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M.2

Mechanics-2

Linear Motion Under a Variable Force Notes

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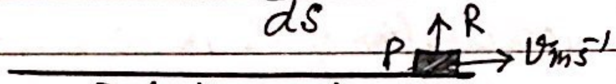
M.2 Linear Motion under a variable force
Notes

- **Newton's Second Law:**

Force $F = ma$

$$F = m \cdot \frac{dv}{dt}$$ [acceleration $a = \frac{dv}{dt}$

or $$F = m \cdot v \frac{dv}{ds}$$ [$\frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt} = v \cdot \frac{dv}{ds}$



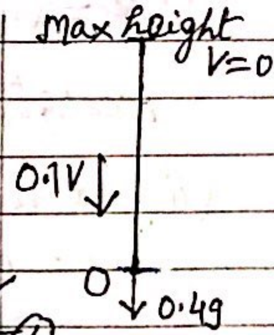
Note: If a particle is moving with a velocity $v\,ms^{-1}$ on a horizontal surface, it loses the contact with it $\Rightarrow R = 0$

Example 1. An object of mass 0.4 kg is projected vertically upwards from the ground, with an initial speed of $16\,ms^{-1}$. A resisting force of magnitude $0.1v$ newtons act on the object during its ascent, where $v\,ms^{-1}$ is the speed of the object at time t s after it starts to move.

(i) Show that $\frac{dv}{dt} = -0.25(v+40)$ --[2]

(ii) Find the value of t at the instant that the object reaches its maximum height. [S-06/05/Q4] --[5]

Solution: Vertically upward direction is +



(i) $$F = ma = m\frac{dv}{dt} = -mg - 0.1v$$

$$\Rightarrow 0.4\frac{dv}{dt} = -0.4 \times 10 - 0.1v$$

$$\Rightarrow \frac{dv}{dt} = -10 - 0.25v$$

$$\Rightarrow \frac{dv}{dt} = -0.25(v+40) \quad \text{— ①}$$

(ii) from ①

$$\frac{1}{-0.25}\int \frac{1}{(v+40)}\,dv = \int dt$$

$$\Rightarrow -4\ln(v+40) = t + c$$

for $t = 0$, $v = 16 \Rightarrow c = -4\ln 56$

$$\Rightarrow -4\ln(v+40) = t - 4\ln 56$$

$$\Rightarrow -4\ln\left(\frac{v+40}{56}\right) = t \quad \text{— ②}$$

for the max. height $v = 0 \Rightarrow$ from ② $t = -4\ln\left(\frac{40}{56}\right) = 4\ln\left(\frac{56}{40}\right) = 1.35$

$$\Rightarrow t = 1.35$$

<u>Example 2.</u> A partical P of mass 0.3 kg is projected vertically upwards from ground at an initial speed $20\,ms^{-1}$. When P is at height x m above the ground, the upwards speed is $v\,ms^{-1}$. It is given that $3v - 90\ln(v+30) + x = A$ where A is a constant.

(i) Differentiate this equation with respect to x and hence show that the acceleration of the particle is $-\frac{1}{3}(v+30)\,ms^{-2}$.

(ii) Find in terms of v, the resisting force acting of the particle.

(iii) Find the time taken for P to reach its maximum height.

[W-09/52/Q7]

<u>Solution:</u> Given $3v - 90\ln(v+30) + x = A$

(i) diff. w.r.t x, $3\frac{dv}{dx} - \frac{90}{(v+30)} \cdot \frac{dv}{dx} + 1 = 0$

$$\Rightarrow \frac{dv}{dx}\left(3 - \frac{90}{v+30}\right) = -1$$

$$\therefore \Rightarrow \frac{dv}{dx} = -\frac{(v+30)}{3v} \quad \text{—①}$$

Now acceleration $a = v \cdot \frac{dv}{dx}$

from ① $v \cdot \frac{dv}{dx} = -\frac{(v+30)}{3v} \times v$

Acceleration: $a = -\frac{1}{3}(v+30)$ ✓ —② $\left[\because a = v \cdot \frac{dv}{dx}\right.$

(ii) We know force $F = ma$

$\Rightarrow F = 0.3 \times -\frac{1}{3}(v+30)$

$= -0.3 \times 10 - \frac{0.3}{3}v$

$F = -mg - 0.1v$

Upward direction is taken as +,

$\therefore$ Resisting force is $0.1v$ ✓ acting downwards.

(iii) from ②

$a = \frac{dv}{dt} = -\frac{1}{3}(v+30)$

$\Rightarrow -3\int \frac{1}{(v+30)}\,dv = \int dt$

$-3\ln(v+30) = t + c$

for $t = 0,\ v = 20$

$\Rightarrow c = -3\ln 50$

$\therefore -3\ln(v+30) = t - 3\ln 50$

or $t = 3\ln\left(\frac{50}{v+30}\right)$ —③

for Max height $v = 0$,

fr ③ $t = 3\ln\left(\frac{50}{30}\right) = 1.532$

$\therefore t = 1.53\,s$ ✓

Example 3. A particle of mass 0.4 Kg is released from rest falls vertically. A resisting force of magnitude $0.08v$ N acts upwards on the particle during its descent, where $v\ ms^{-1}$ is the velocity of the particle at time t s after its release.

(i) Show that the acceleration of the particle is: $(10-0.2v)ms^{-2}$.

(ii) Find the velocity of the particle when $t=15$. [W-07/05/Q4]

Solution: Vertically downwards is considered '+' direction.

(i) $F = ma = 0.4a = 0.4g - 0.08V$

$\Rightarrow a = \underline{(10-0.2V)}$ ✓

(ii) $a = \frac{dv}{dt} = (10-0.2V)$

$\Rightarrow \int \frac{1}{(10-0.2V)} \cdot dV = \int dt$

$\Rightarrow \frac{\ln(10-0.2V)}{-0.2} = t + c$ — ①

Now for $t=0, V=0 \Rightarrow C = -5\ln 10$

fr ① $-5\ln(10-0.2V) = t - 5\ln 10$

$\Rightarrow -5\left[\ln\left(\frac{10-0.2V}{10}\right)\right] = t$

Now $t=15 \rightarrow -5\ln\left(\frac{10-0.2V}{10}\right) = 15$

$\Rightarrow 10 - 0.2V = e^{-3}$

$\Rightarrow 10 - .2V = 10e^{-3}$

$\Rightarrow 0.2V = 10(1-e^{-3})$

$V = 50(1-e^{-3})$

$= 47.5106$

$\therefore$ Req. Velocity $= \underline{47.5\ ms^{-1}}$ ✓

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Example 4. A particle P of mass m kg falls from rest under gravity. There is a resistive force of magnitude mkv^2 N, where v ms^{-1} is the speed of P after it has fallen a distance x m and k is a positive constant.

(a) By solving an appropriate differential equation
Show that, $v^2 = \frac{g}{k}(1-e^{-2kx})$ ... [7]

It is given that $k = 0.01$,
The speed of P when x becomes large approaches V ms^{-1}.

b(i) Find V correct to two decimal places. ... [1]

(ii) Hence find how far P has fallen when its speed is $\frac{1}{2}V$ ms^{-1} ... [2]

9231/FM/SP-20/03/Q3

Downward motion is taken positive (+)

Solution:

$F = ma = mv\frac{dv}{dx} = mg - mkv^2$

$\Rightarrow v\frac{dv}{dx} = g - kv^2$

$\Rightarrow \int \frac{v}{(g-kv^2)}dv = \int dx$

$\Rightarrow \frac{-1}{2k}\ln(g-kv^2) = x + C$ —①

when $x=0, v=0 \Rightarrow C = \frac{-1}{2k}\ln g$

from ① $x = \frac{1}{2k}\ln g - \frac{1}{2k}\ln(g-kv^2)$

$x = \frac{1}{2k}\ln\left(\frac{g}{g-kv^2}\right)$ —②

$\Rightarrow e^{2kx} = g/g-kv^2$

$\Rightarrow v^2 = \frac{g}{k}(1-e^{-2kx})$ ✓ —③

b(i) when $x \to \infty \Rightarrow e^{-2kx} \to 0$

from ③ $V^2 = \frac{g}{k}(1-0) = 1000$ (for $k = 0.01$)

$V^2 = 1000 \Rightarrow \underline{V = 31.62}$ ✓

(continued →)

b(ii) Now vel $= \frac{1}{2}V = \frac{1}{2}\times 31.62$

Now or $v = \left(\frac{31.62}{2}\right)$

from ② when $k = 0.01$

$x = \frac{1}{2\times 0.01}\ln\left(\frac{g}{g - 0.01\times\left(\frac{31.62}{2}\right)^2}\right)$

$x = 50\ln\left(\frac{g}{g-0.01\times 250}\right)$

$= 50\ln 4/3$

$= 14.371$

or $x = 14.4$ m ✓

Example 5. A particle P starts from a fixed point O and moves in a straight line. When the displacement of P from O is x m, its velocity is v ms^{-1} and its acceleration is $\frac{1}{(x+2)}$ ms^{-2}.

(i) Given that $v=2$, when $x=0$,
use integration to show that $v^2 = 2\ln(\frac{1}{2}x+1)+4$. --[4]

(ii) Find the value of v when the acceleration of P is $\frac{1}{4}$ ms^{-2} --[2]

[S-09/05/Q3]

Solution: acceleration $a = v\frac{dv}{dx} = \frac{1}{(x+2)}$ —— ①

(i)

$\Rightarrow \int v\,dx = \int \frac{dx}{(x+2)}$

$\Rightarrow \frac{v^2}{2} = \ln(x+2)+c$ —— ②

$v=2$, when $x=0 \Rightarrow \frac{2^2}{2} = \ln 2 + c$

$\Rightarrow c = 2-\ln 2$

fn ② $\frac{v^2}{2} = \ln(x+2)+2-\ln 2$

$\Rightarrow v^2 = 2\ln(x+2) - 2\ln 2 + 4$

$v^2 = 2\ln(0.5x+1)+4$ ✓ —— ③

(ii) when $a = \frac{1}{4} = \frac{1}{x+2}$ fm ①

$\Rightarrow x = 2$

fm ③ when $x=2$, $v^2 = 2\ln(1+1)+4 = 5.386$

$v = \sqrt{5.386} = 2.32$

$\therefore v = 2.32$ ms^{-1} ✓

Example 6. A particle P starts from rest at a point O and travels in a straight line. The acceleration of P is $(15-6x)\ ms^{-2}$, where x m is the displacement of P from O.

(i) Find the value of x for which P reaches its maximum velocity, and calculate this maximum velocity. ---[5]

(ii) Calculate the acceleration of P when it is at instantaneous rest and $x > 0$. ---[2]

[S-11 | 51 | Q4]

Solution: Given acceleration $a = (15-6x)$

(i) Now velocity is maximum when acc. $a = 15-6x = 0$

$$\Rightarrow x = 2.5 \quad \text{—(1)}$$

Now also $a = v\dfrac{dv}{dx} = 15-6x$

$$\Rightarrow \int v\,dv = \int (15-6x)\,dx$$

$$\Rightarrow \frac{v^2}{2} = 15x - 3x^2 + C \quad \text{—(2)}$$

Now $v=0$ for $x=0 \Rightarrow C = 0$

$\therefore$ from (2)

$$\frac{v^2}{2} = 15x - 3x^2 \quad \text{—(3)}$$

at $x = 2.5$,

$$\frac{v^2}{2} = 15\times 2.5 - 3(2.5)^2 \quad \left[\begin{array}{l}\text{for max } v. \text{ from (1)} \\ x = 2.5\end{array}\right.$$

$$= 37.5 - 18.75 = 18.75$$

$$\Rightarrow v = \underline{6.12\ ms^{-1}} \checkmark$$

(ii) The particle is at instantaneous rest when $v = 0$.

from (3) $15x - 3x^2 = 0$

$$3x(5-x) = 0$$

$$x = 5,\quad \cancel{x=0} \qquad x > 0$$

$\therefore$ acc. at $x = 5$

$$a = 15 - 6\times 5 \qquad (a = 15-6x)$$

$$a = \underline{-15\ ms^{-2}} \checkmark$$

Example 7. A particle P of mass 0.6 kg is projected horizontally with velocity $2\,ms^{-1}$ from a point O on a smooth horizontal surface. A horizontal force of magnitude $0.3x$ N acts on P in the direction OP, where x m is the distance of P from O. Calculate the velocity of P when $x=8$. [S-12/52/Q1] --[4]

Solution: $a = m \cdot v\frac{dv}{dx} = 0.3x$

$$\Rightarrow 0.6\, v\frac{dv}{dx} = 0.3x$$

$$\int 0.6\, v\,dv = \int 0.3x\,dx$$

$$0.6 \times \frac{v^2}{2} = 0.3x^2/2 + c$$

$$\Rightarrow v^2 = x^2/2 + c$$

$$x=0,\ v=2 \Rightarrow c=4$$

$$\Rightarrow v^2 = x^2/2 + 4$$

for $x=8 \Rightarrow$

$$v^2 = 64/2 + 4 = 36$$

$$v = 6\,ms^{-1}$$

Example 8: A particle P of mass 0.25 kg moves in a straight line on a smooth horizontal surface. At time t s the velocity of P is $v\,ms^{-1}$. A variable force of magnitude $3t$ N opposes the motion of P. [S-12/52/Q4]

(i) Given that P comes to rest when $t=3$, find the value of v, when $t=0$. --[4]

(ii) Calculate the distance travelled by P in the interval $0 \le t \le 3$. --[3]

Solution: $a = 0.25\frac{dv}{dt} = -3t$

(i)

$$\int dv = \int -\frac{3}{0.25}t\,dt$$

$$\Rightarrow v = -12 \times t^2/2 + c$$

$$t=3,\ v=0 \Rightarrow 0 = -54 + c$$

$$\Rightarrow c = 54$$

$$\therefore v = -6t^2 + 54 \quad (1)$$

$$\therefore t=0 \Rightarrow v = 54\,ms^{-1}$$

(ii) $v = \frac{dx}{dt} = 54 - 6t^2$ from (1)

$$\Rightarrow \int dx = \int (54 - 6t^2)\,dt$$

$$x = \left[54t - 6t^3/3\right]_0^3$$

$$x = 108\,m$$

Example 9. A small block B of mass 0.2 kg is placed at a fixed point O on a smooth horizontal surface. A horizontal force of magnitude 0.42 N is applied to B. At time t s after the force is first applied, the velocity of B away from O is $v\ ms^{-1}$.

(i) Find the value of v when $t=1$ --- [2]

for $t>1$ an addition force, of magnitude $0.32t$ N and directed towards O, is applied to B. The force of magnitude 0.42 N continue to act as before.

(ii) Find the value of v when $t=2$. --- [3]

For $t>2$ a third force, of magnitude $0.06t^2$ N and directed away from O, is applied to B. The other two forces continue to act as before. --- [3]

(iii) Show that the velocity of B is the same when $t=2$ and when $t=3$.

[S-14 / 53 / Q3]

Solution: $F = ma = 0.2a = 0.42$

$\Rightarrow a = 2.1$

$v = u + at$

$v = 0 + 2.1 \times 1$ (at $t=1$)

$v = \underline{2.1\ ms^{-1}}$ ✓

(ii) when $t>1$

$0.2a = 0.42 - 0.32t$

$\Rightarrow a = 2.1 - 1.6t$

$\frac{dv}{dt} = 2.1 - 1.6t$

$\Rightarrow \int dv = \int (2.1 - 1.6t)\,dt$

$[v]_{2.1}^{v} = [2.1t - 0.8t^2]_1^2$

$\Rightarrow v = 1.8\ ms^{-1}$

$t>2$

(iii) $0.2a = 0.42 - 0.32t + 0.06t^2$

$\Rightarrow \frac{dv}{dt} = [0.42 - 0.32t + 0.06t^2]/0.2$

$\Rightarrow v = [0.42t - 0.16t^2 + 0.02t^3]_0^3 / 0.2$

$v = 1.8\ ms^{-1}$

$\therefore$ No change.

Linear Motion under a variable force.

Example 10. A particle P of mass 0.6 kg is released from rest at a point above ground level and falls vertically. The motion of P is opposed by a force of magnitude $3v$ N, where $v\ ms^{-1}$ is the speed of P. Immediately before P reaches the ground, $v = 1.95$.

(i) Calculate the time after its release when P reaches the ground. ...[5]

P is now projected horizontally with speed $1.95\ ms^{-1}$ across a smooth horizontal surface. The motion of P is again opposed by a force of magnitude $3v$ N, where $v\ ms^{-1}$ is the speed of P.

(ii) Calculate the distance P travels after projection before coming to rest. [S-14/52/Q6] ...[3]

Solution: $F = 0.6\frac{dv}{dt} = 0.6g - 3v$

(i)

$$\Rightarrow \frac{dv}{dt} = 10 - 5v$$

$$\Rightarrow \int \frac{dv}{(10-5v)} = \int dt$$

$$-\frac{1}{5}\ln(10-5v) = t + c \quad \text{——(1)}$$

Now $t=0, v=0 \Rightarrow c = -\frac{1}{5}\ln 10$

$$\text{frm (1)} \quad -\frac{1}{5}\ln\left(\frac{10-5v}{10}\right) = t \quad \text{——(2)}$$

when $v = 1.95$, $t = \underline{0.738\ s}$ frm (2)

(ii) Now

$$F = 0.6\, v\frac{dv}{dx} = -3v$$

$$\Rightarrow \int 0.2\,dv = -\int dx$$

$$\Rightarrow 0.2v = -x + c \quad \text{——(3)}$$

$v = 1.95$ at $x = 0 \Rightarrow c = 0.2 \times 1.95 = 0.39$

frm (3) $\quad 0.2v = -x + 0.39$

Now P comes to rest $\Rightarrow v = 0$

$$0 = -x + 0.39$$

$$\Rightarrow \underline{x = 0.39}$$

Example 11, A particle P of mass 0.2 kg is projected horizontally from a fixed point O, and moves in a straight line on a smooth horizonal surface. A force of magnitude $0.4x$ N acts on P in the direction PO, where x m is the displacement of P from O.

(i) Given that P comes to instantaneous rest when $x = 2.5$, find the initial kinetic energy of P. --[4]

(ii) Find the value of x on the first occasion when speed of P is $2\,ms^{-1}$. S-12/53/Q3 --[2]

Solution: $F = mv\frac{dv}{dx} = 0.2v\frac{dv}{dx} = -0.4x$

(i) $\Rightarrow v\frac{dv}{dx} = -2x$

$\Rightarrow \int v\,dv = -2\int x\,dx$

$\Rightarrow \frac{v^2}{2} = -2\times\frac{x^2}{2} + C$ ——(1)

Given $V = 0$, $x = 2.5 \Rightarrow 0 = -(2.5)^2 + C \Rightarrow C = 6.25$

frm (1) $\frac{v^2}{2} = -x^2 + 6.25$ ——(2)

$\therefore$ Initial K.E at $x = 0$,

frm (2) $\frac{mv^2}{2} = 0 + 0.2\times 6.25$

$\therefore$ $KE = 1.25\,J$ ✓

(ii) $V = 2$ frm (2) $\frac{2^2}{2} = -x^2 + 6.25$

$\Rightarrow x^2 = 4.25$

$x = 2.06$ ✓

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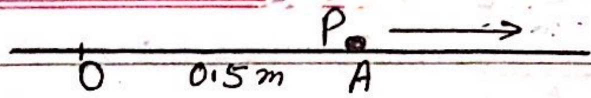
P-11

Example 12.



O and A are fixed point on a horizontal surface, with OA = 0.5 m. A particle P of mass 0.2 kg is projected horizontally with speed $3 ms^{-1}$ from A in the direction OA and moves in a straight line (see diagram). At time t s after projection, the velocity of P is $v\ ms^{-1}$ and its displacement from O is x m. The coefficient of friction between the surface and P is 0.5, and a force of magnitude $0.4/x^2$ N acts on P in the direction PO.

(i) Show that, while the particle is in motion, $v\frac{dv}{dx} = -\left(5+\frac{2}{x^2}\right)$ --[2]

(ii) Calculate the distance travelled by P, before it comes to rest, and show that P does not subsequently move. S-11 | 53 | Q6 --[7]

Solution:

(i) $F = ma = mv\frac{dv}{dx} = 0.2v\frac{dv}{dx} = \text{Friction} - 0.4/x^2$

Force of friction $= \mu R$
$= 0.5 \times mg$

$\Rightarrow 0.2v\frac{dv}{dx} = -0.5\times 0.2g - 0.4/x^2$

$\Rightarrow v\frac{dv}{dx} = -5 - 2x^{-2}$

or $v\frac{dv}{dx} = -\left(5+\frac{2}{x^2}\right)$ —— ①

(ii) $\int v\,dv = -\int(5+2x^{-2})\,dx$

$\frac{v^2}{2} = -5x + \frac{2}{x} + c$ —— ②

$v = 3$ at $x = 0.5 \Rightarrow \frac{3^2}{2} = -5\times 0.5 + 2/0.5 + c$

$\Rightarrow c = 3$

fr ② $\therefore \frac{v^2}{2} = -5x + \frac{2}{x} + 3$ —— ③

P comes to rest $\Rightarrow v = 0$ from ③ $-5x + \frac{2}{x} + 3 = 0$

$5x^2 - 3x - 2 = 0$

$(x-1)(5x+2) = 0$

$x = 1,\ x = -\frac{2}{5}$ ✗

$\therefore$ Distance travelled by P $= 1 - 0.5 = 0.5$ m ✓

fr ③ $\frac{v^2}{2} = -\frac{1}{x}(x-1)(5x+2) > 0 \Rightarrow -\frac{2}{5} \le x < 1 \Rightarrow 0 < x < 1$ ✓

$\therefore$ P does not subsequently move.

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Example 13. A particle P of mass 0.4 kg is released from rest at the top of a smooth plane inclined at 30° to the horizontal. The motion of P down the slope is opposed by a force of magnitude $0.6x$ N, where x m is the distance P travelled down the slope. P comes to rest before reaching the foot of the slope. Calculate

(i) the greatest speed of P during its motion, ---[7]

(ii) the distance travelled by P during it motion. ---[2]

[S-12/51/Q5]

Solution: $F = ma = m \cdot v\frac{dv}{dx}$

(i) $0.4 \cdot v\frac{dv}{dx} = 0.4g\sin 30° - 0.6x$ —— ①

$\Rightarrow \int v\,dv = \int (5 - 1.5x)\,dx$

$v^2/2 = 5x - 1.5x^2/2 + c$

$x = 0, v = 0 \Rightarrow c = 0$

$\therefore v^2/2 = 5x - 1.5x^2/2$ —— ②

for the greatest speed acc. $a = 0$

fr ① $0.4g\sin 30° - 0.6x = 0$

$\Rightarrow 5 - 1.5x = 0$

$x = \frac{5}{1.5} = 10/3$

$\Rightarrow$ from ② at $x = 10/3$

$\frac{v^2}{2} = 5 \times \frac{10}{3} - 1.5\left(\frac{10}{3}\right)^2/2$

$\Rightarrow v = 4.08\ ms^{-1}$ ✓

(ii) for distance travelled $v = 0$

from ② $5x - 1.5x^2/2 = 0$

$x = 6\,2/3 = 6.67$ ✓

Example 14. A force of magnitude $0.4t$ N, applied at an angle $30°$ above the horizontal, acts on a particle P, where t s is the time since the force starts to act. P is at rest on rough horizontal ground when $t=0$. The mass of P is 0.2 kg and the coefficient of friction between P and the ground is μ.

(i) Given that P is about to slip, when $t=2$, find μ and the value of t for the instant when P loses contact with the ground. --[5]

(ii) While P is moving on the ground, it has velocity v ms^{-1} at time t s, show that $\frac{dv}{dt} = 2.165t - 4.330$ --[3]

(iii) Calculate the speed of P when it loses contact with ground. --[4]

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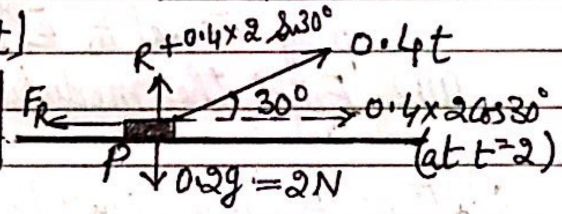
Solution: $R = (0.2g - 0.4t\sin 30°)$

Force at $t=2 \Rightarrow 0.4\times 2$ $[F = 0.4t]$



(i) Vertically at P, at $t=2$: $R + 0.4\times 2\sin 30° = 0.2g$

$\Rightarrow R = 0.2g - 0.4\times 2\sin 30$

$R = 1.6$ N —— ①

at $t=2$, Horizontally at P

Friction: $F = 0.4\times 2\times\cos 30 = 0.6928$ —— ②

$F = \mu R \Rightarrow \mu = \frac{F}{R} = \frac{0.6928}{1.6} = 0.433$

from ① and ②

$\Rightarrow \mu = 0.433$ ✓

(ii) $F = ma = m\frac{dv}{dt}$

$0.2\frac{dv}{dt} = 0.4t\cos 30° - \mu R$

$0.2\frac{dv}{dt} = 0.4t\cos 30 - 0.433(0.2g - 0.4t\sin 30°)$

$0.2\frac{dv}{dt} = 0.4t(\cos 30 + 0.433\sin 30°)$

$\frac{dv}{dt} = 2.165t - 4.33$ —— ③

(iii) P loses the contact with ground when $R = 0$

$R = 0.2g - 0.4t\sin 30° = 0$

$\Rightarrow t = 10$ s —— ④

Now from ③

$\int dv = \int (2.165t - 4.33)\,dt$

$V = 2.165\,t^2/2 - 4.33t + C$

$V = 0,\ t = 2 \Rightarrow C = 4.33$

$\therefore V = 2.165\times t^2/2 - 4.33t + 4.33$ —— ⑤

from ④ P loses the contact with ground at $t = 10$ in ⑤

$V = 2.165\times 10^2/2 - 4.33\times 2 + 4.33$

$V = 69.3$ ✓

Example 15. A particle P of mass 0.4 kg is projected horizontally along a smooth horizontal plane from a point O. After projection the velocity of P is $v\ ms^{-1}$ and its displacement from O is x m. A force of magnitude $8x$ N directed away from O acts on P, and a force of magnitude $(2e^{-x}+4)$ N opposes the motion of P. One end of the light elastic string of natural length 0.5 m is attached to O and other end of the string is attached to P.

(i) Show that $v\frac{dv}{dx} = 20x - 10 - 5e^{-x}$ before the elastic string becomes taut. --[2]

(ii) Given that the initial velocity of P is $6ms^{-1}$, find v when the string first becomes taut. --[3]

When the string is taut, the acceleration of P is proportional to e^{-x}. [W-18/52/Q3]

(iii) Find the modulus of elasticity of the string. --[2]

Solution: $F = ma = 0.4v\frac{dv}{dx} = 8x - (2e^{-x}+4)$

(i) $v\frac{dv}{dx} = 20x - 10 - 5e^{-x}$ ✓

(ii) $\int v\,dv = \int(20x - 10 - 5e^{-x})dx$

$v^2/2 = 10x^2 - 10x + 5e^{-x} + C$

$x=0,\ v=6 \Rightarrow C = 13$

$v^2 = 2[10x^2 - 10x + 5e^{-x} + 13]$

at $x = 0.5 \Rightarrow v^2 = 27$

$\Rightarrow v = \underline{5.2ms^{-1}}$ ✓

(iii) $8x - 4 = T = \frac{\lambda e}{L} = \frac{\lambda(x-0.5)}{0.5}$

$\Rightarrow 8x - 4 = 2\lambda x - \lambda \Rightarrow \underline{\lambda = 4N}$ ✓

$[\dot{F} = (8x-4) - 2e^{-x} = ke^{-x}$

$\Rightarrow T = 8x - 4]$