M-2

Mechanics-2

Motion of a Projectile
Notes

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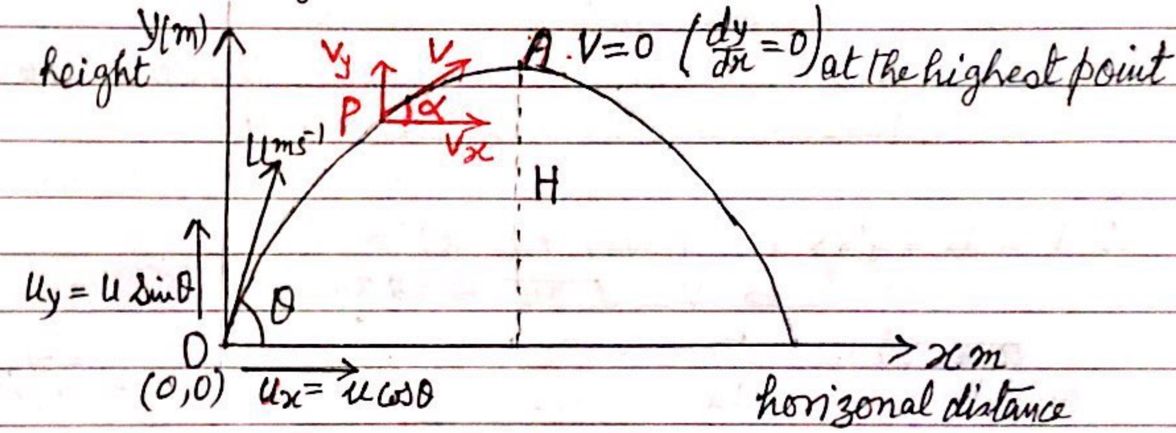
Motion of a Projectile. Notes.

If we throw a small object (ball) in air it takes a parabolic path, calld its trajectory and the object is called projectile. It happens under certain assumptions.
(a) object is a small particle, projectile.
(b) the effect of air on its motion is negligible.
(c) It is not powered, It moves under the gravitation force of earth, has a vertically downwards acceleration $g\,ms^{-2}$ and there is no horizontal acceleration.



Let a particle is thrown with a velocity of $U\,ms^{-1}$ inclined at an angle $\theta°$ with horizontal upwards.
Then, at time t after the projection:

(i) Initially at $t=0$, $x=0$, $y=0$

horizontal velocity $U_x = U\cos\theta$ — Vertical Velocity $U_y = U\sin\theta$

$$U^2 = U_x^2 + U_y^2$$
and $$\tan\theta = \frac{U_y}{U_x}$$

(ii) Let after time t: the velocity of the particle $= V$, angle of inclination $= \alpha$

The horizontal velocity will remain the same $V_x = V\cos\alpha = U\cos\theta$
But vertically, $V\sin\alpha = V_y = U\sin\theta - gt$.
Horizontal Distance, $X = U\cos\theta \times t$
Distance along y axis, $y = U\cos\theta \cdot t - \frac{1}{2}gt^2$

$$V^2 = V_x^2 + V_y^2$$
$$\tan\alpha = \frac{V_y}{V_x}$$

Example 1. A particle is projected with speed $24ms^{-1}$ at an angle of $30°$ above the horizontal. Find the speed and direction of motion of the particle at the instant 4s after projection. [M-19/52/Q6] ---[5]

Solution: $U = 24ms^{-1}$, $\theta = 30°$

Initially Horizontal component $U_x = 24\cos 30° = 12\sqrt{3}$ —— (1)

and Vertical component $U_y = 24\sin 30° = 12$ —— (2)

After $t = 4s$ let the speed $= V ms^{-1}$

and inclination $= \alpha$

Horizontal component $V_x = V\cos\alpha = U\cos.. = 24\cos 30 = 12\sqrt{3}$ — (3)

Vertical component $V_y = U\sin\theta - gt$

or $V_y = 24\sin 30 - 10\times 4 = 12 - 40 = -28$ — (4)

$\therefore V^2 = V_x^2 + V_y^2 \Rightarrow V^2 = (12\sqrt{3})^2 + (-28)^2 = 1216.$

$\Rightarrow V = \sqrt{1216} = \underline{34.9ms^{-1}}$ ✓

and $\tan\alpha = \dfrac{V_y}{V_x} = \dfrac{-28}{12\sqrt{3}} = -1.347 \Rightarrow \alpha = -\tan^{-1}1.347$

$= -53.4°$

or $\alpha = \underline{53.4° \text{ below the horizontal}}$ ✓

Reverse

Example 2: A particle is projected with speed $V ms^{-1}$ at an angle of $\theta°$ above the horizontal. At the instant 4s after projection the speed of the particle is $16ms^{-1}$ and its direction of motion is $30°$ above the horizontal. Find V and θ. --[5]

[S-19/51/Q2]

Solution: Initially horizontal comp. of velocity

$V_x = V\cos\theta$ —— (1)

and vertical comp. $V_y = V\sin\theta$ —— (2)

Now at $t = 4s$, the velocity let $W = 16ms^{-1}$

and direction $\alpha = 30°$ above the horizontal.

The horizontal component $W_x = W\cos\alpha = 16\cos 30 = V\cos\theta$

$\therefore V\cos\theta = 8\sqrt{3}$ —— (3)

and the vertical comp. of vel, $W_y = V\sin\theta - 4g$ [for $t = 4$]

$\Rightarrow W\sin\alpha = V\sin\theta - 4g$

$\Rightarrow 16\sin 30° = V\sin\theta - 4g \Rightarrow V\sin\theta = 48$ — (4)

(continued →)

(Continued →)

(Example 2 →) Now $V_x = V\cos\theta = 8\sqrt{3}$ — ③

$V_y = V\sin\theta = 48$ — ④

$\therefore V^2 = V_x^2 + V_y^2 = (8\sqrt{3})^2 + (48)^2 = 2496$

$\Rightarrow V = \sqrt{2496} = \underline{49.96}$ ✓

and $\tan\theta = \frac{V_y}{V_x} \Rightarrow \tan\theta = \frac{48}{8\sqrt{3}} = 2\sqrt{3}$

$\therefore \theta = \tan^{-1} 2\sqrt{3} = \underline{73.9^\circ}$ ✓

Example 3: A small ball is projected with speed $15\,ms^{-1}$ at an angle of 60° above the horizontal. Find the distance from the point of projection of the ball at the instant when it is travelling horizontally. $\boxed{M-17/52/Q1}$ --- [5]

Solution: Initially $\begin{cases} u = 15\,ms^{-1} \\ \theta = 60^\circ \end{cases}$, $u_x = u\cos\theta = 15\cos 60$

$u_y = u\sin\theta = 15\sin 60.$

The particle travel horizontally at a point when $V_y = 0$ at time 't's → $V_y = u\sin\theta - gt = 0$

$\Rightarrow 15\sin 60 - 10t = 0 \Rightarrow t = \frac{15\sqrt{3}}{10\times 2} = 1.3s$

$\therefore t = 1.3s,$

$x = u\cos\theta \times t = 15\cos 60 \times 1.3 = 9.74$ ✓

and $y = u\sin\theta . t - \frac{1}{2}gt^2$

$= 15\sin 60 \times 1.3 - 5\times(1.3)^2 = 8.4375$

Distance $D = \sqrt{x^2+y^2} = \sqrt{9.74^2 + 8.44^2} = \underline{12.9m}$

Example 4: A small ball B is projected with speed $38\,ms^{-1}$ at an angle of 30° to the horizontal from a point on horizontal ground. Find the speed of B when the path of B makes an angle of 20° above the horizontal. $\boxed{W-18/52/Q1}$ --- [3]

Solution: Given $u = 38\,ms^{-1}$

Initial $\theta = 30^\circ$

$\therefore$ horizontal component $= u_x = u\cos\theta = 38\cos 30$ — ①

Now let the speed $= V$

angle $\alpha = 20^\circ$

horizontal $V_x = V\cos 20$ — ②

from ① & ② $u_x = V_x \Rightarrow V\cos 20 = 38\cos 30 \Rightarrow V = 35\,ms^{-1}$

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Example 5. A particle is projected with speed $20\,ms^{-1}$ at an angle of $60°$ above the horizontal. Calculate the time after projection when the particle is descending at an an angle of $40°$ below the horizontal. [S-17/51/Q1] --[4]

Solution: Initial speed $u = 20\,ms^{-1}$, angle of projection $\theta = 60°$ $\Rightarrow$ $u_x = 20\cos 60$, $u_y = 20\sin 60$

Now let after t's' the particle make angle $\alpha = -40°$

Let the velocity now $= V$.

$$V_x = u_x = 20\cos 60 = 10 \quad \text{--- (1)}$$

and $$V_y = u_y - gt = 20\sin 60 - 10t \quad \text{--- (2)}$$

from (1) & (2) $$\tan(-40) = \frac{V_y}{V_x} = \frac{20\sin 60° - 10t}{10} = -0.84$$

$$\Rightarrow 10t = 17.32 + 0.84 = 25.72$$

$$\Rightarrow t = 2.572\,s \checkmark$$

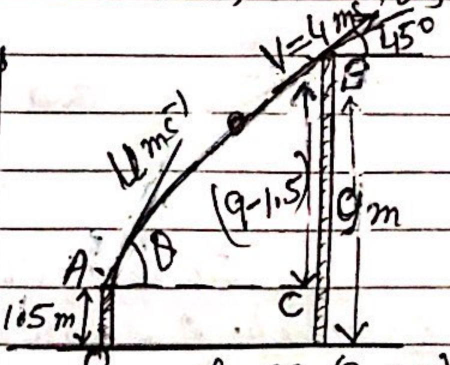
Example 5: A small ball is projected from a point $1.5\,m$ above the horizontal ground. At a point $9\,m$ above the ground the ball is travelling at $45°$ above the horizontal and its velocity is $4\,ms^{-1}$. Find the angle of projection of the ball. [W-17/51/Q2] --[4]

Ball is projected from O.

Solution: $OA = 1.5\,m$, Let ang of Proj $= \theta$

$OB = 9\,m$, Initial Vel $= u$

When at B, $V = 4\,ms^{-1}$, $\alpha = 45°$ above the horizontal



$h = BC = (9-1.5)\,m$

horizontal comp. $u\cos\theta = 4\cos 45$ --- (1) $(u_x = V_x)$

$\therefore V_y = V\sin\theta = 4\sin 45°$ --- (2)

$$V_y^2 = u_y^2 - 2gh$$

$$\rightarrow (4\sin 45°)^2 = (u\sin\theta)^2 - 2\times 10\times(9-1.5) \Rightarrow 8 = (u\sin\theta)^2 - 150$$

$$\Rightarrow u\sin\theta = \sqrt{158} \quad \text{--- (3)}$$

$$\therefore \tan\theta = \frac{u_y}{u_x} = \frac{u\sin\theta}{u\cos\theta} = \frac{\sqrt{158}}{4\cos 45} = \frac{\sqrt{158}}{2\sqrt{2}} = \frac{12.57}{2.83} = 4.44$$

$$\Rightarrow \theta = 77.3° \checkmark$$

(M2)

Motion of a Projectile

A particle P is projected with speed 'U' at an angle 'α' above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t are denoted by x and y respectively.

(a) Derive the equation of trajectory of P in the form.

$$y = x\tan\alpha - \frac{gx^2}{2U^2}\sec^2\alpha.$$

$\boxed{FM/SP-20/03/Q6}$

(b) Find the maximum height. $H = U^2\sin^2\alpha/2g$

(c) Find the time of flight. $t = 2U\sin\alpha/g$

(d) Find the range. $R = U^2\sin 2\alpha/g$

Solution: Initially at $t=0$, Vertical comp. $U_y = U\sin\alpha$

The horizontal component of Velocity $U_x = U\cos\alpha$

After time 't' horizontal displacement $x = Ut\cos\alpha$ — (1)

Let the velocity of the particle at time 't' $= V$

horizontal component. $V_x = U\cos\alpha$

Vertical Component $V_y = U\sin\alpha - gt$ — (2)

Vertical displacement $y = Ut\sin\alpha - \frac{1}{2}gt^2$ — (3)

(a) from (1) $x = Ut\cos\alpha \Rightarrow t = \frac{x}{U\cos\alpha}$ — (4)

from (3) $y = U\sin\alpha \cdot t - \frac{1}{2}g \cdot t^2$

from (3) & (4) $y = \frac{U\sin\alpha \cdot x}{U\cos\alpha} - \frac{1}{2}g \cdot \frac{x^2}{U^2\cos^2\alpha}$

$\Rightarrow y = x\tan\alpha - \frac{gx^2}{2U^2\cos^2\alpha}$

∴ Equation of Path:

or $y = x\tan\alpha - \frac{gx^2}{2U^2} \cdot \sec^2\alpha$,

(b) for greatest height, the vertical component of V, $V_y = 0$

from (2) $\Rightarrow U\sin\alpha - gt = 0$

$\Rightarrow t = \frac{U\sin\alpha}{g}$

(Continued →)

(Continued →) Put $t = U\sin\alpha/g$ in (3)

$y = U \cdot \frac{U\sin\alpha}{g} \cdot \sin\alpha - \frac{1}{2}g\left(\frac{U\sin\alpha}{g}\right)^2$

$= \frac{U^2\sin^2\alpha}{g} - \frac{U^2\sin^2\alpha}{2g}$

∴ The greatest height

$H = \frac{U^2\sin^2\alpha}{2g}$ ✓

(C) Time of Flight:

The Particle hits the ground $\Rightarrow y = 0$

from (3) $y = Ut\sin\alpha - \frac{1}{2}gt^2 = 0$

$\Rightarrow t(U\sin\alpha - \frac{1}{2}gt) = 0$

$\Rightarrow T = t = \frac{2U\sin\alpha}{g}$ ✓ or $t = 0$ ✗

Note: Time of Flight = 2 × time to reach greatest height.

(Continued →)

(d) The Range:

Range of the projectile is value of x for $t = \frac{2u\sin\alpha}{g}$

$\therefore x = ut\cos\alpha$

$= u \times \frac{2u\sin\alpha}{g} \times \cos\alpha$

$= \frac{2u^2\sin\alpha \cdot \cos\alpha}{g} = \frac{u^2\sin 2\alpha}{g}$

$\therefore$ Range $R = \frac{u^2\sin 2\alpha}{g}$

Note: Range is Max. when $\sin 2\alpha = 1 \Rightarrow 2\alpha = 90^\circ \Rightarrow \alpha = 45^\circ$

$\therefore R_{max} = \frac{u^2}{g}$ for $\alpha = 45^\circ$

(e) The greatest height of P above the plane is denoted by H, when P is at a height of $\frac{3}{4}H$, it has travelled a horizontal distance 'd'. [6]
Given that $\tan\alpha = 2$, find in terms of H, the two possible values of d.
[FM/SP-20/03/Q6(b)]

(e) Solution: We know the greatest height

$H = \frac{u^2\sin^2\alpha}{2g}$ — ①

$\left[\tan\alpha = 2 \Rightarrow \sin\alpha = \frac{2}{\sqrt{5}}\right]$

or $= \frac{u^2 \times \frac{4}{5}}{2g}$

$\therefore H = \frac{2u^2}{5g}$ — ②

Now given that $y = \frac{3}{4}H$, $x = d$

$\left(y = x\tan\alpha - \frac{gx^2}{2u^2}\sec^2\alpha\right)$

from ②

$\Rightarrow \frac{3}{4}H = 2d - \frac{g}{2u^2} \times 5d^2$ $[\sec^2\alpha = 5]$

$\Rightarrow \frac{3}{4}H = 2d - \frac{d^2}{H}$ $\because \left[\text{from ②} \; u^2 = \frac{5gH}{2}\right]$

$\Rightarrow 4d^2 - 8dH + 3H^2 = 0$

$(2d - 3H)(2d - H) = 0 \Rightarrow d = \frac{1}{2}H$ or $\frac{3}{2}H$

Example 7: A particle is projected from horizontal ground with speed $u\,ms^{-1}$ and at an angle of $\theta°$ above the horizontal. The greatest height reached by the particle is 10 m and the particle hits the ground at a distance of 40 m from the point of projection. In either order

(i) Find the value of u and θ.

(ii) Find the equation of trajectory, in the form $y = ax - bx^2$, where x m and y m are the horizontal and vertical displacements of the particle from the point of projection.

[W-05/05/Q4]

Solution: The greatest height $H = \dfrac{u^2 \sin^2\theta}{2g} = 10$ —— ①

(i) and the Range $R = \dfrac{u^2 \sin 2\theta}{g} = 40$

or $\dfrac{2u^2 \sin\theta \cos\theta}{g} = 40$ —— ②

from ① & ② (Divide) $\dfrac{u^2 \sin^2\theta}{2g} \times \dfrac{g}{2u^2 \sin\theta \cos\theta} = \dfrac{10}{40}$

$\Rightarrow \tan\theta = 1 \Rightarrow \underline{\theta = 45°}$ —— ③

$\therefore$ from ① $\dfrac{u^2 \sin^2 45°}{2g} = 10 \Rightarrow u^2 = 400$

$\Rightarrow \underline{u = 20\,ms^{-1}}$ —— ④

(ii) Now horizontal component of u.

$u_x = u\cos\theta = 20.\cos 45 = 10\sqrt{2}$ —— ⑤

and Vertical Comp. $u_y = u\sin\theta = 20 \sin 45 = 10\sqrt{2}$ —— ⑥

at time t, $x = u_x \cdot t = 10\sqrt{2} \cdot t$ —— ⑦ fr ⑤

and $y = u_y \cdot t - \frac{1}{2} g t^2$

$y = 10\sqrt{2}\,t - 5t^2$ —— ⑧

[fr ⑦ $t = \dfrac{x}{10\sqrt{2}}$

$y = 10\sqrt{2} \times \dfrac{x}{10\sqrt{2}} - 5 \times \dfrac{x^2}{(10\sqrt{2})^2}$

The equ of Trajectory:

or $\underline{y = x - 0.025x^2}$

Example 8: A particle is projected from a point O at an angle of 35° above the horizontal. At time T s later the particle passes through a point A whose horizontal and vertically upward displacements from O are 8m and 3m respectively.
(i) By using the equation of the particle's trajectory, or otherwise, find the speed of projection of the particle from O and the value of T.
(ii) Find the angle between the direction of motion of the particle at A and the horizontal. W-07/05/Q6

Solution: The equation of particle's trajectory is,
(i) $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$ —①

$\theta = 35°$, $u = ?$, when $x=8$, $y=3$

from ①: $3 = 8\tan 35° - \dfrac{10\times 8^2}{2u^2 \cdot \cos^2 35°}$

$\Rightarrow u = 13.54$ ✓ or $u = 13.5\ ms^{-1}$ ✓

Now Horizontal displacement.

$x = u_x \cdot T \Rightarrow 8 = u\cos\theta \times T$

$\Rightarrow \quad = 13.5\times\cos 35\times T$

$\Rightarrow T = \dfrac{8}{13.5\times\cos 35°} = 0.721$ ✓

$T = 0.721$ ✓

(ii) Let the direction of motion at A when $T = 0.721$ is α, and Velocity $= V$

$V_x = u_x$

$\Rightarrow V_x = u\cos\theta = 13.54\times\cos 35 = 11.09$ —②

and $V_y = u\sin\theta - gT = 13.54\times\sin 35° - 10\times 0.721$

$V_y = 0.5562$ —③

$\therefore \tan\alpha = \dfrac{V_y}{V_x} = \dfrac{0.5562}{11.09} = 0.05$

$\alpha = \tan^{-1} 0.05 = 2.87°$ ✓

Example 9: A particle P is projected from a point O on horizontal ground with speed $V ms^{-1}$ and direction 60° upwards from the horizontal. At t s later the horizontal and vertical displacements from P are x m and y m respectively.

(i) Write down expressions for x and y in terms of V and t and hence show that the equation of the trajectory of P is,

$$y = \sqrt{3}x - \frac{20x^2}{V^2}$$

P passes through the point A at which x = 70 and y = 10, find

(ii) the value of V.

(iii) the direction of motion of P at the instant it passes through A.

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Solution: $V = ?$, $\theta = 60^\circ$

Initially $V_x = V\cos 60 = \frac{V}{2}$ —— (1)

$V_y = V\sin 60 = \frac{\sqrt{3}}{2}V$ —— (2)

Now

fm (1) $x = V_x \cdot t = \frac{V}{2}t = \frac{1}{2}Vt$

or $t = \frac{2x}{V}$ —— (3)

fm (2)

and $y = V_y \cdot t - \frac{1}{2}gt^2 = \frac{\sqrt{3}}{2}Vt - 5t^2$

$y = \frac{\sqrt{3}}{2}Vt - 5t^2$ —— (4)

from (3) $t = \frac{2x}{V}$ Put in (4)

$y = \frac{\sqrt{3}}{2}V \times \frac{2x}{V} - 5\left(\frac{2x}{V}\right)^2$

$\Rightarrow y = \sqrt{3}x - \frac{20x^2}{V^2}$ ✓ —— (5)

(ii) at A, x = 70 and y = 10 fm (5)

$10 = 70\sqrt{3} - \frac{20 \times (70)^2}{V^2}$

$\Rightarrow V^2 = 880.93 \Rightarrow V = 29.68$

or $V = 29.7 ms^{-1}$ ✓

(iii) Let the direction of motion at P is α, and Velocity = W

$W_x = V_x = V\cos\theta = \frac{V}{2}$ fm (1)

or $W_x = \frac{V}{2} = \frac{29.7}{2} = 14.85$ —— (6)

Now A, $x = 70$, time = t

$\therefore t = \frac{2 \times 70}{29.7} = 4.74 s$ ✓ —— (7)

fm (2) $V_y = \frac{\sqrt{3}}{2}V = \frac{\sqrt{3}}{2} \times 29.7 =$

$V_y = 25.72$ —— (8)

$W_y = V_y - g \times t$ fm (7) & (8)

$= 25.72 - 10 \times 4.74$

$= 25.72 - 47.4 = -21.68$

$W_y = -21.68$ —— (9)

$\therefore \tan\alpha = \frac{W_y}{W_x} = \frac{-21.68}{14.85}$

$= -1.46$

$\alpha = -\tan^{-1} 1.46 = -55.6^\circ$

$\therefore \alpha = 55.6^\circ$ below the horizontal

Example 10: A particle is projected from a point O on horizontal ground. The velocity of projection has magnitude $20\,ms^{-1}$ and direction upwards at an angle θ to the horizontal. The particle passes through the point which is 7 m above the ground and 16 m horizontally from O, and hits the ground at point A.

(i) Using the equation of the particle's trajectory, show that the possible values of $\tan\theta$ are $\frac{3}{4}$ and $\frac{17}{4}$.

(ii) Find the distance OA for each of the two possible values of $\tan\theta$.

(iii) Sketch in the same diagram the two possible trajectories.

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Solution: Initial velocity $u = 20\,ms^{-1}$, direction $= \theta$ upwards to horizontal.

Horizontal Component

(i) $u_x = u\cos\theta = 20\cos\theta$

$\therefore \Rightarrow x = u_x \times t$

$x = 20\cos\theta \times t$

$\Rightarrow t = \frac{x}{20\cos\theta}$ —①

Now $u_y = 20\sin\theta$

$\therefore y = u_y t - \frac{1}{2} g t^2$

$\Rightarrow y = 20\sin\theta \cdot t - 5t^2$ —②

using ① $\Rightarrow y = \frac{20\sin\theta \times x}{20\cos\theta} - 5 \times \left(\frac{x}{20\cos\theta}\right)^2$

Equ^n of trajectory:

$y = x\tan\theta - \frac{x^2}{80}(1+\tan^2\theta)$ —③

Passes that $x = 16$ and $y = 7$

fm ③ $7 = 16\tan\theta - \frac{16^2}{80}(1+\tan^2\theta)$

$\Rightarrow 16\tan^2\theta - 80\tan\theta + 51 = 0$

$\Rightarrow \tan\theta = \frac{3}{4}, \frac{17}{4}$ ✓

(ii) A is a point where the particle hits the ground $\rightarrow y = 0$

Case I $y = 0$ and $\tan\theta = 3/4$

fm ③ $0 = x \times \frac{3}{4} - \frac{x^2}{80}\left(1+\frac{9}{16}\right) \Rightarrow x = 38.4\,m$ ✓ or $OA = 38.4\,m$ ✓

(ii) Continued →

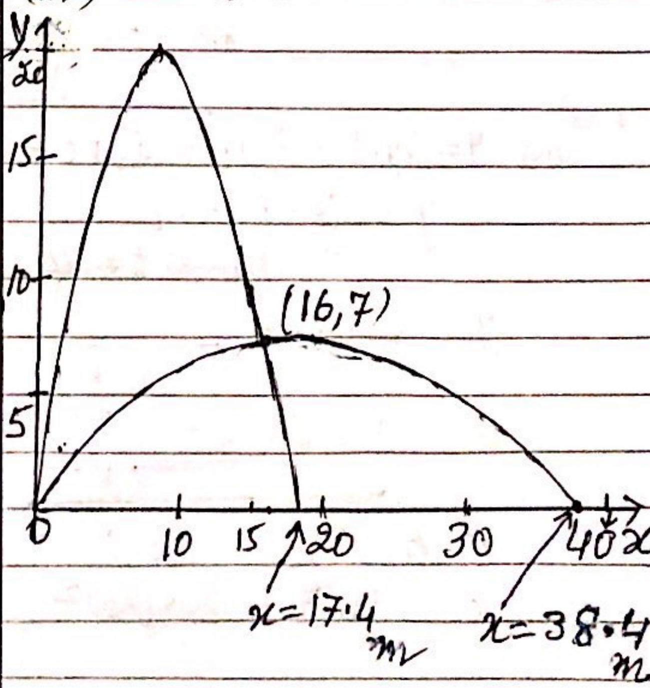
When $y = 0$ and $\tan\theta = \frac{17}{4}$

fm ③ $0 = x \times \frac{17}{4} - \frac{x^2}{80}\left(1+\frac{289}{16}\right)$

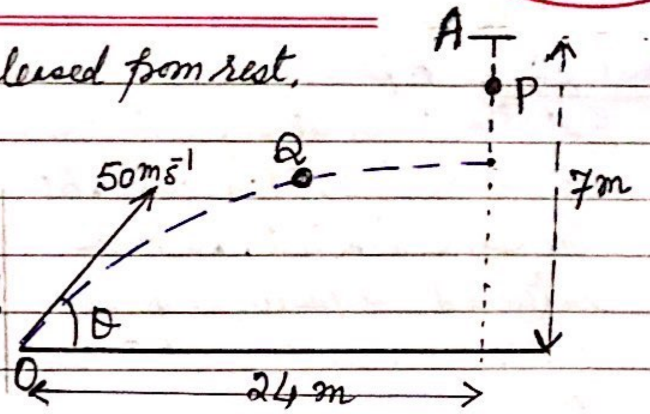
$\Rightarrow x = 17.8$

or $OA = 17.8$ ✓

(iii)



Example 11: A particle P is released from rest, at a point A which is 7 m above horizontal ground. At the same instant that P is released a particle Q is projected from a point O on the ground. The horizontal distance of O from A is 24 m. Particle Q moves in a vertical plane containing O and A, with initial speed $50 ms^{-1}$ and initial direction making an angle θ above the horizontal, where $\tan\theta = \frac{7}{24}$. Show that the particles collide. [W-09/52/Q3]



Solution: For Q, initial speed $u = 50 ms^{-1}$
and angle θ: $\tan\theta = \frac{7}{24} \Rightarrow \sin\theta = \frac{7}{25}$ and $\cos\theta = \frac{24}{25}$

Note:
[Particles P and Q will collide if the sum of the vertical distance travelled by P and Q is 7 m when Q reaches vertically below A.]

Let us consider that may happen at time t's'.

for Q, $u_x = 50\cos\theta$

distance travelled $x = u_x \cdot t$

$\Rightarrow 24 = u\cos\theta \cdot t = \frac{50 \times 24}{25} \times t$

$\Rightarrow t = 0.5 s$.

Now vertically the distance travelled by Q $= y = u_y \cdot t - \frac{1}{2}gt^2$

$= u\sin\theta \times t - 5t^2$

$= 50 \times \frac{7}{25} \times 0.5 - 5 \times (.5)^2$

for Q $y_1 = (7 - 1.25)$ m — ①

And the vertical distance trav. by P $= 0 + \frac{1}{2}gt^2$

$y_2 = 5(0.5)^2 = 1.25$ m — ②

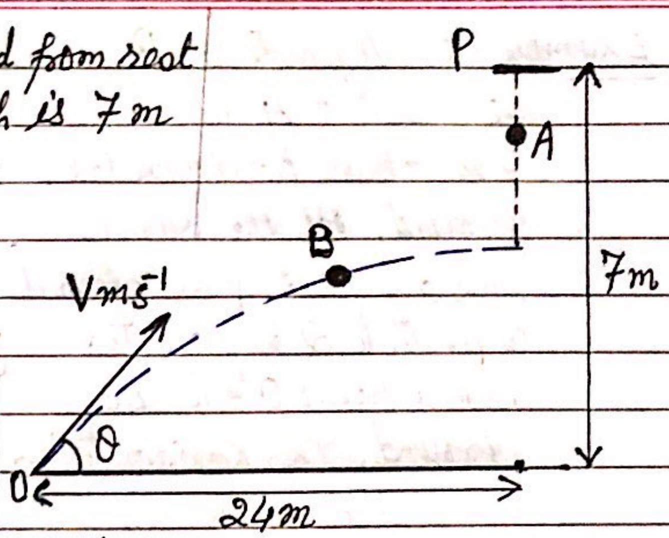
from ① & ②

$y_1 + y_2 = (7 - 1.25) + 1.25$

$\Rightarrow y_1 + y_2 = 7$ m

hence P and Q collide

Example 12: A particle A is released from rest at time t=0, at a point which is 7 m above horizontal ground. At the same instant as A is released, a particle B is projected from a point O on the ground. The horizontal distance of O from P is 24m. Particle B moves in the vertical plane containing O and P, with initial speed $V\,ms^{-1}$ and initial direction making an angle θ above the horizontal.



Write down: (i) an expression for the height of A above the ground at time t's'.

(ii) an expression in terms of V, θ and t for
(a) the horizontal distance of B from O.
(b) the height of B above the ground.

at time $t = T$ the particles A and B collide at a point above the ground.

(iii) ...., Show that $\tan\theta = \frac{7}{24}$ and $VT = 25$

(iv) Deduce that $7V^2 > 3125$.

Solution: Vertical distance travelled by 'A' down at time 't' $= \frac{1}{2}gt^2 = 5t^2$

$\therefore$ height of A above the ground.

(i) $h_1 = 7 - 5t^2$ — ①

for B' $V_x = V\cos\theta$

$\Rightarrow x = V\cos\theta \times t = Vt\cos\theta$ — ②

$y = V_y t - \frac{1}{2}gt^2 = V\sin\theta t - 5t^2$

(ii) or $h_2 = V\sin\theta . t - 5t^2$ — ③

(iii) Now for $t = T$, A & B collide $\Rightarrow h_1 = h_2$

from ① & ③ $7 - 5T^2 = V\sin\theta . T - 5T^2$

$\Rightarrow V\sin\theta . T = 7$ — ④

Now the at $t = T$, $x = 24$

from ② $x = VT\cos\theta = 24$ — ⑤

from ④ & ⑤ $\frac{V\sin\theta . T}{V\cos\theta . T} = \frac{7}{24}$

$\Rightarrow \tan\theta = \frac{7}{24}$ ✓ ($\sin\theta = 7/25$)

from ④ $V\sin\theta . T = 7$

$V \times \frac{7}{25} \times T = 7 \Rightarrow VT = 25$ ✓ — ⑥

(iv) Particles collide above the ground $\Rightarrow h_1 > 0$

from ① $7 - 5T^2 > 0$ | from ⑥ $T = \frac{25}{V}$

$7 - 5 \times \left(\frac{25}{V}\right)^2 > 0$

$\Rightarrow 7V^2 > 3125$ ✓

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Example 13: A stone is projected from a point O on horizontal ground with speed $V\,ms^{-1}$ at angle θ above the horizontal, where $\sin\theta = \frac{3}{5}$. The stone is at its highest point when it has travelled a horizontal distance of 19.2m.

(i) Find the value of V.

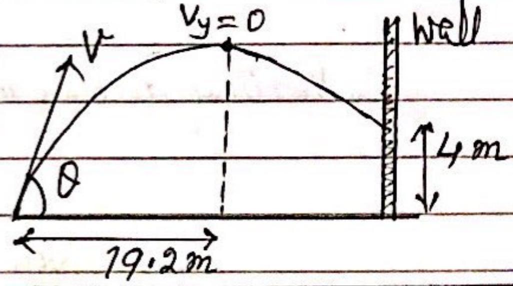
After passing through its highest point the stone strikes a vertical wall at a point 4m above the ground.

(ii) Find the horizontal distance between O and the wall.

At the instant when the stone hits the wall the horizontal component of the stone's velocity is halved in magnitude and reversed in direction. The vertical component of the stones velocity does not change as a result of the stone hitting the wall.

(iii) Find the distance from the wall of the point where the stone reaches the ground.

S-06/05/Q7



Solution: Given $\sin\theta = 3/5 \Rightarrow \cos\theta = 4/5$

$V_x = V\cos\theta$

(i) $x = V\cos\theta \cdot t = 19.2$ given

$\Rightarrow V \times \frac{4}{5} t = 19.2$

$\Rightarrow t = \frac{19.2 \times 5}{4V}$ —— ①

Now

$V_y = V\sin\theta - gt = 0$ at the highest Point

$\Rightarrow V \times \frac{3}{5} - 10t = 0$

fr ① $\frac{3}{5}V - \frac{10 \times 19.2 \times 5}{4V} = 0$

$\Rightarrow V^2 = 400$

$\Rightarrow V = 20$ —— ②

(ii) Particle hits the wall at $h = 4m$

$h = V_y \cdot t - \frac{1}{2}gt^2$

$4 = V\sin\theta \cdot t - 5t^2$

$4 = 20 \times \frac{3}{5}t - 5t^2$

$\Rightarrow 5t^2 - 12t + 4 = 0 \Rightarrow t = 2$ or $\frac{2}{5}$

($\because t = \frac{2}{5}$ is time before reaching the highest point)

Now the distance of wall at $t = 2$

$x = V_x \cdot t$

$= V\cos\theta \times t$

distance of Wall $= 20 \times \frac{4}{5} \times 2 = \underline{32m}$

(iii) Now after hitting the wall

$V_x' = \frac{1}{2} \times V_x = \frac{1}{2}V\cos\theta = \frac{1}{2} \times 20 \times \frac{4}{5}$

New horizontal $V_x' = 8ms^{-1}$ —— ③

and $V_y = V\sin\theta - gt = 20 \times \frac{3}{5} - 10 \times 2$

$V_y = -8$

Particle reaches the ground $h = -$

$h = ut + \frac{1}{2}gt^2$

$-4 = -8t - 5t^2 \Rightarrow 5t^2 + 8t - 4 = 0$

$\Rightarrow t = 0.4s$

$\therefore$ from ③ horizontal distance $= 8 \times 0.4 = 3.\underline{2m}$

Example 14: A small ball is projected from a point O on horizontal ground at an angle of 30° above the horizontal. At time t s after projection the vertically upwards displacement of the ball from O is $(14t - kt^2)$ m, where k is a constant.

(i) State the value of k. ---[1]

(ii) Show that the initial speed of the ball is $28 ms^{-1}$ --[2]

(iii) Find the horizontal displacement of the ball from O, when $t = 3$. --[2]

Solution: Let the initial speed of projection $= u\ ms^{-1}$

angle of projection $= 30°$

Now horizontal component of speed $u_x = u \cdot \cos\theta = u\cos 30°$

$$\Rightarrow u_x = \frac{\sqrt{3} \cdot u}{2} \quad \text{--- (1)}$$

and the vertical component of

Velocity $u_y = u\sin\theta - gt$

$$= u\sin 30 - 10t \quad \text{--- (2)}$$

$\therefore$ Vertical displacement $y = u_y \cdot t - \frac{1}{2}gt^2$

$$= u\sin 30° \cdot t - 5t^2$$

$$y = \frac{1}{2}ut - 5t^2 \quad \text{--- (3)}$$

but given $y = 14t - kt^2$ --- (4)

(i) Comparing (3) & (4) $k = 5$ ✓

(ii) Also comparing (3) & (4) $\frac{1}{2}u = 14 \Rightarrow \underline{u = 28 ms^{-1}}$ ✓ --- (5)

(iii) horizontal displacement $x = u_x \times t$

or $x = u\cos 30° \times t$

$\therefore$ at $= 3$, $x = 28 \times \frac{\sqrt{3}}{2} \times 3$ $\left(\because \text{from (5)}\ u = 28\right)$

$$x = 72.7 m$$

$\therefore$ horizontal displacement $= \underline{72.7 m}$ ✓