19.07.20

M2

Mechanics-2

Linear Motion Under a Variable Force Exercise

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1. A particle P of mass m kg falls from rest under gravity. There is a resistive force of magnitude mkv^2 N, where v m s^{-1} is the speed of P after it has fallen a distance x m and k is a positive constant.

(a) By solving an appropriate differential equation, show that,
$$v^2 = \frac{g}{k}\left(1 - e^{-2kx}\right) \quad \text{---[7]}$$
It is now given that $k = 0.01$,
The speed of P when x becomes large approaches V m s^{-1}.

(b)(i) Find V correct to two decimal places. --[1]

(ii) Hence find how far P has fallen when its speed is $\frac{1}{2}V$ m s^{-1}. --[2]

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2. A smooth horizontal surface has two fixed points O and A which are 0.8 m apart. A particle P of mass 0.25 kg is projected with velocity 3 m s^{-1} horizontally from A in the direction away from O. The velocity of P is v m s^{-1} when the displacement of P from O is x m. A force of magnitude kv^2x^{-2} N opposes the motion of P.

(i) Show that $v\frac{dv}{dx} = -4kv^2x^{-2}$ --[1]

(ii) Express v in terms of k and x. W-19 | 51 | Q3 --[5]

3. A particle P of mass 0.2 kg is projected horizontally from a fixed point O on a smooth horizontal surface. When the displacement of P from O is x m, the velocity of P is v m s^{-1}. A horizontal force of variable magnitude $0.09\sqrt{x}$ N directed away from O acts on P. An additional force of constant magnitude 0.3 N directed towards O acts on P.

(i) Show that $v\frac{dv}{dx} = 0.45\sqrt{x} - 1.5$ ---[2]

(ii) Find the value of x for which the acceleration of P is zero. --[2]

(iii) Given that the minimum value of v is positive, find the set of possible values for the speed of projection. --[5]

W-19 | 52 | Q6

4. A particle P of mass 0.5 kg is attached to a fixed point O by a light elastic string of natural length 1m and modulus of elasticity 16 N. The particle P is projected vertically upwards from O with speed $6\,ms^{-1}$. A resistance force of magnitude $0.1x^2$ N acts on P, when P has displacement x m above O. After projection the upward velocity of P is $v\ ms^{-1}$.

(i) Show that, before the string becomes taut,

$$v\frac{dv}{dx} = -10 - 0.2x^2 \quad \text{---[2]}$$

(ii) Find the velocity of P at the instant the string becomes taut. --[4]

(iii) Find an expression for the acceleration of P while it is moving upwards after the string becomes taut. --[2]

(iv) Verify that P comes to instantaneous rest before the extension of the string is 0.5 m. [S-19/51/Q7] --[4]

5. A particle P of mass 0.5 kg is attached to one end of a light elastic string of natural length 0.8 m and modulus of elasticity 16 N. The other end of the string is attached to fixed point O. The particle P is released from rest at the point 0.8 m vertically below O. When the extension of the string is x m, the downwards velocity of P is $v\ ms^{-1}$ and a force of magnitude $25x^2$ N opposes the motion of P.

(i) Show that, when P is moving downwards,

$$v\frac{dv}{dx} = 10 - 40x - 5x^2 \quad \text{---[2]}$$

(ii) For the instant when P has its greatest downwards speed, find the kinetic energy of P and the elastic potential energy stored in the string. [S-19/52/Q4] --[6]

5' A particle P is projected horizontally from a point O on a rough horizontal surface. The coefficient of friction between the particle and the surface is 0.2. A horizontal force of magnitude $0.06t$ N directed away from O acts on P, where t is the time after projection. P comes to rest when $t = 4$.

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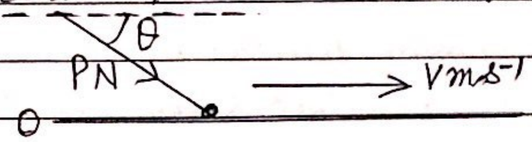
5 (i) The particle begins to move again when $t = 8$. Show that the mass of P is $0{\cdot}24$ kg. --- [2]

(ii) Show that, for $0 \le t \le 4$, $\frac{dv}{dt} = 0{\cdot}25t - 2$, and find the speed of projection of P. -- [5]

(iii) Find the distance from O at which P comes to rest. -- [4]

[M-19/52/Q7]

6. A small object of mass $0{\cdot}2$ kg rests at a point O on a rough horizontal surface. The coefficient of friction between the object and the surface is $0{\cdot}5$. A force of magnitude P N acting at an angle θ below the horizontal is applied to the object. The velocity of the object is v m s^{-1} away from O at t s after the force begins to act. It is given that $\tan\theta = \frac{3}{4}$ and $P = 0{\cdot}4t$ for $0 \le t \le 8$.



(i) Find the value of t when the object starts to move. -- [3]

(ii) Show that, when the force is acting and the object is in motion, $\frac{dv}{dt} = t - 5$ -- [2]

When $t = 8$ the force of magnitude P N ceases to act.

(iii) Find the distance travelled by the object after $t = 8$, before it comes to rest. [M-18/52/Q6] - [5]

7. A partical P of mass $0{\cdot}4$ kg is projected horizontally along a smooth horizontal plane from a point O. At time t s after projection the velocity of P is v m s^{-1}. A force of magnitude $0{\cdot}8t$ N directed away from O acts on P and a force of magnitude $2e^{-t}$ N opposes the motion of P.

(i) Show that $\frac{dv}{dt} = 2t - 5e^{-t}$ -- [2]

(ii) Given that $v = 8$ when $t = 1$, express v in terms of t. -- [3]

(iii) Find the speed of projection of P. -- [2]

[S-18/51/Q3]

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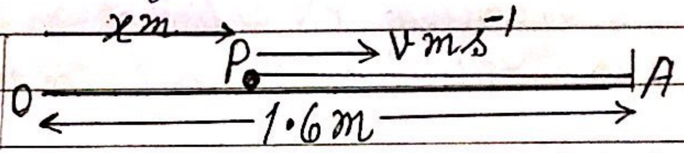
8. A particle P of mass 0.2 kg is released from rest at a point O above horizontal ground. At time t s after its release the velocity of P is $v\ ms^{-1}$ downwards. A vertically downwards force of magnitude $0.6t$ N acts on P. A vertically upwards force of magnitude ke^{-t} N, where k is a constant, also acts on P.

(i) Show that $\frac{dv}{dt} = 10 - 5ke^{-t} + 3t$ --- [2]

(ii) Find the greatest value of k for which P does not initially move upwards. --- [3]

(iii) Given that k = 1, and that P strikes the ground when t = 2, find the height of O above the ground. [S-18/52/Q7] --- [5]

9. A particle P of mass 0.5 kg is projected along a smooth horizontal surface towards a fixed point A.



Initially P is at a point O on the surface, and after projection, P has a displacement from O of x m and velocity $v\ ms^{-1}$. The particle P is connected to A by a light elastic string of natural length 0.8 m and modulus of elasticity 16 N. The distance OA is 1.6 m. The motion of P is resisted by a force of magnitude $24x^2$ N.

(i) Show that $v\frac{dv}{dx} = 32 - 40x - 48x^2$ while P is in motion and the string is stretched. --- [3]

The maximum value of v is 4.5.

(ii) Find the initial value of v. [W-18/51/Q4] --- [5]

10. A particle P of mass 0.4 kg is projected horizontally along a smooth horizontal plane from a point O. After projection the velocity of P is $v\ ms^{-1}$ and its displacement from O is x m. A force of magnitude 8x N directed away from O acts on P, and a force of magnitude $(2e^{-x}+4)$ N opposes the motion of P. One end of the light elastic string of natural length 0.5 m is attached to O and other end of the string is attached to P.

(Continued→)

Linear motion under a variable force.

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10(i) Show that $v.\frac{dv}{dx} = 20x - 10 - 5e^{-x}$ before the elastic string becomes taut. --[2]

(ii) Given that the initial velocity of P is $6\,ms^{-1}$, find v when the string first becomes taut. --[3]

When the string is taut, the acceleration of P is proportional to e^{-x}.

(iii) Find the modulus of elasticity of the string. [W-18/52/Q3] --[2]

11. O and A are fixed points on a rough horizontal surface with OA = 1m. A particle P of mass 0.4kg is projected horizontally with speed $U\,ms^{-1}$ from A in the direction OA and moves in a straight line. After projection, when the displacement of P from O is x m, the velocity of P is $v\,ms^{-1}$. The coefficient of friction between the surface and P is 0.4. A force of magnitude $\frac{0.8}{x}$ N acts on P in the direction PO.

(i) Show that, while the particle is in motion, $v\frac{dv}{dx} = -4 - \frac{2}{x}$ ---[3]

It is given that P comes to instantaneous rest between $x = 2.0$ and $x = 2.1$

(ii) Find the set of possible values of U. [M-17/52/Q6] ---[5]

12. A particle P of mass 0.5 kg is at rest at a point O on a rough horizontal surface. At time $t = 0$, where t is in seconds, a horizontal force acting in a fixed direction is applied to P. At time t s the magnitude of the force is $0.6t^2$ N and the velocity of P away from O is $v\,ms^{-1}$. It is given that P remains at rest at O until $t = 0.5$.

(i) Calculate the coefficient of friction between P and the surface, and show that: $\frac{dv}{dt} = 1.2t^2 - 0.3$ for $t > 0.5$ ---[3]

(ii) Express v in terms of t for $t > 0.5$ --[3]

(iii) Find the displacement of P from O when $t = 1.2$ ---[3]

[S-17/51/Q7]

13. A small object of mass 0.4 kg is released from rest at a point 8 above the ground. The object descends vertically and when its downwards displacement from its initial position is x m the object has velocity $v\ ms^{-1}$. While the object is moving, a force of of magnitude $0.2v^2$ N opposes the motion.

(i) Show that $v\frac{dv}{dx} = 10 - 0.5v^2$ ---[2]

(ii) Express v in terms of x. ---[4]

(iii) Find the increase in the value of v during the final 4 m of the descent of the object. [S-17/52/Q4] ---[2]

14. A particle P of mass 0.2 kg is released from rest at a point O on a rough plane inclined at 60° to the horizontal, and travels down a line of greatest slope. The coefficient of friction between P and the plane is 0.3. A force of magnitude $0.6x$ N acts on P in the direction PO, where x m is the displacement of P from O.

(i) Show that $v\frac{dv}{dx} = 5\sqrt{3} - 1.5 - 3x$, where $v\ ms^{-1}$ is the velocity of P at a displacement x m from O. --[3]

(ii) Find the value of x for which P reaches its maximum velocity, and calculate this maximum velocity. --[4]

(iii) Calculate the magnitude of the acceleration of P immediately after it has first come to instantaneous rest. [W-17/51/Q7] --[4]

15. A particle P of mass 0.2 kg is released from rest at a point O on a smooth horizontal surface. A horizontal force of magnitude te^{-v} N directed away from O acts on P, where $v\ ms^{-1}$ is the velocity of P at time t seconds after release. Find the velocity of P when $t = 2$. [W-17/52/Q1] ---[4]

16. A particle P of mass 0.4 kg is released from rest at a point O on a smooth plane inclined at 30° to the horizontal. P moves down the line of greatest slope through O. The velocity of P is $v\ ms^{-1}$ when its displacement from O is x m. A retarding force of magnitude $0.2v^2$ N act on P in the direction PO. (Continued→)

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16(i) Show that $v\frac{dv}{dx} = 5 - 0.5v^2$ ---[2]

(ii) Express v in terms of x. [W-17/52/Q3] --[4]

17. A particle P of mass 0.2 kg is released from rest at a point O on a plane inclined at 30° to the horizontal. At time t s after its release, P has velocity v ms^{-1} and displacement x m down the plane from O. The coefficient of friction between P and the plane increases as P moves down the plane and equals $0.1x^2$.

(i) Show that $2v\frac{dv}{dx} = 10 - (\sqrt{3})x^2$ --[2]

(ii) Calculate the maximum speed of P. --[5]

(iii) Find the value of x at the point where P comes to rest. --[2]

[M-16/52/Q6]

18. A particle P of mass 0.4 kg is released from rest at a point O on a smooth plane inclined at 30° to the horizontal. When the displacement of P from O is x m down the plane, the velocity of P is v ms^{-1}. A force of magnitude $0.8e^{-x}$ N acts on P up the plane along the line of greatest slope through O.

(i) Show that $v\frac{dv}{dx} = 5 - 2e^{-x}$ --[2]

(ii) Find v when $x = 0.6$ [S-16/51/Q3] ---[4]

19. A particle P of mass 0.4 kg is placed at rest at a point A on a rough horizontal plane. A horizontal force, directed away from A with magnitude 0.6t N, acts on P, where t is the time after P is placed at A. The coefficient of friction between P and the surface is 0.3 and P has displacement from A of x m at time t s.

(i) Show that P starts to move when $t = 2$. Show that when P is in motion it has acceleration $(1.5t - 3)$ ms^{-2} --[3]

(ii) Express the velocity of P in terms of t, for $t \geq 2$. --[4]

(iii) Express x in terms of t, for $t \geq 2$. [S-16/52/Q5] --[3]

20. A small block B of mass 0·25 kg is released from rest at a point O on a smooth horizontal surface. After its release the velocity of B is $v\,ms^{-1}$ when its displacement is x m from O. The force acting on B has magnitude $(2+0\cdot3x^2)$N and is directed horizontally away from O.

(i) Show that $v\frac{dv}{dx} = 1\cdot2x^2+8$ ---[2]

(ii) Find the velocity of B when $x=1\cdot5$ ---[3]

An extra force acts on B after $x=1\cdot5$, It is given that, when $x>1\cdot5$, $v\frac{dv}{dx} = 1\cdot2x^2+6-3x$

(iii) Find the magnitude of extra force and state the direction in which it acts. [W-16/51/Q3] --[2]

21. A particle P of mass 0·4 kg is released from rest at a point O on a smooth plane inclined at 30° to the horizontal. A force of magnitude $3e^{-t}$ N directed up a line of greatest slope acts on P, where t s is the time after release.

(i) Show that $\frac{dv}{dt} = 7\cdot5e^{-t}-5$, where $v\,ms^{-1}$ is the velocity of P up the plane at time t s. ---[2]

(ii) Express v in terms of t. --[3]

(iii) Find the distance of P from O when v has its maximum value. --[3]

[W-16/52/Q5]

22. A particle P of mass 0·1 Kg moves with decreasing speed in a straight line on a smooth horizontal surface. A horizontal resisting force of magnitude $0\cdot2e^{-x}$ N acts on P, where xm is the displacement of P from a fixed point O on the line. The velocity of P is $v\,ms^{-1}$ when its displacement from O is xm.

(i) Show that $v\frac{dv}{dx} = ke^{-x}$, where k is a constant to be found. ---[2]

P passes through O with velocity $2\cdot2ms^{-1}$

(ii) Calculate the value of x at the instant when the velocity of P is $2ms^{-1}$ ---[4]

(iii) Show that the speed of P does not fall below $0\cdot917ms^{-1}$, correct to 3 significant figures. --[2]

[S-15/51/Q6]

23. A force of magnitude of $0.4t$ N, applied at angle $30°$ above the horizontal, acts on a particle P, where t s is the time since the force starts to act. P is at rest on rough horizontal ground when $t=0$. The mass of P is 0.2 kg and the coefficient of friction between P and the ground is μ.

(i) Given that P is about to slip when $t=2$, find μ and the value of t for the instant when P loses contact with the ground. --[5]

(ii) While P is moving on the ground, it has velocity v ms^{-1} at time t s, show that $\frac{dv}{dt} = 2.165t - 4.330$ ---[3]
where the coefficients are correct to 4 significant figures.

(iii) Calculate the speed of P when it loses contact with the ground. --[4]

[S-15/52/Q7]

24. A cyclist and her bicycle have a total mass of 60 kg. The cyclist rides in horizontal straight line, and exerts a constant force in the direction of motion of 150 N. The motion is opposed by a resistance of magnitude $12v$ N, where v ms^{-1} is the cyclist's speed at time t s after passing through a fixed point A.

(i) Show that $5\frac{dv}{dt} = 12.5 - v$ --[2]

(ii) Given that the cyclist passes through A with speed 11.5 ms^{-1}, solve this differential equation to show that $v = 12.5 - e^{-0.2t}$. --[4]

(iii) Express the displacement of the cyclist from A in terms of t. --[3]

[S-15/53/Q6]

25. A particle P moves in a straight line and passes through a point O of the line with velocity 2 ms^{-1}. At time t s after passing through O, the velocity of P is v ms^{-1} and the acceleration of P is given by $e^{-0.5v}$ ms^{-2}. Calculate the velocity of P when $t=1.2$ --[4]

[W-15/51/Q1]

26. A particle P of mass 0.3 kg moves in straight line on a smooth horizontal surface. P passes through a fixed point O of the line with velocity 8 ms^{-1}. A force of magnitude $2x$ N acts on P in the direction PO, where x m is the displacement of P from O.

(i) Show that $v\frac{dv}{dx} = kx$ and state the value of the constant k. --[2]

(continued →)

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26(ii) Find the value of x at the instant when P comes to instantaneous rest. [W-15/51/Q3] --[3]

27. A particle P of mass 0.5 kg is projected vertically upwards from a point on a horizontal surface. A resisting force of magnitude $0.02v^2$ N acts on P, where $v\ ms^{-1}$ is the upward velocity of P when it is at a height of x m above the surface. The initial speed of P is $8 ms^{-1}$.

(i) Show that, while P is moving upwards, $v\frac{dv}{dx} = -10 - 0.04v^2$. --[2]

(ii) Find the greatest height of P above the surface.

(iii) Find the speed of P immediately before it strikes the surface after decending. [W-15/53/Q5]

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Answers

1. $F = ma = mv\frac{dv}{dx} = mg - kv^2$

(a) $\Rightarrow v\frac{dv}{dx} = g - kv^2$

$\Rightarrow \int \frac{v}{(g-kv^2)} dv = \int dx$

$\Rightarrow -\frac{1}{2k}\int \frac{-2kv\,dv}{(g-kv^2)} = x + c$ —①

$\Rightarrow \frac{-1}{2v}\ln(g-kv^2) = x + c$

when $x=0$, $v=0 \Rightarrow c = \frac{-1}{2k}\ln g$

from ①

$x = \frac{1}{2k}\ln g - \frac{1}{2k}\ln(g-kv^2)$

$x = \frac{1}{2k}\ln\left(\frac{g}{g-kv^2}\right)$ —①

$\Rightarrow e^{2kx} = \frac{g}{g-kv^2}$

$\Rightarrow v^2 = \frac{g}{k}\left(1 - e^{-2kx}\right)$ ✓

(b) when $x \to \infty \Rightarrow e^{-2kx} \to 0$

(i)

$V = \frac{g}{0.01}(1-0) = 1000$

$\underline{V = 31.62}$ ✓

(ii) Now $v = \frac{1}{2}V = \frac{1}{2} \times 31.62$

fm ① $x = \frac{1}{2k}\ln\left(\frac{g}{g - 0.01 \times \frac{(31.62)^2}{4}}\right)$

$= \frac{1}{2 \times 0.01}\ln\left(\frac{g}{g - \frac{g}{4}}\right)$

$x = 50.\ln 4/3$

$= 14.371$

$x = \underline{14.4m}$ ✓

2. $F = mv\frac{dv}{dx}$

(i)

$\Rightarrow 0.25v\frac{dv}{dx} = -kv^2x^{-2}$

$\Rightarrow \underline{v\frac{dv}{dx} = -4kv^2x^{-2}}$ ✓

(ii) $\int \frac{dv}{v} = -4k\int -x^{-2}dx$

$\ln v = \frac{4k}{x} + c$ —①

$x = 0.8,\ v = 3 \Rightarrow c = \ln 3 - 5k$

fm ① $\ln v = \frac{4k}{x} + \ln 3 - 5k$

$\Rightarrow v = 3.e^{\left(\frac{4k}{x} - 5k\right)}$ ✓

3. $F = ma = mv\frac{dv}{dx}$

(i)

$\Rightarrow 0.2v\frac{dv}{dx} = 0.09\sqrt{x} - 0.3$

$\Rightarrow \underline{v\frac{dv}{dx} = 0.45\sqrt{x} - 1.5}$ ✓

(ii) $0 = 0.45x^{1/2} - 1.5$

$\Rightarrow \underline{x = 100/9}$ ✓

(iii) $\int v\,dv = \int(0.45x^{1/2} - 1.5)dx$

$\frac{v^2}{2} = \frac{0.45x^{3/2}}{3/2} - 1.5x + C$

$\Rightarrow \frac{v^2}{2} = 0.3x^{3/2} - 1.5x + C$ —①

$v = 0,\ x = 100/9$

$\Rightarrow 0 = 0.3\left(\frac{100}{9}\right)^{3/2} - 1.5\left(\frac{100}{9}\right) + C$

$\Rightarrow C = 50/9$

when $x = 0$, $\frac{v^2}{2} > 50/9 \Rightarrow \underline{v > \frac{10}{3}}$ ✓

Answers

4. (i) $F = m v \frac{dv}{dx}$

(i) $\Rightarrow 0.5 v \frac{dv}{dx} = -0.5g - 0.1x^2$ — ①

$\Rightarrow v\frac{dv}{dx} = -10 - 0.2x^2$ ✓

(ii) $\int v\,dv = \int(-10-0.2x^2)dx$

$\frac{v^2}{2} = -10x - 0.2\frac{x^3}{3} + c$

when $x=0$, $v=6 \Rightarrow c=18$

$\therefore \frac{v^2}{2} = -10x - 0.2\frac{x^3}{3} + 18$ — ①

at $x=1$

$\frac{v^2}{2} = -10 - \frac{0.2}{3} + 18$

$\Rightarrow v^2 = 15.87 \Rightarrow v = 3.98$ ✓

(iii) when string is taut, for 'x'

$L = 1$, $\lambda = 16$, $T = \frac{16(x-1)}{1}$

from ① $0.5 v\frac{dv}{dx} = -0.5g - 0.1x^2 - \frac{16(x-1)}{1}$

$\Rightarrow v\frac{dv}{dx} = -10 - 0.2x^2 - 32x + 32$

or $v\frac{dv}{dx} = 22 - 32x - 0.2x^2$ ✓

(iv) $\int v\,dv = \int(22 - 32x - 0.2x^2)dx$

$\frac{v^2}{2} = 22x - 16x^2 - 0.2\frac{x^3}{3} + k$

$x=1$, $v=3.98 \Rightarrow k=2$,

Now put $x = 1.5$

$\Rightarrow 22\times1.5 - 16\times\frac{(1.5)^3}{3} + 2 = -1.225$

as $\frac{v^2}{2}$ cannot be negative, P comes to rest before the extension of the string is 0.5.

5. Tension due elasticity when extension is $x = \frac{16x}{0.8}$

(i) $F = ma = m v\frac{dv}{dx}$

$\Rightarrow 0.5 v\frac{dv}{dx} = 0.5g - \frac{16x}{0.8} - 25x^2$

$\Rightarrow v\frac{dv}{dx} = 10 - 40x - 50x^2$ ✓ ①

(ii) $\int v\,dv = \int(10 - 40x - 50x^2)dx$ — ②

$\frac{v^2}{2} = 10x - 20x^2 - \frac{50x^3}{3} + (c)$ $v=0$, when $x=0$ $\Rightarrow c=0$

The particle has the greatest downwards speed when. fm ①

$a = v\frac{dv}{dx} = 10 - 40x - 5x^2 = 0$

$\Rightarrow x = 0.2$ or $(x=-1)$ ✗

$K.E = \frac{mv^2}{2} = 0.5\frac{v^2}{2}$ at $x = 0.2$ fm ②

$= 0.5\left[10\times0.2 - 20(.2)^2 - \frac{50\times(.2)^3}{3}\right]$

$K.E. = \frac{8}{15} = 0.533\,J$ ✓

and $E.P.E = \frac{\lambda x^2}{2L} = \frac{16\times0.2^2}{(2\times0.8)}$

$\Rightarrow E.P.E = 0.4\,J$ ✓

5 (i) $F = \mu R = \mu mg$

$\Rightarrow .06t = \mu mg$ at $t=8$ ✓

$\Rightarrow .06\times8 = 0.2mg \Rightarrow m = 0.24\,kg$

(ii) $F = ma = m.\frac{dv}{dt} = 0.06t - .2mg$

$\Rightarrow 0.24\frac{dv}{dt} = 0.06t - 0.2\times0.24g$

$\Rightarrow \frac{dv}{dt} = 0.25t - 2$

$\int dv = \int(0.25t - 2)dt$

$\Rightarrow v = 0.25t^2/2 - 2t + c$ put $v=0$, $t=4$ $\Rightarrow c=6$

$\Rightarrow$ Initial Velocity $= 6\,ms^{-1}$ ✓

(Continued →)

Answers

5 (iii)

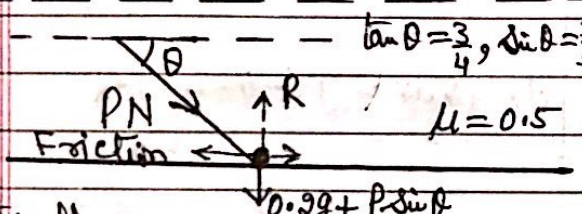
$x = \int (0.25t^2/2 - 2t + 6)\,dt$

$= 0.25t^3/6 - t^2 + 6t + (K)$

at $t = 4$ $\quad (K = 0)$

$x = OP = 32/3 = 10\frac{2}{3} = 10.7\,m$

6. $\tan\theta = \frac{3}{4}$, $\sin\theta = \frac{3}{5}$



$\mu = 0.5$

Vertically:

$R = mg + P\sin\theta = 0.2g + 0.4t\sin\theta$

(i) $\Rightarrow R = 2 + 0.4 \times \frac{3}{5}t = (2 + 0.24t)$

$F = \mu R = 0.5(2 + 0.24t) = 1 + 0.12t$

Horizontally - initially

$0.4t\cos\theta = 1 + 0.12t$

$\Rightarrow 0.4 \times 0.8t = 1 + 0.12t \quad [\cos\theta = \frac{4}{5}]$

$\Rightarrow t = 5$ ✓

(ii) $F = ma = m\frac{dv}{dt} = P_x - \mu$

$\Rightarrow 0.2\frac{dv}{dt} = 0.4t\cos\theta - (1 + 0.12t)$

$0.2\frac{dv}{dt} = 0.4 \times 0.8t - 1 - 0.12t$

$\Rightarrow \frac{dv}{dt} = t - 5$ ✓

(iii) $\int dv = \int (t - 5)\,dt$

$\Rightarrow v = t^2/2 - 5t + c$

$v = 0,\ t = 5 \Rightarrow c = 12.5$

$v = t^2/2 - 5t + 12.5$ ✓

at $t = 8$, $v = 8^2/2 - 5 \times 8 + 12.5 = 4.5$

When P ceases to act $F = \mu R$

$\Rightarrow ma = -0.5 \times 0.2g$

$\Rightarrow a = -0.5 \times 0.2g / 0.2$

$a = -5\,ms^{-2}$

$0 = v^2 + 2aS$

$\Rightarrow 0 = (4.5)^2 - 2 \times 5 \times S \Rightarrow S = 20.25/10 = 2.025\,m$ ✓

7. $m = 0.4\,kg$

$F = ma = 0.8t - 2e^{-t}$

(i) $\Rightarrow 0.4 \times \frac{dv}{dt} = 0.8t - 2e^{-t}$

$\Rightarrow \frac{dv}{dt} = 2t - 5e^{-t}$ ✓

(ii) $\int dv = \int (2t - 5e^{-t})\,dt$

$v = t^2 + 5e^{-t} + c$

$t = 1, v = 8 \Rightarrow c = 5.16$

$\therefore v = t^2 + 5e^{-t} + 5.16$ ✓ — ①

or $v = t^2 + 5e^{-t} + 7 - 5e^{-1}$ ✓

(iii) from ① at $t = 0$

$v = 0 + 5 + 5.16 = 10.2\,ms^{-1}$ ✓

8. $F = ma \quad m = 0.2\,kg$

(i) $0.2\frac{dv}{dt} = mg + 0.6t + ke^{-t}$

$= 0.2g + 0.6t + ke^{-t}$

$\Rightarrow \frac{dv}{dt} = 10 + 3t - 5ke^{-t}$ ✓

(ii) $\frac{dv}{dt} = 10 - 5ke^{0} = 0 \quad (\text{at } t = 0)$

$\Rightarrow k = 2$ ✓

(iii) $\int dv = \int (10 + 3t - 5 \times 1 \cdot e^{-t})\,dt \quad [k = 1]$

$\Rightarrow v = 10t + 3t^2/2 + 5e^{-t} + c$

$[v = 0, t = 0 \Rightarrow c = -5]$

$\Rightarrow v = 10t - 3t^2/2 + 5e^{-t} - 5$

$\int dx = \int (10t - 3t^2/2 + 5e^{-t} - 5)\,dt$

$\Rightarrow x = 5t^2 + t^3/2 - 5e^{-t} - 5t + C$

$x = 0, t = 0 \Rightarrow C = 5$

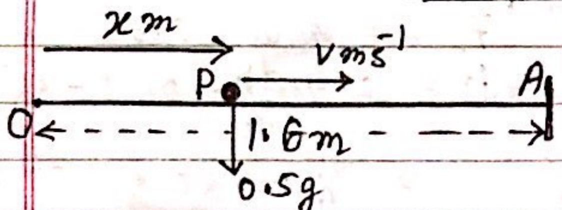
$\Rightarrow x = 5t^2 + t^3/2 - 5e^{-t} - 5t + 5$

for $t = 2$, $x = 18.3\,m$

$\therefore$ Height $= 18.3\,m$ ✓

Answers

9.



(i) $T = \frac{\lambda e}{L} = 16(1.6-0.8-x)/0.8$

$\Rightarrow T = 16 - 20x$ ✓

$F = ma = T - 24x^2$

$\Rightarrow 0.5\, v\frac{dv}{dx} = 16 - 20x - 24x^2$

$\Rightarrow v\frac{dv}{dx} = 32 - 40x - 48x^2$ ✓

(ii) for Max Vel $\rightarrow v\frac{dv}{dx} = 0$

$0 = 32 - 40x - 48x^2$

$\Rightarrow x = 0.5\,m$ ✓ (for Max V)

Now $\int v\,dv = \int (32 - 40x - 48x^2)\,dx$

$\Rightarrow \frac{V^2}{2} = 32x - \frac{40x^2}{2} - \frac{48x^3}{3} + c$

Max vel $V = 4.5$ at $x = 0.5$

$\Rightarrow 4.5^2/2 = 32\times0.5 - 20\times(0.5)^2 - 16(0.5)^3 + c$

$\Rightarrow c = 1.125$

$\Rightarrow V^2 = 64x - 40x^2 - 32x^3 + 2\times1.125$

Initially $x = 0$

$V^2 = 0 + 2.25$

$V = 1.5\,ms^{-1}$ ✓

10 (i) $F = ma = 0.4\,v\frac{dv}{dx} = 8x - (2e^{-x} + 4)$ — ①

$v\frac{dv}{dx} = 20x - 10 - 5e^{-x}$

(ii) $\int v\,dv = \int (20x - 10 - 5e^{-x})\,dx$

$v^2/2 = 10x^2 - 10x + 5e^{-x}$

$\left[\frac{v^2}{2}\right]_6^v = \left[10x^2 - 10x + 5e^{-x}\right]_0^{0.5}$

$\Rightarrow v = 5.2$ ✓

(iii) $8x - 4 = T = \frac{\lambda e}{L} = \frac{\lambda(x-0.5)}{0.5}$

$\Rightarrow 8x - 4 = 2\lambda x - \lambda \Rightarrow \lambda = 4N$ ✓

11(i) Friction $= \mu R = 0.4\times0.49$

Now $F = ma = m\cdot\frac{dv}{dx}\times v$

$\Rightarrow 0.4\,v\frac{dv}{dx} = -0.4\times0.49 - 0.8/x$

$\Rightarrow v\frac{dv}{dx} = -4 - 2/x$ ✓

(ii) $\int v\,dv = \int (-4 - 2/x)\,dx$

$\frac{v^2}{2} = -4x - 2\ln x + C$ — ①

$v = U$ when $x = 1$

$\Rightarrow C = U^2/4 + 4$

fn ① $\frac{v^2}{2} = -4x - 2\ln x + \frac{U^2}{2} + 4$ — ②

when $x = 2$ fn ② $v = 0$

$0 = -4\times2 - 2\ln 2 + \frac{U^2}{2} + 4$

$\Rightarrow U^2 = 10.7(725)$

$\Rightarrow U = 3.282$ ✓

~~Also at~~ $x = 2.1,\ v = 0$

$0 = -4\times2.1 - 2\ln 2.1 + \frac{U^2}{2} + 4$

$\Rightarrow U^2 = 11.7677$

$\Rightarrow U = 3.43$

$\therefore 3.28 < U < 3.43$

$\because$ Now $\left[F - T = (8x-4) - 2e^{-x} - T = k e^{-x} \right.$

$\left. \because \Rightarrow T = 8x - 4 \right]$

M2

$F = 0.6t^2 N$

Answers

12. (i) Force of friction $F = 0.6 \times 0.5^2$

$R = mg = 0.5g$

$\Rightarrow \mu = \frac{F}{R} = \frac{0.6 \times 0.5^2}{0.5g} = 0.03$ ✓

Now force at $P = ma = 0.5\frac{dv}{dt}$

or $0.5\frac{dv}{dt} = 0.6t^2 - 0.03 \times 0.5g$

$\Rightarrow \frac{dv}{dt} = 1.2t^2 - 0.3$ ✓ for $t > 0.5$

(ii) $\int dv = \int (1.2t^2 - 0.3)dt$

$v = 0.4t^3 - 0.3t + c$

$t = 0.5, v = 0 \Rightarrow c = 0.1$

$\therefore v = 0.4t^3 - 0.3t + 0.1$ ✓

(iii) $\int dx = \int (0.4t^3 - 0.3t + 0.1)dt$

$x = 0.1t^4 - 0.15t^2 + 0.1t + c$

$t = .5, x = 0, \Rightarrow c = -0.01875$

$\therefore$ at $t = 1.2$, $x = 0.0926$ ✓

13. (i) $F = ma = mg - 0.2v^2$

$\Rightarrow 0.4a = 0.4g - 0.2v^2$

$\Rightarrow v\frac{dv}{dx} = 10 - 0.5v^2$

(ii) $\int \frac{v\,dv}{(10 - 0.5v^2)} = \int dx$

$-\ln(10 - 0.5v^2) = x + c$

$x = 0, v = 0 \Rightarrow c = -\ln 10$

$\Rightarrow -\ln(10 - 0.5v^2) = x - \ln 10$

$\Rightarrow v = \sqrt{(20 - 20e^{-x})}$

(iii) Increase from 8m to 4m

$= \sqrt{20 - 20e^{-8}} - \sqrt{20 - 20e^{-4}}$

$= 0.0404\ ms^{-1}$.

14. (i) $F = ma = mv\frac{dv}{dx}$

$0.2v\frac{dv}{dx} = 0.2g\sin 60° - 0.3 \times 0.2g\cos 60° - 0.6x$

$\Rightarrow v\frac{dv}{dx} = 5\sqrt{3} - 1.5 - 3x$ ✓ —— ①

(ii) from ① for max vel. $a = v\frac{dv}{dx} = 0$

$\Rightarrow 5\sqrt{3} - 1.5 - 3x = 0$

$\Rightarrow x = 2.39$ ✓

also from ① $\int v\,dv = \int (5\sqrt{3} - 1.5 - 3x)dx$

$\Rightarrow v^2/2 = 5\sqrt{3}x - 1.5x - 3x^2/2 + c$ —— ②

$v = 0, x = 0 \Rightarrow c = 0$

at $x = 2.39 \Rightarrow v = 4.13$ ✓ from ②

(iii) $0 = 5\sqrt{3}x - 1.5x - \frac{3x^2}{2}$

$\Rightarrow x = 4.77$

from ① $a = 5\sqrt{3} - 1.5 - 3 \times 4.77$

Magnitude of $a = 7.16\ ms^{-1}$

15. $F = ma = m\frac{dv}{dt} = te^{-v}$

$\Rightarrow 0.2\frac{dv}{dt} = te^{-v}$

$\Rightarrow \int e^v dt = 5\int t\,dt$

$\Rightarrow e^v = 5 t^2/2 + c$ —— ①

$t = 0, v = 0 \Rightarrow c = 1$

fr ① $e^v = 5t^2/2 + 1$

$\therefore v = \ln(2.5t^2 + 1)$

$v(2) = \ln(11) = 2.4$

$v = 2.4\ ms^{-1}$ ✓

Answers

16. $F = ma = m\frac{dv}{dx}\times v$

(i) $\Rightarrow 0.4v\frac{dv}{dx} = 0.4g\sin 30 - 0.2v^2$

$v\frac{dv}{dx} = \underline{5 - 0.5v^2}$ ✓

(ii) $\int \frac{v\,dv}{(5-0.5v^2)} = \int dx$

$\Rightarrow -\ln(5-0.5v^2) = x + c$ — ①

$x=0, v=0 \Rightarrow c = -\ln 5$

In ①

$-\ln(5-0.5v^2) = x - \ln 5$

$\Rightarrow 5 - 0.5v^2 = 5e^{-x}$

$v = \sqrt{(10-10e^{-x})}$ ✓

17. $F = ma = mv\frac{dv}{dx}$

(i) $\Rightarrow 0.2v\frac{dv}{dx} = 0.2g\sin 30^\circ - 0.1x^2(0.2g\cos 30^\circ)$

$\Rightarrow 2v\frac{dv}{dx} = \underline{10 - \sqrt{3}x^2}$ ✓ — ①

(ii) $2\int v\,dv = \int (10-\sqrt{3}x^2)\,dx$

$\Rightarrow v^2 = 10x - \sqrt{3}x^3/3$ — ②

from ① for Max speed $10 - \sqrt{3}x^2 = 0$

$\Rightarrow x = 2.4$

for ② $v^2 = 10\times 2.4 - \sqrt{3}\times(2.4)^3/3$

Max $v = \underline{4\ (.002)}$ ✓

(iii) from ② $0 = 10x - \sqrt{3}x^3/3$

$\Rightarrow \underline{x = 4.16}$ ✓

18. $F = ma = mv\frac{dv}{dx}$

(i) $\Rightarrow 0.4v\frac{dv}{dx} = 0.4\sin 30^\circ - 0.8e^{-x}$

$v\frac{dv}{dx} = \underline{5 - 2e^{-x}}$ ✓

(ii) $\int v\,dv = \int (5-2e^{-x})\,dx$

$\frac{v^2}{2} = 5x + 2e^{-x} + C$

$v=0, x=0 \Rightarrow C = -2$

$\frac{v^2}{2} = 5x + 2e^{-x} - 2$ — ①

at $x = 0.6$

$\Rightarrow v = 2.05$ ✓ for ①

19(i) $0.4a = 0.6t - 0.3\times 0.4g$

at $t = 2$

$a = (1.5t - 3)$ ✓

(ii) $v = \int (1.5t-3)\,dt$

$v = 0.75t^2 - 3t + C$

$v=0, t=2 \Rightarrow C = 3$

$v = \underline{0.75t^2 - 3t + 3}$ ✓

(iii) $x = \int (0.75t^2 - 3t + 3)\,dt$

$x = 0.25t^3 - 1.5t^2 + 3t + (k)$

$x=0, t=2 \Rightarrow k = -2$

$x = 0.25t^3 - 1.5t^2 + 3t - 2$ ✓

20(i) $0.25v\frac{dv}{dx} = 2 + 0.3x^2$ — ①

$\Rightarrow v\frac{dv}{dx} = \underline{1.2x^2 + 8}$ ✓

(ii) $\int v\,dv = \int (1.2x^2 + 8)\,dx$

$\frac{v^2}{2} = 0.4x^3 + 8x\ (+C)$ $\begin{cases} x=0 \\ v=0 \\ C=0 \end{cases}$

at $x = 1.5$

$\underline{v = 5.17}$ ✓

(iii) Now Given $v\frac{dv}{dx} = 1.2x^2 + 6 - 3x$

$\Rightarrow 0.25v\frac{dv}{dx} = 0.3x^2 + 1.5 - 0.75x$ — ②

from ① − ② Force $= (0.5 + 0.75x)$ N towards O.

Answers

21. (i) $0.4\frac{dv}{dt} = 3e^{-t} - 0.4g\sin 30^\circ$
$\Rightarrow \frac{dv}{dt} = 7.5e^{-t} - 5$ ✓ —(1)

(ii) $\int_0^v dv = \int_0^t (7.5e^{-t} - 5)\,dt$
$\Rightarrow \quad v = -7.5e^{-t} - 5t + c$
$v = 0,\ t = 0 \Rightarrow c = 7.5$
$\therefore\ v = 7.5 - 7.5e^{-t} - 5t$ ✓

(iii) Solve $\frac{dv}{dt} = 0 \Rightarrow$ frm (1) $t = 0.405$
$\int_0^x dx = \int_0^{0.405} (7.5 - 7.5e^{-t} - 5t)\,dt$
$\Rightarrow x = 0.13\,m$ ✓

22 (i) $0.1v\frac{dv}{dx} = -0.2e^{-x}$
$\Rightarrow v\frac{dv}{dx} = -2e^{-x}$
$\therefore k = -2$ ✓

(ii) $\int v\,dv = \int -2e^{-x}\,dx$
$\Rightarrow \frac{v^2}{2} = 2e^{-x} + c$ —(1)
Given $v = 2.2$ for $x = 0$
$\Rightarrow c = \frac{2.2^2}{2} - 2e^0 = 0.42$
frm (1) $\frac{v^2}{2} = 2e^{-x} + 0.42$
for $v = 2 \Rightarrow \frac{2^2}{2} = 2e^{-x} + 0.42$
$\Rightarrow x = 0.236$ ✓

(iii) $\frac{v^2}{2} = 2e^{-\infty} + 0.42$ when $x \to \infty$
$\Rightarrow v = 0.917\,ms^{-1}$ ✓

23 (i) $R = 0.2g - 0.4 \times 2\sin 30$
$F_R = 0.4 \times 2\cos 30$
$\mu = F_R/R = 0.433$ ✓
Now $0.2g = 0.4t\sin 30$
$\Rightarrow t = 10$ ✓

(ii) $0.2\frac{dv}{dt} = 0.4t\cos 30^\circ - 0.433(0.2g - 0.4t\sin 30^\circ)$
$\Rightarrow \frac{dv}{dt} = 2.165t - 4.33$ ✓

(iii) $\int dv = \int (2.165t - 4.33)\,dt$
$v = 2.165t^2/2 - 4.33t + c$
$v = 0,\ t = 2 \Rightarrow c = 4.33$
$v = 2.165 \times t^2/2 - 4.33t + 4.33$
at $t = 10\,s$
$v = 2.165 \times 10^2/2 - 4.33 \times 10 + 4.33$
$\Rightarrow v = 69.3$ ✓

24 (i) $60\frac{dv}{dt} = 150 - 12v$
$\Rightarrow 5\frac{dv}{dt} = 12.5 - v$ ✓

(ii) $\int \frac{5\,dv}{(12.5 - v)} = \int dt$
$\Rightarrow -5\ln(12.5 - v) = t + c$ —(1)
$t = 0,\ v = 11.5 \Rightarrow c = 0$
$\therefore -5\ln(12.5 - v) = t$
$\Rightarrow v = 12.5 - e^{-0.2t}$ ✓

(iii) $x = \int (12.5 - e^{-0.2t})\,dt$
$x = 12.5t - e^{-0.2t}/-0.2 + c$
$t = 0,\ x = 0 \Rightarrow c = -5$
$\therefore x = 12.5t + 5e^{-0.2t} - 5$ ✓

Answers

25. $\frac{dv}{dt} = e^{-0.5}$

$\int \frac{1}{e^{-0.5v}} dt = \int dt$

$\int e^{0.5v} dt = t + c$

$\Rightarrow \frac{e^{0.5v}}{0.5} = t + c$

$t = 0, v = 2 \Rightarrow c = 2e$

$\Rightarrow 2e^{0.5v} = t + 2e$

$\therefore$ at $t = 1.2$

$2e^{0.5v} = 1.2 + 2e$

$\Rightarrow e^{0.5v} = 0.6 + e$

$\Rightarrow \frac{1}{2}v = \ln(.6 + e)$

$= 1.2$

$\Rightarrow \underline{v = 2.4}$ ✓

26 (i) $0.3v\frac{dv}{dx} = -2x$

$v\frac{dv}{dx} = -\frac{20}{3}x$

$\Rightarrow K = -\frac{20}{3} = \underline{-6\frac{2}{3}}$ ✓

(ii) $\int_8^0 v\,dv = -\frac{20}{3}\int_0^x x\,dx$

$\left[\frac{v^2}{2}\right]_8^0 = -\frac{20}{3}\cdot\frac{x^2}{2}$

$-32 = -\frac{20}{3}\frac{x^2}{2}$

$x^2 = \frac{96}{10} = 9.6$

$\underline{x = 3.1}$ ✓

27. $0.5v\frac{dv}{dx} = -0.5g - 0.02v^2$

(i)

$v\frac{dv}{dx} = \underline{-10 - 0.04v^2}$ ✓

(ii) $\int_8^0 \frac{v}{(-10 - 0.04v^2)} dv = \int_0^x dx$

$x = \left[-\ln(10 + 0.4v^2)/0.08\right]_8^0$

$x = \underline{2.85}$ ✓

(iii) $v\frac{dv}{dx} = 10 - 0.04v^2$

$\int_0^v \frac{v}{(10 - 0.04v^2)} dv = \int_0^{2.85} dx$

$\ln\left[(10 - 0.04v^2)/10\right] - 0.08 = 2.85$

$\Rightarrow \underline{v = 7.14\ ms^{-1}}$ ✓

←— X —→