 1J}$ AG ✓

(iii) $12e/1.2 = 1.6g\sin30°$

$\Rightarrow e = 0.8$

$1.6V^2/2 + 12\times0.8^2/2.4 =$

$1.6g\times0.6\sin30° + 12\times0.2^2/2.4$

$\Rightarrow \underline{V = 1.5\, ms^{-1}}$

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M.2

Answers

23. $l_0=0.4m,\ mg=5N,\ \lambda=?$

(i) $T=mg=\lambda e/l_0 \Rightarrow 5=\frac{(0.6-0.4)\lambda}{0.4}$

$\Rightarrow \lambda=10N$

(ii) $10(0.2^2/2\times0.4)+\frac{5}{g}v^2/2=0.3\times5$

$\Rightarrow v=2ms^{-1}$ ✓

(iii) $10e^2/2\times0.4=5(e+0.1)$

$\Rightarrow e=0.483$ ✓

24. $m=0.3kg,\ \lambda=12N,\ l_0=l$

$r=2l\,m,\ v=4ms^{-1}$

$T=mv^2/r=0.3\times4^2/2l$ — ①

Also $T=\frac{\lambda e}{l_0}=\frac{12(2l-l)}{l}$

$\Rightarrow T=12N$ — ②

from ① & ② $\frac{0.3\times4^2}{2l}=12$

$\Rightarrow l=0.2m$ ✓

25. $r=0.9m,\ \lambda=8N,\ l_0=1.2m,\ m=0.2kg$

(i) $\boxed{EE=8(0.9\pi-1.2)^2/2\times1.2=8.83}$

Now $8.83=0.2g\times0.9+0.2v^2/2+8(0.9\pi-1.2)^2/2\times1.2$

$\Rightarrow v=8.29ms^{-1}$ ✓

(ii) $\theta=1.2/0.9=4/3\,rad\ (=76.4^\circ)$

$8.83=0.2g\times0.9+0.2g\cos\theta+0.2v^2/2$

$\Rightarrow v=8.13ms^{-1}$ ✓

26. $m=0.5kg,\ \lambda=24N,\ l_0=0.6m$

(i) $\boxed{T=\lambda e/l_0 \Rightarrow 5=\frac{24e}{0.6}\Rightarrow e=0.125}$

$\boxed{\therefore AP=l_0+e=0.6+0.125=0.725m}$ ✓

(ii) $0.5g\times0.725=\frac{24\times0.125^2}{2\times0.6}+\frac{0.5v^2}{2}$

$\Rightarrow v=3.64ms^{-1}$ ✓

27. $l_0=0.7m,\ m=0.3kg,\ \lambda=?$

$EE(B)=\frac{\lambda\times(1.2-0.7)^2}{2\times0.7}=\frac{5\lambda}{28}$

or $EE=\frac{\lambda\times(0.9-0.7)^2}{2\times0.7}=\frac{\lambda}{35}$

$\Rightarrow \frac{\lambda\times0.5^2}{2\times0.7}-\frac{\lambda\times0.2^2}{2\times0.7}=\frac{0.3\times4^2}{2}$

$\Rightarrow \lambda=16N$ ✓

28. $m=0.3kg,\ l_0=0.9m,\ \lambda=18N$

(i) $T=mg=\frac{\lambda e}{l_0}\Rightarrow 0.3g=\frac{18e}{0.9}\Rightarrow e=0.15m$ ✓

(ii)(a) $12=18\cdot ext/0.9$ and $ht=3-0.9-ext$

$\Rightarrow ext=0.6 \Rightarrow ht=3-0.9-0.6$

$\Rightarrow ht=1.5m$ ✓

(b) $\frac{0.3\times6^2}{2\times0.9}-\frac{0.3u^2}{2}+0.3g(0.6-0.15)$

$=\frac{18\times0.6^2}{2\times0.9}-\frac{18\times0.15^2}{2\times0.9}$

$\Rightarrow \left(\frac{0.3u^2}{2}=3.375\right)$

$0.3v^2=0.3u^2+0.3g(3-0.6-0.9)$

or $v^2=u^2+2g(3-0.6-0.9)$

$\Rightarrow v=7.25ms^{-1}$ ✓

29. $l_0=0.5m,\ \lambda=30N,\ OP=0.8$

(i) $T=mg=\frac{\lambda e}{l_0}\Rightarrow mg=30(0.8-0.5)/0.5$

$\Rightarrow m=1.8kg$ ✓

(ii) $EE=30(1.2-0.5)^2/2\times0.5$

$1.8v^2/2=30(1.2-0.5)^2/2\times0.5-1.8(1.2-0.5)g$

$\Rightarrow v=1.53ms^{-1}$ ✓

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Answers

30. $l_0 = 0.4m$, $\lambda = 20N$, $m = 0.5Kg$

(i) $T = \frac{\lambda x}{l_0} \Rightarrow 0.5g \sin 30^\circ = \frac{20x}{0.4}$

$\Rightarrow x = 0.05m$ ✓

(ii) $\frac{20(0.05)^2}{2\times 0.4} + 0.5\times 5^2/2 =$

$20e^2/2\times 0.4 + \frac{0.5\times 2^2}{2}$

$-0.5(e-0.05)g\sin 30^\circ$

$\Rightarrow 2.5e^2 - 2.5e - 5.1875 = 0$

$\Rightarrow e = 0.508$ ✓

31. (i) C of M is 0.1m from AB.

Moment about D: $T \cdot CD = W \times$ (D from M along AB)

$T \times 0.05 = 20(0.25 - 0.1)$

$T = 60 N$ ✓

(ii) $T = \lambda e/l_0 \Rightarrow 60 = 48e/0.6$

($m = 2.5Kg$) $\Rightarrow e = 0.75m$

$2.5 v^2/g + 48(0.75)^2/2\times 0.6$

$= 2.5g(0.75 + 0.6)$

$\Rightarrow v = 3ms^{-1}$ ✓

32. $\lambda = 20N$, $l_0 = 0.8m$

(i) $T = 20(0.8\sin\theta)/0.8 = 20\sin\theta$

(ii) No friction (so perpendicular)

(iii) $20\sin\theta(0.8\cos\theta) =$

$8(0.6\sin\theta) + 2(1.2\sin\theta)$

$\Rightarrow \theta = 63.3^\circ$ ✓

33. $m = 0.2Kg$, $l_0 = 0.75$, $\lambda = 21N$.

(i) $0.2g = R + 21\times(0.8-0.75)/0.75$

$\Rightarrow R = 0.6N$ ✓

(ii) $\frac{21e}{0.75} \times \frac{0.8}{(e+0.75)} = 0.2g$

$\Rightarrow e = 0.0735$ ✓

(iii) $\frac{0.2(3)^2}{2} + \frac{21(0.05)^2}{2\times 0.75}$

$= \frac{0.2V^2}{2} + \frac{21\times 0.0735^2}{1.5}$

$\Rightarrow V = 2.93ms^{-1}$!

34. $M = ?$, $l_0 = 0.8m$, $\lambda = 12.5$

(i) $Mg = \frac{\lambda e}{l_0} \Rightarrow Mg = \frac{12.5e}{0.8}$

$\Rightarrow e = 0.64M$ ✓

(ii) $Mg(0.8+e) = \frac{M\times 4.4^2}{2} + \frac{12.5e^2}{(2\times 0.8)}$

$\Rightarrow$

$10M(0.8 + 0.64M) = 9.68M$

$+ 12.5\frac{(0.64M)^2}{1.6}$

$\Rightarrow 8 + 6.4M = 9.68 + 3.2M$

$\Rightarrow M = 0.525$ ✓

(iii)

$0.525gd = \frac{12.5(d-0.8)^2}{2\times 0.8}$

$\Rightarrow 0.672d = d^2 - 1.6d + 0.64$

$\Rightarrow d = 1.94$ ✓

←— X —— X —→

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