16.7.20

M.2

Mechanics-2

Hook's Law
Notes

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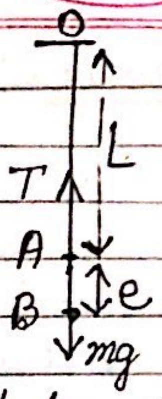
Hook's Law

Notes. P. 1



- **Hook's Law:**
  The tension 'T' in an elastic spring (or string) is proportional to extension 'e' produced.

$$T \propto e$$

or $T = \dfrac{\lambda e}{L}$ where 'λ' is the modulus of elastity of the string, its unit is Newton 'N'

'e' is the extension produced.
'L' is the natural length of the string.

Example 1. A particle P of mass 0.3 Kg is attached to one end of a light elastic string of naturel length 0.9 m and modulus of elasticity 1 N. The other other end of the string is attached to a fixed point O, which is 3 m above the ground. ...[2]
Find the extension of string when P is in equilibrium position.

S-15 | 51 | Q5(i)

Solution: $T = mg = \dfrac{\lambda e}{L}$

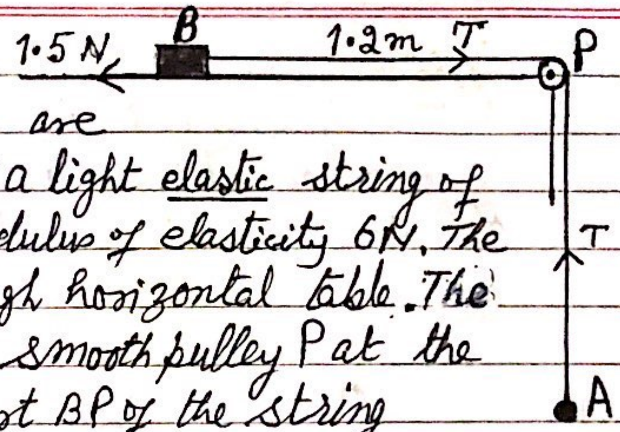
$$0.3g = \frac{18 \times e}{0.9}$$

$$\begin{cases} m = 0.3\,Kg \\ L = 0.9\,m \\ \lambda = 1\ N \end{cases}$$

$$\Rightarrow e = \frac{(0.3 \times 10) \times .9}{18} = 0.15$$

Extension $e = 0.15$

Example 2:



A particle A and a block B are attached to opposite ends of a light elastic string of natural length 2m and modulus of elasticity 6N. The block is at rest on a rough horizontal table. The string passes over a small smooth pulley P at the edge of the table, with part BP of the string horizontal and of length 1.2m. The frictional force acting on B is 1.5 N and the system is in equilibrium. Find the distance PA. [S-08/05/Q1] ---[3]

Solution: Natural length of string $L = 2m$, $\lambda = 6N$, $F = 1.5 N$

Let the distance $PA = x$ m.

$$\Rightarrow BP + PA = (L + e)$$
$$\Rightarrow 1.2 + PA = (2 + e)$$
$$\Rightarrow PA = (e + 0.8) \quad \text{---(i)}$$

{ e is the extention in the string.

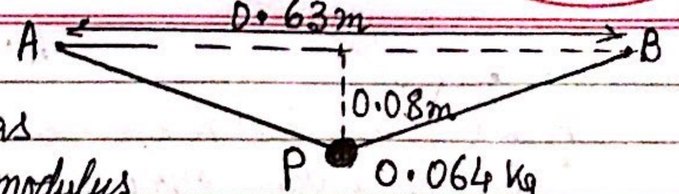
Using Hook's Law

$$F = T = \frac{\lambda e}{L}$$

$$\Rightarrow 1.5 = \frac{6e}{2} \Rightarrow e = 0.5$$

from (i) $PA = 0.5 + 0.8 = 1.3$

or $\underline{PA = 1.3\ m}$



Example 3.

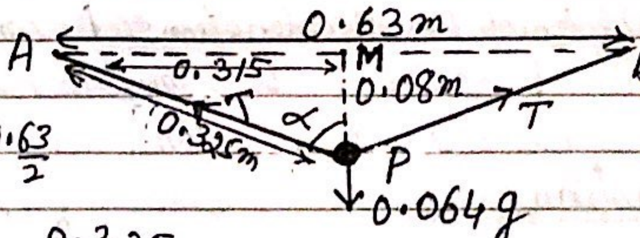
A light elastic string has natural length 0.6 m and modulus of elasticity λ N. The ends of the string are attached to fixed points A and B, which are at the same horizontal level and 0.63 m apart. A particle P of mass 0.064 kg is attached to the mid-point of the string and hangs in equilibrium at a point 0.08 m below AB. Find

(i) the tension in the string,

(ii) the value of λ.

S-06 | 05 | Q1 ...[3]

Solution.



$L = 0.6m$, $\lambda = ?$

M is mid point of AB, $AM = \frac{0.63}{2}$

$AM = 0.315$

$AP = \sqrt{0.315^2 + 0.08^2} = 0.325$

(i) Vertical components of forces at

$mg = 2T\cos\alpha \Rightarrow 0.064g = 2T \times (0.08/0.325)$

$\Rightarrow \underline{T = 1.3 N}$ ✓

(ii) Extension in the string AP, $e = (0.325 - 0.6/2) = 0.025m$

Now Using Hook's law for AP, natural length of $AP = \frac{0.6}{2} = 0.3$

$T = \frac{\lambda e}{L} \Rightarrow 1.3 = \frac{\lambda \times 0.025}{0.3}$

$\Rightarrow \lambda = 15.6$

Hence $\lambda = \underline{15.6 N}$ ✓

M-2

P-4.

Example 4.

A and B are two fixed points on a smooth horizontal table. The distance AB is 2.5m. An elastic string of natural length 0.6m and modulus of elasticity 24N has one end attached to the table at A, and the other end attached to a particle P of mass 0.95 kg. Another elastic string of natural length 0.9m and modulus of elasticity 18N has one end attached to the table at B, and the other end attached to P. The particle P is held at rest at the mid point of AB.

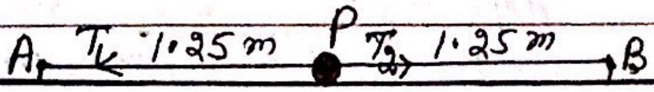
(i) Find the tensions in the strings. ---[3]

The particle is released from rest.

(ii) Find the acceleration of P immediately after its release. ---[2]

(iii) P reaches its maximum speed at the point C. Find the distance AC. [S-07/05/Q6] ---[4]

Solution:



(i) for string AP, $L=0.6$, $e=1.25-0.6=0.65$, $\lambda=24N$

Using Hook's Law $T=\frac{\lambda e}{L} \Rightarrow T_1=\frac{24\times0.65}{0.6}=26N \Rightarrow \underline{T_1=26N}$

Now for string BP, $L=0.9m$, $e=1.25-0.9=0.35$, $\lambda=18N$

$\therefore T=\frac{\lambda e}{L} \Rightarrow T_2=\frac{18\times0.35}{0.9}=7N \Rightarrow \underline{T_2=7N}$

(ii) $F=ma \Rightarrow T_1-T_2=0.95a$

$\Rightarrow 26-7=0.95a$

$\Rightarrow \underline{a=20\,ms^{-2}}$

(iii) Total extention in both the strings $=2.5-(0.6+0.9)=1m$

Let at the time max speed let the extention in string $PA=x$

extension in string $PB=1-x$

and $T_1=T_2$

$24\cdot x/0.6 = 18\cdot(1-x)/0.9 \Rightarrow x=\frac{1}{3} \Rightarrow x=0.333\,m$

$\therefore$ Distance $AC=L+x=0.6+0.333=\underline{0.933m}$

M.2

Hook's Law

P.5

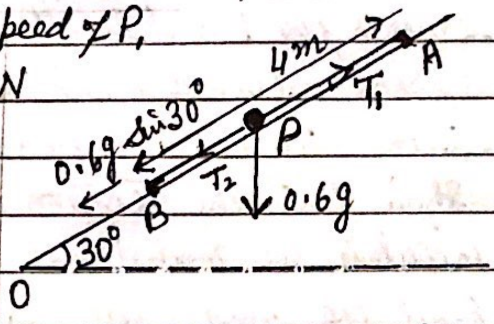
[S-12/ 53/ Q5]

Example 5. A light elastic string has natural length 3m and modulus of elasticity 45 N. A particle of mass 0·6 Kg is attached to the mid point of the string. The ends of the string are attached to fixed points A and B which lie on a line of greatest slope of a smooth plane inclined at 30° to the horizontal. The distance AB is 4 m, and A is higher than B.

(i) Calculate the distance AP when P rests on the slope in equilibrium. [3]

P is released from rest at the point between A and B where AP = 2·5 m.

(ii) Find the maximum speed of P.



Solution: natural length $L = 3m$; $\lambda = 45N$

$AB = 4m$

Total extension $= 4-3 = 1m$

Let the extension during equilibrium at P in the AP portion $= e$ $\Longrightarrow$ $AP = (L/2 + e)$ — ①

∴ Extension in BP part $= (1-e)$

∴ downward component of weight $= 0·6g \sin 30°$

$T_1 = T_2 + 0·6g \sin 30°$ $\quad \left[T = \frac{\lambda e}{L}\right]$

$\Rightarrow \frac{45e}{1·5} = \frac{45(1-e)}{1·5} + 0·6g \sin 30$ $\quad$ [Natural length of string AP $= \frac{L}{2} = 1·5$]

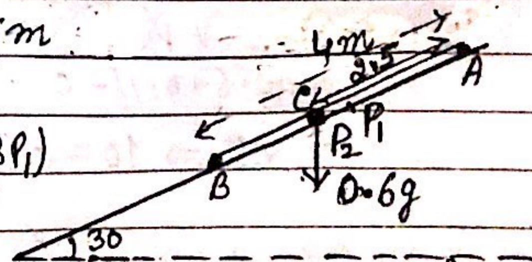
$\Rightarrow 30e = 30(1-e) + 3 \Rightarrow e = 0·55m$

$\therefore AP = \frac{L}{2} + e = \frac{3}{2} + 0·55 = 2·05m$

$\Rightarrow AP = 2·05m$

(ii) $AP_1 = 2·05$ when P is in equilibrium

Now P is released at P_2, $\left\{ AP_2 = 2·5m, \; L = 1·5, \; e = 1 \right.$



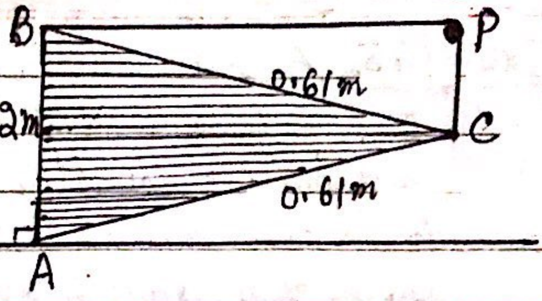
EPE at P_2 = EPE at P_1 (AP_1) + EPE (BP_1) + GPE (P_1P_2) + KE

$(e = 2·5 - 1·5)$

$\frac{45 \times 1^2}{2 \times 1·5} = \frac{45 \times 0·55^2}{2 \times 1·5} + \frac{45 \times 0·45^2}{2 \times 1·5} + 0·6g \times 0·45 \sin 30° + \frac{0·6 V^2}{2}$

$\Rightarrow V = 4·5 ms^{-1}$ $\quad$ $[P_1P_2 = 2·5 - 2·05 = 0·45]$

Example 6.



ABC is a uniform triangular lamina of weight 19N, with $AB = 0.22m$ and $AC = BC = 0.61m$.
The plane of the lamina is vertical.
A rests on a rough horizontal surface, and AB is vertical.
The equilibrium of the lamina is maintained by a light elastic string of natural length 0.7m which passes over a small smooth peg P and is attached to B and C. The portion of the string attached to B is horizontal, and the portion of the string attached to C is vertical.

(i) Show that the tension in the string is 10N. ---[3]

(ii) Calculate the modulus of elasticity of the string ---[2]

(iii) Find the magnitude and direction of the force exerted by the surface on the lamina at A. $\boxed{S\text{-}11 \mid 51 \mid Q5}$ ---[3]

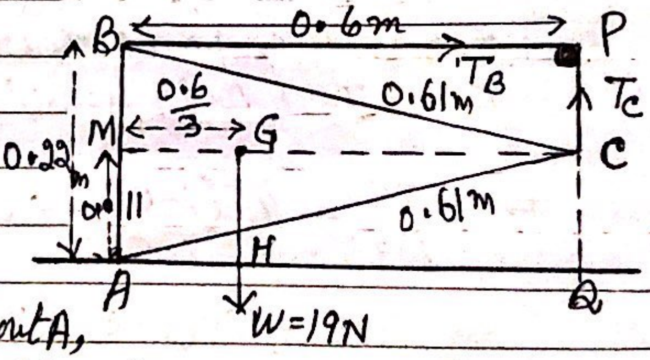
Solution: Let M is the mid point of AB

$AM = 0.22/2 = 0.11$

$BP = AQ = MC\ \sqrt{0.61^2 - 0.11^2} = 0.6$

Let G is the COM of lamina

$GM = \frac{0.6}{3} = AH$



(i) Taking moment of the forces about A,

$W \times AH + T_B \times AB = T_C \times AQ$ $\quad [T_B = T_C = T \text{ let}]$

$19 \times \frac{0.6}{3} + T \times 0.22 = T \times 0.6$

$\Rightarrow T = 10N$ ✓

(ii) $e = (BP + PC - L)$, $\lambda = ?$, $L = 0.7$

$= (0.6 + 0.11 - 0.7)$

$T = \frac{\lambda e}{L} \Rightarrow 10 = \frac{\lambda(0.6 + 0.11 - 0.7)}{0.7}$

$\Rightarrow \lambda = 700N$ ✓

(iii) $F_H = 10$, $F_V = 19 - 10$

$F = \sqrt{10^2 + 9^2} = 13.5N$ ✓

$\alpha = \tan^{-1}\left(\frac{9}{10}\right)$

$\alpha = 42°$ with horizontal. ✓

Example 7. One end of a light elastic string of natural length 0.3m and modulus of elasticity 6N is attached to a fixed point O on a smooth horizontal plane. The other end of the string is attached to a particle P of mass 0.2 kg, which moves on the plane in a circular path with centre O. The angular speed of P is ω rad s^{-1}.

(i) For the case $\omega = 5$, calculate the extension of the string. ---[4]

(ii) Express the extension of the string in terms of ω, and hence find the set of possible value of ω. S-11/53 Q5 --[4]

Solution: $L = 0.3\,m$, $\lambda = 6N$, $m = 0.2\,kg$

(i) $T = \frac{\lambda e}{L} = \frac{6e}{0.3}$ —— ①

also $T = m\omega^2 r = 0.2 \times 5^2 \times (0.3 + e)$ — ② $(\omega = 5)$

for ① & ② $\frac{6e}{0.3} = 0.2 \times 5^2 \times (0.3 + e)$

$\Rightarrow \underline{e = 0.1}$ ✓

(ii) $T = 0.2\omega^2(0.3 + e) = 6e/0.3$

$\Rightarrow e = \frac{0.6\omega^2}{(20 - 0.2\omega^2)} > 0$

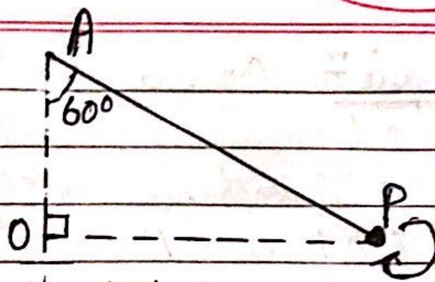
$\Rightarrow 20 - 0.2\omega^2 > 0$

$\Rightarrow 0.2\omega^2 < 20$

$\Rightarrow \omega^2 < 100$

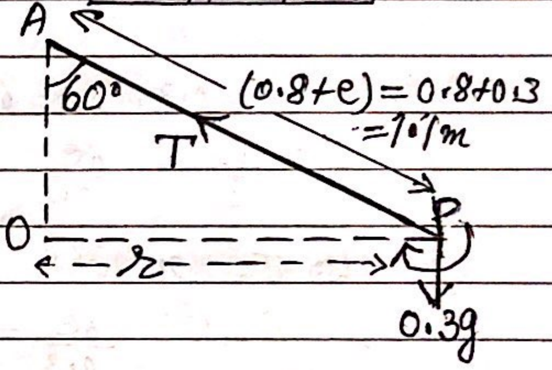
$\Rightarrow \underline{0 < \omega < 10}$ ✓

Example 8:



A particle P of mass 0.3kg is attached to a fixed elastic string of natural length 0.8m and modulus of elasticity 16 N. The particle P moves in a horizontal circle, which has centre O. It is given that AO is vertical and that angle OAP is 60°, calculate the speed of P.

[M-19/52/Q4] --[6]



Solution: $m=0.3$ Kg, $\lambda=16N$

natural length of string $L=0.8m$

Let extension $=e$

$AP=(0.8+e)$ —— (i)

Vertically at P,

$T\cos 60 = 0.3g$

$\Rightarrow \quad T = 6N$ —— (ii)

Also $T=\dfrac{\lambda e}{L} \Rightarrow 6=\dfrac{16\times e}{0.8} \Rightarrow e=0.3$ —— (iii)

from (i) and (iii) $AP=0.8+0.3=1.1$ ✓

$r=OP=AP\sin 60^\circ = 1.1\times\sin 60$ ✓ —— (iv)

Now Horizontally at P.

$T\sin 60 = mv^2/r$

$\Rightarrow 6\sin 60 = 0.3\times v^2/1.1\sin 60$

$\Rightarrow \underline{v=4.06\,ms^{-1}}$ ✓

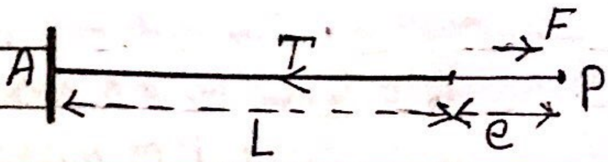
M2

Hook's Law

Work and energy

P-9

Work done:



Given is an elastic string (or spring) of natural length 'L', modulus of elasticity 'λ', one end is fixed at a point A and a force F is applied on the other end P to produce and extension 'e' in it in the direction AP and a tension in the string 'T'.

Then the work done in stretching the string to produce in extension 'e'.

$$\text{Work done} = \frac{\lambda \cdot e^2}{2L}$$ ✓

Elastic Potential Energy (EPE):

Elastic potential energy is the work done in stretching (or compressing) a string (or spring).

$$EPE = \frac{\lambda \cdot e^2}{2L}$$ ✓

Example 9. A particle of mass 0.3 kg is attached to one end of a light elastic string of natural length 0.6m and modulus of elasticity 9N. The other end of the string is attached to a fixed point O on a smooth horizontal surface. The particle is projected horizontally from O with speed $4ms^{-1}$. F. the greatest distance of the particle from O. ...[3]

W-19/52/Q1

Solution: $E.P.E = K.E$

$$\frac{\lambda \cdot e^2}{2L} = \frac{1}{2}mv^2$$

$$\Rightarrow \frac{9e^2}{2 \times 0.6} = \frac{1}{2} \times 0.3 \times 4^2$$

$$\Rightarrow e = 0.566\,m$$

∴ distance of the particle from O $= L + e$

$$= 0.6 + 0.566$$

$$= 1.166$$

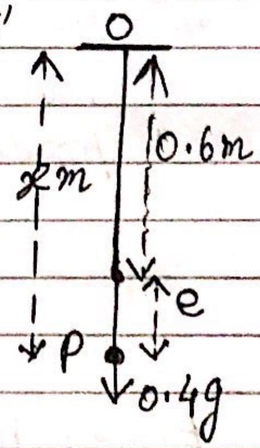
$$d = 1.17\,m$$ ✓

Example 10. One end of a light elastic string is attached to a fixed point O. The other end of the string is attached to a particle P of mass 0.4 Kg. The string has natural length 0.6m and modulus of elasticity 24N. The particle is released from rest at O. Find the two possible values of the distance OP for which the particle has speed $1.5 ms^{-1}$. ---[6]

S-18/52/Q2

Solution: $m = 0.4$ Kg, $L = 0.6$ m, $\lambda = 24$ N, $V = 1.5 ms^{-1}$

when P is released from O; let OP = x m



At O, Gravitational P.E = (K.E + E.P.E) at P

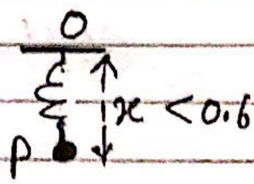
$$mgh = \frac{1}{2}mV^2 + \frac{\lambda e^2}{2L}$$

$$\Rightarrow 0.4 \times g \times x = \frac{1}{2} \times 0.4 \times 1.5^2 + \frac{24(x-0.6)^2}{2 \times 0.6}$$

$$\Rightarrow 20x^2 - 28x + 7.65 = 0$$

$$\Rightarrow x = 1.0279 \quad \text{or} \quad 0.372 < 0.6 \text{ (Rejected)}$$

or $OP = 1.0279$ —— (1)



There is another posibility of the particle 'P' has speed $1.5 ms^{-1}$, falling freely under gravity, the string is not taut (OP < 0.6),

Then EPE = 0

$$P.E = K.E$$

$$mgh = \frac{1}{2}mV^2$$

$$0.4 \times 10 \times x = \frac{0.4 \times 1.5^2}{2}$$

$$\Rightarrow x = \frac{1.5^2}{20}$$

$$= 0.1125 \text{ m} \quad —— (2)$$

or Alternatively

$\because (V^2 = u^2 + 2gS)$

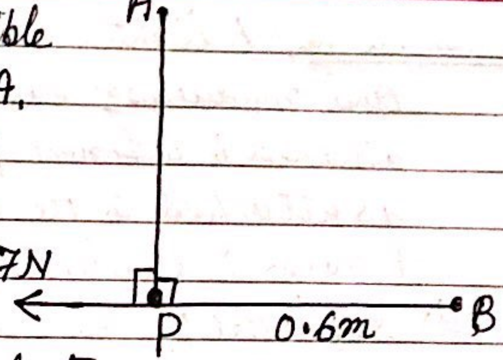
$$1.5^2 = 0 + 2g \times x$$

$$x = \frac{2.25}{20}$$

$$OP = 0.1125 m$$

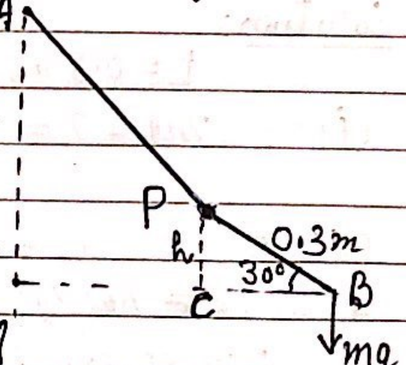
$\therefore$ OP = 1.0279m or 0.1125m from (1) & (2)

Example 11. One end of a light inextensible string is attached to a fixed point A. The other end of the string is attached to a particle P of mass m kg which hangs vertically below A. The particle is also attached to one end of a light elastic string of natural length 0.25m. The other end of the string is attached to a point B which is 0.6m from P and on the same horizontal level as P. Equilibrium is maintained by a horizontal force of magnitude 7N applied to P.



(i) Calculate the modulus of elasticity of the string. ---[2]

P is released from rest by removing the 7N force. In its subsequent motion P first comes to instantaneous rest at at a point where BP = 0.3m and the elastic string makes angle 30° with the horizontal.



(ii) Find the value of m. ---[4]

S-17/51/Q2

<u>Solution:</u> $L = 0.25m$, $\lambda = ?$, $e = (0.6 - 0.25) = 0.35$

$F = T = \lambda e / L$

$\Rightarrow 7 = \dfrac{\lambda \times 0.35}{0.25} \Rightarrow \underline{\lambda = 5N}$ ✓

(ii) Gain in Graviation P.E = Loss in EPE

$mgh = \text{change in EPE} \left(\dfrac{\lambda e_1^2}{2L} - \dfrac{\lambda e_2^2}{2L}\right)$ [PC = h, $h = 0.3\sin 30°$]

$mg \times 0.3\sin 30 = \dfrac{5 \times 0.35^2}{2 \times 0.25} - \dfrac{5 \times 0.05^2}{2 \times 0.25}$

$e_1 = 0.6 - .25 = 0.35$

New $e_2 = 0.3 - 0.25 = 0.05$

$\Rightarrow m = \underline{0.8}$ ✓

M.2

Example 12, A light elastic string has natural length 0·8 m and modulus of elasticity 16 N. One end of the string is attached to a fixed point O, and a particle P of mass 0·4 kg is attached to the other end of the string. The particle P hangs in equilibrium vertically below O.

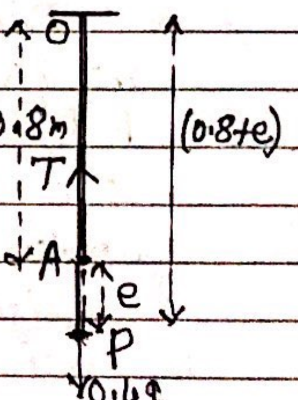
(i) Show that the extension of the string is 0·2 m. ---[2]

P is projected vertically downwards from the equilibrium position. P first comes to instantaneous rest at the point where OP = 1·4 m

(ii) Calculate the speed at which P is projected. --[3]

(iii) Find the speed of P at the first instant when subsequently becomes slack. S-14/51/Q3 --[2]

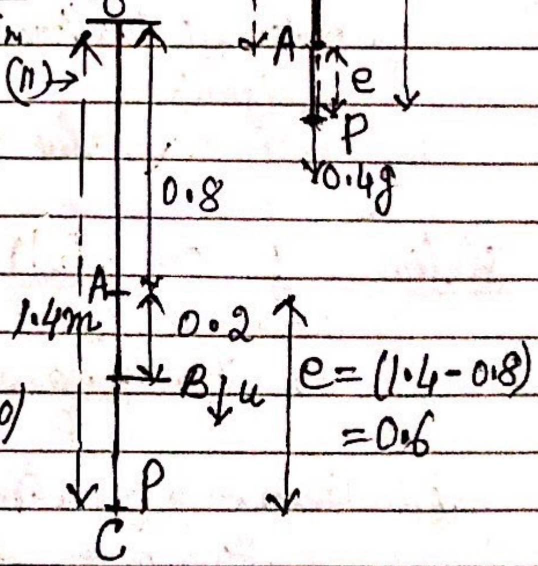
Solution:



$L = 0{\cdot}8\ m,\ \lambda = 16N,\ m = 0{\cdot}4\ kg$

(i) $mg = T = \lambda e/L \Rightarrow 0{\cdot}4g = 16e/0{\cdot}8$

$\Rightarrow \underline{e = 0{\cdot}2\ m}$ ✓



(ii) Now when $OC = 1{\cdot}4m$, $e = 1{\cdot}4 - 0{\cdot}8 = 0{\cdot}6m$

at C, $EE = \lambda e^2/2L = 16 \times 0{\cdot}6^2/2 \times 0{\cdot}8$

Now at B, P is projected downwards with velocity u (let)

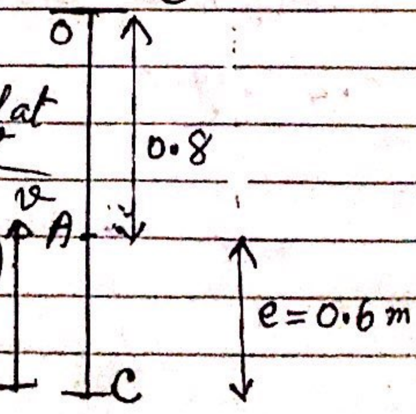
K.E + EPE at B + Gravitation PE loss from B→C

= EPE at C

$\Rightarrow 0{\cdot}4u^2/2 + 16 \times 0{\cdot}2^2/2 \times 0{\cdot}8 + 4g(1{\cdot}4 - 1{\cdot}0)$

$= 16 \times 0{\cdot}6^2/2 \times 0{\cdot}8$

$\Rightarrow u = \underline{2{\cdot}83\ ms^{-1}}$ ✓



(iii) String will be slack at A, $OA = 0{\cdot}8m = L$, let speed at A = V

EPE at C = K.E. at A + Grav P.E

$\Rightarrow 16 \times 0{\cdot}6^2/2 \times 0{\cdot}8 = 0{\cdot}4V^2/2 + 0{\cdot}4g(1{\cdot}4 - 0{\cdot}8)$

$\Rightarrow V = \underline{2{\cdot}45\ ms^{-1}}$ ✓

Example 13. A elastic string has modulus of elasticity 5 N and natural length 1·5 m. One end of the string is attached to a fixed point O and particle P of mass 0·1 kg is attached to the other end of the string. P is released from rest at the point 2·4 m vertically below O. Calculate the speed of P at the instant the string first becomes slack. [S-14/53/Q1] [3]

Solution: $\lambda = 5N$, $L = 1{\cdot}5\,m$, $e = (2{\cdot}4 - 1{\cdot}5) = 0{\cdot}9\,m$

String will be slack at A (OA = 1·5)

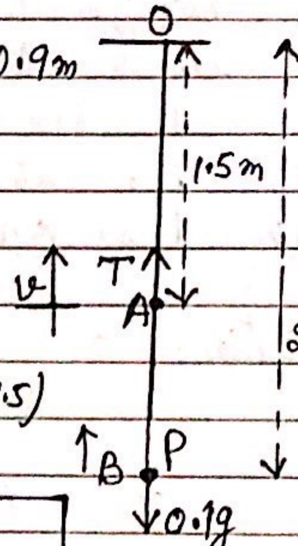
Let the speed at A is $v\,ms^{-1}$

EPE at B = K.E. at A + P.E. at A

$\lambda e^2/2L = mv^2/2 + mgh$

$5 \cdot (2{\cdot}4 - 1{\cdot}5)^2 / 2 \times 1{\cdot}5 = 0{\cdot}1 v^2/2 + 0{\cdot}1g(2{\cdot}4 - 1{\cdot}5)$

$\Rightarrow \underline{v = 3\,ms^{-1}}$ ✓



Example 14: A particle P of mass 0·4 kg is attached to one end of a light elastic string of natural length 1·2 m and modulus of elasticity 19·2 N. The other end of the string is attached to a fixed point A. The particle P is released from rest at the point 2·7 m vertically above A. Calculate

(i) the initial acceleration of P. ---[3]

(ii) the speed of P when it reaches A. [S-13/51/Q2] ---[4]

Solution: $\lambda = 19{\cdot}2$, $m = 0{\cdot}4\,kg$

$L = 1{\cdot}2$, $e = (2{\cdot}7 - 1{\cdot}2) = 1{\cdot}5$

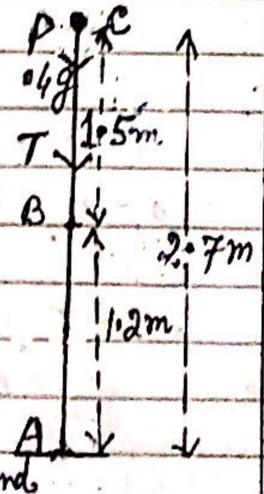
$T = \lambda e/L = 19{\cdot}2 \times 1{\cdot}5 / 1{\cdot}2$

(i) $\Rightarrow T = 24N$

$mg + T = ma$

$0{\cdot}4g + 24 = 0{\cdot}4a$

$\Rightarrow a = 70\,ms^{-2}$



(ii) K.E at A = E.P.E + G.PE at C

$\Rightarrow \frac{mv^2}{2} = \frac{\lambda e^2}{2L} + mgh$

$\frac{0{\cdot}4v^2}{2} = \frac{19{\cdot}2(2{\cdot}7 - 1{\cdot}2)^2}{2 \times 1{\cdot}2} + 0{\cdot}4g \times 2{\cdot}7$

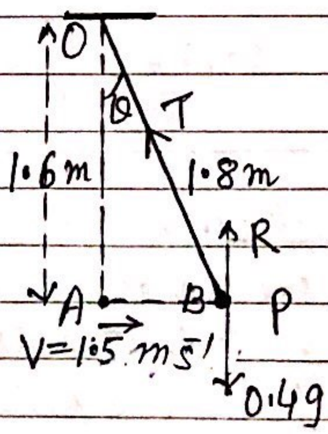
$\Rightarrow V = \underline{12\,ms^{-1}}$ ✓

Example 15: A particle P of mass 0·2 kg is attached to one end of a light elastic string of natural length 1·6 m and modulus of elasticity 18 N. The other end of the string is attached to a fixed point O, which is 1·6 m above a smooth horizontal surface. P is placed on the surface vertically below O and is then projected horizontally. P moves with initial speed 1·5 ms⁻¹ in a straight line on the surface. Show that when OP = 1·8 m,

(i) P is at instantaneous rest. ---[3]

(ii) P is on the point of losing contact with the surface. --[4]

[S-13/52/Q3]



Solution: $m = 0\cdot2\,kg$, $\lambda = 18\,N$

$L = 1\cdot6\,m$, $\underset{at\,P}{e} = (1\cdot8 - 1\cdot6)$

Speed at A, $v = 1\cdot5\ ms^{-1}$

(i) EE at P $= \lambda e^2/2L$

$= 18\times(1\cdot8-1\cdot6)^2/2\times1\cdot6$

K.E at A = EE at P + K.E at B.

$\frac{mv^2}{2} = \lambda e^2/2L + (K.E_B)$

$\frac{0\cdot2\times1\cdot5^2}{2} = \frac{18(1\cdot8-1\cdot6)^2}{2\times1\cdot6} + K.E_B$

$\Rightarrow KE_B = 0 \Rightarrow \underline{V_B = 0}$ ✓

∴ P is at instantaneous rest at B.

(ii) $T = \lambda e/L = 18\times(1\cdot8-1\cdot6)/1\cdot6$

$\Rightarrow T = 2\cdot25\,N.$

Vertically at B, $T\cos\theta + R = mg$

$2\cdot25\times\frac{1\cdot6}{1\cdot8} + R = 0\cdot2g \Rightarrow \underline{R=0}$ ✓

∴ at B the particle P is on the point of losing contact with the surface.

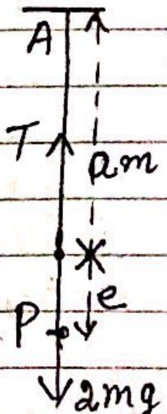
M-2

Example 16: A light elastic string has natural length a and modulus of elasticity $24mg$. One end of the string is attached to a fixed point A. The other end of the string is attached to a particle of mass $2m$.

(a) Find in terms of a, the extension of the string when the particle hangs freely in equilibrium below A. --[2]

(b) The particle is released from rest at A.
Find in terms of a, the distance of the particle below A, --[6]
when it first comes to instanteneous rest. | 9231 | SP-20 | 03 | Q2 |

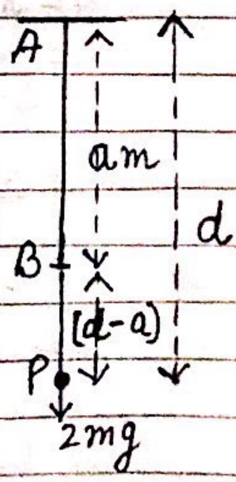
Solution: $T = \frac{\lambda}{L} . e$; $L = a$; $\lambda = 24mg$



(a) $\Rightarrow T = 2mg = \frac{24mg . e}{a}$

$\Rightarrow \underline{e = \frac{a}{12}}$ ✓

(b) Let 'd' is be distance below A.
Gravitional potential Energy (GPA) of the particle at A $= 2mgd$ —— ①



Elastic Potential Energy (EPE) at A
$= \frac{\lambda e^2}{2L} = 2mg.(d-a)^2/2a$ —— ②

No change in K.E (at A and P)

at P Loss in GPA = Gain in EPE

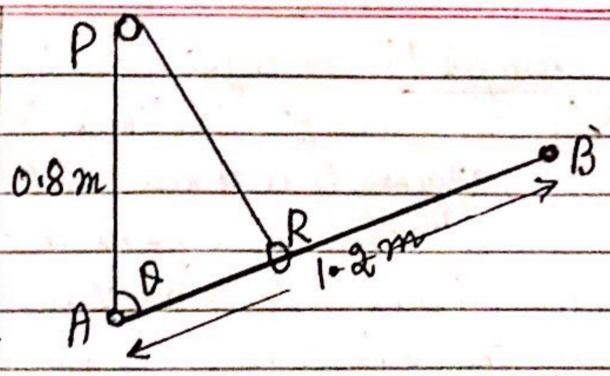
$\Rightarrow 2mgd = 24mg(d-a)^2/2a$ from ① & ②

$\Rightarrow 6(d-a)^2 = ad$

$\Rightarrow d = \frac{3}{2}a$ or $\frac{2}{3}a^{\times}$ (as $d > a$)

$\therefore$ Required distance $\underline{d = \frac{3}{2}a}$ ✓

Example 17.



A uniform rigid rod AB of length 1.2m and weight 8N has a particle of weight 2N attached at B. The end A of the rod is freely hinged to a fixed point. One end of the elastic string of natural length 0.8m and modulus of elasticity 20N is attached to the hinge. The string passes over a small smooth pully P fixed 0.8m vertically above the hinge. The other end of the string is attached to small light smooth ring R which can slide on the rod. The system is in equilibrium with the rod inclined at an angle θ with the vertical.

(i) Show that the tension in the string is $20 \sin\theta$ N. ---[1]

(ii) Explain why the part of the string attached to the ring is perp to the rod. [1]

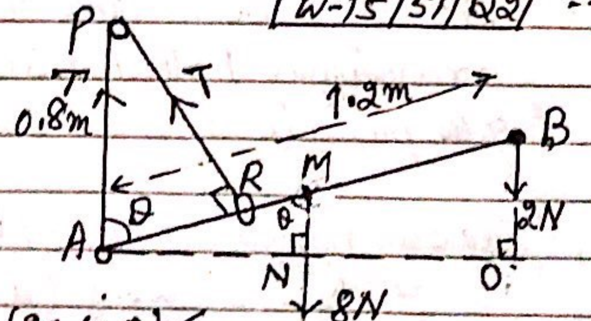
(iii) Find θ. [W-15/51/Q2] ---[3]

Solution: $L = 0.8$, $\lambda = 20$ N

$e = (PA + PR - 0.8)$

$e = (0.8 + 0.8\sin\theta - 0.8)$

$e = 0.8\sin\theta$



Now $T = \dfrac{\lambda e}{L} = \dfrac{20 \times 0.8\sin\theta}{0.8} \Rightarrow \underline{T = (20\sin\theta)}$

(ii) No friction (so perpendicular)

(iii) Taking Moment about the point 'A',

$T \times AR = 8 \times AN + 2 \times AO$

$(20\sin\theta) \times 0.8\cos\theta = 8 \times (0.6\sin\theta) + 2 \times (1.2\sin\theta)$ $\left[\because AM = \dfrac{AB}{2} = \dfrac{1.2}{2} = 0.6\right]$

$\Rightarrow \underline{\theta = 63.3^\circ}$

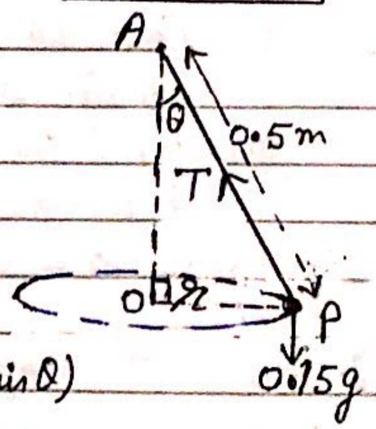
Hook's Law

Example 18. A particle P of mass 0·15 Kg is attached to one end of a light elastic string of natusel length 0·4m and modulus of elasticity 12 N. The other end of the string is attached to a fixed point A. The particle P moves in a horizontal circle which has its centre vertically below A, with the string inclined $\theta°$ to the vertical and AP = 0·5m.

(i) Find the angular speed of P and the value of θ. ---[5]

(ii) Calculate the difference between the elastic potential energy stored in the string and the kinetic energy of P. ----[4]

[S-17/51/Q6]



Solution: $m = 0{\cdot}15$ Kg, $L = 0{\cdot}4m$, $\lambda = 12N$

(i) $T = \dfrac{\lambda e}{L} = \dfrac{12 \times 0{\cdot}1}{0{\cdot}4}$ $\left\{ \begin{aligned} e &= 0{\cdot}5 - 0{\cdot}4 \\ &= 0{\cdot}1 \end{aligned} \right.$

$\Rightarrow T = 3N$ ✓

radius $r = OP = 0{\cdot}5 \sin\theta$

Horizontal components at P,

$T \sin\theta = m\omega^2 r \Rightarrow 3\sin\theta = 0{\cdot}15\,\omega^2 \cdot (0{\cdot}5 \sin\theta)$

$\Rightarrow \underline{\omega = 6{\cdot}32 \text{ rad}s^{-1}}$ ✓

Vertically at P,

$T\cos\theta = mg \Rightarrow 3\cos\theta = 0{\cdot}15g$

$\Rightarrow \cos\theta = 0{\cdot}5$

$\Rightarrow \underline{\theta = 60°}$ ✓

(ii) $v = r\omega = 0{\cdot}5 \sin 60° \times 6{\cdot}32$

$K{\cdot}E = \frac{1}{2} m v^2$

$= 0{\cdot}15 \times (6{\cdot}32 \times 0{\cdot}5 \sin 60°)^2$

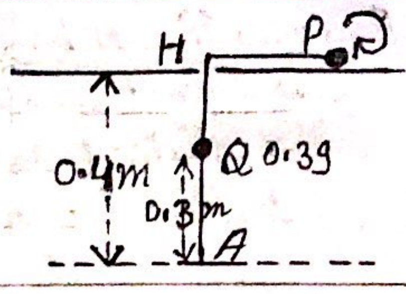
$= 0{\cdot}5625 J$ ✓

$EPE = \dfrac{\lambda e^2}{2L} = \dfrac{12 \times (0{\cdot}1)^2}{2 \times 0{\cdot}4}$

$\therefore$ Difference $= 0{\cdot}5625 - \dfrac{12 \times (0{\cdot}1)^2}{2 \times 0{\cdot}4} = 0{\cdot}4125 J$

$\therefore$ Req. Difference $= \underline{0{\cdot}4125 J}$ ✓

Example 19. A particle P of mass 0.2 kg is attached to one end of a light inextensible string of length 0.6 m. The other end of the string is attached to a particle Q of mass 0.3 kg. The string passes through a small hole H in a small horizontal surface. A light elastic string of natural length 0.3 m and modulus of elasticity 15N joins Q to a fixed point A which is 0.4 m vertically below H. The particle P moves on the surface in horizontal circle with centre H.



(i) Calculate the greatest possible speed of P for which the elastic string is not extended. ---[4]

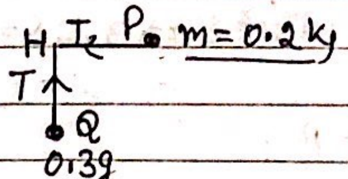
(ii) Find the distance HP given that the angular speed of P is 8 rad s⁻¹. [S-18/51/Q6] ---[5]

Solution:

(i) $r = 0.6 - (QH) = 0.6 - (0.4 - 0.3)$

$\Rightarrow r = 0.5\,m$

In HQ → $T = mg = 0.3g$ — ①



as P moves in circle,

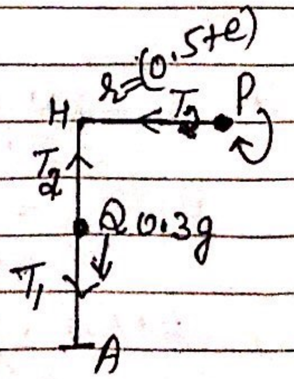
In HP → $T = mv^2/r$

$T = 0.2V^2/0.5$ — ②

from ① & ② $0.3g = 0.2V^2/0.5$

$\Rightarrow V = 2.74\,ms^{-1}$ ✓

(ii) $HP = r = (0.5 + e)$



$T_1 = \frac{\lambda e}{L} = \frac{15e}{0.3} = 50e$ — ③

also $T_2 = m\omega^2 r = 0.2 \times 8^2 \times (0.5 + e)$ — ④

$T_2 = T_1 + 0.3g$

$\Rightarrow 0.2 \times 8^2 \times (.5 + e) = 50e + 3$

$\Rightarrow e = (6.4 - 3)/(50 - 12.8) \Rightarrow e = 0.0914$ ✓

$HP = 0.5 + 0.0914 = 0.5914$

$\Rightarrow HP = 0.591$ ✓