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M-2

Mechanics - 2

Motion of a Projectile
Exercise

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FM | M2

Motion of a Projectile

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1. A partical P is projected with speed U at angle α above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t are denoted by x and y respectively.

(a) Derive the equation of the trajectory of P in the form

$$y = x \tan\alpha - \frac{gx^2}{2U^2}\sec^2\alpha$$ ---[3]

(b) The greatest height of P above the plane is denoted by H. When P is at a height of $\frac{3}{4}H$, it has travelled a horizontal distance d.

Given that $\tan\alpha = 2$, find in terms of H, the two possible values of d. [9231 FM SP-20 03 Q6] --[6]

2. A particle is projected from a point on horizontal ground with speed $15\,ms^{-1}$ at an angle of $\theta°$ above the horizontal. The particle strikes the ground 2s after projection.

(i) Find θ ---[2]

(ii) Calculate the time after projection at which the direction of motion of the particle is 20° below the horizontal. ---[4] [W-19 51 Q2] [9709]

3. A small B is projected with speed $30\,ms^{-1}$ at an angle of 60° above the horizontal from a point O. At time t s after projection the horizontal and vertically upwards displacements of B from O are x m and y m respectively.

(i) Express x and y in terms of t and hence find the equation of the trajectory of the ball. ---[4]

(ii) Find the value of x for which OB makes an angle of 45° above the horizontal. [9709 W-19 51 Q4] --[3]

4. A small ball is projected from a point O on horizontal ground at an angle of 30° above the horizontal. At t s after projection the horizontal and vertically upwards displacements of the ball from O are x m and y m respectively. It is given that $x = 40t$. (Continued →)

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(continued →)

4 (i) Calculate the initial speed of the ball, and express y in terms of t. --[3]

(ii) Hence find the equation of the trajectory of the ball. [W-19|52|Q2] --[2]

5. A particle is projected from a point O on horizontal ground with speed $V\ ms^{-1}$ at an angle of 60° above the horizontal. At the instant 3s after projection the direction of motion of the particle is 30° below the horizontal.

(i) Find V ---[3]

(ii) Calculate the distance of the particle from O at the instant 3s after projection. [W-19|52|Q4] --[3]

6. A particle is projected with speed $V\ ms^{-1}$ at an angle of θ° above the horizontal. At the instant 4s after projection the speed of the particle is $16\ ms^{-1}$ and its direction of motion is 30° above the horizontal. Find V and θ. [S-19|51|Q2] ---[5]

7. A small ball is projected with speed $25\ ms^{-1}$ at an angle of 30° above the horizontal from a point O on the horizontal ground. At time ts after projection the horizontal and vertically upwards displacements of the ball from O are x m and y m respectively.

(i) Express x and y in terms of t and hence find the equation of the trajectory of the ball. ---[4]

(ii) Find x for the position of the ball when its path makes an angle of 15° below the horizontal. [S-19|51|Q4] ---[4]

8. A small ball is projected from a point O on horizontal ground at an angle of 30° above the horizontal. At time ts after projection the vertically upwards displacement of the ball from O is $(14t - Kt^2)$ m, where K is a constant. (i) State the value of K. ---[1]

(ii) Show that the initial speed of the ball is $28\ ms^{-1}$. --[2]

(iii) Find the horizontal displacement of the ball from O when $t = 3$. --[2]

[S-19|52|Q1]

9. A particle is projected with speed 15 ms^{-1} at an angle of $\theta°$ above the horizontal. At the instant 4s after projection the speed of the particle is 30 ms^{-1}.

(i) Find θ. ---[4]

(ii) Show that at the instant 4s after projection of the particle the particle is 33.75 m below the level of the point of projection and find the direction of motion at this instant. S-19|52|Q6 ---[4]

10. A particle is projected with speed 24 ms^{-1} at an angle of 30° above the horizontal. Find the speed and direction of motion of the particle at the instant 4s after projection. ---[5] M-19|52|Q1

11. A small ball is projected from a point O on horizontal ground. At time t s after projection the horizontal and vertically upwards displacements of the ball from O are x m and y m respectively, where $x = 4t$ and $y = 6t - 5t^2$

(i) Find the equation of the trajectory of the ball. ---[2]

(ii) Hence or otherwise calculate the angle of projection of the ball and its initial speed. M-19|52|Q3 ---[4]

12. An object is projected with speed 15 ms^{-1} at an angle of 35° above the horizontal from a point on horizontal ground. Find the speed and direction of motion of the object at time 2s after the instant of projection. M-18|52|Q2 ---[5]

13. A particle P is projected from a point O on horizontal ground. At the instant t s after projection, the horizontal and vertically upwards displacements of P from O are x m and y m respectively. The equation of trajectory of P is $y = 3x - 0.05x^2$

(i) Find the angle of projection and the initial speed of P. ---[3]

(ii) Find the coordinates of P at an instant when OP makes an angle of 45° with the horizontal. ---[2]

(Continued→)

(continued→)

13(iii) For the instant when P is at its greatest height above the ground, calculate this height and the corresponding value of t. --[4]
[M-18/52/Q4]

14. A small ball B is projected from a point O on horizontal ground. The initial velocity of B has horizontal and vertically upwards components of $18 ms^{-1}$ and $25 ms^{-1}$ respectively. For instant 4s after projection, find the speed and direction of motion of B. ---[4]
[S-18/51/Q1]

15. A small object is projected from a point O with speed $V ms^{-1}$ at an angle of 45° above the horizontal. At time t s after projection, the horizontal and vertically upwards displacements of the object from O are x m and y m respectively.

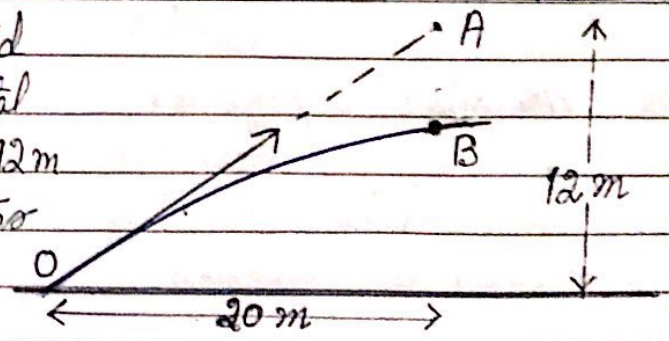
(i) Express x and y in terms of t, and hence find the equation of the path. --[4]

The object passes through the point with coordinates (24, 18)

(ii) Find V --[2]

(iii) The object passes through two points which are 22.5 m above the level of O. Find the values of x for these points. --[3]
[S-18/51/Q4]

16. A small ball B is projected from a point O on horizontal ground towards a point A 12 m above the ground. 0.9 s after projection B has travelled a horizontal distance of 20 m and is vertically below A.



(i) Find the angle and the speed of projection of B. ---[4]

(ii) Calculate the distance AB, when B is vertically below A. ---[2]
[S-18/52/Q1]

16. A small ball is projected with speed $15 ms^{-1}$ at an angle of 60° above the horizontal. Find the distance from the point of projection of the ball at the instant when it is travelling horizontally. --[5]
[M-17/52/Q1]

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17. A particle P is project from a point O on horizontal ground with initial speed $20 ms^{-1}$ and angle of projection 30°. At the instant t s after projection, the horizontal and vertically upwards displacements of P from O are x m and y m respectively.

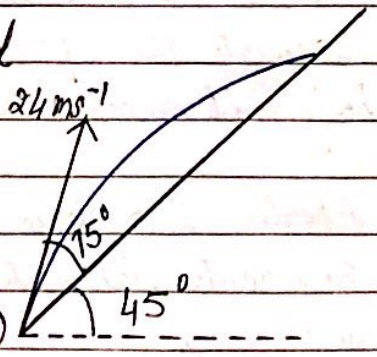
(i) Express x and y in terms of t and hence find the equation of the trajectory of P. ---[4]

P is at the same height above the ground at two points which are a horizontal distance apart of 15 m,

(ii) Calculate this height. [S-18|52|Q4] ---[3]

18. A small ball B is projected with speed $30 ms^{-1}$ at an angle of 60° to the horizontal from a point on horizontal ground. Find the time after projection when the speed of B is $25 ms^{-1}$ for the second time. [W-18|51|Q1] ---[4]

19. A small object is projected with speed $24 ms^{-1}$ from a point O at the foot of a plane inclined at 45° to the horizontal. The angle of projection of the object is 15° above a line of greatest slope of the plane. At time t s after projection, the horizontal and vertically upwards displacements of the object from O are x m and y m respectively.



(i) Express x and y in terms of t, and hence find the value of t for the instant when the object strikes the plane. ---[4]

(ii) Express the vertical height of the object above the plane in terms of t and hence find the greatest vertical height of the object above the plane. [W-18|51|Q7] ---[5]

20. A small ball B is projected with speed $38 ms^{-1}$ at an angle of 30° to the horizontal from a point on horizontal ground. Find the speed of B when the path of B makes an angle of 20° above the horizontal. [W-18|52|Q1] ---[3]

21. A small object is projected horizontally with speed V ms^{-1} from a point O above horizontal ground. At time t s after projection, the horizontal and vertically upwards displacements of the object from O are x m and y m respectively.

(i) Express x and y in terms of t and hence show that the equation of the path of the object is $y = \frac{5x^2}{V^2}$. ---[3]

The object passes through the points with coordinates $(a, -a)$ and $(a^2, -16a)$, where a is a positive constant.

(ii) Find the value of a. ---[3]

(iii) Given that the object strikes the ground at the point where $x = 5a$, find the height of O above the ground. [W-18/52/Q4] ---[2]

22. A particle P is projected with speed 20 ms^{-1} at an angle of 60° below the horizontal, from a point O which is 30 m above horizontal ground.

(i) Calculate the time taken by P to reach the ground. ---[3]

(ii) Calculate the speed and direction of motion of P immediately before it reaches the ground. [M-17/52/Q3] ---[4]

23. A particle is projected with speed 20 ms^{-1} at an angle of 60° above the horizontal. Calculate the time after projection when the particle is descending at an angle of 40° below the horizontal. [S-17/51/Q1] ---[4]

24. A particle is projected from a point O on horizontal ground. The initial components of the velocity of the particle are 10 ms^{-1} horizontally and 15 ms^{-1} vertically. At time t s after projection, the horizontal and vertically upwards displacements of the particle from O are x m and y m respectively.

(i) Express x and y in terms of t, and hence find the equation of the trajectory of the particle. ---[4]

The horizontal ground is at the top of a vertical cliff. The point O is at a distance d m from the edge of the cliff. The particle is project towards the edge of the cliff and --- (continued→)

(Continued →)

24. does not strike the ground before it passes over the edge of the cliff.

(ii) Show that d is less than 30. [S-17/51/Q4] ---[2]

(iii) Find the value of x when the particle is 14m below the level of O. ---[2]

25. A particle P is projected from a point O with speed V ms^{-1}. At time t s after projection the horizontal and vertically upwards displacements of P from O are x m and y m respectively. The equation of the trajectory of P is $y = 2x - \frac{25x^2}{V^2}$

(i) Write down the value of $\tan\theta$, where θ is the angle of projection of P. ---[1]

when t = 4, P passes through the point A where x = y = a.

(ii) Calculate V and a. [S-17/52/Q7] ---[5]

(iii) Find the direction of motion of P when it passes through A. ---[3]

26. A small ball is projected from a point 1·5 m above horizontal ground. At a point 9m above the ground the ball is travelling at 45° above the horizontal and its velocity is 4ms^{-1}. Find the angle of projection of the ball. [W-17/51/Q2] ---[4]

27. A particle P is projected with speed 25ms^{-1} at an angle of 30° above the horizontal from a point O on horizontal ground. At time t s after projection the horizontal and vertically upwards displacements of P from O are x m and y m respectively.

(i) Express x and y in terms of t and hence show that the equation of the trajectory of P is, $y = \frac{x}{\sqrt{3}} - \frac{4x^2}{375}$ ---[4]

(ii) Find the horizontal distance between the two points at which P is 5 m above the ground. ---[3]

[W-17/51/Q4]

28. A small ball B is projected from a point O which is h m above a horizontal plane. At time 2s after projection, B has speed 18 ms^{-1} and is moving in the direction 30° above the horizontal.

(i) Find the initial speed and the angle of projection of B. ---[4]

B has speed 38 ms^{-1} immediately before it strikes the plane,

(ii) Calculate h. --[2]

B bounces when it strikes the plane, and leaves the plane with speed 20 ms^{-1} but its horizontal component of velocity unchaged.

(iii) Find the total time which elapse between the initial projection of B and the instant when it slikes the plane for the second time. [W-17|52|Q7]---[5]

29. A particle P is projected from a point on horizontal ground. At the instant 2s after projection, the particle has travelled a horizontal distance of 30 m and at its greatest height above the ground. Find the initial speed and the angle of projection of the particle. ---[5]

[M-16|52|Q1]

30. A stone is thrown with speed 9 ms^{-1} at an angle of 60° above the horizontal from a point on horizontal ground. Find the distance between the two points at which the path of the stone makes an angle 45° with the horizontal. [M-16|52|Q3]--[5]

31. A small ball is projected with speed 16 ms^{-1} at an angle 45° above the horizontal from a point on horizontal ground. Calculate the period of time, before the ball lands, for which the speed of the balls is less than 12 ms^{-1}. [S-16|51|Q1]

32. A particle is projected at an angle of θ° below the horizontal from a point at the top of a vertical cliff 26 m high. The particle strikes horizontal ground at a distance 8 m from the foot of the cliff 2s after the instant of projection. Find

(continued →)

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(Continued →)

32(i) the speed of projection of the particle and the value of θ. --[6]

(ii) the direction of motion of the particle immediately before it strikes the ground. S-16/51/Q5 --[3]

33. A small ball B is projected with speed 12 ms^{-1} at an angle of 30° above the horizontal from a point O on horizontal ground. At the instant 0.8 s after projection, B is 0.5 m vertically above the top of a vertical post.

(i) Calculate the height of the top of the post above the ground. --[3]

(ii) Show that B is at its greatest height 0.2 s before passing over the post. S-16/52/Q1 --[2]

33. The point O is 8 m above a horizontal plane. A particle P is projected from O. After projection, the horizontal and vertically upwards displacements of P from O are x m and y m respectively. The equation of trajectory of P is, $y = 2x - x^2$

(i) Find the value of x for the point P, where it strikes the plane. --[2]

(ii) Find the angle and speed of projection of P. --[3]

(iii) Calculate the speed of P immediately before it strikes the plane. --[2]

S-16/52/Q3

34. A particle P is projected with speed 35 ms^{-1} from a point O on a horizontal plane. In the subsequent motion, the horizontal and vertically upwards displacements of P from O are x m and y m respectively. The equation of the trajectory of P is, $y = kx - \frac{(1+k^2)x^2}{245}$, where k is a constant. P passes through the points A(14, a) and B(42, 2a), where a is a constant.

(i) Calculate the two possible values of k and hence show that the larger of the two possible angles of projection is 63.435, correct to 3 d.p. --[5]

For the larger angle of projection, calculate W-16/51/Q7

(ii) the time after projection when P passes through A. --[2]

(iii) the speed and direction of motion of P when it passes through B. --[4]

34'. A particle is projected with speed 25 ms^{-1} at an angle of 50° above the horizontal. Calculate the time after projection when the particle has speed 18 ms^{-1} and is rising. --[4] W-15/53/Q1

35. A stone S is thrown horizontally from top T of a high tower. At the same instant 1·6s after S is thrown, the line ST makes an angle of 30° below the horizontal. Find the speed with which S is thrown. [W-16|52|Q1] --[3]

36. A particle P is projected with speed 20 ms^{-1} at an angle of 30° above the horizontal from a point O on the horizontal ground. P subsequently bounces when it first strikes the ground at the point A.

(i) Find the time after projection when P first strikes the ground, and the distance OA. --[3]

When P bounces at A the horizontal component of the velocity of P is unchanged. The vertical component of velocity is 8 ms^{-1} immediately after bouncing. P strikes the ground for the second time at B where it remains at rest.

(ii) Calculate the first and last times after projection at which the speed of P is 18 ms^{-1}. [W-16|52|Q4] --[5]

37. A stone is projected from a point O on horizontal ground. The equation of the trajectory of the stone is,

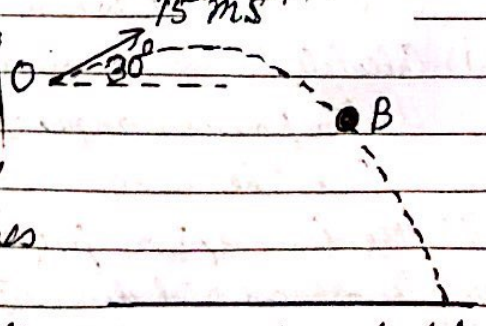
$$y = 1.2x - 0.15x^2$$

where x m and y m are respectively the horizontal and vertically upwards displacements of the stone from O. Find

(i) the greatest height of the stone. [S-15|51|Q2] --[2]

(ii) the distance from O of the point where the stone strikes the ground. --[2]

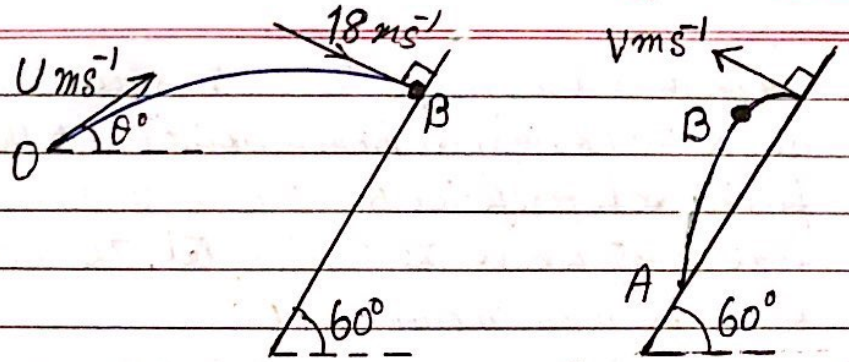
38. A small ball B is projected from a point O above horizontal ground, with initial speed 15 ms^{-1} at an angle of projection of 30° above the horizontal. The ball strikes the ground 3s after projection.



(i) Calculate the speed and direction of motion of the ball immediately before it strikes the ground. --[5]

(ii) Find the height of O above the ground. [S-15|51|Q4] --[2]

39.



A small ball B is projected with speed $U ms^{-1}$ at an angle of $\theta°$ above the horizontal from a point O. At time 2s after the instant of projection, B strikes a smooth wall which slopes at 60° to the horizontal. The speed of B is $18 ms^{-1}$ and its direction of motion is perpendicular to the wall at the instant of impact. B bounces off the wall with the speed $V ms^{-1}$ in a direction perpendicular to the wall. At time 0.8 s after B bounces off the wall, B strikes the wall again at a lower point A.

(i) Find U and θ. ---[5]

(ii) By considering the motion of B after it bounces off the wall, calculate V. S-15/52/Q6 ---[4]

40. A particle P is projected with speed $V ms^{-1}$ at an angle of 60° above the horizontal from a point O on horizontal ground. P is moving at an angle of 45° above the horizontal at the instant 1.5 s after projection.

(i) Find V. ---[3]

(ii) Hence calculate the horizontal and vertical displacement of P from O at the instant 1.5 s after projection. S-15/53/Q2 --[2]

41. A small ball is projected from a point 1.5 m above horizontal ground with speed $29 ms^{-1}$ at an angle of 30° above the horizontal.

(i) Show B strikes the ground 3s after projection. ---[2]

(ii) Find the speed and direction of motion of B immediately before it strikes the ground. ---[4]

S-15/53/Q4

42. A particle P is projected with speed V ms^{-1} at an angle of 60° above the horizontal from a point O. At the instant 1s later a particle Q is projected from O with the same initial speed at an angle of 45° above the horizontal. The two particles collide when Q has been in motion for t s.

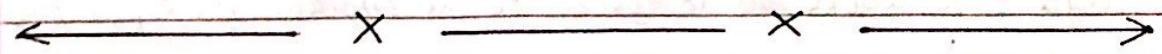
(i) Show that $t = 2.414$, correct to 3 decimal places. ---[3]

(ii) Find the value of V ---[4]

The collision occurs after P has passed through the highest point of its trajectory.

(iii) Calculate the vertical distance of P below its greatest height when P and Q collide. [W-15/51/Q7] ---[4]

43. A particle P is projected with speed V ms^{-1} at an angle of θ° above the horizontal from a point O on the horizontal ground. At the instant 4s after projection the particle passes through the point A, where OA = 40m and the line OA makes an angle of 30° with the horizontal. Calculate V and θ. [W-15/53/Q3] ---[5]



Motion of a Projectile

Answers

1. $x = u\cos\alpha \cdot t$

(a) $y = u\sin\alpha \cdot t - \frac{1}{2}gt^2$

Eliminate t,

$y = u\sin\alpha \cdot \frac{x}{u\cos\alpha} - \frac{1}{2}g\left(\frac{x}{u\cos\alpha}\right)^2$

$\therefore y = x\tan\alpha - \frac{gx^2}{2u^2}\cdot\sec^2\alpha$ ✓ —①

(b) For greatest height, using vertical motion, $H = \frac{(u\sin\alpha)^2}{2g}$ —②

Now $\tan\alpha = 2 \Rightarrow \sin\alpha = \frac{2}{\sqrt{5}}$

$H = \frac{2u^2}{5g}$ —③

from ① $\frac{3}{4}H = 2d - \frac{1}{2u^2} g \times 5d^2$ [for $x = d$

(from ③ $u^2 = \frac{5gH}{2}$)

$\Rightarrow \frac{3}{4}H = 2d - \frac{d^2}{H}$

$\Rightarrow 4d^2 - 8dH + 3H^2 = 0$

$(2d - 3H)(2d - H) = 0$

$\Rightarrow d = \frac{1}{2}H ; \frac{3}{2}H$ ✓

2. (i) $V = u + gt$ for Vertical Motion

$-15\sin\theta = 15\sin\theta - 2g$

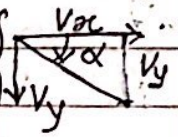
$\Rightarrow \sin\theta = \frac{2}{3} \Rightarrow \theta = \sin^{-1}2/3$

$\Rightarrow \theta = 41.8°$ ✓

(ii) Give $\alpha = 20°$

$V_x = 15\cos\theta = 15\cos 41.8$

$V_y = ?$



Vertically

$V_y = V_x \tan\alpha$ $\left\{\alpha = 20°\right.$

$= 15\cos 41.8° \times \tan 20$

$V_y = 4.07$ —①

Also, Vertically $V_y = u_y - gt$

$4.07 = 15\sin 41.8 - 10t$

$\Rightarrow t = 1.41\,s$ ✓

3. $x = 30\cos 60° \cdot t \Rightarrow t = \frac{x}{30\cos 60°}$ —①

(i) $y = 30\sin 60° \cdot t - \frac{gt^2}{2}$

$y = \frac{30\sin 60 \times x}{30\cos 60} - \frac{5x^2}{(30\cos 60)^2}$ from ①

$y = 1.73x - \frac{x^2}{45}$ ✓ —②

(ii) $\tan 45° = \frac{y}{x}$

$\Rightarrow y = x$

from ② $x = 1.732x - \frac{x^2}{45}$

or $1 = 1.732 - \frac{x}{45}$

$\Rightarrow \frac{x}{45} = 0.732$

$\Rightarrow x = 0.732 \times 45 = 32.94$

$\therefore x = 32.9$ ✓

4. Let the initial speed = V, $\theta = 30°$

(i) $\therefore$ at any time t,

$x = V\cos 30 \times t = 40t$ given

$\Rightarrow V = \frac{40}{\cos 30} = 46.2\,ms^{-1}$ ✓

and $y = V\sin 30 \cdot t - \frac{1}{2}gt^2$

$= 46.2 \times \frac{1}{2}t - \frac{1}{2}gt^2$

$y = 23.1t - 5t^2$ ✓ —①

(ii) Given $x = 40t \Rightarrow t = \frac{x}{40}$ Put in ①

$y = 23.1 \times \frac{x}{40} - 5 \cdot \frac{x^2}{1600}$

$\Rightarrow y = 0.577 - \frac{x^2}{320}$ ✓

5. (i) Vertical comp. $V_y = V\sin 60 - 3g$

Horizontal comp $V_x = V\sin 60$

$\therefore \tan\alpha = \frac{V\sin 60 - 3g}{V\cos 60} = \tan 30°$

$\Rightarrow V = 15\sqrt{3} = 26\,ms^{-1}$ ✓

(ii) $y = 26\sin 60 \times 3 - \frac{1}{2}g \times 3^2$

$x = 26\cos 60 \times 3$

$\therefore D^2 = x^2 + y^2 = (26\cos 60 \times 3)^2 + \left(26\sin 60 \times 3 - \frac{9 \times 3}{2}\right)^2$

$D = 45\,m$ ✓

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Answers

6. At $t=4$s

$V_x = V\cos\theta = 16\cdot\cos 30 = 8\sqrt{3}$ —①

$V_y = 16\sin 30^\circ = V\sin\theta - g\times 4$

$\Rightarrow V\sin\theta = 16\sin 30 + 4g$

$\Rightarrow 8\,V\sin\theta = 48$ —②

from ① & ② $\tan\theta = \frac{V_y}{V_x} = \frac{48}{8\sqrt{3}} = 2\sqrt{3}$

$\theta = \tan^{-1} 2\sqrt{3} = 73.9^\circ$ ✓

and $V^2 = V_x^2 + V_y^2 = (8\sqrt{3})^2 + (48)^2$

$V = 49.96$

or $V = 50\ ms^{-1}$ ✓

7. $x = 25\cos 30\cdot t \Rightarrow t = \frac{x}{25\cos 30}$ —①

$y = 25\sin 30\,t - \frac{gt^2}{2}$

from ① $\Rightarrow y = 25\sin 30\cdot\frac{x}{25\cos 30} - \frac{g}{2}\left(\frac{x}{25\cos 30}\right)^2$

$\Rightarrow y = \frac{x}{\sqrt{3}} - \frac{4x^2}{375}$ —②

or $y = 0.577x - 0.0107x^2$ ✓

(ii) $\frac{V_y}{V_x} = \tan 15^\circ \Rightarrow \frac{V_y}{25\cos 30} = \tan 15$

$\Rightarrow V_y = 12.5\sqrt{3}\times\tan 15 = 5.8$ (downward)

Now Vertically

$v = u + gt$

$-5.8 = 25\sin 30 - 10t$

$\Rightarrow t = 1.83$

$\therefore x = 25\cos 30\times 1.83 = 39.6$ ✓

Alternative method:

diff ② $\frac{dy}{dx} = \frac{1}{\sqrt{3}} - \frac{8x}{375} = -\tan 15^\circ$

$\Rightarrow x = 39.6$ ✓

8(i) $k = g/2 = 5$ ✓

(ii) $V\sin 30 = 14 \Rightarrow V = 28\ ms^{-1}$ ✓

(iii) $x = V\cos\theta\times t = 28\cdot\cos 30\times 3$

$\Rightarrow x = 72.7$m ✓

9. $V_x = 15\cos\theta$ —①

(i) $V_y = 15\sin\theta - 4g$

$V^2 = V_x^2 + V_y^2$

$30^2 = (15\cos\theta)^2 + (15\sin\theta - 40)^2$

$\Rightarrow 225 - 1200\sin\theta + 1600 = 900$

$\Rightarrow \sin\theta = \frac{925}{1200} = 0.77$

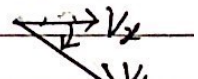
$\therefore \theta = \sin^{-1}(0.77) = 50.4^\circ$ ✓

(ii) Vertically, $s = V\sin\theta\cdot t - \frac{1}{2}gt^2$

$s = 15\sin 50.4\times 4 - \frac{1}{2}\times 10\times 4^2$

$s = -33.75$m ✓ $\therefore$ 33.75m below ✓

Component Horizontally



$V_x = V\cos\theta = V_1\cos\alpha$

$15\times\cos 50.4 = 30\times\cos\alpha$

$\cos\alpha = \frac{15\cos 50.4}{30} = 0.3187$

$\alpha = 71.4^\circ$ below the horizontal. ✓

10. $V_x = 24\cos 30^\circ = 12\sqrt{3}$

$V_y = 24\sin 30 - 4g = -28$

$V^2 = (12\sqrt{3})^2 + (-28)^2 = 1216$

$\Rightarrow V = 34.9\ ms^{-1}$ ✓

$\tan\alpha = \frac{V_y}{V_x} = \frac{-28}{12\sqrt{3}} = -1.347$

$\alpha = -\tan^{-1} 1.347 = -53.4^\circ$

or $\alpha = 53.4^\circ$ below the horizontal. ✓

11.(i) $x = 4t \Rightarrow t = x/4$ —①

$y = 6t - 5t^2$

from ① $y = 6\frac{x}{4} - 5\left(\frac{x}{4}\right)^2$

$y = 1.5x - 0.3125x^2$ ✓ —②

(ii) Eqn of trajectory

$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$ —③

from ② & ③ $\tan\theta = 1.5 \Rightarrow \theta = 56.3^\circ$ ✓

from ①

$V\cdot\cos\theta = 4$

$V = \frac{4}{\cos 56.3} = 7.21\ ms^{-1}$ ✓

Answers

12. $V_x = V\cos\theta = 15\cos 35^\circ = 12.287$

$V_y = 15\sin 35^\circ - 29 = -11.396$

$V = \sqrt{(12.287)^2 + (-11.396)^2}$

$V = 16.8\ ms^{-1}$ ✓

$\tan\theta = \frac{V_y}{V_x} = \frac{-11.396}{12.287} = -0.927$

$\theta = 42.8$ below the horizontal ✓

13. $y = 3x - 0.05x^2$ — ①

$y = x\tan\theta - gx^2/2u^2\cos^2\alpha$ — ②

(i) Comparing $\tan\theta = 3$

$\theta = 71.6^\circ$ ✓

and $0.05 = \frac{g}{2u^2\cos^2 71.6}$

$\Rightarrow V = 10\sqrt{10} = 31.6\ ms^{-1}$ ✓

(ii) when $\theta = 45^\circ$, $y = x$

from ① $x = 3x - 0.05x^2$

$\Rightarrow 1 = 3 - 0.05x$

$\Rightarrow x = \frac{2}{0.05} = 40$ ✓

and $y = 40$ ✓

(iii) diff ①, $\frac{dy}{dx} = 3 - 0.1x = 0$ at highest point

$\Rightarrow x = 30$

from ① $y = 3\times 30 - 0.05\times 30^2 = 45$

$\Rightarrow H = 45$ ✓

Now $x = V\cos\theta \times t$

$30 = 31.6\times\cos 71.6 \times t$

$\Rightarrow t = 3.01$ ✓

14. Horizontal Comp. $V_x = V\cos\theta = 18$ — ①

Vertical. $V_y = 25 - 49 = -15$ — ②

$V^2 = V_x^2 + V_y^2 = 18^2 + (-15)^2 = 549$

$V = 23.4\ ms^{-1}$

$\tan\theta = \frac{V_y}{V_x} = \frac{-15}{18} = -0.833$

$\therefore \theta = 39.8^\circ$ below the horizontal

15. $x = V\cos 45\cdot t \Rightarrow t = \frac{x}{V\cos 45^\circ}$ — ①

(i) $y = V\sin 45\cdot t - \frac{gt^2}{2}$ — ②

from ①, $y = \frac{V\sin 45^\circ \times x}{V\cos 45} - \frac{g}{2}\left(\frac{x}{V\cos 45^\circ}\right)^2$

$\Rightarrow y = x - \frac{10x^2}{V^2}$ — ③

(ii) ③ passes through (24, 18)

$\Rightarrow 18 = 24 - \frac{10\times 24^2}{V^2}$

$\Rightarrow V = 31\ ms^{-1}$ ✓

(iii) $22.5 = x - \frac{10x^2}{960}$

$\Rightarrow x^2 - 96x + 2160 = 0$

$\Rightarrow x = 36, 60$ ✓

16. (i) $\tan\theta = \frac{12}{20} \Rightarrow \theta (= 30.96) = 31^\circ$ ✓

at $t = 0.9$, $x = V\cos\theta \times t = 20$

$V\cos 31^\circ \times 0.9 = 20$

$\Rightarrow V = \frac{20}{0.9\times\cos 31^\circ} = 25.9\ ms^{-1}$ ✓

(ii) $H = 25.9\sin 31\times 0.9 - g\times\frac{0.9^2}{2} = 7.948$

$\therefore AB = 12 - 7.948 = 4.05\ m$ ✓

16′ It travel horizontally, when $V_y = 0$

$V_y = V\sin\theta - gt \Rightarrow 0 = V\sin 60 - gt$

$\Rightarrow t = 3\sqrt{3}/4 = 1.30$ ✓

$x = 15\cos 60\times 1.3 = 9.7428$

$y = H = \frac{u^2\sin^2\theta}{2g} = \frac{(15\sin 60)^2}{2g}$

$\Rightarrow y = 8.4375$

$D = \sqrt{9.742^2 + 8.442^2} = 12.9$

$D = 12.9\ m$ ✓

17(i) $x = 20\cos 30\cdot t \rightarrow x = 10\sqrt{3}\,t$

$\Rightarrow t = \frac{x}{10\sqrt{3}}$ — ①

$y = (20\sin 30)t - \frac{1}{2}gt^2$

$y = 10t - 5t^2$

$y = \frac{x}{\sqrt{3}} - \frac{x^2}{60}$ ✓ (from ①)

(Continued →)

Answers

17(ii) $y = \frac{x}{\sqrt{3}} - \frac{x^2}{60}$

Now $\frac{x}{\sqrt{3}} - \frac{x^2}{60} = \frac{(x+15)}{\sqrt{3}} - \frac{(x+15)^2}{60}$

$\Rightarrow x = 9.821$

$\therefore y = 4.06\ m$ ✓

---

18. Horizontal $V_x = 30\cos 60 = 15$

Vertical $V_y = ?$

$V^2 = V_x^2 + V_y^2 = 25^2$

or $15^2 + V_y^2 = 25^2$

$V_y = \pm 20$

Now second time $= t$, $V_y = -20$

$V_y = 30\sin 60 - gt$

$\Rightarrow -20 = 30 \times \frac{\sqrt{3}}{2} - 10t$

$10t = 15\sqrt{3} + 20 = 45.98$

$\therefore t = 4.598 = 4.6\,s$ ✓

---

19. $x = (24\cos 60°)t$ ——①

(i) $y = (24\sin 60)t - \frac{g}{2}t^2$ ——②

when the object strikes the plane inclined at $45° \Rightarrow x = y$

fm ① & ② $(24\cos 60)t = (24\sin 60)t - \frac{gt^2}{2}$

$\Rightarrow 5t^2 - 12\sqrt{3}t + 12t = 0$

$t[5t - 12(\sqrt{3}-1)] = 0$

$t = \frac{12(\sqrt{3}-1)}{5} = 1.76$ ✓

(ii) $h = (24\sin 60)t - \frac{gt^2}{2} - (24\cos 60)t$

$\frac{dh}{dt} = 24(\sin 60 - \cos 60) - gt = 0$

$\Rightarrow t = 0.878$

$\Rightarrow h = 3.86\ m$ ✓

---

20. $V\cos 20 = 38\cos 30°$

$\Rightarrow V = 35\ ms^{-1}$ ✓

---

21. $x = Vt \Rightarrow t = \frac{x}{V}$ ——①

and $y = -\frac{1}{2}gt^2$

(i) $y = -\frac{1}{2}g\left(\frac{x}{V}\right)^2$ fm①

$y = -\frac{5x^2}{V^2}$ ✓ ——②

(ii) ②passes through $(a, -a)$

$\Rightarrow -a = -\frac{5a^2}{V^2}$ ——③

②also passes through $(a^2, -16a)$

$\Rightarrow -16a = -5\frac{(a^2)^2}{V^2}$ ——④

fm ③ & ④ $a^2 = 16 \Rightarrow a = 4$ as $a > 0$

(iii) from ③ $V^2 = 5a = 5 \times 4 = 20$

given $x = 5a = 5 \times 4 = 20$

fm ② $\therefore y = -5\frac{(20)^2}{20} = -100$

$\therefore$ height $= 100\ m$ ✓

---

22. $\therefore y = V_y t + \frac{1}{2}gt^2$

(i) $30 = (20\sin 60)t + 5t^2$

$\Rightarrow 5t^2 + 10\sqrt{3}t - 30 = 0$

$\Rightarrow t = 1.27$ ✓

(ii) $V_y^2 = (20\sin 60)^2 + 2g \times 30$

$\Rightarrow V_y = 30$ ——①

and $V_x = 20\cos 60 = 10$ ——②

$\therefore V^2 = V_x^2 + V_y^2 = 30^2 + 10^2 = 1000$

$V = 31.6\ ms^{-1}$ ✓

and $\tan\theta = \frac{V_y}{V_x} = \frac{30}{10} = 3$

$\Rightarrow \theta = \tan^{-1}3 = 71.6°$ with the horizontal.

---

23. $V_x = 20\cos 60 = 10$

$V_y = 20\sin 60 - gt$ } descending θ is –ve

$-\tan 40 = \frac{V_y}{V_x} = \frac{20\sin 60 - 10t}{10} = -0.84$

$\Rightarrow 10t = 17.32 + 8.4 = 25.72$

$\therefore t = 2.57\,s$ ✓

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Answers

24. $x = 10t \Rightarrow t = x/10$ —①

(i) $y = 15t - \frac{1}{2}gt^2$

$= 15(\frac{x}{10}) - 5(\frac{x}{10})^2$ fm①

$\Rightarrow y = 1.5x - 0.05x^2$ ✓ —②

(ii) To cross the horizontal ground again.

fm②, $y = 0 = 1.5x - 0.05x^2$

$\Rightarrow x = 30$ ✓ $\Rightarrow d < 30$

(iii) $-14 = 1.5x - 0.05x^2$ from②

$\Rightarrow x = 37.5$ ✓

25. (i) $y = 2x - \frac{25x^2}{V^2}$ —①

We know $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$ —②

fm①&② $\tan\theta = 2$ ✓ $\Rightarrow \theta = 63.4349°$

(ii) ① Passes through (a, a) at $t = 4$

$\Rightarrow a = 2a - \frac{25a^2}{V^2} \Rightarrow V^2 = 25a$ —③

Also $x = V\cos\theta \cdot t$ at $t = 4$

$\Rightarrow a = V\cdot\cos 63.4349 \times 4$ —④

fm③&④ $V^2 = 25 \times 4 \times V\cos 63.4349$

$V = 44.7$ ✓

fm④ $a = 80$ ✓

(iii) $V_y = 44.7213\sin 63.4349 - 4g$

$= 0$

$\therefore \tan\alpha = \frac{V_y}{V_x} = \frac{0}{V_x} = 0$

$\alpha = \tan^{-1} 0 = 0$ ✓

26. $V\cos\theta = 4\cos 45$ —①

$(4\sin 45°)^2 = (V\sin\theta)^2 - 2g(9 - 1.5)$

$\Rightarrow V\sin\theta = \sqrt{158}$ —②

fm①&② $\therefore \tan\theta = \frac{\sqrt{158}}{4\cos 45°}$

$\therefore \theta = 77.3°$ ✓

27. $x = 25\cos 30\, t \Rightarrow t = \frac{x}{25\cos 30°}$ —①

(i) and $y = (25\sin 30°)t - \frac{g}{2}t^2$

fm① $= \frac{25\cdot\sin 30\, x}{25\cos 30} - 5\left(\frac{x}{25\cos 30}\right)^2$

$\Rightarrow y = \frac{x}{\sqrt{3}} - \frac{4x^2}{375}$ —②

(ii) $5 = \frac{x}{\sqrt{3}} - \frac{4x^2}{375}$

$\Rightarrow 4x^2 - 216.5x + 1875 = 0$

$\Rightarrow x = 43.3\ ;\ 10.8$

$\therefore$ Distance $= 43.3 - 10.8 = 32.5$ m ✓

28. Vertical comp. $19\sin 30 = V_y - 2g$

$\Rightarrow V_y = 29$ —①

and $V_x = 18\cos 30$

$= 9\sqrt{3}$ —②

(i) $\tan\theta = \frac{V_y}{V_x} = \frac{29}{9\sqrt{3}} = 1.86$

$\theta = 61.7°$ ✓

$V^2 = V_y^2 + V_x^2 = 29^2 + (9\sqrt{3})^2 = 1084$

$V = 32.9\,(24)\ m\,s^{-1}$ ✓

(ii) On the Plane (just before)

$V_{y_1}^2 = 38^2 - (18\cos 30)^2 = 1444 - 243$

$= 1201$ —③

also

on plane $V_{y_1}^2 = V_y^2 + 2gh$

$1201 = 29^2 + 20h$ fm①&②

$h = \frac{360}{20} = 18$ ✓

(iii) $-V_{y_1} = 29 - gt$

$\Rightarrow -\sqrt{1201} = 29 - 9t \Rightarrow t = 6.36$ ✓

$V_{y_2} = \sqrt{20^2 - (18\cos 30)^2} = 12.53$

$\therefore -12.53 = 12.53 - gt'$

$t' = 2.506$

$\therefore T = 6.366 + 2.506$

$= 8.87$ ✓

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Answers

29. $x = V\cos\theta \cdot t = 30$ given

or $V\cos\theta \times 2 = 30$

$\Rightarrow V\cos\theta = 15$ —①

Vertical velocity at the highest point $0 = V\sin\theta - g\times 2$

$\Rightarrow V\sin\theta = 20$ —②

$V^2 = 15^2 + 20^2$. frm ① & ②

$V = 25\,ms^{-1}$ ✓

and $\tan\theta = \frac{V\sin\theta}{V\cos\theta} = \frac{20}{15} = 1.33$

$\theta = 53.1^\circ$ ✓

30. $V_x = 9\cos 60 = 4.5$

$V_y = \pm 4.5$

$-4.5 = 4.5 - gt$ $[v = u - gt]$

$\Rightarrow t = 0.9$

distance $= V_x \times t = 4.5 \times 0.9$

$= 4.05\ m$ ✓

31. $V_y^2 = V^2 - V_x^2$

$= 12^2 - (16\cos 45)^2$

$\Rightarrow V_y = 4$

Now $-4 = 4 - gt$

$\Rightarrow t = 0.85$ ✓

32. (i) $x = V\cos\theta \cdot t \Rightarrow 8 = V\cos\theta \times 2$

$\Rightarrow V\cos\theta = 4$ —①

$h = ut + \frac{1}{2}gt^2$

Vertically $-26 = -2V\sin\theta - \frac{1}{2}\times 2^2 \times 9$

$\Rightarrow V\sin\theta = 3$ —②

$V^2 = 3^2 + 4^2 \Rightarrow V = 5$ ✓

and $\tan\theta = \frac{3}{4} \Rightarrow \theta = 36.9$ ✓

(ii) $V_y = 3 + 29 = 23$

$\tan\alpha = \frac{23}{4} \Rightarrow \alpha = 80.1^\circ$ ✓

with the horizontal.

33. (i) $-8 = 2x - x^2$

$\Rightarrow x^2 + 2x - 8 = 0 \Rightarrow x = 4$ ✓

(ii) $\tan\theta = 2 \Rightarrow \theta = 63.4^\circ$ ✓ frm ①

also $\frac{-gx^2}{2V^2\cos^2\theta} = -x^2$ frm ① & ②

$\Rightarrow 2V^2\cos^2\theta = 9$ $\left[\begin{array}{l}\tan\theta = 2\\ \cos\theta = \frac{1}{\sqrt{5}}\end{array}\right.$

$2V^2 \times \frac{1}{5} = 10$

$\Rightarrow V^2 = 25 \Rightarrow V = 5\,ms^{-1}$ ✓

(iii) $V^2 = \left((V\cos\theta)^2 + (V\sin\theta)^2\right) + 2g\times 8$

$= V^2 + 160$

$= 5^2 + 160 = 185$

$V = 13.6\,ms^{-1}$

34. $y = kx - \frac{(1+k^2)x^2}{245}$ —①

(i) A(14, a) lies on ①

$a = 14k - 0.8(1+k^2)$ —②

B(42, 2a) lies on ①

$2a = 42k - 7.2(1+k^2)$ —③

frm ② & ③

$42k - 7.2(1+k^2) = 2[14k - 0.8(1+k^2)]$

$\Rightarrow k = \frac{1}{2}$ or 2 ✓

$\theta = \tan^{-1}k = \tan^{-1}2 = 63.435^\circ$ ✓

(ii) for A(14, a) $\Rightarrow x = 14 = 35\cos\theta \cdot t$

$\Rightarrow t = \frac{14}{35\times\cos 63.435}$

$= 0.894\ s$

(iii) $V_y = 35\sin 63.4 - g\left[\frac{42}{35\cos 63.4}\right]$

$\tan\alpha = \frac{41.495}{35\cos 63.4} \Rightarrow \alpha = 15.9^\circ$ above horizontal.

$V^2 = 41.495^2 + (35\cos 63.4)^2$

$\Rightarrow V = 16.3\,ms^{-1}$ ✓

Or $2a = 48$

$V^2 = 35^2 - 2g\times 48 \Rightarrow V = 16.3\,ms^{-1}$

$\cos\alpha = 35\cos 63.4/16.3 \Rightarrow \alpha = 15.9^\circ$ ✓

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$V_y^2 = V^2 - V_x^2$

34. $V_y^2 = 18^2 - (25\cos 50)^2$

$\Rightarrow V_y = 8.1095658$

$V_y = u\sin\theta - gt$

$8.1095658 = 25\sin 50 - gt$

$\Rightarrow t = 1.1\,s$ ✓

35. $y = \frac{1}{2}gt^2 = \frac{1}{2}g\times(1.6)^2 = 12.8$

$x = Vt = 1.6V$

$\therefore \tan 30 = \frac{y}{x} = \frac{12.8}{1.6V}$

$\Rightarrow V = 13.9\ ms^{-1}$ ✓

36. $-20\sin 30 = 20\sin 30 - gT$

(i) $\Rightarrow T = 2\,s$ ✓

$OA = V\cos\theta\cdot T = 20\cos 30^\circ \times 2$

$= 34.6\ m$ ✓

(ii) $V_y^2 = 18^2 - (20\cos 30)^2$

$\Rightarrow V_y = \pm 4.899$

Now $4.899 = 20\sin 30 - gt$

$\Rightarrow t = 0.51\,s$ ✓

also $-4.899 = 8 - gt$

$\Rightarrow t = 1.29$

$\therefore T = 2 + 1.29 = 3.295$ ✓

37. $y = 1.2x - 0.15x^2$ — (1)

(i) $\frac{dy}{dx} = 1.2 - 2\times 0.15x = 0$ for Max height

$\Rightarrow x = 4$

from $y = H = 1.2\times 4 - 0.15\times 4^2$

$\Rightarrow H = 2.4\ m$ ✓

(ii) Twice 'x' for greatest height $= 2x$

$\therefore$ Distance $= 2\times 4 = 8\ m$ ✓

38. $V_x = 15\cos 30^\circ = 12.99$

(i) $V_y = 15\sin 30^\circ - 3g = -22.5$ [for $t=3$]

$V^2 = V_x^2 + V_y^2 = (12.99)^2 + (-22.5)^2$

$\Rightarrow V = 26\ ms^{-1}$ ✓

$\tan\theta = \frac{-22.5}{12.99} \Rightarrow \theta = 60^\circ$ To the horizontal

38(ii) $y = (15\sin 30)\times t - g\times\frac{3^2}{2}$

Height $= 22.5\ m$ ✓

39.(i) $V_x = u\cos\theta = 18\cos 30 = 9\sqrt{3}$ — (1)

$V_y = u\sin\theta - 2g = -18\sin 30$

$\Rightarrow u\sin\theta = 11$ — (2)

$U^2 = u_x^2 + u_y^2 = (9\sqrt{3})^2 + 11^2 = 364$

$U = 19.1\ ms^{-1}$ ✓

$\tan\theta = \frac{u_y}{u_x} = \frac{11}{9\sqrt{3}} = 0.705$

$\Rightarrow \theta = 35.2^\circ$ ✓

(ii) $X = V\cos 30\times 0.8$

$y = -0.8V\sin 30^\circ + \frac{1}{2}g(0.8)^2$

$\tan\alpha = \frac{Y}{X} = \frac{3.2 - 0.4V}{0.8V\cos 30} = \tan 60$

$\Rightarrow 3.2 - 0.4V = 0.8\times\sqrt{3}\times\frac{\sqrt{3}}{2}V$

$\Rightarrow 1.6V = 3.2 \Rightarrow V = 2$ ✓

40. $V_y = V\sin 60 - 1.5g$

(i) $V_x = V\cos 60$

When $\theta = 45^\circ \Rightarrow V_y = V_x$

$\Rightarrow V\cos 60 = V\sin 60 - 1.5g$

$V\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right) = 15$

$V = \frac{15\times 2}{(\sqrt{3}-1)} = 41\ ms^{-1}$

(ii) $x = V\cos 60\cdot t$

$X = 41\cos 60\times 1.5 = 30.7\ m$ ✓

and $y = 41\sin 60\times 1.5 - \frac{1}{2}g(1.5)^2$

or $y = 42\ m$ ✓

41(i) $-1.5 = 29\sin 30\,t - \frac{gt^2}{2} \Rightarrow t = 3$ ✓

(ii) $V_y = 29\sin 30 - 3g = -15.5$

$V^2 = (29\cos 30)^2 + 15.5^2 \Rightarrow V = 29.5$ ✓

$\tan\theta = \frac{3g - 29\sin 30}{29\cos 30} \Rightarrow \theta = 31.7^\circ$ ✓ with horizontal.

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Answers

42. (i) $x = V\cos 45 \cdot t = V\cos 60 \cdot (t+1)$

$\Rightarrow t = 2.414$ ✓

(ii) $y = V\sin 45 \cdot t - \frac{gt^2}{2}$

$= V\sin 60(t+1) - \frac{g(t+1)^2}{2}$

$\Rightarrow V\{\sin 60 \times 3.414 - \sin 45 (2.414)\}$

$= 5\{(3.414)^2 - (2.414)^2\}$

$\Rightarrow V = 23.3$ ✓

(iii) Greatest $H = \frac{23.3^2 \sin^2 60}{29}$

$= 20.2$

$h = 23.3 \sin 60 \cdot (3.414) - \frac{g(3.414)^2}{2}$

$\Rightarrow h = 10.67$

$\therefore$ Req. distance $= 20.2 - 10.67$

$= 9.53\ m$ ✓

---

43. $x = 4V\cos\theta = 40\cos 30$

$\Rightarrow V\cos\theta = 10\cos 30° \quad —①$

and $y = 4V\sin\theta - \frac{4^2 \cdot g}{2} = 40 \sin 30$

$\Rightarrow V\sin\theta = 25 \quad —②$

Sq and add ① & ②

$V^2 = (10\cos\theta)^2 + 25^2$

$\Rightarrow V = 26.5\ m\,s^{-1}$

and $\tan\theta = \frac{25}{10\cos 30} = 2.8868$ ✓

$\Rightarrow \theta = 70.9°$